1 Necessity of Noise Modeling

- A quantum system has to involve some incoherence processes (measurements, random perturbations, etc), which ultimately makes the system probabilistic. So a noisy quantum system produces a random distribution of quantum states $|\psi_i\rangle$ with probability p_i due to imprecise controls and environmental noise [ding2020].
- Start by modeling noise as a probability distribution over pure quantum states, $\{p_i, |\psi_i\rangle\}$ [ding2020].
- Noise is pervasive, difficult to characterize/understand, and can have multiple sources, but you cannot consider practical quantum computing without considerations for effects of noise on the quantum system.
- Noise modeling informs how noise disrupts correctness during computation. The better they are, the better research can facilitate simulations, because actual quantum computers are few and far between.
- Determining specific noise requires physical experiments, which is not feasible for most users. Realistic noise models are not feasible to simulate at scale, so noise is estimated and these simulation techniques make simplifying assumptions.
- quantum noise is context dependent, so estimating error for one gate does not necessarily model larger systems with more qubits. Qubit errors cannot be considered independently. This makes noise impossible to simulate. This impacts benchmarking quantum computers, as error rates may increase with larger systems due to increase complexity (i.e. extrapolating information from running a program on small system does not straightforwardly inform how it runs on a larger system).

2 Basic of Noise Characterization

- Different errors that noise can introduce (leading to the different noise models) [resch21]:
  - undesired **environment** interactions or decay of quantum states
    - Ideal scenario is complete isolation from environment, which is not possible, so the goal is increased coherence time.
    - Measured with: coherence time, dephasing time, and relaxation time
    - Performing operations on a quantum system requires external input to the system, so this also increases risk of environmental interactions (coherence-controllability tradeoff).
    - these interactions can also lead to unitary errors that cause unitary rotation of quantum states
  - undesired **qubit qubit** interactions

    * qubits can become entangled and their states correlated. Entanglement is only useful when users desire it.
    * Unexpected entanglement of qubits lead to mixture of quantum states and decoherence/degredation of quantum state (cross-talk), when what we actually want to happen with cross-talk is just a unitary evolution of the state in order to retain information in the quantum state rather than losing the information.
    * Ancilla (extra) qubit interactions (with data qubit) is a technique used for error correction, where error information (syndromes) is extracted by measuring ancilla. However, this itself can lead to cross-talk between ancilla and data qubit, where data qubit loses quantum information to the ancilla and its state is changed.
  - imprecise **control** of operations
    * perfect application of gates gives us precise correct quantum states;
    * imperfect calibrations lead to slightly different rotations, which doesn't directly destroy quantum states but rather evolves it to an unwanted state. This affects the ability of error correction techniques.
  - **Leakage** to other quantum states:
    * Computation works on the assumption that qubits are encoded in **computational subspace**, where only two possible states are represented ($|0\rangle$ *or* $|1\rangle$).
    * quantum systems that we use as qubits actually have more states than that.
    * Leakage: when a qubit enters one of the other states outside of subspace.
    * Seepage: when a qubit returns to computational subspace.
    * Leakage and seepage can be caused by one of the other error sources.

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3 Some Noise Models

- Stochastic Pauli Noise
  - Simple and easy to correct for [wallman16]
  - models undesired environment interactions
  - SPN is implemented by inserting a X, Y, or Z gate into a circuit at random. Certain error rates for each gives us depolarizing noise or symmetric depolarizing noise.
  - SPN can be considered inaccurate and overly optimistic, can provide reasonable approximations of certain conditions [resch21]
  - Improving on SPN can involve augmenting Pauli gates with Clifford group operators (H, S, and CNOT) and Pauli measurements (i.e. simply using a larger set of gates for the random insertion)

- Coherent Noise
- Amplitude/Phase Damping
- Overview of which physical noise can correspond to which noise model [resch21]:

Physical Noise Source	Noise Model
Environment Interaction	Stochastic Pauli Noise, Amplitude/Phase Damping, Pauli Measurements
Imperfect Control	Coherent Over/Under Rotations

