1 Equations in r coordinates

$$0 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\phi + \frac{1}{\epsilon_0}\left(-en_{\mathrm{e}} + Z_{\mathrm{i}}en_{\mathrm{i}} + Z_{\mathrm{b}}en_{\mathrm{b}} + Z_{\mathrm{b}}egn_{\mathrm{b}}^{\mathrm{rp}}\right) \tag{1.1}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot{A}_\theta = \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}rA_\theta + \mu_0(-en_{\mathrm{e}}u_{\mathrm{e}\theta} + Z_{\mathrm{i}}en_{\mathrm{i}}u_{\mathrm{i}\theta} + Z_{\mathrm{b}}en_{\mathrm{b}}u_{\mathrm{b}\theta}) \tag{1.2}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot{A}_\phi = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}A_\phi + \mu_0(-en_{\mathrm{e}}u_{\mathrm{e}\phi} + Z_{\mathrm{i}}en_{\mathrm{i}}u_{\mathrm{i}\phi} + Z_{\mathrm{b}}en_{\mathrm{b}}u_{\mathrm{b}\phi}) \tag{1.3}$$

$$\frac{\partial}{\partial t}A_\phi = \dot{A}_\phi \tag{1.4}$$

$$\frac{\partial}{\partial t}A_\theta = \dot{A}_\theta \tag{1.5}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}} = -\frac{1}{r}\frac{\partial}{\partial r}rn_{\mathrm{e}}u_{\mathrm{e}r} + \nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}\left(n_{01}+n_{02}+n_{03}\right) - \nu_{\mathrm{L}}(n_{\mathrm{e}}-n_{\mathrm{ediv}})$$
$$+ (1-f_{\mathrm{CX}})\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} \tag{1.6}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}}u_{\mathrm{e}r} = -\frac{1}{r}\frac{\partial}{\partial r}ru_{\mathrm{e}r}n_{\mathrm{e}}u_{\mathrm{e}r} + \frac{1}{r}u_{\mathrm{e}\theta}n_{\mathrm{e}}u_{\mathrm{e}\theta} - \frac{\partial}{\partial r}\frac{n_{\mathrm{e}}T_{\mathrm{e}}}{m_{\mathrm{e}}}$$
$$- \frac{e}{m_{\mathrm{e}}}n_{\mathrm{e}}E_r - \frac{e}{m_{\mathrm{e}}}B_\phi n_{\mathrm{e}}u_{\mathrm{e}\theta} + \frac{e}{m_{\mathrm{e}}}B_\theta n_{\mathrm{e}}u_{\mathrm{e}\phi} \tag{1.7}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}}u_{\mathrm{e}\theta} = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 u_{\mathrm{e}r}n_{\mathrm{e}}u_{\mathrm{e}\theta} + \frac{1}{r^2}\frac{\partial}{\partial r}r^3 n_{\mathrm{e}}\mu_{\mathrm{e}}\frac{\partial}{\partial r}\frac{u_{\mathrm{e}\theta}}{r} - \frac{e}{m_{\mathrm{e}}}n_{\mathrm{e}}E_\theta + \frac{e}{m_{\mathrm{e}}}B_\phi n_{\mathrm{e}}u_{\mathrm{e}r}$$
$$- \nu_{\mathrm{NCe}}n_{\mathrm{e}}u_{\mathrm{e}\theta} - \nu_{\mathrm{ei1}}n_{\mathrm{e}}(u_{\mathrm{e}\theta}-u_{\mathrm{i}\theta}) - \nu_{\mathrm{ei2}}n_{\mathrm{e}}(u_{\mathrm{e}\phi}-u_{\mathrm{i}\phi})$$
$$- \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be1}}n_{\mathrm{b}}(u_{\mathrm{e}\theta}-u_{\mathrm{b}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be2}}n_{\mathrm{b}}(u_{\mathrm{e}\phi}-u_{\mathrm{b}\phi})$$
$$- \frac{e^2 B_\phi^2 D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}n_{\mathrm{e}}\left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi}u_{\mathrm{e}\phi} + \frac{B}{B_\phi}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}}r + \frac{1-\alpha_{\mathrm{e}}}{eB_\phi}\frac{\partial}{\partial r}T_{\mathrm{e}} + \frac{E_r}{B_\phi}\right]$$
$$- 2\nu_{\mathrm{L}}n_{\mathrm{e}}u_{\mathrm{e}\theta} - \nu_{\mathrm{0e}}n_{\mathrm{e}}u_{\mathrm{e}\theta} + F_{\mathrm{e}\theta}^{\mathrm{neo}} \tag{1.8}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}}u_{\mathrm{e}\phi} = -\frac{1}{r}\frac{\partial}{\partial r}ru_{\mathrm{e}r}n_{\mathrm{e}}u_{\mathrm{e}\phi} + \frac{1}{r}\frac{\partial}{\partial r}rn_{\mathrm{e}}\mu_{\mathrm{e}}\frac{\partial}{\partial r}u_{\mathrm{e}\phi} - \frac{e}{m_{\mathrm{e}}}n_{\mathrm{e}}E_\phi - \frac{e}{m_{\mathrm{e}}}B_\theta n_{\mathrm{e}}u_{\mathrm{e}r}$$
$$- \nu_{\mathrm{ei3}}n_{\mathrm{e}}(u_{\mathrm{e}\phi}-u_{\mathrm{i}\phi}) - \nu_{\mathrm{ei2}}n_{\mathrm{e}}(u_{\mathrm{e}\theta}-u_{\mathrm{i}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be2}}n_{\mathrm{b}}(u_{\mathrm{e}\theta}-u_{\mathrm{b}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be3}}n_{\mathrm{b}}(u_{\mathrm{e}\phi}-u_{\mathrm{b}\phi})$$
$$+ \frac{e^2 B_\phi^2 D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}\frac{B_\theta}{B_\phi}n_{\mathrm{e}}\left[u_{\mathrm{e}\theta} - \frac{B_\theta}{B_\phi}u_{\mathrm{e}\phi} + \frac{B}{B_\phi}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}}r + \frac{1-\alpha_{\mathrm{e}}}{eB_\phi}\frac{\partial}{\partial r}T_{\mathrm{e}} + \frac{E_r}{B_\phi}\right]$$

