Chapter 2 Solutions

1 Linear Algebra

1.1 Bases and linear independence

1. We see that
$$a_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + a_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + a_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
has solution $a_1 = a_2 = -a_3 \neq 0$. Therefore the vectors are linearly dependent.

2. Choosing the basis $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the matrix representation of A with respect to this basis is
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
Taking instead the basis $|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, the matrix representation of A with respect to this basis is
$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

3.