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A GOAL PROGRAMMING MODEL FOR ACADEMIC RESOURCE ALLOCATION*

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The rapid rate of technological development and the growing complexity of society in recent years have brought renewed awareness of the importance of higher education. The rapid expansion of higher education, both in size and quality, requires systematic approaches and dynamic planning for efficient resource allocation on the part of the university administrators. This paper presents a goal programming model for an optimum allocation of resources in an institution of higher learning.

The rapid rate of technological development and the growing complexity of society have brought about renewed awareness of the importance of higher education. Never in the history of our country has society directed such attention toward the broad area of higher education. Today, higher education is a ten billion dollar a year enterprise which includes six million students, faculty, and staff.[1]

Contrary to the early 1960's when there were generous supports of higher education from many state legislatures, the federal government, and the public in general, the period from the late 1960's has been characterized by severe financial stringency for many institutions. There may be many reasons for this trend, such as gradual reduction of federal research grants and a faster rate of expenditure increase in higher education than the rate of increase in state revenues. However, another important reason seems to be the switch of high priorities from higher education to the more pressing social problems which require immediate attention of the government.

The rising expenditure of higher education has caused lawmakers and the public to develop a keener and more critical view of the operational efficiency of educational institutions. Institutions can no longer request prodigious sums of money from the legislature without clear justification in terms of viable goals, alternatives, and expected results. One of the most important functions of the university administrator is to acquire the ever increasing operational funds. The increasing financial pressure has greatly enhanced the importance of efficient resource allocation on the part of the institutions.

Although management science and mathematical models are developed and taught within the confines of academies, the application of these techniques for their own operation has been generally neglected [1]. Perhaps in the past, academic planning and operational efficiency were of no significant importance; this is, of course, no longer the case. It is necessary for many institutions to develop a dynamic and systematic planning model for efficient resource allocation for their survival.

In recognition of the importance of planning and rational decision processes, many universities have been and are in the process of establishing formal long-range planning

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[1] Annual current expenditure (1962–63 dollars) by institutions of higher education rose from \$3.6 billion in 1954–55 to an estimated \$9.7 billion in 1964–65—an increase of 169 percent. It is expected that current expenditures will reach \$20.1 billion in 1974–75—an increase of 107 percent over the 1964–65 figure [6].

models as well as utilizing scientific decision techniques. For example, the application of Planning-Programming-Budgeting System (PPBS) to academic planning has become popular in the past several years. The purpose of the PPBS approach in the university is to achieve a more efficient resource allocation in achieving its objectives over a long-term planning horizon. Under this system, long-range planning is established for program elements such as colleges, departments, and curricula. With the limited available resources for pursuing university goals, the administrators have to eliminate some goals, postpone some, and reduce others in scale in order to fit desirable goals into practical and feasible objectives.

The PPBS is a managerial process. This system, however, does not automatically provide solutions to problems. In order to implement the system, or in addition to the formal PPB system, or even in absence of such a system, many universities have developed various analytical models.

The majority of models have focused upon a specific segment of the total institution in great depth [21], [22], [24], [26], [29], [30]. With a number of specialized models such as prediction of student population [8], [17], [23], [25], faculty growth [2], [9], [22], [27], [38], facility requirements [6], [16], [18], [19], [36], expenditure analysis [31], [34], etc., the overall planning of the institution is attempted. The larger models have attempted to encompass functions of the total university system as well as to analyze interactions among major components [3], [7], [11], [12], [13], [20], [32], [33], [35], [37].

The various models introduced thus far vary considerably in their mathematical sophistication, functions, methods, purposes, subjects, and data. However, the majority of the models attempt to reduce some degree of uncertainty based primarily upon the past trend or data.

The crucial issue in the adminstration of higher education does not end at operational efficiency, but it embodies the very purpose, function, and concept of each institution. In essence, then, the operational policy is based on the combined philosophy of many conflicting ideas of the university community, funders, administrators, faculty, students, and staff. The present situation in many universities is one of compromise and tense coexistence among all parties involved [35]. Any effective model, therefore, must be capable of reflecting the administrators' judgment about the priority of desired goals within the constraints of the existing situation. Most models introduced thus far fail to meet this requirement. The goal programming approach appears to be the most appropriate technique in developing a model to attain multiple, competitive, and often conflicting goals with varying priorities. It is the purpose of this paper to present a goal programming model for an optimum allocation of resources in institutions of higher learning. Although it is possible to formulate a complex, multi-time period model that serves the purpose of long-range planning for the entire university, the scope of this study is limited to the planning of one college within the university. In addition, the planning horizon under consideration is limited to one year. This limited scope allows a clear presentation of the methodology, development of the model, and application potential of the study. Once the basic model is completed for a year, it can be extended, although not a simple task, for a longer planning horizon by forecasting parameter changes.[2] The aggregative university model can be designed when models for major components are established and their interactions are identified. However, this is more easily said than done.

