Nash Equilibrium

Rational decision makers must choose the best action based on their beliefs about other players formed from previous games. However, we also assume that she views each play of a game in isolation. We also assume that these beliefs are always correct.

A Nash Equilibrium is an action profile a^* with property that no player i can do better by choosing an action other than a^*_i, given that every other player j adheres to a^*_j.

This is like a steady state, a social norm. No player has any reason to choose a different action. Also, the second assumption needn't always be correct and this concept can be applied to other situations if we feel it makes sense.

In Payoff function terms, a^* is a Nash equilibrium if for any player i, and their action a_i , a^* is at least as good according to i's preferences as the action profile (a_i ,a^*_{-i}). Hence $u_i(a^*) >= u_i(a_i, a^*_{-i})$

Existence of Nash Equilibrium is not necessary, and there can also be more than one. The theory also gives no way to find when we have reached an equilibrium.

Prisoner's Dilemma

(Fink, Fink) is a Nash Equilibrium. No matter what the other player chooses, you're always better off choosing Fink. The incentive to free ride eliminates the mutually desirable option. This is true for selfish players, but what happens if they care about the other person's outcome as well? (are altruistic)

An equilibrium may be strict (deviating from it makes your situation worse) or non-strict. Only non-strict equilibrium are required as per the definition.

Best Response Functions

Complicated games necessitate these to judge equilibrium states. The set of player i's best actions when the list of other player's actions are a_{-i} is $B_i(a_{-i}) = \{a_i \text{ in } A_i : u_i(a_i, a_{-i}) >= u_i(a'_I, a_{-i}) \text{ for all } a'_i \text{ in } A_i \}$

This function is set-valued. Nash equilibrium in terms of BRF is any action profile, where a^*_I is in $B_i(a^*_{-i})$ for any player i. Plotting a graph of the BRF functions, one can find the Nash Equilibrium by seeing the intersection points.

Strict Domination

A situation where a particular player's action a'' is superior to another action a' regardless of what the other players do. The opposite is an action that is strictly dominated, ie. It is never in the BRF.

Of course, a'' may not always be the best action, but it is always better than the action a'.

Weak Domination

An action a'' weakly dominates an action a' if it is always atleast as good as a' in all situations and is better than a' for some actions of the other players. These cannot exist in strict Nash Equilibrium

Symmetric Games

Where each player has the same set of actions, and each player's evaluation of the outcome depends only on her actions and her opponents' actions, and not on her being player 1 or 2, that is $u_1(a_1, a_2) = u_2(a_2, a_1)$

Symmetric Nash Equilibrium require that all players use the same action.