Problem Set #4

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# 1 Part One

In the following excercises we are examining the relationship between radio station buyers and their targets. Essentially we want to understand the characteristics that make a merger between radio stations more likely. In this example, we examine a one to one matching model (one buyer and one target) and we assume there is one market per year in the years 2007 and 2008.

To undertake this analysis, we analyze the payoffs for each match that took place and we compare these payoffs to all other matches that could have taken place. The idea behind this is that we assume that the actual matches that took place should be more preferred than any other matches that could have taken place. We use these assumptions to build our score function. This score function represents the number of observations correctly predicted by our model. By maximizing the score function we can obtain the parameters of interest in the model.

### 1.1 Model

In the first model we assume that we have the following payoff structure:

$$f_m(b,t) = x_{1bm}y_{1tm} + \alpha x_{2bm}y_{1tm} + \beta distance_{btm} + \epsilon_{btm}$$

In this case the score function takes the following form:

$$Q(\beta) = \sum_{y=1}^{Y} \sum_{b=1}^{M_y - 1} \sum_{b'=b+1}^{M_y} 1[f(b, t) + f(b', t') \ge f(b', t) + f(b, t')]$$

If the expression holds, then our indicator function becomes 1, if not then we give it a 0. We then sum over all possible matches to obtain the score. In our excercise we want to obtain the parameters on corporate owenership  $\alpha$  and distance  $\beta$ .

In this model have 4 equations each with 3 terms, so we have a total of 12 terms per inequality. To analyze the model, we need our data to be set up in arrays that represent each inequality. The total amount of possible matches is 2,421. After setting up the data we should have a matrix of sorts (multiple stacked arrays) with the dimensions 2421 x 12.

For ease of analysis, I define the score function as:

$$Q(\beta) = \sum_{y=1}^{Y} \sum_{b=1}^{M_y - 1} \sum_{b'=b+1}^{M_y} 1[f(b, t) + f(b', t') - f(b', t) - f(b, t') \ge 0]$$

I have organized the data within each array to be in the correct order. As such, when defining the score function I sum and subtract term by term within each array to determine whether the total is positive. If it is positive I assign that array a value of -1 and continue through the analysis. To find the maximum score and the parameters that maximize that score, I use the "Nelder-Mead" minimization routine. To obtain the maximum I must have the score be a negative number.

### 1.2 Results

For my initial guesses of  $\alpha$  and  $\beta$  I choose 1 and 1. The optimization routine converges and finds the maximum score of 2285 while the parameters  $\alpha$  and  $\beta$  equal 1.825 and -0.65 respectively.

These results indicate that corporate ownership increases the probability of a match relative to non-corporate ownership while an increase in distance between the buyer and target reduces the probability of such a match. These results seem to make sense. Corporate ownership probably increases the payoffs from a merger. This could be due to the fact that corporations are more likely to engage in mergers and be profitable from such matches. They may have more scale or resources with which to aid their targets. An increase in distance between buyer and target should also likely decrease the probability of the merger. The further the distance, the higher the costs to maintain the relationship for both buyer and target.

# 2 Part Two

### 2.1 Model

The second model has a similar structure, however, this time we include prices, we estimate a coefficient on population, and include the HHI index and estimate its coefficient. Including prices in the model, allows us to identify buyer and target characteristics of the merger.

The payoff structure is as follows:

$$f_m(b,t) = \delta x_{1bm} y_{1tm} + \alpha x_{2bm} y_{1tm} + \gamma HHI_{tm} + \beta distance_{btm} + \epsilon_{btm}$$

We still have 4 equations but we now have four terms. So we should have at least 16 terms per array. In my data, I included the prices in the array in order to preserve the same style of model I implemented in the first section for the score estimator. In this model, my "matrix" is 2421 x 20.

By including the price information, we have the following score function:

$$Q(\beta) = \sum_{y=1}^{Y} \sum_{b=1}^{M_y-1} \sum_{b'=b+1}^{M_y} 1[f(b,t) - f(b,t') \ge p_{bt} - p_{b't'} \land f(b',t') - f(b',t) \ge p_{b't'} - p_{bt}]$$

Again for ease of analysis, I define the following score function:

$$Q(\beta) = \sum_{y=1}^{Y} \sum_{b=1}^{M_y-1} \sum_{b'=b+1}^{M_y} 1[f(b,t) - f(b,t') - p_{bt} + p_{b't'} \ge 0 \land f(b',t') - f(b',t) - p_{b't'} + p_{bt} \ge 0]$$

This time if both conditions hold, I give the indicator function a value of -1 and continue summing by arrays in order to find the maximum score and the necessary parameters.

## 2.2 Results

For my initial guesses of  $\delta$ ,  $\alpha$ ,  $\gamma$ , and  $\beta$  I use the values: (1, 2, 1, -1). The function converges with a maximum score of 1886. The values of the parameters  $\delta$ ,  $\alpha$ ,  $\gamma$ , and  $\beta$  that maximize this score are: (2.55623088, 2.13923887, -0.04588474, -0.21368366) respectively.

In this model, the population and corporate ownership characteristics again positively impact the probability of the merger. As before, an increase in distance negatively impacts the probability of the merger occurring. The HHI measure which measures market concetration negatively impacts the probability of the merger. This result also makes sense as buyers are less likely to be able to compete effectively in highly saturated markets.