$$- 2\nu_{\mathrm{L}} n_{\mathrm{e}} u_{\mathrm{e}\phi} - \nu_{0\mathrm{e}} n_{\mathrm{e}} u_{\mathrm{e}\phi} \tag{1.9}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_{\mathrm{e}} T_{\mathrm{e}} = -\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\mathrm{e}r} n_{\mathrm{e}} T_{\mathrm{e}} - n_{\mathrm{e}}\chi_{\mathrm{e}}\frac{\partial}{\partial r} T_{\mathrm{e}} \right) + u_{\mathrm{e}r}\frac{\partial}{\partial r} n_{\mathrm{e}} T_{\mathrm{e}} - e E_{\theta} n_{\mathrm{e}} u_{\mathrm{e}\theta} - e E_{\phi} n_{\mathrm{e}} u_{\mathrm{e}\phi}$$

$$- \frac{3}{2}\nu_{\mathrm{Tei}} n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{i}}) - \nu_{\mathrm{L}} T_{\mathrm{e}}(n_{\mathrm{e}} - n_{\mathrm{ediv}}) - \frac{3}{2}\nu_{\mathrm{L}T_{\mathrm{e}}} n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{ediv}})$$

$$- E_{\mathrm{H}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}}(n_{01} + n_{02} + n_{03}) + f_{\mathrm{ei}} P_{\mathrm{b}} + P_{\alpha \mathrm{e}} + P_{\mathrm{RFe}} - P_{\mathrm{Br}} \tag{1.10}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} = -\frac{1}{r}\frac{\partial}{\partial r} r n_{\mathrm{i}} u_{\mathrm{i}r} + \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}}\frac{n_{\mathrm{e}}}{n_{01} + nnb + n_{03}}(n_{01} + n_{02} + n_{03}) - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}}(n_{\mathrm{e}} - n_{\mathrm{ediv}})$$

