Paper Review and Implementation of

Generating Fuzzy Rules by Learning from Examples (Li-Xin Wang and Jerry M. Mendel)

EE658A COURSE PROJECT

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ACKNOWLEDGEMENT

I would like to take this opportunity to express my gratitude to Course Instructor Prof. Nischal K. Verma and Course TA Mohd. Aquib for their continued support and guidance. Also, I would like to thank my parents, without whose constant motivation and support, I would never be able to complete this project.

AGENDA

INTRODUCTION TO FUZZY LOGIC AND FUZZY SYSTEMS

- Deals with uncertainties due to imprecision, ambiguity and vagueness
- Key agent of Computational Intelligence
- Introduced by Prof. Lotfi A Zadeh in 1965
- Multivalued logic
- Involves soft or partial truth

MOTIVATION

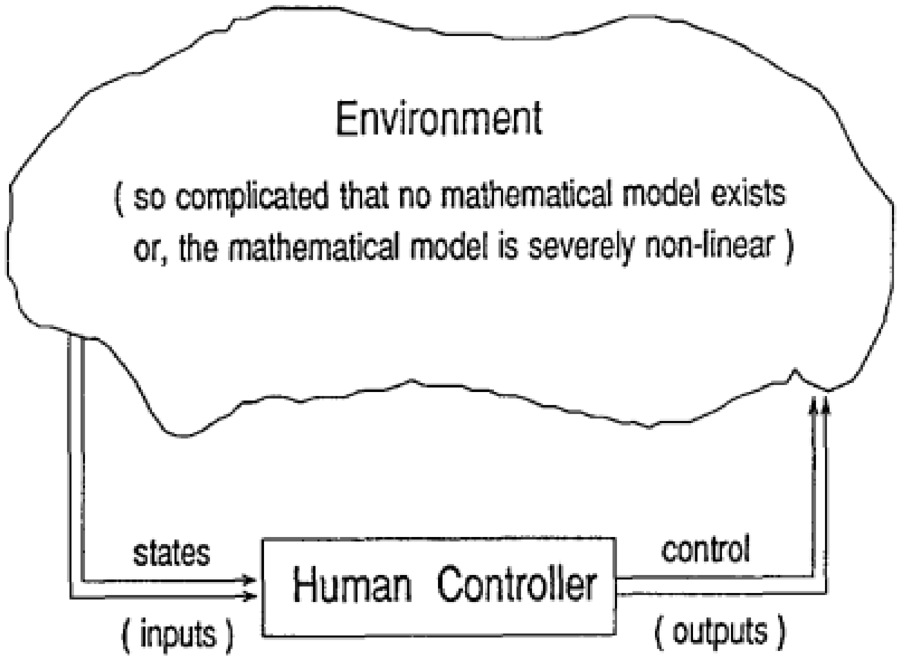
Most contemporary intelligent control and signal processing approaches were heuristic in nature – they used some *ad hoc method* to combine standard control or signal processing methods with expert systems.

Limitations of existing approaches:

1. Problem specificity of the approach
2. Difficulty in theoretical analysis

INTRODUCTION

TASK: Design a control system to replace the human controller



INFORMATION: 2 kinds of information are available to us.

A. Linguistic rules: Experience of the human controller

B. Numerical data: Sampled successful input-output pairs

Neither of the above alone is usually sufficient for designing a successful control system.

CONTRIBUTION

Previous work has shown that **fuzzy control** is an effective way to utilize **linguistic rules**, whereas **neural control** is suited for using **numerical data**.

Through this paper, the authors propose a new approach – **numerical-fuzzy control.**

The key ideas of this approach are –

1. Generating fuzzy rules from numerical data pairs.
2. Collecting the numerical fuzzy rules and linguistic fuzzy rules into a common fuzzy rule base.
3. Designing a control or signal processing system based on the combined fuzzy rule base.

The paper has also evaluated the proposed approach against existing neural and fuzzy approaches, on two benchmark problems – the **Truck Backer-Upper Control problem** and the **Mackey-Class chaotic time series prediction problem.**

APPROACH

The paper proposes a **5-stage approach** for generating fuzzy rules from numerical data and using these rules along with linguistic fuzzy rules to obtain a mapping from input space to output space.