4 Reference Formulas and Definitions

- Coherence time: time that a quantum system can stay undisturbed (i.e. isolated from interactions with environment)
- Qubit relaxation time: time it takes for loss of energy from system.
- Dephasing time: time it takes to polarize qubit that is in superposition (i.e. from $|0\rangle$ *and* $|1\rangle$, to $|0\rangle$ *or* $|1\rangle$)
- coherence-controllability tradeoff: the conflicting requirements for quantum computation are quantum states needing isolation to remain intact and quantum states must have interactions with control mechanisms in order to yield useful computation [resch21]
- Cross-talk: unexpecteed qubit-qubit interaction (entanglement) that mixes their states and leads to decoherence.
- Quantum Circuit Model: For an n-qubit quantum system,

  $$|\psi\rangle = \sum_{b \in \{0,1\}^n} \alpha_b |b\rangle$$

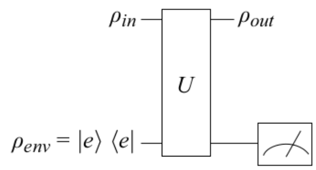
  Coefficient α_b is the amplitude of basis bit-string b.
- The joint state of two separate quantum systems is represented with *tensor product*, $|\psi_0\rangle \otimes |\psi_1\rangle$
- trace of a matrix is the sum of its diagonal elements, i.e. diagonal sum. $|e_i\rangle$ is the basis vector with 1 at the $i^{(th)}$ index, and 0 everywhere else:

  $$\mathbf{tr}(A) = \sum_i A_{ii} = \sum_i |e_i\rangle A |e_i\rangle$$

- density matrix representation, ρ, is how mixed state is represented:

  $$\rho = \sum_i p_i |\psi_i\rangle |\psi_i\rangle$$

- trace and density matrix helps define the fidelity metric for quantifying the quality of a quantum state: $\mathbf{tr}(\rho\sigma)$, where σ is actual quantum state, versus the density matrix or "correct" quantum state [resch21].
- Unitary coupling transformation U represents the impact on noise/the environment on a quantum state. U is applied to both environment and syste, $\rho_{\text{env}} \otimes \rho_{\text{in}}$



- linear map: $\rho \to \mathcal{E}(\rho)$
- The unitary operator: $\mathcal{E}(\rho) = U\rho U^{\dagger}$
- operator form for entire unitary coupling evolution, with environment, U, and measurement. U acts on both system and environment.

$$\rho_{\text{in}} \to \rho_{\text{out}} = \mathbf{tr}_{\text{env}}(U(\rho_{\text{env}} \otimes \rho_{\text{in}})U^{\dagger})$$

5 Paper Overviews

- Salonik Resch and Ulya R. Karpuzcu. 2021. Benchmarking Quantum Computers and the Impact of Quantum Noise. ACM Comput. Surv. 54, 7, Article 142 (September 2022), 35 pages.
  - Details benchmarking quantum computers from a computer architecture perspective and the challenges, as well as the significance and complexity of things that have to be considered when benchmarking, such as noise model (and ability to simulate and/or characterize), target application, and performance metrics. The authors categorize some physical noise sources (environment or other qubits), and overviews some noise models (Stochastic Pauli Noise, Coherent Noise, Amplitude/Phase Damping) and references other related works that cover these models. They also go over different metrics and how different ones may be preferred (process fidelity, average gate fidelity/infidelity, trace distance, Hellinger fidelity). They run benchmarks on a few quantum benchmarks and use the process fidelity metric to illustrate the impact of noise.
- M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2012). Chapter 9.
  - (classical) Fidelity is defined as the distance between probability distributions, p and q:

$$F(p_x, q_x) = \sum_x \sqrt{p_x q_x}$$

– Trace Distance for two quantum states:

$$D(\rho, \sigma) = \frac{1}{2}\mathbf{tr}|\rho - \sigma|$$

– (quantum) Fidelity of two states:

$$F(\rho, \sigma) = \mathbf{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}$$

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