$$+ \nu_{\mathrm{b}} n_{\mathrm{b}} + \nu_{\mathrm{b}} g n_{\mathrm{b}}^{\mathrm{rp}} - f_{\mathrm{CX}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} + \nu_{\mathrm{bL}} n_{\mathrm{b}} + S_{\mathrm{LC}} - \nu_{\mathrm{OL}} n_{\mathrm{i}} \tag{1.11}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}r} = -\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}r} + \frac{1}{r} u_{\mathrm{i}\theta} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \frac{\partial}{\partial r}\frac{n_{\mathrm{i}} T_{\mathrm{i}}}{m_{\mathrm{i}}}$$

$$+ Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_r + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} B_{\phi} n_{\mathrm{i}} u_{\mathrm{i}\theta} - Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} B_{\theta} n_{\mathrm{i}} u_{\mathrm{i}\phi} \tag{1.12}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}\theta} = -\frac{1}{r^2}\frac{\partial}{\partial r} r^2 u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}\theta} + \frac{1}{r^2}\frac{\partial}{\partial r} r^3 n_{\mathrm{i}}\mu_{\mathrm{i}}\frac{\partial}{\partial r}\frac{u_{\mathrm{i}\theta}}{r} + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_{\theta} - Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} B_{\phi} n_{\mathrm{i}} u_{\mathrm{i}r}$$

$$- \nu_{\mathrm{NCi}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei1}} n_{\mathrm{e}}(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}) - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei2}} n_{\mathrm{e}}(u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}}\nu_{\mathrm{bi}} n_{\mathrm{b}}(u_{\mathrm{i}\theta} - u_{\mathrm{b}\theta})$$

$$+ \frac{e^2 B_{\phi}^2 D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} n_{\mathrm{e}}\left[u_{\mathrm{e}\theta} - \frac{B_{\theta}}{B_{\phi}} u_{\mathrm{e}\phi} + \frac{B}{B_{\phi}}\left\langle \frac{\omega}{m} \right\rangle_{\mathrm{e}} r + \frac{1 - \alpha_{\mathrm{e}}}{e B_{\phi}}\frac{\partial}{\partial r} T_{\mathrm{e}} + \frac{E_r}{B_{\phi}} \right]$$

$$- 2\nu_{\mathrm{L}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \nu_{0\mathrm{i}} n_{\mathrm{i}} u_{\mathrm{i}\theta} - \nu_{\mathrm{CX}} n_{\mathrm{i}} u_{\mathrm{i}\theta} + S_{\mathrm{LC}\theta} - \nu_{\mathrm{OL}} n_{\mathrm{i}} u_{\mathrm{i}\theta} + F_{\mathrm{i}\theta}^{\mathrm{neo}} + T_{\mathrm{i}\theta} \tag{1.13}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} u_{\mathrm{i}\phi} = -\frac{1}{r}\frac{\partial}{\partial r} r u_{\mathrm{i}r} n_{\mathrm{i}} u_{\mathrm{i}\phi} + \frac{1}{r}\frac{\partial}{\partial r} r n_{\mathrm{i}}\mu_{\mathrm{i}}\frac{\partial}{\partial r} u_{\mathrm{i}\phi} + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} E_{\phi} + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}} B_{\theta} n_{\mathrm{i}} u_{\mathrm{i}r}$$

$$- \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei3}} n_{\mathrm{e}}(u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei2}} n_{\mathrm{e}}(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}}\nu_{\mathrm{bi}} n_{\mathrm{b}}(u_{\mathrm{i}\phi} - u_{\mathrm{b}\phi})$$

