

Assignment 8

CSCI 411

1.

$$S \rightarrow SS$$

$$S \rightarrow (S)$$

$$S \rightarrow \varepsilon$$

a. $((00))$

$$\begin{aligned} S &\rightarrow (S) \\ &\rightarrow ((S)) \\ &\rightarrow ((SS)) \\ &\rightarrow ((S)(S)) \\ &\rightarrow ((\varepsilon)(\varepsilon)) \\ &\rightarrow ((00)) \rightarrow ((00)) \end{aligned}$$

b. $0(0)$

$$\begin{aligned} S &\rightarrow SS \\ &\rightarrow (S)(S) \\ &\rightarrow (S)(S) \\ &\rightarrow (\varepsilon)(\varepsilon) \\ &\rightarrow 0(0) \rightarrow 0(0) \end{aligned}$$

2.

$$S \rightarrow 0A \mid 1B \mid \varepsilon$$

$$A \rightarrow 0S \mid 1C$$

$$B \rightarrow 1S \mid 0C$$

$$C \rightarrow 0B \mid 1A$$

a. 000110

$$\begin{aligned} S &\rightarrow 0A \\ &\rightarrow 00S \\ &\rightarrow 000A \\ &\rightarrow 0001C \\ &\rightarrow 00011A \\ &\rightarrow 000110S \\ &\rightarrow 000110\varepsilon \rightarrow 000110 \end{aligned}$$

b. 11001001

$$\begin{aligned} S &\rightarrow 1B \\ &\rightarrow 11S \\ &\rightarrow 110A \\ &\rightarrow 1100S \\ &\rightarrow 11001B \\ &\rightarrow 110010C \\ &\rightarrow 1100100B \\ &\rightarrow 11001001S \\ &\rightarrow 11001001\varepsilon \rightarrow 11001001 \end{aligned}$$

3.

$$S \rightarrow S + S \mid S * S \mid (S) \mid N$$

$$N \rightarrow 0.FD \mid PD.FD \mid PD$$

$$D \rightarrow 0D \mid 1D \mid 2D \mid 3D \mid 4D \mid 5D \mid 6D \mid 7D \mid 8D \mid 9D \mid \epsilon$$

$$F \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

$$P \rightarrow 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

a. $12.3 + 3 * 0.22$

$$S \rightarrow S + S$$

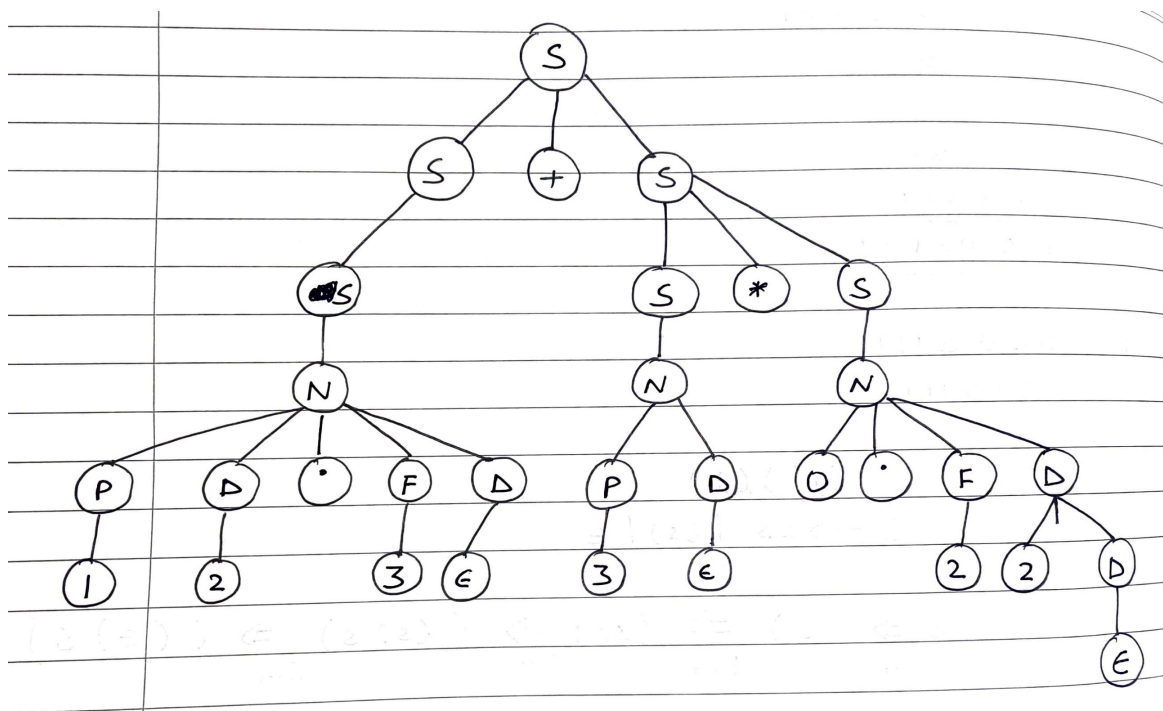
$$\rightarrow S + S * S$$

$$\rightarrow N + N * N$$

$$\rightarrow PD.FD + PD * 0.FD$$

$$\rightarrow 12.3\epsilon + 3\epsilon * 0.22D$$

$$\rightarrow 12.3\epsilon + 3\epsilon * 0.22\epsilon \rightarrow 12.3 + 3 * 0.22$$



