Chapter 2 Solutions

1   Linear Algebra

1.1   Bases and linear independence

1. We see that

$$a_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + a_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + a_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

   has solution $a_1 = a_2 = -a_3 \neq 0$. Therefore the vectors are linearly dependent.

2. Choosing the basis $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the matrix representation of A with respect to this basis is

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

   Taking instead the basis $|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, the matrix representation of A with respect to this basis is

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

3. Using equation (2.12) repeatedly, we have

$$BA|v_i\rangle = B(\sum_j A_{ji}|w_j\rangle)$$

$$= \sum_j A_{ji}(B|w_j\rangle)$$

$$= \sum_j A_{ji} \sum_k B_{kj}|x_k\rangle$$

$$= \sum_k (\sum_j B_{kj}A_{ji})|x_k\rangle$$

   Also by (2.12), we have

$$BA|v_i\rangle = \sum_k (BA)_{ki}|x_k\rangle$$

Comparing the two expressions, we find

$$(BA)_{ki} = \sum_j B_{kj} A_{ji},$$

which is the matrix product of the matrix representations for B and A.

4.