$$- \frac{e^2 B_{\phi}^2 D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}}\frac{B_{\theta}}{B_{\phi}} n_{\mathrm{e}}\left[u_{\mathrm{e}\theta} - \frac{B_{\theta}}{B_{\phi}} u_{\mathrm{e}\phi} + \frac{B}{B_{\phi}}\left\langle \frac{\omega}{m} \right\rangle_{\mathrm{e}} r + \frac{1 - \alpha_{\mathrm{e}}}{e B_{\phi}}\frac{\partial}{\partial r} T_{\mathrm{e}} + \frac{E_r}{B_{\phi}} \right]$$

$$- 2\nu_{\mathrm{L}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{0\mathrm{i}} n_{\mathrm{i}} u_{\mathrm{i}\phi} - \nu_{\mathrm{CX}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + S_{\mathrm{LC}\phi} - \nu_{\mathrm{OL}} n_{\mathrm{i}} u_{\mathrm{i}\phi} + T_{\mathrm{i}\phi} \tag{1.14}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_{\mathrm{i}} T_{\mathrm{i}} = -\frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{5}{2} u_{\mathrm{i}r} n_{\mathrm{i}} T_{\mathrm{i}} - n_{\mathrm{i}}\chi_{\mathrm{i}}\frac{\partial}{\partial r} T_{\mathrm{i}} \right) + u_{\mathrm{i}r}\frac{\partial}{\partial r} n_{\mathrm{i}} T_{\mathrm{i}} + Z_{\mathrm{i}} e E_{\theta} n_{\mathrm{i}} u_{\mathrm{i}\theta} + Z_{\mathrm{i}} e E_{\phi} n_{\mathrm{i}} u_{\mathrm{i}\phi}$$

$$- \frac{3}{2}\nu_{\mathrm{Tei}} n_{\mathrm{e}}(T_{\mathrm{i}} - T_{\mathrm{e}}) + m_{\mathrm{b}} v_{\mathrm{b}}\frac{B_{\theta} u_{\mathrm{i}\theta} + B_{\phi} u_{\mathrm{i}\phi}}{B}\frac{P_{\mathrm{b}\|}}{E_{\mathrm{b}}} - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} T_{\mathrm{i}}(n_{\mathrm{e}} - n_{\mathrm{ediv}})$$

$$- \frac{3}{2}\nu_{\mathrm{L}T_{\mathrm{i}}} n_{\mathrm{i}}(T_{\mathrm{i}} - T_{\mathrm{idiv}}) + \frac{3}{2}\frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}}\frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}}(n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03})$$

$$-\frac{3}{2}\nu_{\mathrm{CX}}n_{\mathrm{i}}(T_{\mathrm{i}}-T_{01})+(1-f_{\mathrm{ei}})P_{\mathrm{b}}+P_{\alpha\mathrm{i}}+P_{\mathrm{RFi}} \tag{1.15}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}}=\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}}-g\frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}}-\nu_{\mathrm{b}}n_{\mathrm{b}}-\nu_{\mathrm{bL}}n_{\mathrm{b}}-\nu_{\mathrm{eff}}g(\sqrt{\delta}n_{\mathrm{b}}-n_{\mathrm{b}}^{\mathrm{rp}})+\frac{1}{r}\frac{\partial}{\partial r}rf_{t}D_{\mathrm{b}}\frac{\partial}{\partial r}n_{\mathrm{b}} \tag{1.16}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}}u_{\mathrm{b}\theta}=Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}n_{\mathrm{b}}E_{\theta}-\nu_{\mathrm{be1}}n_{\mathrm{b}}(u_{\mathrm{b}\theta}-u_{\mathrm{e}\theta})-\nu_{\mathrm{be2}}n_{\mathrm{b}}(u_{\mathrm{b}\phi}-u_{\mathrm{e}\phi})-\nu_{\mathrm{bi}}n_{\mathrm{b}}(u_{\mathrm{b}\theta}-u_{\mathrm{i}\theta})$$