[2] For a multi-year planning model based on the goal programming approach, see [15].

The Goal Programming Approach

Goal programming (GP) is a special extension of linear programming [4], [5], [10], [14]. This method is capable of handling decision problems which deal with a single goal with multiple subgoals as well as problems with multiple goals with multiple subgoals [10]. In the conventional linear programming method, the objective function is undimensional—either to maximize profits (effectiveness) or to minimize costs (sacrifice). The GP model handles multiple goals in multiple dimensions. Therefore, there is no dimensional limitation of the objective function.

Often, goals set by the decision maker are achievable only at the expense of other goals. Furthermore, these goals are incommensurable. Thus, there is a need to establish a hierarchy of importance among these incompatible goals so that the low order goals are considered only after the higher order goals are satisfied or have reached the point beyond which no further improvements are desirable. If the decision maker can provide an ordinal ranking of goals in terms of their contributions or importance to the organization, the problem can be solved by GP.

In GP, instead of trying to maximize or minimize the objective criterion directly, the deviations between goals and what can be achieved within the given set of constraints are to be minimized. In the simplex algorithm of LP, such deviations are called "slack" variables. These deviational variables take on a new significance in GP. The deviational variable is represented in two dimensions, both positive and negative deviations from each subgoal or goal. Then, the objective function becomes the minimization of these deviations, based on the relative importance or preemptive priority weights assigned to them. The objective function, however, may also include real variables with ordinary or preemptive weights in addition to the deviational variables.

The primary characteristic of GP is that it allows for an ordinal solution. Stated differently, management may be unable to obtain information on the cost or value of a goal or a subgoal, but often upper or lower limits may be stated for each subgoal. Usually the manager has judgment to determine the priority of the desired attainment of each goal or subgoal and rank them in ordinal sequence. Economically speaking, the manager works with the problem of the allocation of scarce resources. Obviously, it is not always possible to achieve every goal to the extent desired by management. Thus, with or without GP the manager attaches a certain priority to the achievement of a certain goal. The true value of GP is, therefore, the solution of problems involving multiple, conflicting goals according to the manager's priority structure.

The general GP model can be mathematically expressed as [10]:

$$\text{Minimize} \quad Z = \textstyle\sum_{i=1}^{m} (d_i^+ + d_i^-)$$

$$\text{subject to} \quad Ax - Id^+ + Id^- = b, \qquad x, d^+, d^- \geqq 0,$$

where m goals are expressed by an m component column vector b $(b_1, b_2, \cdots, b_m)$, A is an $m \times n$ matrix which expresses the relationship between goals and subgoals, x represents variables involved in the subgoals $(x_1, x_2, \cdots, x_n)$, d^+ and d^- are m-component vectors for the variable representing deviations from goals, and I is an identity matrix in m dimensions.

The manager must analyze each one of the m goals considered in the model in terms of whether over- or under-achievement of the goal is satisfactory. If over-achievement is acceptable, d_i^+ can be eliminated from the objective function. On the other hand, if under-achievement is satisfactory, d_i^- should not be included in the objective function. If the exact achievement of the goal is desired, both d_i^+ and d_i^- must be represented in the objective function.

The deviational variables d_i^+ and d_i^- must be ranked according to their preemptive priority weights, from the most important to the least important. In this way the low order goals are considered only after the higher order goals are achieved as desired. If goals are classified in k ranks, the preemptive priority factor M_j $(j = 1, 2, \cdots, k)$ should be assigned to the deviational variables, d_i^+ and d_i^-. The priority factors have the relationship of $M_j >>> M_{j-1}$ $(j = 1, 2, \cdots, k)$, which implies that the multiplication of n, however large it may be, cannot make M_{j-1} greater than or equal to M_j. Of course, it is possible to refine goals even further by the means of decomposing the deviational variables. To do this, additional constraints and additional priority factors are required.