Step 1: Dividing the Input and Output spaces into fuzzy regions

Step 2: Generating fuzzy rules from given data pairs

Step 3: Assigning a degree to each rule

Step 4: Creating a combined **Fuzzy Rule Base**

Step 5: Determine a mapping based on the combined Fuzzy Rule Base

STEP 1: DIVIDING INPUT AND OUTPUT SPACES INTO FUZZY REGIONS

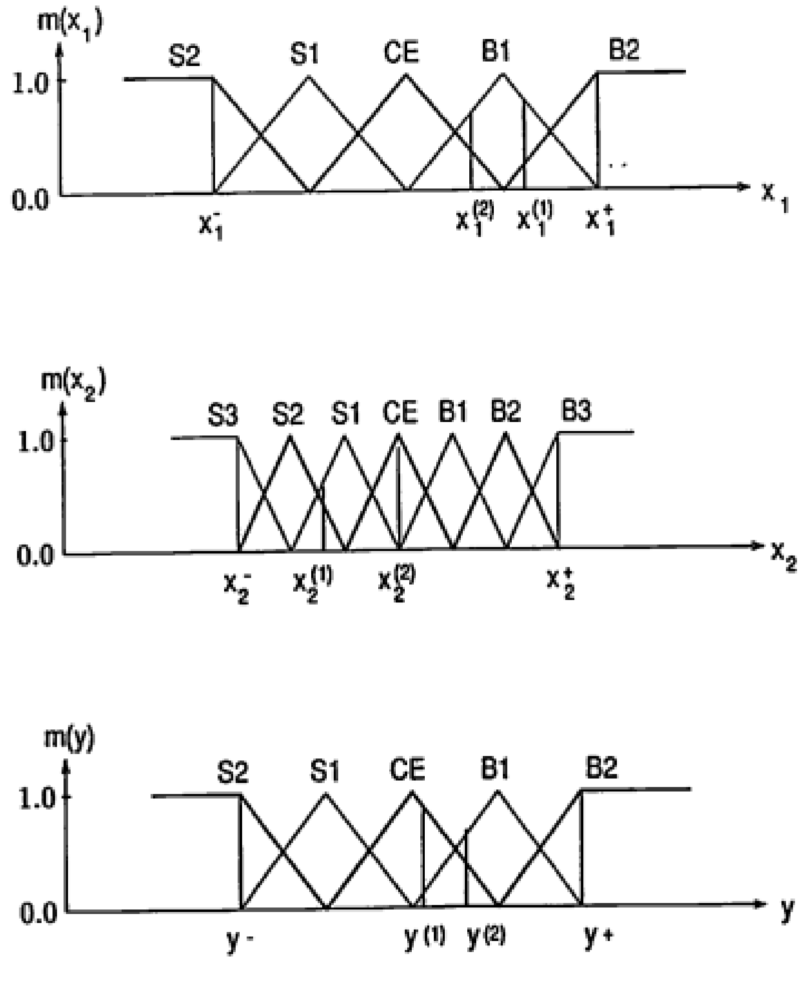
Let's suppose that we are given a set of desired input-output pairs $\left\{\left(x_1^{(i)},\ x_2^{(i)};\ y^{(i)}\right)\right\}_{i=1}^{N}$.

We need to generate fuzzy rules from these pairs and use those rules to determine a mapping $f\colon (x_1, x_2) \ \rightarrow \ y$.

First, let us assume that the **domain intervals** of x_1, x_2 and y are $[x_1^-,\ x_1^+]$, $[x_2^-,\ x_2^+]$ and $[y^-, y^+]$.

We divide each domain interval into $2N+1$ regions ($S_N, S_{N-1}, ..$ $...S_2, S_1, CE, B_1, B_2, ... B_{N-1}, B_N$) and assign each region a fuzzy membership function.

Note: N can be different for each domain interval. The lengths and shape of the regions' membership functions may also vary.



STEP 2: GENERATING FUZZY RULES FROM GIVEN DATA PAIRS

1. Determine the degrees of $x_1^{(i)}$, $x_2^{(i)}$ and $y^{(i)}$ in each of the defined regions.
2. Assign $x_1^{(i)}$, $x_2^{(i)}$ and $y^{(i)}$ to the region with maximum degree.
3. Obtain one "AND" rule from $\left(x_1^{(i)},\ x_2^{(i)};\ y^{(i)}\right)$.

Example: Let's say, degree of $x_1^{(i)}$ is maximum in region S1, degree of $x_2^{(i)}$ is maximum in region CE, and degree of corresponding $y^{(i)}$ is maximum in region S2. Then, from this input-output pair, we obtain the following rule:

IF x_1 *is S1 AND* x_2 is CE, THEN y *is S2.*

STEP 3: ASSIGNING A DEGREE TO EACH RULE

Since, each data pair generates a rule, and there are lot of data pairs, hence there will be **conflicting** and **redundant** rules, i.e., many rules with the same *antecedents* (IF part) having same or different *consequents* (THEN part).

To resolve this issue, we assign a degree to each rule and accept only the rule with maximum degree from many rules having the same antecedents.

Example: If RULE1 is "IF x_1 is S1 AND x_2 is CE, THEN *y is S2", then we assign a degree to RULE1 as*

$$D(RULE1) = m_{S1}(x_1)\, m_{CE}(x_2)\, m_{S2}(y)$$

Note: *Often we also multiply the "usefulness" $m^{(i)}$ of a given data pair (annotated by a human expert) in the expression for degree of a rule. Hence, we can redefine the degree of RULE1 as*

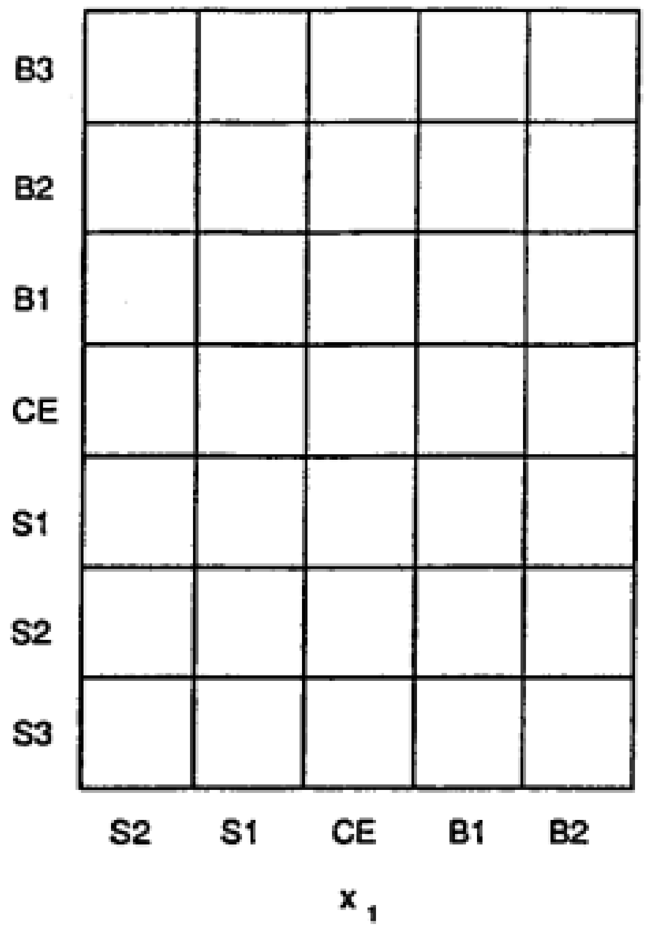
$$D(RULE1) = m_{S1}(x_1)\, m_{CE}(x_2)\, m_{S2}(y)\, m^{(1)}$$

STEP 4: CREATING A COMBINED FUZZY RULE BASE

In this step, we combine the fuzzy rules obtained in Step 3, and linguistic fuzzy rules annotated by human experts into a common framework – *the combined fuzzy rule base*.