$$-\nu_{0\mathrm{b}}n_{\mathrm{b}}u_{\mathrm{b}\theta}-\nu_{\mathrm{CX}}n_{\mathrm{b}}u_{\mathrm{b}\theta}+v_{\mathrm{b}\theta}\frac{P_{\mathrm{b}\|}}{E_{\mathrm{b}}}-\nu_{\mathrm{bL}}n_{\mathrm{b}}u_{\mathrm{b}\theta}-\nu_{\mathrm{eff}}g\sqrt{\delta}n_{\mathrm{b}}u_{\mathrm{b}\theta}$$

$$+\frac{1}{r}\frac{\partial}{\partial r}ru_{\mathrm{b}\theta}f_{t}D_{\mathrm{b}}\frac{\partial}{\partial r}n_{\mathrm{b}} \tag{1.17}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}}u_{\mathrm{b}\phi}=Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}n_{\mathrm{b}}E_{\phi}-\nu_{\mathrm{be3}}n_{\mathrm{b}}(u_{\mathrm{b}\phi}-u_{\mathrm{e}\phi})-\nu_{\mathrm{be2}}n_{\mathrm{b}}(u_{\mathrm{b}\theta}-u_{\mathrm{e}\theta})-\nu_{\mathrm{bi}}n_{\mathrm{b}}(u_{\mathrm{b}\phi}-u_{\mathrm{i}\phi})$$

$$-\nu_{0\mathrm{b}}n_{\mathrm{b}}u_{\mathrm{b}\phi}-\nu_{\mathrm{CX}}n_{\mathrm{b}}u_{\mathrm{b}\phi}+v_{\mathrm{b}\phi}\frac{P_{\mathrm{b}\|}}{E_{\mathrm{b}}}-\nu_{\mathrm{bL}}n_{\mathrm{b}}u_{\mathrm{b}\phi}-\nu_{\mathrm{eff}}g\sqrt{\delta}n_{\mathrm{b}}u_{\mathrm{b}\phi}$$

$$+\frac{1}{r}\frac{\partial}{\partial r}ru_{\mathrm{b}\phi}f_{t}D_{\mathrm{b}}\frac{\partial}{\partial r}n_{\mathrm{b}} \tag{1.18}$$

$$\frac{\partial}{\partial t}n_{01}=\frac{1}{r}\frac{\partial}{\partial r}rD_{01}\frac{\partial}{\partial r}n_{01}-\frac{1}{Z_{\mathrm{i}}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}n_{01}-\nu_{\mathrm{CX}}\frac{n_{\mathrm{i}}}{n_{01}+n_{02}}n_{01}$$

$$+\gamma_{0}\frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}}(n_{\mathrm{e}}-n_{\mathrm{ediv}})+\frac{1}{r}\frac{\partial}{\partial r}r\Gamma^{\mathrm{puff}}\Big|_{\mathrm{b}} \tag{1.19}$$

$$\frac{\partial}{\partial t}n_{02}=\frac{1}{r}\frac{\partial}{\partial r}rD_{02}\frac{\partial}{\partial r}n_{02}-\frac{1}{Z_{\mathrm{i}}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}n_{02}+\nu_{\mathrm{CX}}\frac{n_{\mathrm{i}}}{n_{01}+n_{02}}n_{01} \tag{1.20}$$

$$\frac{\partial}{\partial t}n_{03}=\frac{1}{r}\frac{\partial}{\partial r}rD_{03}\frac{\partial}{\partial r}n_{03}-\frac{1}{Z_{\mathrm{i}}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}n_{03}+f_{\mathrm{CX}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} \tag{1.21}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}}^{\mathrm{rp}}=\frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}}+\nu_{\mathrm{eff}}(\sqrt{\delta}n_{\mathrm{b}}-n_{\mathrm{b}}^{\mathrm{rp}})-\nu_{\mathrm{b}}n_{\mathrm{b}}^{\mathrm{rp}}-\frac{1}{r}\frac{\partial}{\partial r}\left(ru_{\mathrm{b}}^{\mathrm{rp}}n_{\mathrm{b}}^{\mathrm{rp}}-rD_{\mathrm{b}}^{\mathrm{rp}}\frac{\partial}{\partial r}n_{\mathrm{b}}^{\mathrm{rp}}\right) \tag{1.22}$$