One more step in the procedure to be considered is the weighting of those deviational variables at the same priority level, i.e., variables with the same M_j coefficient. The criterion to be used here is the minimization of the opportunity cost or regret. This implies that the coefficient of regret σ_i, which is positive, must be assigned to the individual deviational variables on the same goal level. The coefficient σ_i simply represents the relative amount of unsatisfactory deviation from the goal.

The General Model

A general resource allocation planning model of a college will be first introduced. The application of the model is demonstrated in the numerical example section. For the development of a GP model, the following variables, constants, and constraints should be examined:

Variables

x_1 = number of graduate research assistants,
x_2 = number of graduate teaching assistants,
x_3 = number of instructors,
x_4 = number of assistant professors without terminal degree,[3]
x_5 = number of associate professors without terminal degree,
x_6 = number of full professors without terminal degree,
x_7 = number of part-time faculty without terminal degree,
x_8 = number of special professors without terminal degree,
x_9 = number of staff,
y_1 = number of assistant professors with terminal degree,
y_2 = number of associate professors with terminal degree,
y_3 = number of full professors with terminal degree,
y_4 = number of part-time faculty with terminal degree,
y_5 = number of special faculty with terminal degree,
w_2 = total payroll increase from prior year, comprised of faculty, staff, and graduate assistant salary increases.

Constants

a_1 = percentage of the academic staff that is classified as full-time faculty,
a_2 = percentage of academic staff at the undergraduate level with terminal degree,
a_3 = percentage of academic staff at the graduate level with terminal degree,
a_4 = estimated number of undergraduate student credit hours required per session,

[3] The terminal degree represents Ph.D., D.B.A., J.D., and LL.B.

a_5 = estimated number of graduate student credit hours required per session,
a_6 = desired undergraduate faculty/student ratio,
a_7 = desired graduate faculty/student ratio,
a_8 = desired faculty/staff ratio,
a_9 = desired faculty/graduate research assistant ratio,
b_{14} = projected undergraduate student enrollment for the coming academic year,
b_{15} = projected graduate student enrollment for the coming academic year,
b_{16} = desired percentage increase in salary for graduate assistants,
b_{17} = desired percentage increase in salary for faculty,
b_{18} = desired percentage increase in salary for staff.

Maximum teaching loads, desired proportion of each faculty type, and average annual salary defined as:

TABLE 1

Variable	Desired Proportion	Teaching Loads		Salary
		Undergraduate	Graduate	
x_1	c_1	b_1	b'_1	s_1
x_2	c_2	b_2	b'_2	s_1
x_3	c_3	b_3	b'_3	s_2
x_4	c_4	b_4	b'_4	s_3
x_5	c_5	b_5	b'_5	s_4
x_6	c_6	b_6	b'_6	s_5
x_7	c_7	b_7	b'_7	s_6
x_8	c_8	b_8	b'_8	s_7
x_9	—	—	—	s_8
y_1	c_9	b_9	b'_9	s_3
y_2	c_{10}	b_{10}	b'_{10}	s_4
y_3	c_{11}	b_{11}	b'_{11}	s_5
y_4	c_{12}	b_{12}	b'_{12}	s_6
y_5	c_{13}	b_{13}	b'_{13}	s_7

Constraints

A. *Accreditation.* (a) A certain percentage of the academic staff must be full-time faculty.

$$(1) \qquad \Big(\sum_{i=3}^{6} x_i + x_8 + \sum_{i=1}^{3} y_i + y_5\Big)\Big/\Big(\sum_{i=2}^{8} x_i + \sum_{i=1}^{5} y_i\Big) \geqq a_1 ,$$

where it is assumed that the denominator is positive in all constraints.

(b) A given percentage of the faculty available for undergraduate and graduate teaching duties are usually required to possess the terminal degree. If we assume for this model that x_2 through x_7 and y_1 through y_3 are available for undergraduate teaching assignments, and x_8 and y_1 through y_5 are available for graduate teaching responsibilities, we may write:

$$(2) \qquad \begin{aligned} \sum_{i=1}^{3} y_i\Big/\Big(\sum_{i=2}^{7} x_i + \sum_{i=1}^{3} y_i\Big) &\geqq a_2 , \\ \sum_{i=1}^{5} y_i\Big/\Big(x_8 + \sum_{i=1}^{5} y_i\Big) &\geqq a_3 . \end{aligned}$$

(c) There is usually a maximum number of student credit hours per session (for both graduate and undergraduate) that a faculty member may teach. It is not necessary to formulate a separate constraint for this requirement, since it is easily incorporated into later constraints by selecting appropriate desired class sizes and teaching loads.