Assuming that linguistic rules are also assigned a degree, in case of a conflict, only the rule with a higher degree will be accepted.

We know from Step 2 that fuzzy rules obtained from numerical data pairs will always be "AND" rules. However, linguistic fuzzy rules may be "OR" rules, in which case, all the boxes corresponding to the antecedents of the rule need to be filled with the output of the rule.



STEP 5: DETERMINE A MAPPING BASED ON THE FUZZY RULE BASE

Now for any new data pair (x_1, x_2), we need to determine the output control y. We use the following defuzzification strategy to achieve this:

1. For each fuzzy rule, we multiply the degrees of x_1 and x_2 in the antecedents' corresponding regions to determine the degree of the output control corresponding to (x_1, x_2).

$$m_{O^i}^{i} = m_{I_1^i}(x_1)\, m_{I_2^i}(x_2)$$

2. Now, we use the ***centroid defuzzification formula*** to determine the output control y.

$$y = \frac{\sum_{i=1}^{K} m_{O^i}^{i} \bar{y}^i}{\sum_{i=1}^{K} m_{O^i}^{i}}$$

FUZZY SYSTEM AS AN "UNIVERSAL APPROXIMATOR"

The proposed five-step approach generates a mapping from input space to output space. For the general n —input one output case, the mapping can be represented by

$$y = \frac{\sum_{i=1}^{K} \bar{y}^i m^i}{\sum_{i=1}^{K} m^i}; \quad \text{where } m^i = \prod_{1 \le j \le n} [m_j^i(x_j)]$$

The generated fuzzy system can approximate any real continuous function g defined on the compact set Q and thus acts as a ***universal approximator*** where the compact set Q is defined as

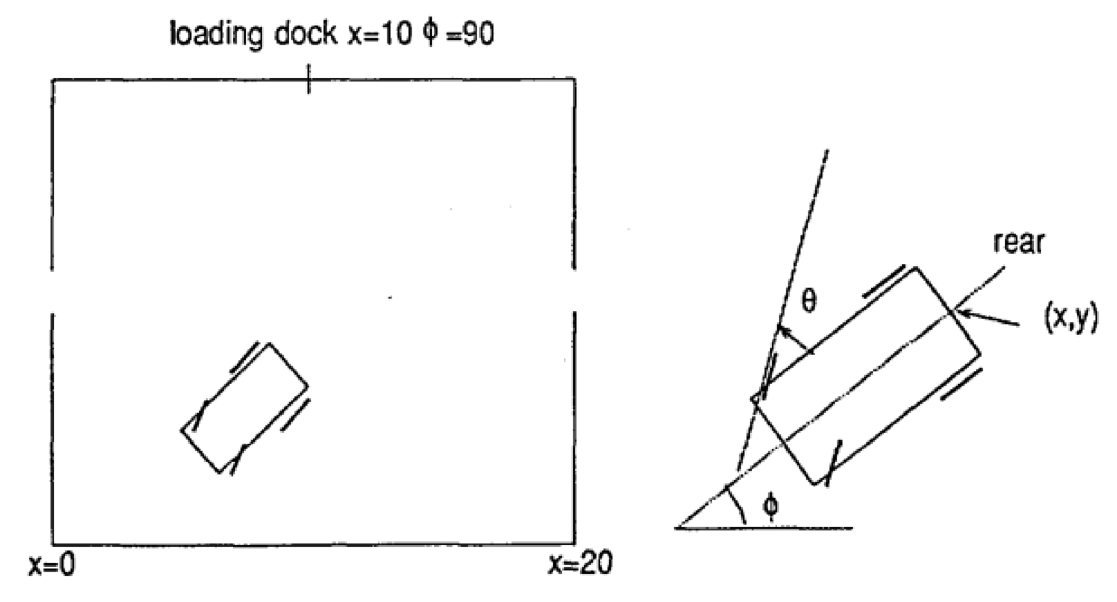
$$Q = [a_1, b_1] \times [a_2, b_2] \times [a_3, b_3] \times \cdots \times [a_n, b_n] \subset \mathbb{R}^n$$

TRUCK BACKER-UPPER CONTROL PROBLEM

System defined by 3 state variables (x, y, ϕ) and control $\theta.$ y is not considered as input variable.

14 data tables (sequences of desired trajectories with different initial configurations) and membership functions of each region of x, ϕ and θ obtained from [2].

Loading dock at $(x, \phi°) = (10, 90)$. Truck is backed up from initial state to loading dock configuration, unit distance traversed based on output control θ in every step.

