Lecture 28: Review

Reading: Relevant chapters in ISLR.

STATS 202: Data mining and analysis

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Unsupervised learning

- In unsupervised learning, all the variables are on equal standing, no such thing as an input and response.
- **Two sets of methods:**
  1. PCA: find the main directions of variation in the data
  2. Clustering: find meaningful groups of samples
     - Hierarchical clustering (single, complete, or average linkage).
     - K-means clustering.

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- What is a biplot, how is it interpreted?
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- What is the effect of rescaling variables?

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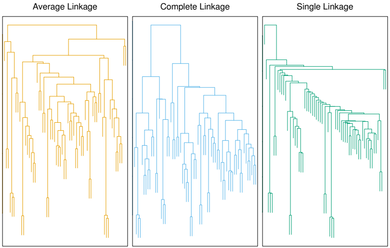
- The algorithm starts from some assignment, and is guaranteed to decrease this average distance.
- This find a local minimum, not necessarily a global minimum, so we typically repeat the algorithm from many different random starting points.

Hierarchical clustering



- Agglomerative algorithm produces a *dendrogram*.
- At each step we join the two clusters that are "closest":
  - **Complete:** distance between clusters is maximal distance between any pair of points.
  - **Single:** distance between clusters is minimal distance.
  - **Average:** distance between clusters is the average distance.
- Height of a branching point = distance between clusters joined.

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Two classes of problem:

- Regression: y_i is numerical

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- Classification: y_i is categorical

$$0 - 1 \text{ loss} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(y_i \neq \hat{y}_i).$$

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Both the MSE for regression, and the 0-1 loss for classification can be computed:

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We want to minimize the error on a very large test set which is sampled from the same process as the training data. This is called the *test error*.

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The expected test MSE of $\hat{f}$ has the following decomposition for any fixed x:

$$E[(y - \hat{f}(x))^2] = \underbrace{E[(\hat{f}(x) - E\hat{f}(x))^2]}_{\text{Var}(\hat{f}(x)) > 0} + \underbrace{(E[\hat{f}(x) - f(x)])^2}_{\text{Square bias of } \hat{f}(x). > 0} + \text{Var}(\epsilon)$$

Variance: Increases with the flexibility of the model
Bias: Decreases as the flexibility of the model increases

Regression methods

- Nearest neighbors regression
- Multiple linear regression

Classification methods

- Nearest neighbors classification
- Logistic regression
- LDA and QDA

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5. How does rescaling or transforming the variables affect the method?
6. In what situations does this method work well? What are its limitations?

Evaluating a classification method

We have talked about the 0-1 loss:

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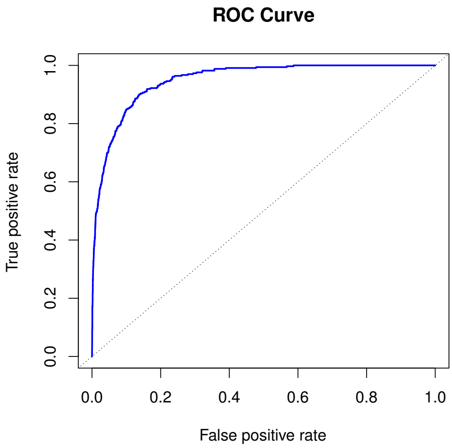
A much more informative summary of the error is a **confusion matrix**:

		Predicted class		
		− or Null	+ or Non-null	Total
True class	− or Null	True Neg. (TN)	False Pos. (FP)	N
	+ or Non-null	False Neg. (FN)	True Pos. (TP)	P
	Total	N*	P*	

The ROC curve



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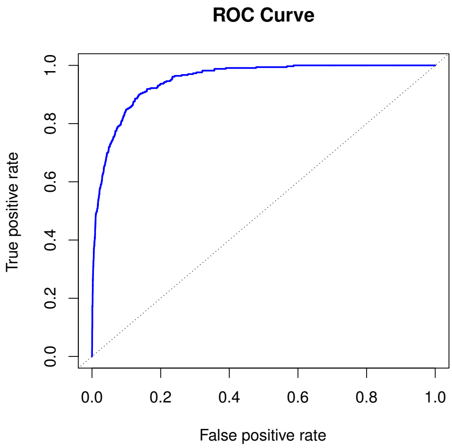

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- The area under the curve (AUC) measures the quality of the classifier:
  - 0.5 is the AUC for a random classifier
  - The closer AUC is to 1, the better.

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- No approach is superior to all others.
- What are the main differences? How do the bias and variance of the test error estimates compare? Which methods depend on the random seed?

The Bootstrap

- **Main idea:** If we have enough data, the empirical distribution is similar to the actual distribution of the data.
- Resampling with replacement allows us to obtain datasets mimicing how original data was selected.