B. *Total Number of Academic Staff*. One of the most important determinants of the number of academic staff requirements is the estimated number of student credit hours (both graduate and undergraduate) needed per session. With this information plus the maximum desired teaching loads of faculty members, the requirement of academic staff can be determined.

$$\begin{aligned} &\textstyle\sum_{i=2}^{7} b_i x_i + \sum_{i=1}^{5} b_{i+8}\, y_i \geqq a_5 \qquad \text{(undergraduate)}, \\ &\textstyle\sum_{i=2}^{7} b'_i x_i + \sum_{i=1}^{5} b'_{i+8}\, y_i \geqq a_5 \qquad \text{(graduate)}. \end{aligned} \tag{3}$$

Another aspect to be considered in the determination of academic staff requirements is the desired faculty/student ratio.

$$\begin{aligned} (\textstyle\sum_{i=2}^{7} x_i + \sum_{i=1}^{3} y_i)/b_{14} &\geqq a_6 \qquad \text{(undergraduate)}, \\ (x_8 + \textstyle\sum_{i=1}^{5} y_i)/b_{15} &\geqq a_7 \qquad \text{(graduate)}. \end{aligned} \tag{4}$$

C. *Distribution of Academic Staff*. It is necessary to impose some constraints on the distribution of the academic faculty. If there were no constraints, the model would call for the most productive type of faculty in terms of teaching load, salary, and accreditation, i.e., instructors and the assistant professors with terminal degrees. In this model, we assume that the college desires to minimize the number of faculty without terminal coverage and to maximize those with terminal degrees.

$$\begin{aligned} \textstyle\prod_{i=2}^{8} c_i \cdot T &\leqq \textstyle\prod_{i=2}^{8} x_i\,, \\ c_{12} \cdot T &\leqq y_4\,, \\ \textstyle\prod_{i=1}^{3} c_i \cdot T &\geqq \textstyle\prod_{i=1}^{3} y_i\,, \\ c_{13} \cdot T &\geqq y_5\,, \end{aligned} \tag{5}$$

where "$\prod$" represents "product" of the indicated terms and "T" represents $\sum_{i=2}^{8} x_i + \sum_{i=1}^{5} y_i$.

D. *Number of Staff*. Due to the ever-increasing amount of stenographic services required by the academic staff, it is imperative, if backlogs and bottlenecks are to be avoided, that an adequate staff be provided. This objective may be incorporated into the model by designing a constraint which reflects a desired faculty/staff ratio.

$$(\textstyle\sum_{i=2}^{8} x_i + \sum_{i=1}^{5} y_i)/x_9 \geqq a_8\,. \tag{6}$$

E. *Number of Graduate Research Assistants*. To provide adequate research support for the academic staff, it is desired to assign graduate research assistants to faculty members. This can be handled by introducing a constraint for desired faculty/graduate research assistant ratio

$$(\textstyle\sum_{i=3}^{8} x_i + \sum_{i=1}^{5} y_i)/x_1 \geqq a_9\,. \tag{7}$$

F. *Salary Increase*. To maintain an adequate staff, it is necessary to provide periodic salary increases. Any academic community must be cognizant of the fact that there exists a keen competition for members of its faculty. One of the most viable means of meeting this competition is to offer salary increases according to the policy of the institution. The payroll increase constraint is:

$$b_{16}(s_1 \textstyle\sum_{i=1}^{2} x_i) + b_{17}(s_2 x_3 + \sum_{i=3}^{7} s_i x_{i+1} + \sum_{i=3}^{7} s_i y_{i-2}) + b_{18}(s_8 x_9) \leqq w. \tag{8}$$

G. *The Total Payroll Budget*. The increase in the salaries of the faculty, the staff and graduate assistants represents only one facet of the entire budget. The total payroll budget is a major concern in a situation where limited resources are involved. The total payroll constraint can be expressed:

$$(9)\qquad s_1 \sum_{i=1}^{2} x_i + s_2 x_3 + \sum_{i=3}^{7} s_i x_{i+1} + \sum_{i=3}^{7} s_i y_{i-2} + s_8 y_9 + w = p,$$

where p represents the total payroll budget.