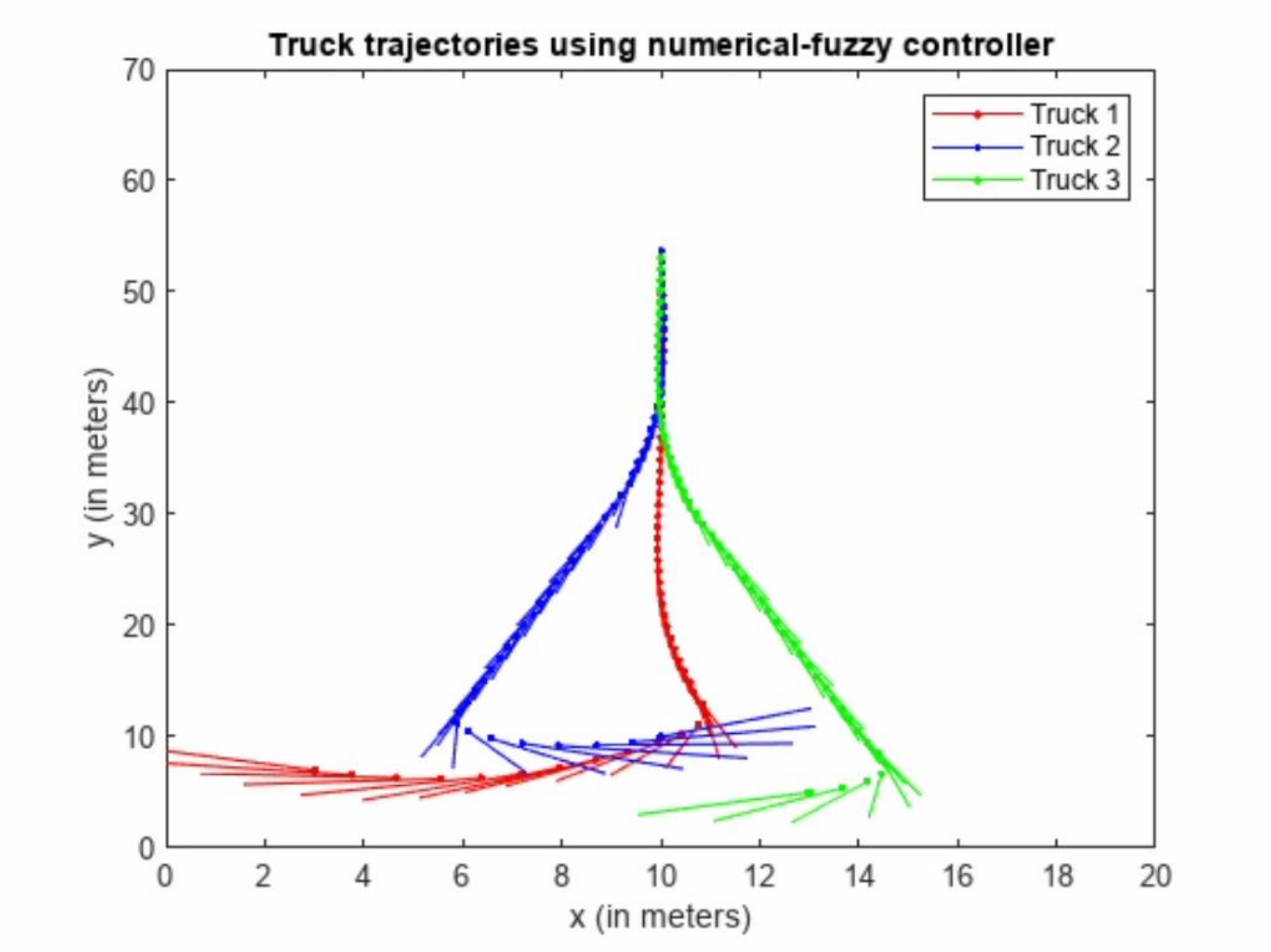


Kinematics of truck governed by approximate equations (13) – (15) listed in [1].

EXAMPLE 1 (WITH COMPLETE DATA AND NO LINGUISTIC RULES)

$\phi \backslash x$	S2	S1	CE	B1	B2
S3	S3	S3	X	X	X
S2	S2	S3	S3	S3	X
S1	B1	S1	S2	S3	S3
CE	B2	B2	CE	S2	S2
B1	B3	B3	B2	B1	S1
B2	X	B3	B3	B3	B2
B3	X	X	X	B3	B3

Fuzzy rule base for example 1



video showing trajectory of trucks for example 1

EXAMPLE 2 (WITH TRUNCATED DATA AND LINGUISTIC RULES)

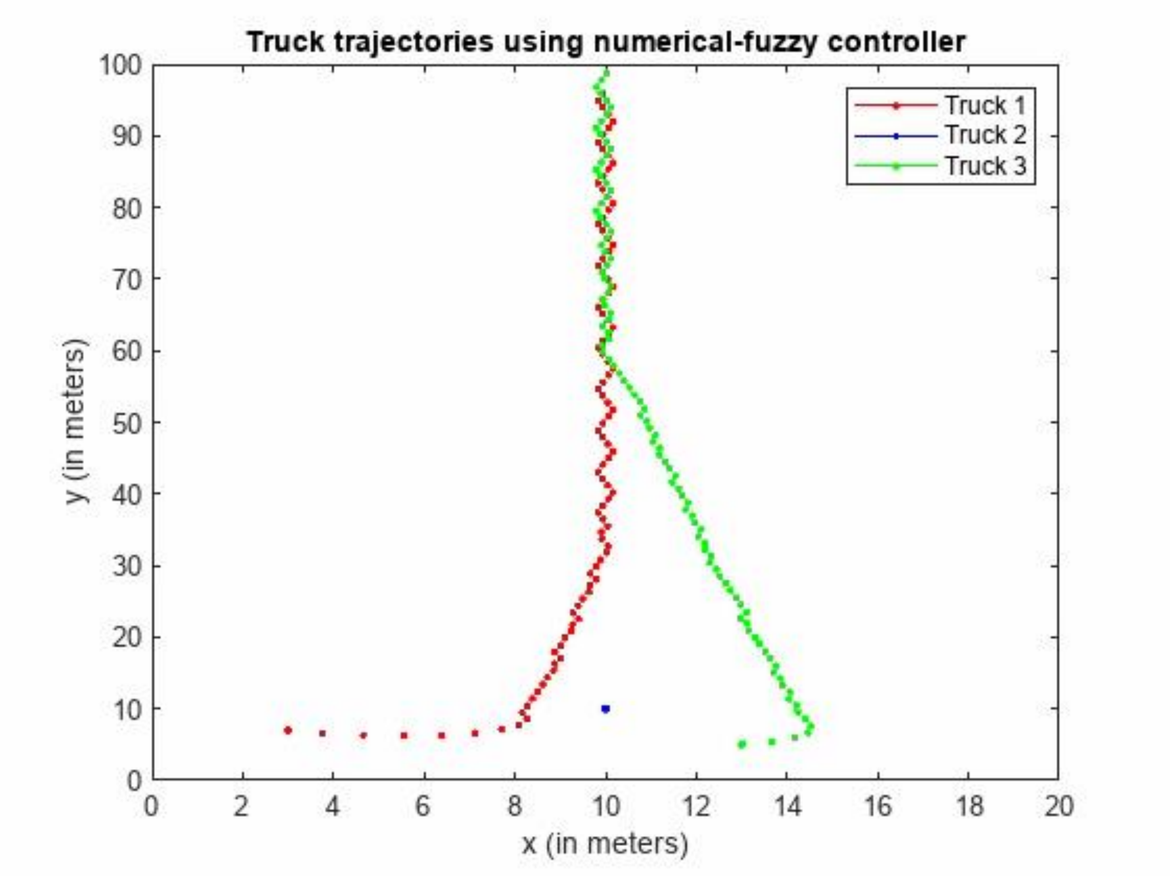
$\phi \backslash x$	S2	S1	CE	B1	B2
S3	S3	S3	X	X	X
S2	S2	S3	X	S3	X
S1	B2	S3	X	S3	X
CE	B2	B2	X	S2	S2
B1	X	B3	X	B1	S2
B2	X	B3	X	B3	B2
B3	X	X	X	B3	B3

