COMP163 Homework Assignment 1: Due Monday, September 20, 2021, 11:59PM

Reading: Please read Chapter 1 in the Text, and also review convex hulls in the "Lecture Notes" and in "Computational Geometry: A User's Guide" on Convex Hulls.

General Information: When describing an algorithm, do not forget to analyse its running time and explain why the algorithm is correct. Although you may discuss these problems in the preliminary stages with others, work submitted should be done individually and written in your own words. If you have any discussions with others (students, friends, TAs, faculty, ...) relative to a homework problem or if you gain information from a written (or video/audio) source other than your own notes from lecture, you are expected to identify your collaborator/source.

Problems:

1. Left Turn – just persuade yourself but do *not* submit: Given points $A = (x_1, y_1)$, $B = (x_2, y_2)$, and $C = (x_3, y_3)$, the determinant $D =$

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

   gives twice the signed area of triangle ΔABC where the sign is $+$ if and only if A, B, C appear in counterclockwise order on the boundary of ΔABC. In other words, A, B, C forms a left turn if and only if $D > 0$. Use analytic geometry to derive and verify this fact to yourself.

   You do *NOT* need to hand-in a written solution to this problem!

2. Un-ordered Divide and Conquer Convex Hull

   (a) Let P_1 and P_2 be n_1- and n_2-vertex convex polygons, respectively, with no vertices in common. Prove that the number of lines of support common to P_1 and P_2 is at most $2 * min\{n_1, n_2\}$ and that this bound is achievable. [How many common lines of support can they have if P_1 and P_2 do not intersect? How many if P_1 is interior to P_2]

   (b) Given two arbitrary convex polygons P_1 and P_2 with n_1 and n_2 vertices each (their boundaries may intersect one, two, or more times; they may be disjoint; one may be contained within the other), specify as efficient an algorithm as you can to compute the convex hull of $P \cup Q$. Analyse its complexity.

   (c) Use this algorithm to generate a divide-and-conquer algorithm without presorting for finding the convex hull of an arbitrary set of n points. Analyse complexity.

3. Reprise of in-class exercise
   Given a set S of n planar points in general position and in arbitrary order, provide and analyse the most efficient algorithm that you can for constructing:

   - a simple monotone polygon R whose vertices are exactly the points in S and which contains the line segment between the point in S of smallest x-coordinate and the point in S of largest x-coordinate. [Can you do this in $\Theta(n \log n)$ time?]

- a simple starshaped polygon P whose vertices are exactly the points in S for which the point of S of smallest x-coordinate "sees" every point of P. [Can you do this in $\Theta(n \log n)$ time?]

Now assume that you are given a series of query points q that are NOT in S. For each of P and R, provide and analyze an algorithm that will test whether each query point q is interior or exterior to that polygon. [Can you do this in $\Theta(\log n)$ time?]