

## 1 Variable kernel w/ grad-h

Suppose we know acceleration, time derviative of energy, coefficient viscosity, and the next timestep at the initial time. We integrate each particle as follows:

1. Calculate the position, and predict the velocity and energy:

$$\mathbf{r}_i = \mathbf{r}_i^{(0)} + \mathbf{v}_i^{(0)} \Delta t + \frac{1}{2} \mathbf{a}_i^{(0)} \Delta t^2 \quad (1)$$

$$\mathbf{v}_i^{(1/2)} = \mathbf{v}_i^{(0)} + \frac{1}{2} \mathbf{a}_i^{(0)} \Delta t \quad (2)$$

$$u_i^{(1/2)} = u_i^{(0)} + \frac{1}{2} \dot{u}_i^{(0)} \Delta t \quad (3)$$

$$\tilde{\mathbf{v}}_i = \mathbf{v}_i^{(0)} + \mathbf{a}_i^{(0)} \Delta t \quad (4)$$

$$\tilde{\mathbf{u}}_i = \mathbf{u}_i^{(0)} + \dot{\mathbf{u}}_i^{(0)} \Delta t \quad (5)$$

$$\tilde{\alpha}_i = \alpha_i^{(0)} + \dot{\alpha}_i^{(0)} \Delta t. \quad (6)$$

2. Calculate the density, kernel-support length and grad-h term of each particle by an iterative method:

- (a) Estimate the density by the following expression:

$$\rho_i = \sum_j m_j W_i \quad (7)$$

$$\Omega_i = 1 + \frac{1}{D} \frac{H_i}{\rho_i} \sum_j m_j \frac{\partial W_i}{\partial H_i} \quad (8)$$

where  $W_i = W(r_{ij}, H_i)$ ,  $r_{ij} = |\mathbf{r}_{ij}|$ , and  $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ . The kernel function is expressed as

$$W(r, H) = H^{-D} w(r/H), \quad (9)$$

$$\frac{\partial W(r, H)}{\partial H} = -H^{-(D+1)} \left[ Dw(q) + q \frac{\partial w}{\partial q} \right]. \quad (10)$$

In the case of  $D = 1$ ,

$$w(q) = \frac{5}{4} (1 - q)_+^3 (1 + 3q) \quad (11)$$

$$\frac{\partial w(q)}{\partial q} = \frac{15}{4} [(1 - q)_+^3 - (1 - q)_+^2 (1 + 3q)], \quad (12)$$

and in the case of  $D = 2, 3$ ,

$$w(q) = C_W (1 - q)_+^4 (1 + 4q) \quad (13)$$

$$\frac{\partial w(q)}{\partial q} = 4C_W [(1 - q)_+^4 - (1 - q)_+^3 (1 + 4q)], \quad (14)$$

where  $C_W = 7/\pi$  ( $D = 2$ ), and  $C_W = 21/(2\pi)$  ( $D = 3$ ).

(b) calculate the kernel length as follow:

$$H_i = (H/h) \times \eta \left( \frac{m_i}{\rho_i} \right)^{1/D}, \quad (15)$$

where  $h$  is the kernel length,  $\eta = 1.6$ , and  $H/h = 1.620185(D = 1)$ ,  $1.897367(D = 2)$ , and  $1.93492(D = 3)$ .

(c) Return to step (2a) if this is the first time.

(d) Calculate divergence and rotation of  $\mathbf{v}$ :

$$\nabla \cdot \tilde{\mathbf{v}}_i = \frac{1}{\Omega_i \rho_i} \sum \tilde{\mathbf{v}}_{ij} \cdot \left[ m_j \frac{\partial W_i}{\partial r_{ij}} \frac{\mathbf{r}_{ij}}{r_{ij}} \right], \quad (16)$$

$$\nabla \times \tilde{\mathbf{v}}_i = \frac{1}{\Omega_i \rho_i} \sum \tilde{\mathbf{v}}_{ij} \times \left[ m_j \frac{\partial W_i}{\partial r_{ij}} \frac{\mathbf{r}_{ij}}{r_{ij}} \right]. \quad (17)$$

The derivative of the kernel function is as follows:

$$\frac{\partial W(r, H)}{\partial r} = H^{-(D+1)} \frac{\partial w(r/H)}{\partial (r/H)}. \quad (18)$$