Fuzzy rule base for example 2
(with truncated data)

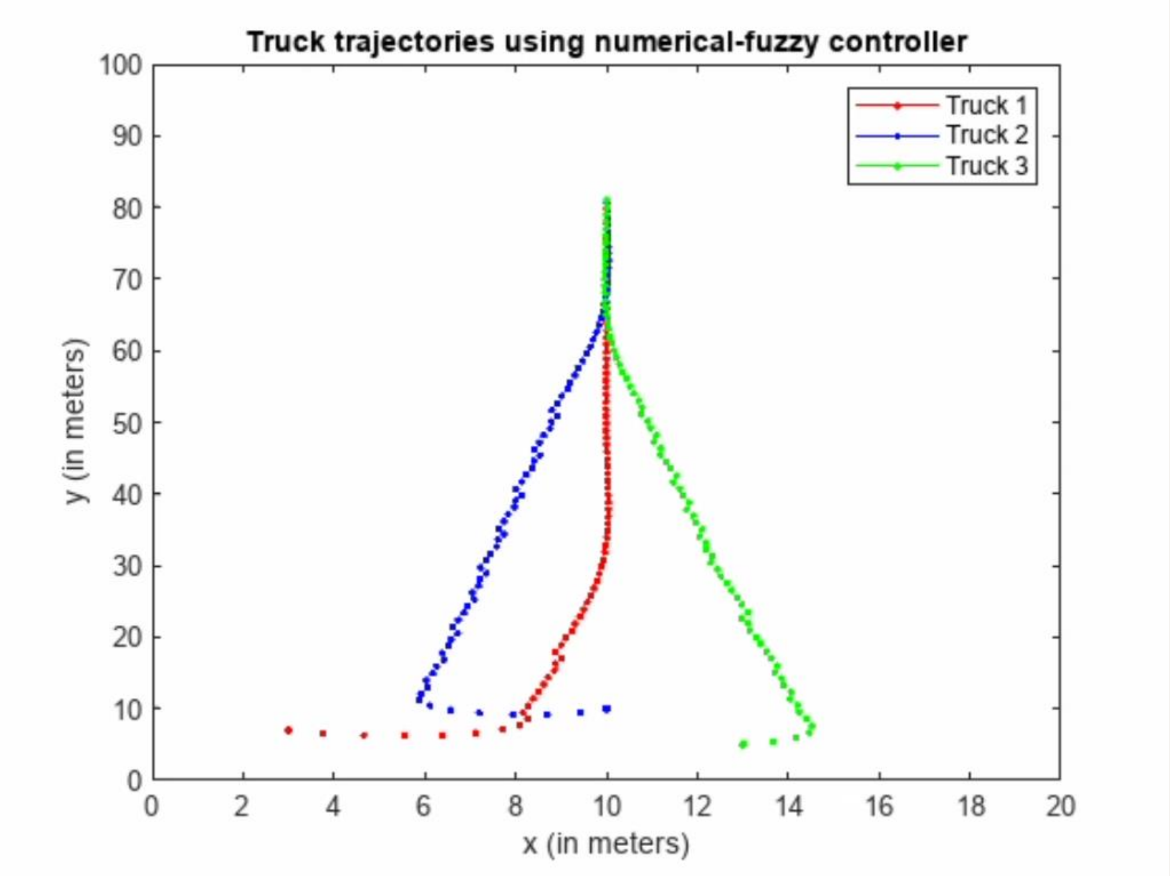
$\phi \backslash x$	S2	S1	CE	B1	B2
S3	S3	S3	X	X	X
S2	S2	S3	X	S3	X
S1	B2	S3	S2	S3	X
CE	B2	B2	CE	S2	S2
B1	X	B3	B2	B1	S2
B2	X	B3	B3	B3	B2
B3	X	X	X	B3	B3

Fuzzy rule base for example 2
(with added linguistic rules)

EXAMPLE 2 (WITH TRUNCATED DATA AND LINGUISTIC RULES)



video showing trajectory of trucks for example 2 (with truncated data)



video showing trajectory of trucks for example 2 (with truncated data and added linguistic rules)