Objective Function

The objective function is to minimize deviations, either negative or positive, from set goals with certain "preemptive" priority factors assigned by the dean of the college in accordance with the university policies, existing conditions, and his judgment.

A Numerical Example

A simplified numerical example will be presented to demonstrate the application of the general model.[4] Let us assume that the Dean of the College of Business in a university provided the following priority structure for academic goals and information on constants:

Priority Structures

M_7 = Maintain the necessary requirements for accreditation by AACSB.

M_6 = Assure adequate salary increases for the academic staff, graduate assistants, and general staff.

M_5 = Assure adequate number of faculty by meeting desired faculty/student ratios and by having instruction available for the needed student credit hours. The graduate faculty/student requirements are considered to be twice as important as the undergraduate requirement (note the weights at M_5 level in the objective function).

M_4 = Attain a desirable distribution of the academic staff with respect to rank.

M_3 = Maintain desired faculty/staff ratio.

M_2 = Maintain desired faculty/graduate research assistant ratio.

M_1 = Minimize cost.

TABLE 2

Teaching Loads, Average Salaries, Desired Proportions of Total Staff

Variable	Teaching Load		Desired Proportion		Salary
	Undergraduate	Graduate	Maximum	Minimum	
x_1	0	0	—	—	$3,000
x_2	6	0	7%	—	3,000
x_3	12	0	7	—	8,000
x_4	9	0	15	—	13,000
x_5	9	0	5	—	15,000
x_6	6	0	2	—	17,000
x_7	3	0	1	—	2,000
x_8	0	3	—	1%	30,000
x_9	—	—	—	—	4,000
y_1	6	3	—	21	13,000
y_2	6	3	—	14	15,000
y_3	3	3	—	23	17,000
y_4	0	3	2	—	2,000
y_5	0	3	—	2	30,000

[4] This example is based on actual operational data at the College of Business, Virginia Polytechnic Institute and State University. We would like to thank Dean H. H. Mitchell for his help in preparing this study. The average salaries of the faculty are, however, arbitrarily determined.

The constraints in the model needed for accreditation were given the highest priority by the dean of the college. These goals will be considered first in the goal programming model, followed by the lower priority goals.

Constraints for Accreditation

It is required that 75 percent of the academic staff be full-time faculty according to AACSB. Since in our model x_3 to x_6, x_8, y_1 to y_3 and y_5 are considered full-time, we may write:

$$(10)\quad \sum_{i=3}^{6} x_i + x_8 + \sum_{i=1}^{3} y_i + y_5 - 0.75\left(\sum_{i=1}^{8} x_i + \sum_{i=1}^{5} y_i\right) + d_1^- - d_1^+ = 0.$$

It is also required that at least 40 percent of the academic teaching staff at the undergraduate level possess terminal coverage. This is expressed as:

$$(11)\quad \sum_{i=1}^{3} y_i - 0.40\left[\sum_{i=2}^{7} x_i + \sum_{i=1}^{3} y_i\right] + d_2^- - d_2^+ = 0.$$

At least 75 percent of the academic staff teaching graduate studies are required to possess terminal coverage. This is expressed as:

$$(12)\quad \sum_{i=1}^{5} y_i - 0.75\left[x_8 + \sum_{i=1}^{6} y_i\right] + d_3^- - d_3^+ = 0.$$

Constraints for Number of Academic Staff

To determine the faculty requirement, it is necessary to forecast the total number of student credit hours of instruction needed. In this example, the projected student enrollment is 1,820, the average number of credit hours/student taken at the college is 10, and the desired class size is set at 20. Therefore, 910 total student credit hours can be calculated by means of the following formula:

(Projected enrollment)·(Number of Credit hours/student)/(Desired class size)

$$(13)\quad 6x_2 + 12x_3 + 9x_4 + 9x_5 + 6x_6 + 3x_7 + 6y_1 + 6y_2 + 3y_3 + d_4^- - d_4^+ = 910.$$

