1 Relaxation to a fixed profile

1.1 Relaxation equation

- Variables
  - Profile funcition: F
  - Fixed profile: F_0
  - Relaxation time: τ
- Equation

$$\frac{\mathrm{d}F}{\mathrm{d}t} = -\frac{F - F_0}{\tau} \tag{1.1}$$

- Finite difference method

$$\frac{F_{i+1} - F_i}{\Delta t} = -\frac{\delta F_{i+1} + (1-\delta)F_i - F_0}{\tau} \tag{1.2}$$

$$= -\frac{\delta F_{i+1}}{\tau} - \frac{(1-\delta)F_i}{\tau} + \frac{F_0}{\tau} \tag{1.3}$$

$$F_{i+1} - Fi = \left[-\delta\frac{\Delta t}{\tau}\right] F_{i+1} + \left[-(1-\delta)\frac{\Delta t}{\tau}\right] F_i + \frac{\Delta t}{\tau} F_0 \tag{1.4}$$

$$\left[1 + \delta\frac{\Delta t}{\tau}\right] F_{i+1} = \left[1 - (1-\delta)\frac{\Delta t}{\tau}\right] F_i + \frac{\Delta t}{\tau} F_0 \tag{1.5}$$

$$F_{i+1} = \frac{1 - (1-\delta)\Delta t/\tau}{1 + \delta\Delta t/\tau} F_i + \frac{\Delta t/\tau}{1 + \delta\Delta t/\tau} F_0 \tag{1.6}$$