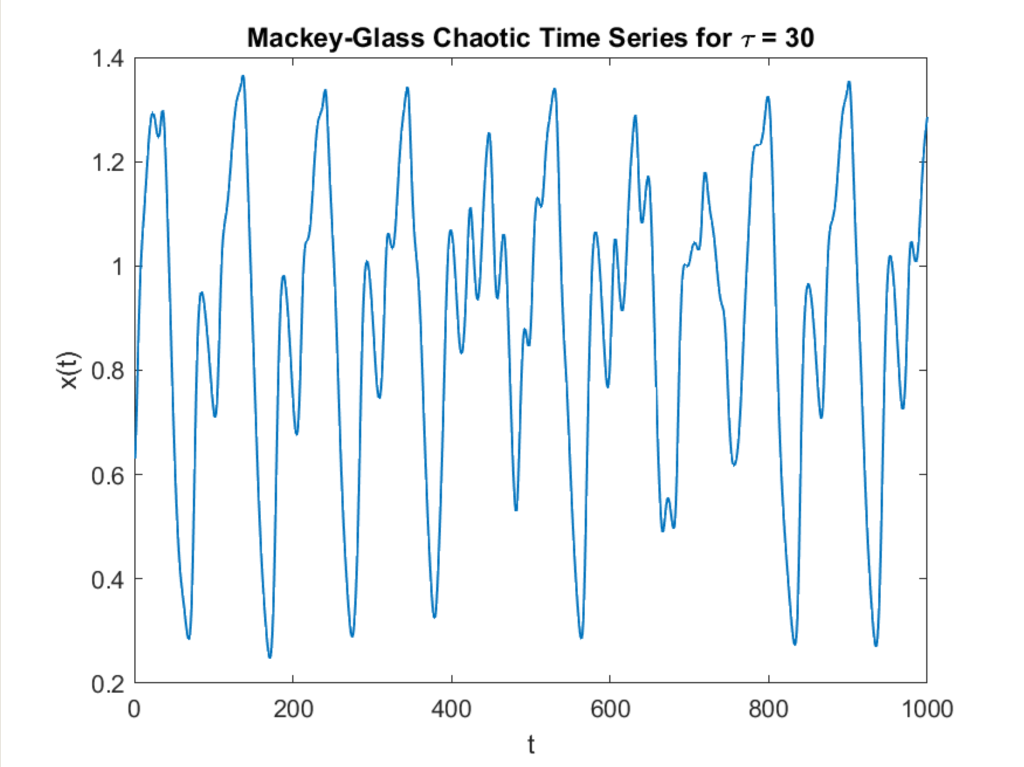
MACKEY-GLASS CHAOTIC TIME SERIES PREDICTION

A section of the Mackey-Glass chaotic time series was generated from the delay-differential equation mentioned in the paper [1, 18].

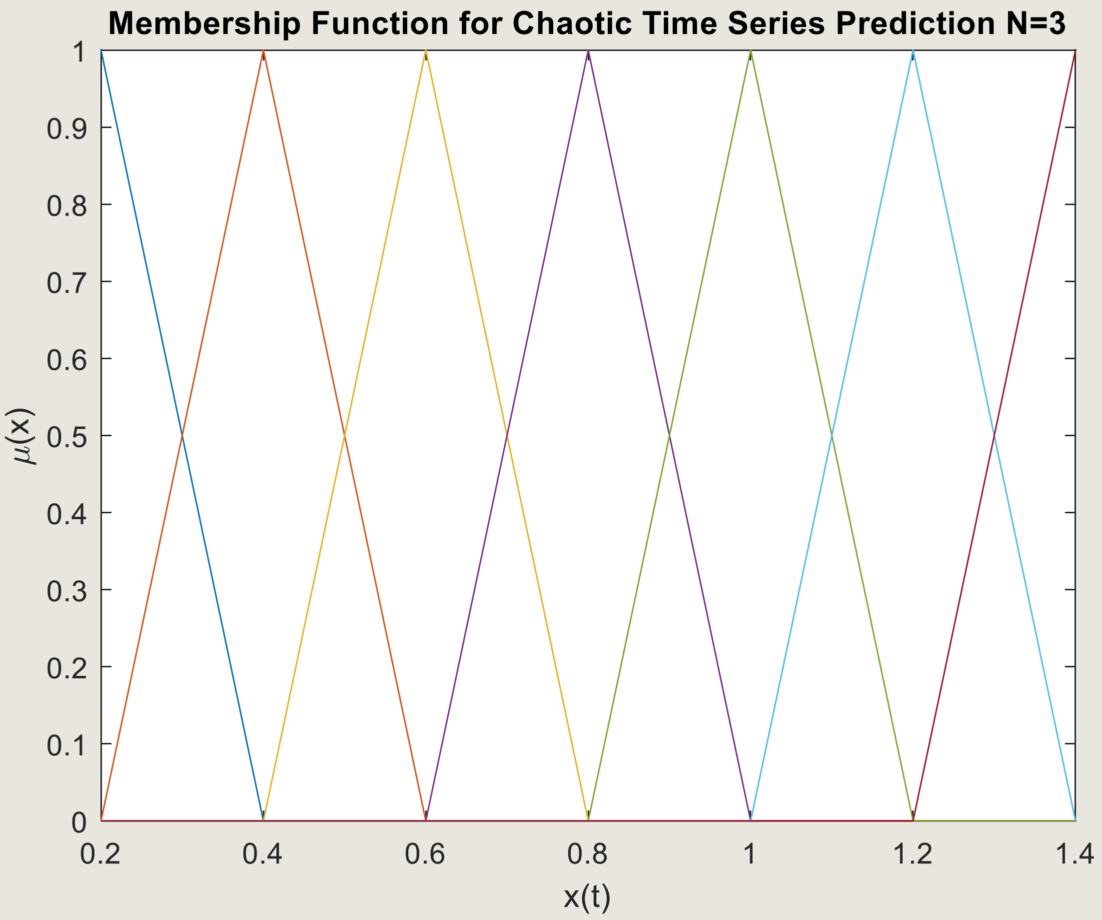
Given $m = 9, l = 1, 1000$ data pairs were generated. The numeric fuzzy approach was then applied to create a fuzzy rule base and predict the sequence $x(701)$ to $x(1000)$.