For the graduate student credit hours of instruction, we forecast 100 hours per session. The procedure is similar to the undergraduate forecast and the constraint becomes:

$$(14)\quad 3x_8 + 3y_1 + 3y_2 + 3y_3 + 3y_4 + 3y_5 + d_5^- - d_5^+ = 100.$$

The next aspect to be considered in the determination of the required academic staff is the desired faculty/student ratio at both the graduate and undergraduate level. The forecasted enrollments in the next year at undergraduate and graduate levels are 1,820 and 100, respectively. The desired undergraduate faculty/student ratio is about 1/20 and the desired graduate faculty/student ratio is about 1/10.[5] These constraints then become, for the undergraduate requirement:

$$(15)\quad \sum_{i=2}^{7} x_i + \sum_{i=1}^{3} y_i + d_6^- - d_6^+ = (0.05)(1{,}820) = 91$$

and for the graduate faculty:

$$(16)\quad x_8 + \sum_{i=1}^{5} y_i + d_7^- - d_7^+ = (0.10)(100) = 10.$$

Constraints for the Distribution of Academic Staff

It is necessary to impose some constraints on the distribution of the academic faculty according to the desired proportion of the total faculty for each type of staff.

[5] The desired faculty/student ratio is based on the AACSB accreditation regulations and the dean's academic policy.

$$
(17)\quad
\begin{aligned}
0.07T - x_2 + d_8^- - d_8^+ &= 0,\\
0.07T - x_3 + d_9^- - d_9^+ &= 0,\\
0.15T - x_4 + d_{10}^- - d_{10}^+ &= 0,\\
0.05T - x_5 + d_{11}^- - d_{11}^+ &= 0,\\
0.02T - x_6 + d_{12}^- - d_{12}^+ &= 0,\\
0.01T - x_7 + d_{13}^- - d_{13}^+ &= 0,\\
0.01T - x_8 + d_{14}^- - d_{14}^+ &= 0,\\
0.21T - y_1 + d_{15}^- - d_{15}^+ &= 0,\\
0.14T - y_2 + d_{16}^- - d_{16}^+ &= 0,\\
0.23T - y_3 + d_{17}^- - d_{17}^+ &= 0,\\
0.02T - y_4 + d_{18}^- - d_{18}^+ &= 0,\\
0.02T - y_5 + d_{19}^- - d_{18}^+ &= 0,
\end{aligned}
$$

where $T = \sum_{i=2}^{8} x_i + \sum_{i=1}^{5} y_i$.

Number of Staff. In order to insure adequate staff for clerical and administrative work, the desired faculty/staff ratio is set at 4 to 1 by the dean. The constraint is then:

$$(18)\quad T - 4x_9 + d_{20}^- - d_{20}^+ = 0.$$

Number of Graduate Research Assistants. We set the desired faculty/graduate research assistant ratio at 5 to 1. Hence, the constraint is:

$$(19)\quad \textstyle\sum_{i=3}^{8} x_i + \sum_{i=1}^{5} y_i - 5x_1 + d_{21}^- - d_{21}^+ = 0.$$

Cost of Academic Staff, Graduate Assistants, and Staff

The total salary increase constraint can be expressed as:

$$
(20)\quad
\begin{aligned}
&0.06[3{,}000\textstyle\sum_{i=1}^{2} x_i] + 0.08(8{,}000x_3 + 13{,}000x_4 + 15{,}000x_5\\
&\quad + 17{,}000x_6 + 2{,}000x_7 + 30{,}000x_8 + 13{,}000y_1 + 15{,}000y_2 + 17{,}000y_3\\
&\quad + 2{,}000y_4 + 30{,}000y_5) + 0.06(4{,}000x_9) - w + d_{22}^- - d_{22}^+ = 0,
\end{aligned}
$$

where there is a 6 percent increase for graduate students and staff and an 8 percent increase for faculty.

The total payroll constraint for the entire college will be:

$$
(21)\quad
\begin{aligned}
&3{,}000x_1 + 3{,}000x_2 + 8{,}000x_3 + 13{,}000x_4 + 5{,}000x_5 + 17{,}000x_6\\
&\quad + 2{,}000x_7 + 30{,}000x_8 + 13{,}000y_1 + 15{,}000y_2 + 17{,}000y_3\\
&\quad + 2{,}000y_4 + 30{,}000y_5 + 4{,}000x_9 + w + d_{23}^- - d_{23}^+ = 0.
\end{aligned}
$$

