

# CS201

Mathematics For Computer  
Science  
Indian Institute of Technology, Kanpur

Group Number: 11

Anjali Rana (190147),  
Atreya Goswami (190201),  
Lipi Jain (190457)

# Assignment 1

Date of Submission:  
September 26 2020

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## Question 1

Let  $S = \{(a, b, c) | a, b, c \in \mathbb{Z}\}$  be the set of all triplets of integers. Show that  $|S| = \aleph_0$ .

## Solution

## Question 2

For any  $a, b, c, d \notin \{-\infty, \infty\}$ , show that  $|[a, b]| = |[c, d]|$  where  $[x, y]$  is the set of all real numbers between  $x$  and  $y$

### Solution

Your solution goes here.

### Question 3

Show that  $|[0, 1]| = \aleph_1$  where  $[0, 1]$  is the set of all real numbers between 0 and 1.

### Solution

Your solution goes here.

## Question 4

Show that  $|\{0, 1\}^*| = \aleph_1$  where  $\{0, 1\}^*$  is the set of all binary strings of infinite length.

## Solution

Your solution goes here.

## Question 5

Suppose  $R$  is a partial order on  $A$  and  $S$  be a partial order on  $B$ . Let  $L$  be a binary relation on  $A \times B$  defined as  $(a, b)L(a', b')$  iff

- $a \neq a'$  and  $aRa'$
- $a = a'$  and  $bSb'$ .

Show that  $L$  is also a partial order on  $A \times B$ . Is it a total order?

## Solution

Your solution goes here.

## Question 6

Let  $R$  be a binary relation on  $\mathbb{N}$  defined as  $aRb$  if  $b = 2^k a$  where  $k$  is a non-negative integer. Show that  $R$  is a partial order on  $\mathbb{N}$ .

## Solution

Your solution goes here.

## Question 7

Let  $n$  be a positive integer. Consider the relation  $\equiv_n$  on  $\mathbb{Z}$  such that  $a \equiv_n b \iff a = b \pmod n$ . Show that  $\equiv_n$  is an equivalence relation on  $\mathbb{Z}$ . What are the equivalence classes?

## Solution

Your solution goes here.

## Question 8

Consider the relation  $S$  on  $\mathbb{N}$  such that  $aSb \iff ab$  is a perfect square. Show that  $S$  is an equivalence relation on  $\mathbb{N}$ . What are the equivalence classes?

### Solution

Your solution goes here.



## Question 9

There was an ambiguity in the definition of a well-ordering in the lectures. It is clarified here.

A well-ordering  $R$  on set  $A$  is a partial order such that for every subset  $B \subseteq A$ ,  $B$  has an element  $m$  such that  $mRb$  for every  $b \in B$ .

In lecture 6, a partial order is shown to be a well-ordering twice: once during proof of the implication that Axiom of Choice implies Zorn's Lemma, and next during proof of the implication that Zorn's Lemma implies Well-Ordering Principle. Redo both these proofs in light of the above clarification.

## Solution

Your solution goes here.