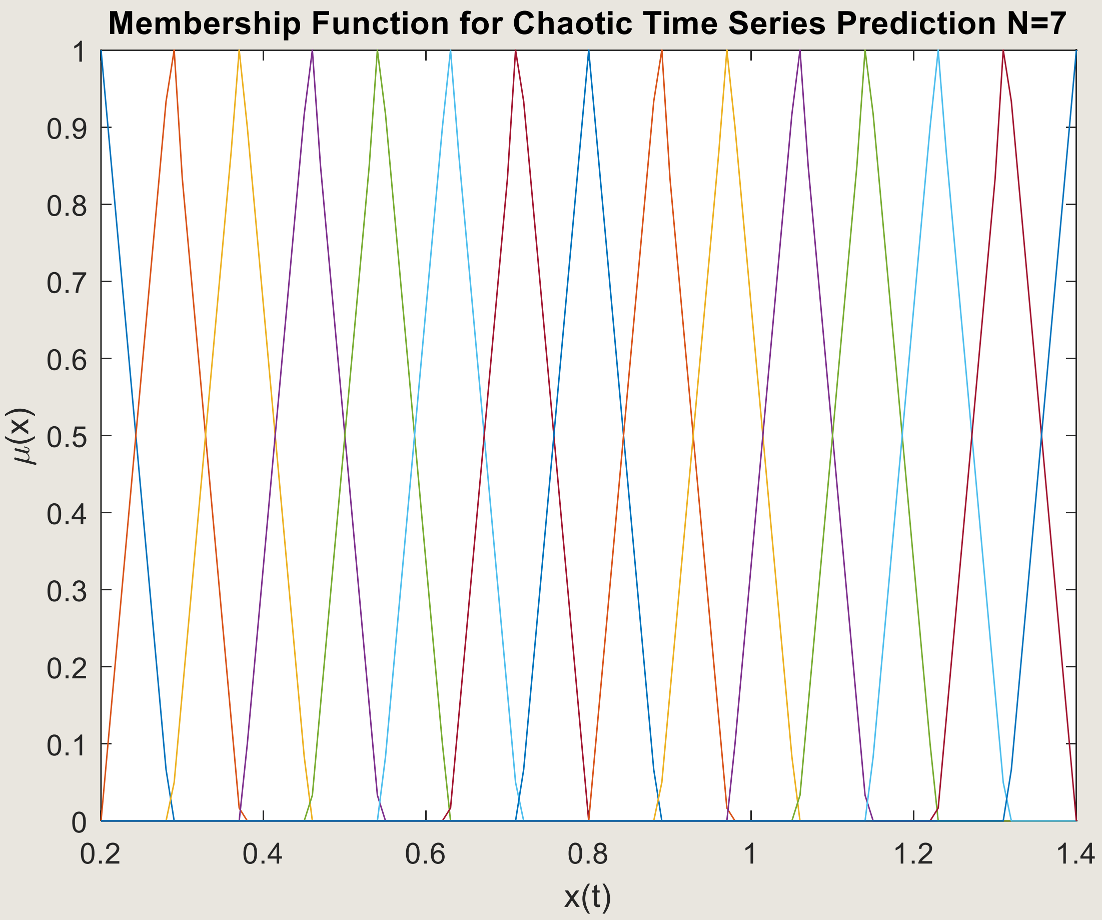
In example 1, we used 700 data pairs $x(1)$ to $x(700)$ for training. In example 2, we used 200 data pairs $x(501)$ to $x(700)$ for training.

The predictions were made for $N = 3, 7, 14$, where the x-space was divided into $2N + 1$ fuzzy regions.