Objective Function

$$
(22)\quad
\begin{aligned}
\text{Min.}\quad Z = {}& M_7\textstyle\sum_{i=1}^{3} d_i^- + M_6 d_{22}^- + 2M_5 d_5^- + 2M_5 d_7^- + M_5 d_4^-\\
&+ M_5 d_6^- + M_4\textstyle\sum_{i=8}^{13} d_i^- + M_4 d_{18}^- + M_4\sum_{i=14}^{17} d_i^+ + M_4 d_{19}^+\\
&+ M_3 d_{20}^+ + M_2 d_{21}^+ + M_1 d_{23}^+ .
\end{aligned}
$$

Solution

The GP model provides three types of solutions: (1) identification of the input (resource) requirements to attain all the desired goals; (2) the degree of goal attainments with the given inputs; and (3) the degree of goal attainments under various combinations of inputs and goal structures. This paper presents three separate solutions in order to demonstrate the capability of the model. The solution is based on the GP computer program written by the authors.

A. *The First Run.* In the first run, the above problem is solved to determine the input requirements necessary to achieve all the goals presented by the dean.[6] The results of the first run are presented below.

TABLE 3

Goal Attainment	
Accreditation	Achieved
Salary increase	Achieved
Faculty/student ratios	Achieved
Faculty/staff ratio	Achieved
Faculty distribution	Achieved
Faculty/graduate assistant ratio	Achieved
Minimize cost	$2,471,000
Variables	
$x_1 = 32$	$x_8 = 1$
$x_2 = 10$	$x_9 = 38$
$x_3 = 10$	$y_1 = 42$
$x_4 = 22$	$y_2 = 20$
$x_5 = 7$	$y_3 = 34$
$x_6 = 0$	$y_4 = 0$
$x_7 = 1$	$y_5 = 3$
$w = \$176,000$	

The solution of the first run indicates that all goals are achieved at the total cost of $2,471,000. Since the minimization of cost is treated as the goal with the lowest priority factor, the solution identifies the input requirements necessary to attain all the goals.

Although the result of the above solution provides valuable information to the administrator, there are a couple of points to be evaluated. First, as is usually the case, the desired faculty distribution may be impossible to obtain in reality. Second, the total cost derived by the first solution may far exceed the amount of funds the dean is able to obtain.[7]

Suppose, for example, that 1 percent of the academic staff are professors with no terminal coverage. The optimum solution called for zero. There is nothing that can be done about this situation so the constraint for this type of academic staff must be changed to read: $0.01T - x_6 - d_{12}^- + d_{12}^+ = 0$. Further, suppose the administrator believes his maximum allocation of funds will be $1,850,000. Or in fact, suppose this is all the funds allocated to the college. This forces the right-hand side of equation (2)

[6] The first draft of this numerical example was presented at the 11th American Meeting of the Institute of Management Sciences, 1970.

[7] The dean of the college may have administrative options of cutting enrollments or phasing out programs to balance the budget and desired academic goals. This is based on the specific conditions of the particular university or college. The dean has, however, indirect means to control enrollment. For example, he can add more difficult courses as part of the requirements or decrease such courses. Such action will affect the loads placed on other departments or colleges in the university.

to become \$1,850,000 instead of 0 and we are no longer considering the cost minimization as the lowest priority.

B. *The Second Run.* In the second run, the dean treats the avoidance of deficit operation as the second priority goal, after meeting accreditation requirements. Also, he adjusts the constraint concerning faculty distribution of full professors with no terminal degrees. The new objective function and the solution are presented below.

Objective Function

$$Z = M_7\sum_{i=1}^{3} d_i^- + M_6 d_{23}^+ + M_5 d_{22}^- + 2M_4 d_5^- + 2M_4 d_7^- + M_4 d_4^-$$
$$(23) \qquad + M_4 d_6^- + M_3\sum_{i=8}^{11} d_i^- + M_3 d_{13}^- + M_3 d_{18}^- + M_3 d_{12}^+ + M_3\sum_{i=14}^{17} d_i^+$$
$$+ M_3 d_{19}^+ + M_2 d_{20}^+ + M_1 d_{21}^+ .$$

Goal Attainment

Accreditation	Achieved
Avoid deficit	Achieved
Salary increase	Achieved
Faculty/student ratio	Achieved
Faculty distribution	Not Achieved—several ranks were not represented in this solution
Faculty/staff ratio	Not Achieved—no staff
Faculty/graduate research assistant ratio	Not Achieved

Variables

$x_1 = 0$	$x_8 = 0$
$x_2 = 9$	$x_9 = 0$
$x_3 = 20$	$y_1 = 28$
$x_4 = 20$	$y_2 = 18$
$x_5 = 7$	$y_3 = 30$
$x_6 = 1$	$y_4 = 0$
$x_7 = 1$	$y_5 = 0$
$w_1 = 135{,}000$	
cost = \$1,850,000	

The result of the second run indicates that with \$1,850,000 appropriated to the college the dean is unable to achieve all the desired goals. In fact, due to the priority structure of the goals, there is no fund available to hire any clerical staff after achieving the higher priority goals. Also, the desired faculty/graduate research assistant ratio was not attained.

