Exponential Family *(canonical form)*

$$f(y;\theta) = exp[y\theta - b(\theta) + c(y)]$$

- Binomial Distribution:

$$\begin{aligned}
f(y;p) &= \binom{n}{y} p^y (1-p)^{n-y} \\
&= exp\left[ln\left(\binom{n}{y} p^y (1-p)^{n-y}\right)\right] \\
&= exp\left[ln\left(\binom{n}{y}\right) + yln(p) + (n-y)ln(1-p)\right] \\
&= exp\left[ln\left(\binom{n}{y}\right) + yln(p) + nln(1-p) - yln(1-p)\right] \\
&= exp\left[ln\left(\binom{n}{y}\right) + yln\left(\frac{p}{1-p}\right) + nln(1-p)\right] \\
&= exp\left[yln\left(\frac{p}{1-p}\right) + nln(1-p) + ln\left(\binom{n}{y}\right)\right] \\
&= exp[y\theta - b(\theta) + c(y)]
\end{aligned}$$

With $\theta = ln\left(\frac{p}{1-p}\right)$

$b(\theta) = nln\big(1 + exp(\theta)\big)$

$c(y) = ln\left(\binom{n}{y}\right)$

** Note:

$$\begin{aligned}
nln\big(1 + exp(\theta)\big) &= nln\left(1 + exp\left(ln\left(\frac{p}{1-p}\right)\right)\right) \\
&= nln\left(1 + \left(\frac{p}{1-p}\right)\right) = nln\left(\frac{1}{1-p}\right) = nln(1) - nln(1-p) \\
&= -nln(1-p)
\end{aligned}$$

- <u>Poisson Distribution:</u>

$$
\begin{aligned}
f(y;\lambda) &= \frac{\lambda^y e^{-\lambda}}{y!} \\
&= exp\left[ln\left(\frac{\lambda^y e^{-\lambda}}{y!}\right)\right] \\
&= exp[yln(\lambda) - \lambda - ln(y!)] \\
&= exp[y\theta - b(\theta) + c(y)]
\end{aligned}
$$

With $\theta = ln(\lambda)$

$b(\theta) = e^{\theta}$

$c(y) = -ln(y!)$

Exponential Dispersion Family *(canonical form)*

$$f(y;\theta,\phi)=exp\left[\frac{y\theta-b(\theta)}{a(\phi)}+c(y,\phi)\right]$$

- Normal Distribution:

$$\begin{aligned}
f(y;\mu,\sigma^2)&=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(y-\mu)^2}{2\sigma^2}}\\
&=exp\left[ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(y-\mu)^2}{2\sigma^2}}\right)\right]\\
&=exp\left[ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)-\frac{(y-\mu)^2}{2\sigma^2}\right]\\
&=exp\left[ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)-\frac{(y^2-2y\mu+\mu^2)}{2\sigma^2}\right]\\
&=exp\left[\frac{(2y\mu-\mu^2-y^2)}{2\sigma^2}-\frac{1}{2}ln(2\pi\sigma^2)\right]\\
&=exp\left[\frac{y\mu-\frac{1}{2}\mu^2}{\sigma^2}-\left(\frac{y^2}{2\sigma^2}+\frac{1}{2}ln(2\pi\sigma^2)\right)\right]
\end{aligned}$$

With $\theta=\mu$

$b(\theta)=\frac{1}{2}\theta^2$

$a(\phi)=\sigma^2$

$c(y,\phi)=-\left(\frac{y^2}{2\sigma^2}+\frac{1}{2}ln(2\pi\sigma^2)\right)$