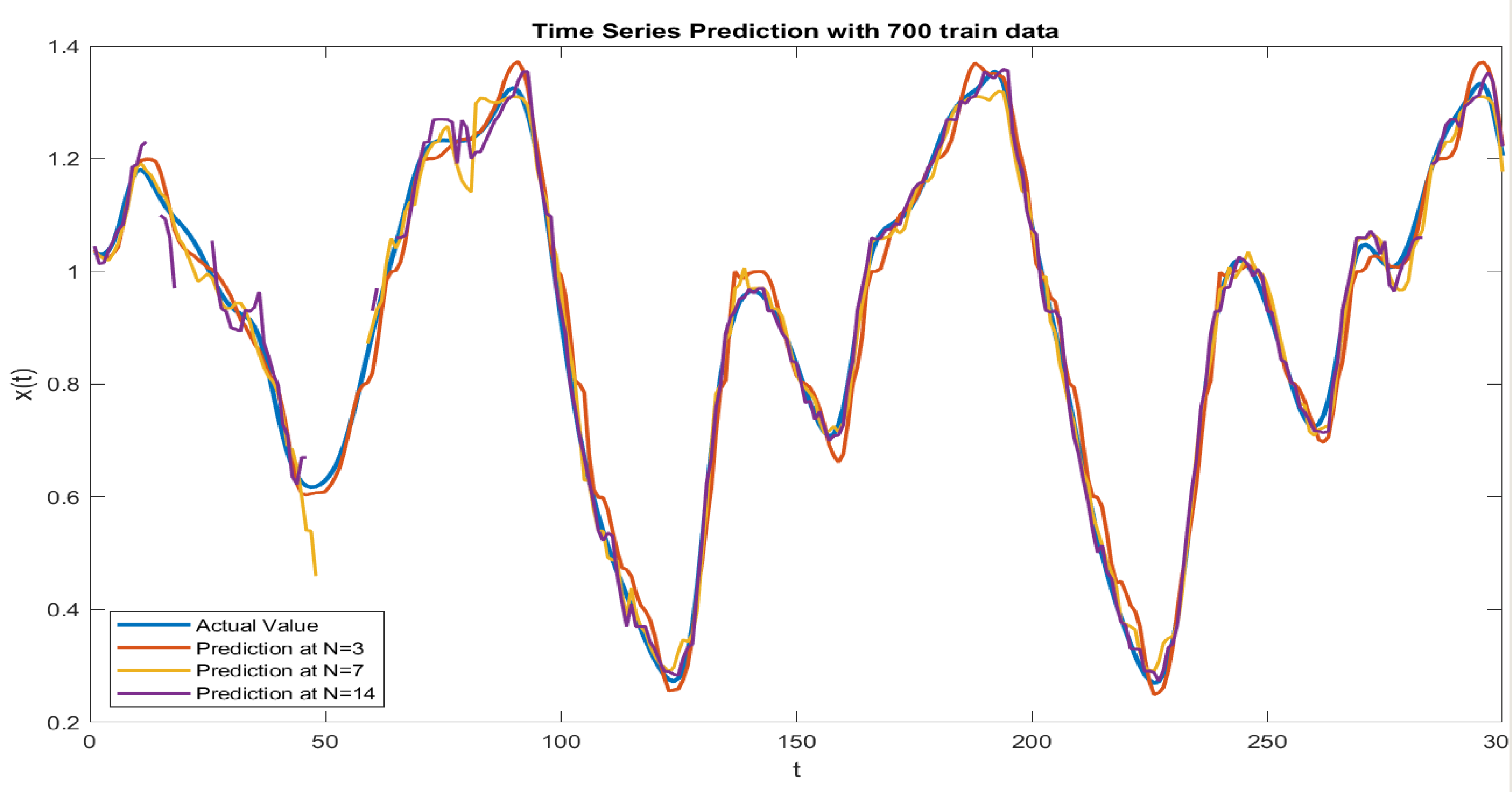


MEMBERSHIP FUNCTIONS FOR FUZZY REGIONS $(N = 3, 7)$

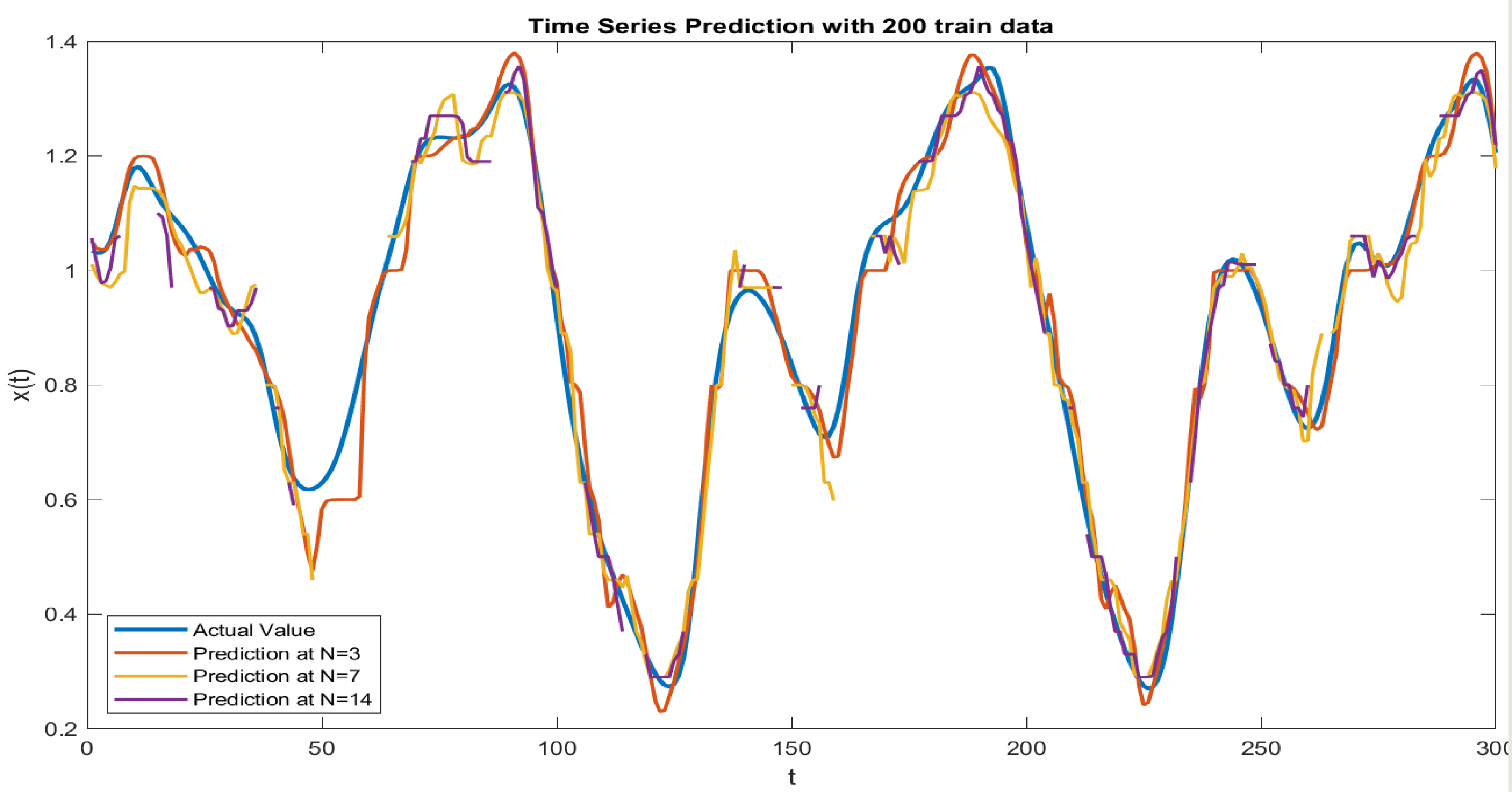




EXAMPLE 1: MACKEY-GLASS CHAOTIC TIME SERIES PREDICTION



EXAMPLE 2: MACKEY-GLASS CHAOTIC TIME SERIES PREDICTION



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1. L-X Wang and Jerry M Mendel. “Generating fuzzy rules by learning from examples”. In: *IEEE Transactions on systems, man, and cybernetics* 22.6 (1992), pp. 1414– 1427.

2. Li-Xin Wang. “Generating fuzzy rules from numerical data, with applications”. In: *University of Southern California, tech. report* (1991).

THANK YOU