b. $4 * (9 + 13.0) * 75.89$

$S \rightarrow S * S * S$

$\rightarrow N * (S) * N$

$\rightarrow PD * (S + S) * PD.FD$

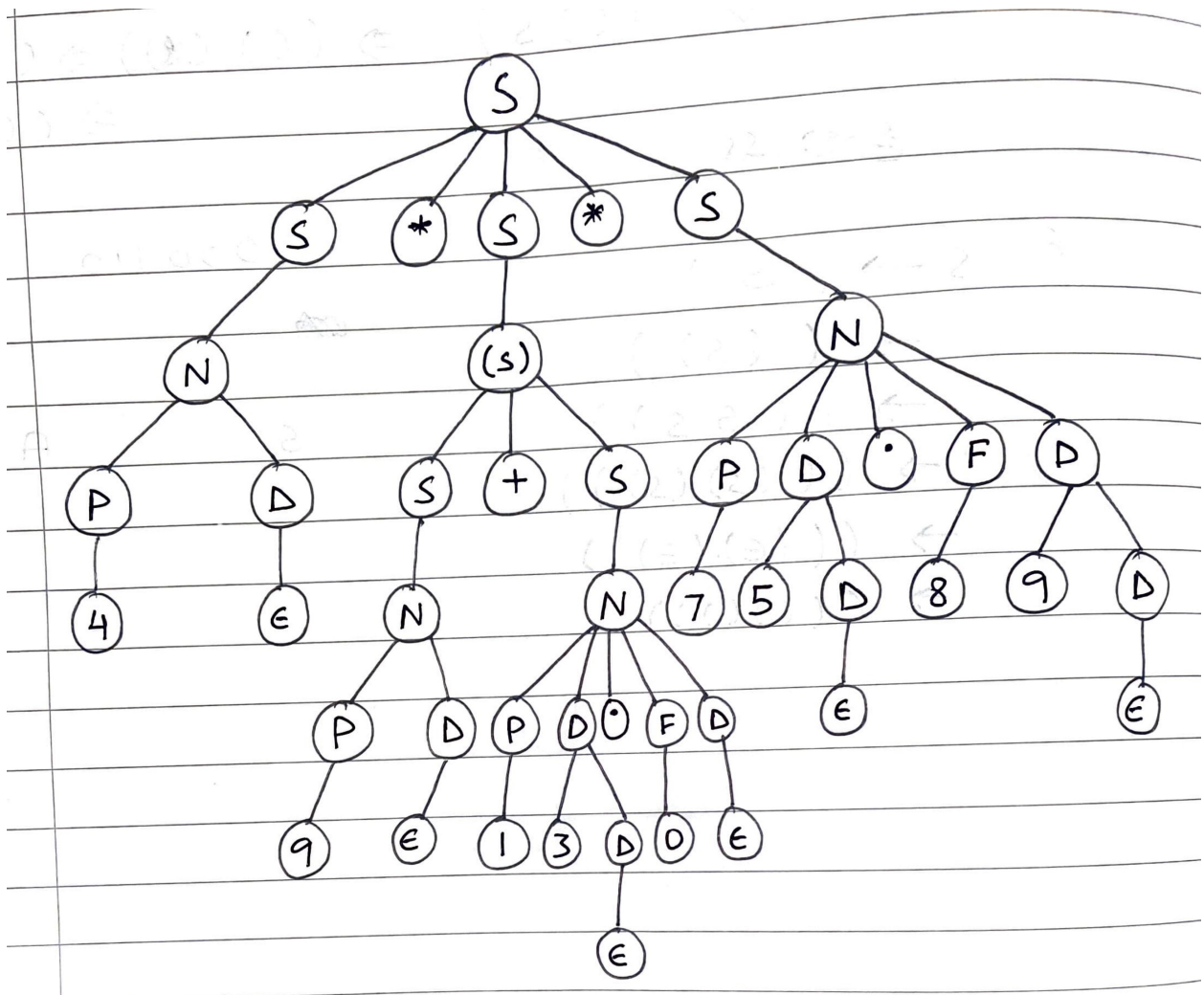
$\rightarrow 4\epsilon * (N + N) * 75D.89D$

