1 Delaunay Triangulation, Voronoi Diagram, and Gabriel Graph

Relate the Delaunay triangulation and Voronoi diagram to edges that appear in the Gabriel Graph. (Hint: The Gabriel Graph is a subgraph of the Delaunay triangulation). Don't all edges of a Delaunay triangle cross a Voronoi edge? Prove it in both directions.

($\Rightarrow$) For two points $p_i, p_j \in S$, $\overline{p_ip_j}$ is the edge that connects them, and C_{ij} is the circle with $\overline{p_ip_j}$ as the diameter. We say that C_{ij} is an empty circle with no points of S in its interior and therefore $\overline{p_ip_j} \in \mathcal{GG}(S)$; we then consider a third point $p_k \in S$, where $p_k \neq p_i, p_j$ and p_k is not interior to C_{ij}. Either

1. p_r is on the circumference of C_{ij}. Therefore, by definition[1], $\Delta p_ip_jp_j$ is a face in the Delaunay triangulation, and so $\overline{p_ip_j} \in \mathcal{DT}(S)$ (as do the other edges formed by the three points).
2. p_r is exterior to C_{ij}. Therefore, by definition[2], $\overline{p_ip_j} \in \mathcal{DT}(S)$.

The $\mathcal{DT}(S)$ is the dual of the $\mathcal{VD}(S)$; by definition, the vertices of $\mathcal{DT}(S)$ are Voronoi sites and the edges of $\mathcal{VD}(S)$ are those that connect those circumcenters in $\mathcal{DT}(S)$ that share a triangle edge (and so, cross that edge). We can conclude that for $\overline{p_ip_j} \in \mathcal{GG}(S)$ if and only if $\overline{p_ip_j} \in \mathcal{DT}(S)$, because $mathcalGG(S) \subseteq \mathcal{DT}(S)$.

Since $\mathcal{GG}(S) \subseteq \mathcal{DT}(S)$, we can find $\mathcal{GG}(S)$ very easily. First, compute the $\mathcal{DT}(S)$, which can be done in $O(n \log n)$ time and $O(n)$ space[3]. Then, We can walk the edges of $\mathcal{DT}(S)$ in $O(n)$ time, and check each if the circle with just that edge, $\overline{p_ip_j}$, as the diameter contains an interior point or not. Since this was an edge in the $\mathcal{DT}$, there is only one potential point that could be interior, the third point in the triangle; so we only need to check if the neighbors of p_i, p_j are interior to C_{ij}. If it has interior points, then discard that edge; otherwise, it is an edge in $\mathcal{GG}(S)$. The total runtime is $O(n \log n)$ due to computing the $\mathcal{DT}(S)$, which is optimal and faster than checking every pair of points with every other point for being interior, which would come out to be $O(n^3)$ time.

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[1] Theorem 9.6.i [2] pg. 198
[2] Theorem 9.6.ii [2] pg. 198
[3] Theorem 9.12 [2] pg. 206

2 Euclidean Norm

norm formula is
L-dist is distance from 2 points

a. d

3 Double Wedges

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4 Project

I emailed Diane about a project on line sweeping.

References

[1] Jake and Diane's office hours, classmates: Stephanie, Alex, Anju with homework problem discussions.

[2] Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars. 2008. Computational Geometry: Algorithms and Applications (3rd ed. ed.). Springer-Verlag TELOS, Santa Clara, CA, USA.

[3] H. Edelsbrunner and L.J Guibas, "Topological Sweep an Arrangement". Journal of Computer and System Sciences. 38:164-194. 1989.