Week 4: Mixed Nash Equilibria

Week 4: Mixed Nash Equilibria

Mixed Strategy

Each player has a set of pure strategies, a mixed strategy is a probability distribution over this set. It is chosen such that the probability of choosing any strategy from the set is 1, ie. $\sum_{s_i \in S_i} \sigma_i(s_i) = 1$. A pure strategy will now be called a degenerate mixed strategy

Mixed Extension

- The set of all mixed strategies of a player
- $\Delta(S_i) = \left\{(\sigma_{i1}, \sigma_{i2} ...) \in \mathbb{R}^m : \sigma_{ij} \geq 0 \text{ for } j = 1, 2... \text{ and } \sum_{j=1}^{m} \sigma_{ij} = 1\right\}$

Payoff Function

- $U_i(\sigma_1, \sigma_2, ...) = \sum \sigma(s_1, s_2, ...) u_i(s_1, s_2, ...)$
- From now, we will use u instead of U always for convinience

Mixed Strategy Nash Equilibrium

- Given a strategic game $\Gamma =< N, (S_i), (u_i) >$, a mixed strategy profile (MSP) $(\sigma_1^*, \sigma_2^*, ...)$ is called a NE if $\forall i \in N$,
  $u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$
- If BRF is given by $b_i(\sigma_{-i})$, then clearly a MSP is an NE iff $\sigma_i^* \in b_i(\sigma_{-i}^*) \forall i \in N$

Properties of Mixed Strategies

Convex Combination

- A convex combination of n reals y_i is a weighted sum of the form $\sum(\lambda_i y_i) where \sum(\lambda_i) = 1$
- For a strategic game Γ, $u_i(\sigma_i, \sigma_{-i}) = \sum \sigma_i(s_i) u_i(s_i, \sigma_{-i})$
- This means that the payoff a player under mixed strategy can be computed as convex combination of payoffs for all pure strategies

The Maximum value of a Convex Combination, is the Maximum Value itself

- so $\max u_i(\sigma_i, \sigma_{-i}) = \max u_i(s_i, \sigma_{-i})$
- Furthermore, $\rho_i \in \arg\max u_i(\sigma_i, \sigma_{-i})$
  iff $\rho_i(x) = 0 \forall x \notin \arg\max u_i(s_i, \sigma_{-i})$
- Basically there exists a degenerate mixed strategy which can maximise the payoff function

Conditions for a Profile to be MSNE

- The support of a mixed strategy $\delta(\sigma_i) = s_i \in S_i : \sigma_i(s_i) > 0$
- So for a MSP to be a MSNE iff $\forall i \in N$,

1. $u_i(s_i, \sigma^*_{-i})\text{is the same}\forall s_i \in \delta(\sigma^*_i)$
2. $u_i(s_i, \sigma^*_{-i} \geq u_i(s'_i, \sigma^*_{-i})\forall s_i \in \delta(\sigma^*_i); \forall s'_i \notin \delta(\sigma^*_i)$

- So any pure strategy with popular probability in the MSNE profile has the same payoff
- And for considerations of an NE, we only need to consider the effects of pure strategy deviations
- A MSNE with degenerate strategies only implies that these pure strategies form a pure strategy NE

Domination

- Strict domination : $u_i(\sigma_i, \sigma_{-i}) > u_i(\sigma'_i, \sigma_-i)$
- If the relation is always $\geq$ then it is called very weak domination, and if it is sometimes $\geq$ and sometimes > then it is called weak domination
- The MSP consisting of everyone's dominant actions forms a dominant MSNE
- Strictly dominant MS like Strictly dominant MSNE is unique