3. Calculate the pressure ( $\tilde{P}_i$ ) and sound speed ( $\tilde{c}_{s,i}$ ) as follows:

$$P_i = (\gamma - 1) \rho_i \tilde{u}_i \quad (19)$$

$$c_{s,i} = \left( \gamma \frac{P_i}{\rho_i} \right)^{1/2}. \quad (20)$$

4. Calculate the Balsara switch:

$$f_i = \frac{|\nabla \cdot \tilde{\mathbf{v}}_i|}{|\nabla \cdot \tilde{\mathbf{v}}_i| + |\nabla \times \tilde{\mathbf{v}}_i| + 0.0001 \tilde{c}_{s,i} / H_i}. \quad (21)$$

5. Calculate the acceleration and the time derivative of the energy:

$$\mathbf{a}_i = - \sum \left( \frac{\tilde{P}_i}{\Omega_i \rho_i^2} + \frac{\tilde{P}_j}{\Omega_j \rho_j^2} + f_{ij} \Pi_{ij} \right) \left[ \frac{m_j}{2} \left( \frac{\partial W_i}{\partial r_{ij}} + \frac{\partial W_j}{\partial r_{ij}} \right) \frac{\mathbf{r}_{ij}}{r_{ij}} \right], \quad (22)$$

$$\dot{\mathbf{u}}_i = \sum \left( \frac{\tilde{P}_i}{\Omega_i \rho_i^2} + \frac{f_{ij} \Pi_{ij}}{2} \right) \left[ \frac{m_j}{2} \left( \frac{\partial W_i}{\partial r_{ij}} + \frac{\partial W_j}{\partial r_{ij}} \right) \frac{\mathbf{r}_{ij}}{r_{ij}} \right] \cdot \tilde{\mathbf{v}}_{ij}, \quad (23)$$

where  $f_{ij} = (f_i + f_j)/2$ , and  $\Pi_{ij}$  is an artificial viscosity. The artificial viscosity is expressed as follows:

$$\Pi_{ij} = - \frac{\alpha}{2} \frac{v_{ij}^{\text{sig}} w_{ij}}{\rho_{ij}} \quad (24)$$

$$v_{ij}^{\text{sig}} = c_{s,i} + c_{s,j} - 3w_{ij} \quad (25)$$

$$w_{ij} = \begin{cases} \frac{\mathbf{r}_{ij} \cdot \tilde{\mathbf{v}}_{ij}}{|\mathbf{r}_{ij}|} & (\mathbf{r}_{ij} \cdot \tilde{\mathbf{v}}_{ij} < 0) \\ 0 & (\mathbf{r}_{ij} \cdot \tilde{\mathbf{v}}_{ij} \geq 0) \end{cases} \quad (26)$$

where  $\alpha = 1.0$  and  $\rho_{ij} = (\rho_i + \rho_j)/2$ .

6. Calculate  $\dot{\alpha}$ :

$$\dot{\alpha}_i = -\frac{\tilde{\alpha}_i - \alpha_{\min}}{h_i/(0.25\tilde{c}_{s,i})} + S_i \quad (27)$$

$$S_i = \max [-(\nabla \cdot \tilde{\mathbf{v}}_i)(\alpha_{\max} - \tilde{\alpha}_i), 0] \quad (28)$$

7. Correct the velocity and energy:

$$\mathbf{v}_i = \mathbf{v}_i^{(1/2)} + \frac{1}{2}\mathbf{a}_i\Delta t \quad (29)$$

$$u_i = u_i^{(1/2)} + \frac{1}{2}\dot{u}_i\Delta t \quad (30)$$

$$\alpha_i = \alpha_i^{(1/2)} + \frac{1}{2}\dot{\alpha}_i\Delta t. \quad (31)$$

8. Calculate the next timestep:

$$\Delta t = C \min_i \left( \frac{2H_i}{\max_j v_{ij}^{\text{sig}}} \right), \quad (32)$$

where  $C = 0.15$ .