2 Relations Between Fields and Potentials

$$E_{r}=-\frac{\partial}{\partial r}\phi \tag{2.1}$$

$$E_{\theta}=-\frac{\partial}{\partial t}A_{\theta}=-\dot{A}_{\theta} \tag{2.2}$$

$$E_{\phi}=-\frac{\partial}{\partial t}A_{\phi}=-\dot{A}_{\phi} \tag{2.3}$$

$$B_\theta = -\frac{\partial}{\partial r} A_\phi \tag{2.4}$$

$$B_\phi = \frac{1}{r} \frac{\partial}{\partial r} r A_\theta \tag{2.5}$$

3 Equations in $s(= r^2)$ coordinates

$$0 = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} \phi + \frac{1}{\epsilon_0} \left(-e n_e + Z_i e n_i + Z_b e n_b + Z_b e g n_b^{rp} \right) \tag{3.1}$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} r \dot{A}_\theta = \left(4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} r A_\theta - 4 \frac{\partial}{\partial s} r A_\theta \right) + \mu_0 (-e r n_e u_{e\theta} + Z_i e r n_i u_{i\theta} + Z_b e r n_b u_{b\theta}) \tag{3.2}$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\phi = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} A_\phi + \mu_0 (-e n_e u_{e\phi} + Z_i e n_i u_{i\phi} + Z_b e n_b u_{b\phi}) \tag{3.3}$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \tag{3.4}$$

$$\frac{\partial}{\partial t} r A_\theta = r \dot{A}_\theta \tag{3.5}$$

$$\frac{\partial}{\partial t} n_e = -2 \frac{\partial}{\partial s} r n_e u_{er} + \nu_{ionize} \frac{n_e}{n_{01} + n_{02} + n_{03}} \left(n_{01} + n_{02} + n_{03} \right) - \nu_L (n_e - n_{ediv})$$
$$+ (1 - f_{CX}) \frac{P_b}{E_b} \tag{3.6}$$

$$\frac{\partial}{\partial t} r n_e u_{er} = -2 r \frac{\partial}{\partial s} u_{er} r n_e u_{er} + \frac{u_{e\theta}}{r} r n_e u_{e\theta} - 2 s \frac{\partial}{\partial s} \frac{n_e T_e}{m_e}$$
$$+ 2 \frac{e}{m_e} s n_e \frac{\partial}{\partial s} \phi - \frac{e}{m_e} B_\phi r n_e u_{e\theta} - 2 \frac{e}{m_e} s \frac{\partial A_\phi}{\partial s} n_e u_{e\phi} \tag{3.7}$$

$$\frac{\partial}{\partial t} r n_e u_{e\theta} = -2 \frac{\partial}{\partial s} r u_{er} r n_e u_{e\theta} + \left(4 \frac{\partial}{\partial s} s \mu_e \frac{\partial}{\partial s} r n_e u_{e\theta} - 4 \frac{\partial}{\partial s} s \mu_e r u_{e\theta} \frac{\partial n_e}{\partial s} - 4 \frac{\partial}{\partial s} \mu_e r n_e u_{e\theta} \right)$$
$$+ \frac{e}{m_e} n_e r \dot{A}_\theta + \frac{e}{m_e} B_\phi r n_e u_{er} - \nu_{NCe} r n_e u_{e\theta} - \nu_{ei1} r n_e (u_{e\theta} - u_{i\theta}) + 2 s \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} n_e (u_{e\phi} - u_{i\phi})$$
$$- \frac{m_b}{m_e} \nu_{be1} r n_b (u_{e\theta} - u_{b\theta}) + 2 \frac{m_b}{m_e} s \frac{\partial A_\phi}{\partial s} \frac{\nu_{be2}}{B_\theta} n_e (u_{e\phi} - u_{b\phi})$$
$$- \frac{e^2 B_\phi^2 D_e}{m_e T_e} n_e \left[r u_{e\theta} + 2 \frac{\partial A_\phi}{\partial s} \frac{s}{B_\phi} u_{e\phi} + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2 s \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial s} T_e - 2 s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right]$$
$$- 2 \nu_L r n_e u_{e\theta} - \nu_{0e} r n_e u_{e\theta} + r F_{e\theta}^{neo} \tag{3.8}$$

