CS201

Mathematics For Computer Science

Indian Institute of Technology, Kanpur

Group Number: 11

Anjali Rana (190147),
Atreya Goswami (190201),
Lipi Jain (190457)

Assignment 1

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Question 1

Let $S = \{(a, b, c)|a, b, c \in \mathbb{Z}\}$ be the set of all triplets of integers. Show that $|S| = \aleph_0$.

Solution

Question 2

For any $a, b, c, d \notin \{-\infty, \infty\}$, show that $|[a, b]| = |[c, d]|$ where $[x, y]$ is the set of all real numbers between x and y

Solution

Your solution goes here.

Question 3

Show that $|[0,1]| = \aleph_1$ where $[0,1]$ is the set of all real numbers between 0 and 1.

Solution

Your solution goes here.

Question 4

Show that $|\{0,1\}^*| = \aleph_1$ where $\{0,1\}^*$ is the set of all binary strings of infinite length.

Solution

Your solution goes here.

Question 5

Suppose R is a partial order on A and S be a partial order on B. Let L be a binary relation on $A \times B$ defined as $(a, b)L(a', b')$ iff

- $a \neq a'$ and aRa'
- $a = a'$ and bSb'.

Show that L is also a partial order on $A \times B$. Is it a total order?

Solution

Your solution goes here.

Question 6

Let R be a binary relation on $\mathbb{N}$ defined as aRb if $b = 2^k a$ where k is a non-negative integer. Show that R is a partial order on $\mathbb{N}$.

Solution

Your solution goes here.

Question 7

Let n be a positive integer. Consider the relation $\equiv_n$ on $\mathbb{Z}$ such that $a \equiv_n b \iff a = b \mod n$. Show that $\equiv_n$ is an equivalence relation on $\mathbb{Z}$. What are the equivalence classes?

Solution

Your solution goes here.

Question 8

Consider the relation S on $\mathbb{N}$ such that $aSb \iff ab$ is a perfect square. Show that S is an equivalence relation on $\mathbb{N}$. What are the equivalence classes?

Solution

Your solution goes here.

Question 9

There was an ambiguity in the definition of a well-ordering in the lectures. It is clarified here.

A well-ordering R on set A is a partial order such that for every subset $B \subseteq A$, B has an element m such that mRb for every $b \in B$.

In lecture 6, a partial order is shown to be a well-ordering twice: once during proof of the implication that Axiom of Choice implies Zorn's Lemma, and next during proof of the implication that Zorn's Lemma implies Well-Ordering Principle. Redo both these proofs in light of the above clarification.

Solution

Your solution goes here.