9. Return to step 1.

## 2 Test

1. Shock tube
2. Strong shock
3. Point like explosion
4. Kelvin-Helmholtz instability

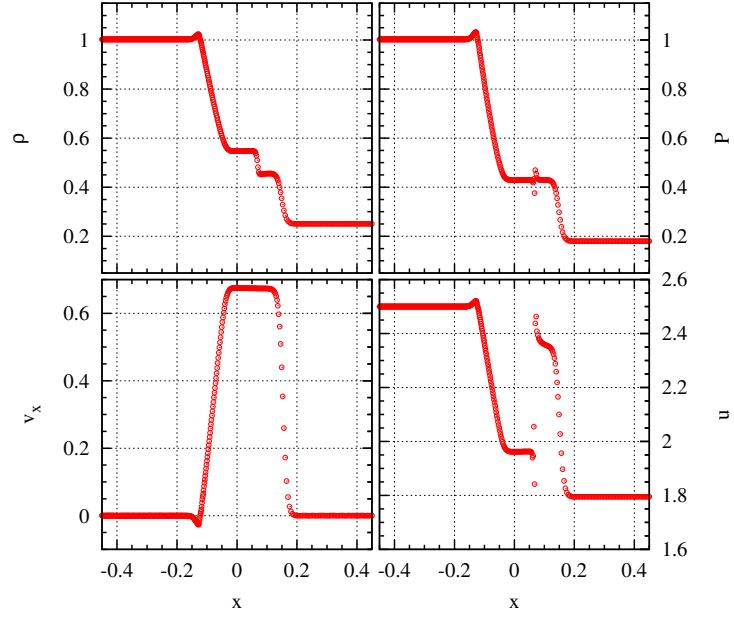


Figure 1: Shock tube ( $\gamma = 1.4$ ,  $\alpha = 2.0$ ,  $\eta = 1.6$ ).

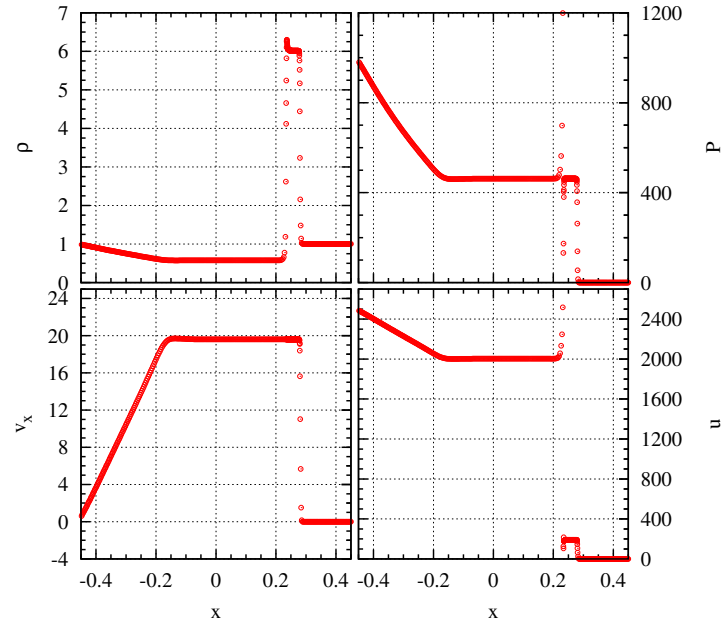


Figure 2: Strong shock ( $\gamma = 1.4$ ,  $\alpha = 2.0$ ,  $\eta = 1.6$ ).

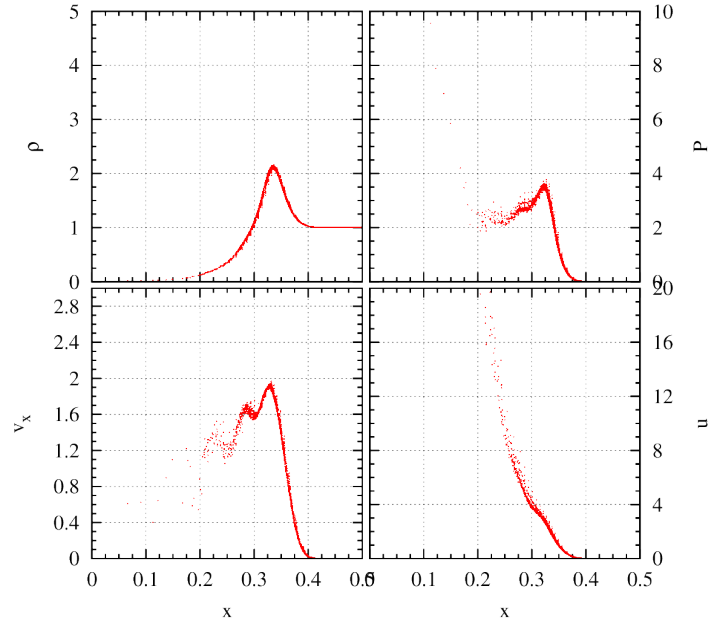


Figure 3: Point like explosion ( $\gamma = 5/3$ ,  $\alpha = 3.0$ ,  $\eta = 1.6$ ).

Figure 4: KH instability ( $\gamma = 5/3$ ,  $\alpha = 3.0$ ,  $\eta = 1.6$ ).

Figure 5: Evrard test.