$\rightarrow 4\epsilon * (PD + PD.FD) * 75\epsilon.89\epsilon$

$\rightarrow 4\epsilon * (9\epsilon + 13D.0\epsilon) * 75\epsilon.89\epsilon$

$\rightarrow 4\epsilon * (9\epsilon + 13\epsilon.0\epsilon) * 75\epsilon.89\epsilon$

$\rightarrow 4 * (9 + 13.0) * 75.89$



4.

a.

$$S \rightarrow 0S \mid 1S \mid 11$$

b.

$$S \rightarrow A11A$$

$$A \rightarrow 0A \mid 1A \mid 0 \mid 1 \mid \varepsilon$$

c.

$$S \rightarrow aSa \mid bSb \mid cSc \mid a \mid b \mid c \mid \varepsilon$$

d.

Case 1: $i = j$,

$$S_1 \rightarrow AB$$

$$A \rightarrow aAb \mid ab \mid \varepsilon$$

$$B \rightarrow cB \mid c \mid \varepsilon$$

Case 2: $i = k$,

$$S_2 \rightarrow aS_2c \mid C$$

$$C \rightarrow bC \mid b \mid \varepsilon$$

Using Case 1 & 2,

$$S \rightarrow S_1 \mid S_2$$

$$\begin{aligned}
 S_1 &\rightarrow AB \\
 A &\rightarrow aAb \mid ab \mid \varepsilon \\
 B &\rightarrow cB \mid c \mid \varepsilon \\
 C &\rightarrow bC \mid b \mid \varepsilon
 \end{aligned}$$

e.

$$\begin{aligned}
 S &\rightarrow AB \\
 A &\rightarrow aC \mid aCaC \\
 B &\rightarrow CaCaCaCA \mid \varepsilon \\
 C &\rightarrow bC \mid cC
 \end{aligned}$$

f.

$$\begin{aligned}
 S &\rightarrow aSc \mid A \\
 A &\rightarrow bAc \mid \varepsilon
 \end{aligned}$$

g.

$$\begin{aligned}
 S &\rightarrow A \\
 A &\rightarrow (A) (A) \mid A + A \mid A \mid B
 \end{aligned}$$

5.

a.

$$S \rightarrow aAc \mid bbB \mid aa$$

$$A \rightarrow cSc \mid C$$

$$B \rightarrow A \mid Bbb$$

$$C \rightarrow C \mid a \mid b \mid \varepsilon$$

Step 1: Remove NULL productions

$$S \rightarrow aAc \mid bbB \mid aa$$

$$B \rightarrow A \mid Bbb$$

$$A \rightarrow cSc \mid C \mid \varepsilon$$

$$C \rightarrow a \mid b$$

$$S \rightarrow aAc \mid ac \mid bbB \mid aa$$

$$B \rightarrow \varepsilon \mid Bbb$$

$$A \rightarrow cSc \mid C$$

$$C \rightarrow a \mid b$$

$$S \rightarrow aAc \mid ac \mid bb \mid bbB \mid aa$$

$$B \rightarrow bb \mid Bbb$$

$$A \rightarrow cSc \mid C$$

$$C \rightarrow a \mid b$$

Step 2: Eliminate unit rules

$$S \rightarrow aAc \mid ac \mid bb \mid bbB \mid aa$$

$$B \rightarrow bb \mid Bbb$$

$$A \rightarrow cSc \mid a \mid b$$

$$C \rightarrow a \mid b$$

Step 3: Simplify grammar, remove C

$$S \rightarrow aAc \mid ac \mid bb \mid bbB \mid aa$$

$$B \rightarrow bb \mid Bbb$$

$$A \rightarrow cSc \mid a \mid b$$

b.

$$S \rightarrow 0AB0 \mid 1C1 \mid C0A \mid \varepsilon$$

$$A \rightarrow C \mid AB \mid 1 \mid 001$$

$$B \rightarrow SB1 \mid 0$$

$$C \rightarrow SAC \mid C1SSC$$

Step 1: Removal of ε is not possible in initial variable. Hence, we skip removal of null production.

Step 2: Eliminate unit rules

$$S \rightarrow 0AB0 \mid 1C1 \mid C0A \mid \varepsilon$$

$$A \rightarrow SAC \mid C1SSC \mid AB \mid 1 \mid 001$$

$$B \rightarrow SB1 \mid 0$$

$$C \rightarrow SAC \mid C1SSC$$

Step 3: Simplify grammar, C is not terminating (removal necessary from all productions)

$$S \rightarrow 0AB0 \mid \varepsilon$$

$$A \rightarrow AB \mid 1 \mid 001$$

$$B \rightarrow SB1 \mid 0$$