Now, let us suppose that the dean of the college presented the result of the second run to the president of the university and that he was successful in obtaining an additional \$120,000. Based on the result of the second computer run, the dean is aware of the fact that he should assign higher priorities to the faculty/staff and faculty/graduate research assistant ratios for an efficient operation of the college.

C. *The Third Run.* In the third run, the dean again assigned the highest priority to the accreditation requirements, and the second priority factor on the cost minimization to \$1,970,000. To insure an adequate staff support he assigned the third priority to the faculty/staff ratio and the fourth priority to the faculty/graduate research assistant ratio. The faculty/student ratio was assigned the sixth priority, followed by the faculty distribution ratios given the lowest priority factor.

The objective function for the third program is:

$$\begin{aligned}(24)\qquad \text{Min.}\quad Z = {} & M_7\textstyle\sum_{i=1}^{3} d_i^- + M_6 d_{23}^+ + M_5 d_{22}^- + M_4 d_{20}^+ + M_3 d_{21}^+ + 2M_2 d_5^- \\ & + 2M_2 d_7^- + M_2 d_4^- + M_2 d_6^- + M_1 \textstyle\sum_{i=8}^{11} d_i^- + M_1 d_{13}^- \\ & + M_1 d_{18}^- + M_1 d_{12}^+ + M_1 \textstyle\sum_{i=14}^{17} d_i^+ + M_1 d_{19}^+ .\end{aligned}$$

The results of the program are shown below.

TABLE 4

Accreditation	Achieved
Salary increase	Achieved
Faculty/staff ratio	Achieved
Faculty/graduate research assistant ratio	Achieved
Faculty/student ratios	Achieved
Faculty distribution	Not Achieved—again several ranks were not presented in this solution

Variables

$x_1 = 26$	$x_9 = 32$
$x_2 = 9$	$y_1 = 27$
$x_3 = 22$	$y_2 = 18$
$x_4 = 19$	$y_3 = 26$
$x_5 = 6$	$y_4 = 0$
$x_6 = 1$	$y_5 = 0$
$x_7 = 0$	$w = 144{,}000$
$x_8 = 0$	cost = \$1,970,000

As is apparent from the result above, the most important academic goals of the college are met by restructuring the priority levels and by acquiring an additional \$120,000.[8]

Conclusion

Virtually all models developed for university management have focused upon the analysis of input (resource) requirements. They have generally neglected or often ignored the system outputs, unique institutional values, and bureaucratic decision structures. However, these are important environmental factors which greatly influence the decision process. In this study the GP approach is utilized because it allows the optimization of goal attainments while permitting an explicit consideration of the existing decision environment.

Developing and solving the GP model points out where some goals cannot be achieved under the desired policy and, hence, where tradeoff must occur due to limited resources. Furthermore, the model allows the administrator to review critically the priority structure in view of the solution derived by the model. Indeed, the most important property of the GP model is its great flexibility which allows model simulation with numerous variations of constraints and priority structures of goals.

The GP approach is not the ultimate solution for all budgeting and planning problems in an academy. It requires that administrators be capable of defining, quantifying, and ordering objectives. The GP model simply provides the best solution under the

[8] The authors are happy to report that the dean of their college has used the information generated by this study in his academic planning.

given constraints and priority structure. Therefore, some research questions concerning the identification, definition and ranking of goals still remain. There is the need for future research to develop a systematic methodology to generate such information.

The purpose of this study is to demonstrate the application potential of GP to complex decision problems in university management. The model presented is a simple illustration. No doubt, each constraint requires an indepth analysis and it may well be a research area in itself. Furthermore, departmental interactions, boundary conditions, the administrator's own preferences, and the bureaucratic decision structure are important areas which require continuing research. It is hoped that this paper will provide a guide for developing more complete models closer to reality which will perhaps encompass an entire university or a university system.

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