$$\frac{\partial}{\partial t}n_{\mathrm{e}}u_{\mathrm{e}\phi} = -2\frac{\partial}{\partial s}ru_{\mathrm{er}}n_{\mathrm{e}}u_{\mathrm{e}\phi} + \left(4\frac{\partial}{\partial s}s\mu_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}u_{\mathrm{e}\phi} - 4\frac{\partial}{\partial s}s\mu_{\mathrm{e}}u_{\mathrm{e}\phi}\frac{\partial n_{\mathrm{e}}}{\partial s}\right)$$

$$+ \frac{e}{m_{\mathrm{e}}}n_{\mathrm{e}}\dot{A}_{\phi} + 2\frac{e}{m_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}rn_{\mathrm{e}}u_{\mathrm{er}} - \nu_{\mathrm{ei3}}n_{\mathrm{e}}(u_{\mathrm{e}\phi} - u_{\mathrm{i}\phi}) + 2\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{ei2}}}{B_{\theta}}rn_{\mathrm{e}}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta})$$

$$+ 2\frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{be2}}}{B_{\theta}}rn_{\mathrm{e}}(u_{\mathrm{e}\theta} - u_{\mathrm{i}\theta}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{e}}}\nu_{\mathrm{be3}}n_{\mathrm{b}}(u_{\mathrm{e}\phi} - u_{\mathrm{b}\phi})$$

$$- 2\frac{e^2 B_{\phi}D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}\frac{\partial A_{\phi}}{\partial s}\left[rn_{\mathrm{e}}u_{\mathrm{e}\theta} + sn_{\mathrm{e}}\frac{B}{B_{\phi}}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}} + 2s\frac{1-\alpha_{\mathrm{e}}}{eB_{\phi}}\left(\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - T_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}\right)\right.$$

$$\left. - 2sn_{\mathrm{e}}\frac{1}{B_{\phi}}\frac{\partial}{\partial s}\phi\right] - \frac{e^2 B_{\phi}^2 D_{\mathrm{e}}}{m_{\mathrm{e}}T_{\mathrm{e}}}\left(\frac{B_{\theta}}{B_{\phi}}\right)^2 n_{\mathrm{e}}u_{\mathrm{e}\phi} - 2\nu_{\mathrm{L}}n_{\mathrm{e}}u_{\mathrm{e}\phi} - \nu_{\mathrm{0e}}n_{\mathrm{e}}u_{\mathrm{e}\phi} \tag{3.9}$$

$$\frac{\partial}{\partial t}\frac{3}{2}n_{\mathrm{e}}T_{\mathrm{e}} = -5\frac{\partial}{\partial s}ru_{\mathrm{er}}n_{\mathrm{e}}T_{\mathrm{e}} + \left(4\frac{\partial}{\partial s}s\chi_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - 4\frac{\partial}{\partial s}s\chi_{\mathrm{e}}T_{\mathrm{e}}\frac{\partial n_{\mathrm{e}}}{\partial s}\right) + 2ru_{\mathrm{er}}\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}}$$

$$- e\frac{E_{\theta}}{r}rn_{\mathrm{e}}u_{\mathrm{e}\theta} - eE_{\phi}n_{\mathrm{e}}u_{\mathrm{e}\phi} - \frac{3}{2}\nu_{\mathrm{Tei}}n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{i}})$$

$$- \nu_{\mathrm{L}}T_{\mathrm{e}}(n_{\mathrm{e}} - n_{\mathrm{ediv}}) - \frac{3}{2}\nu_{\mathrm{L}T_{\mathrm{e}}}n_{\mathrm{e}}(T_{\mathrm{e}} - T_{\mathrm{ediv}})$$

$$- E_{\mathrm{H}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}\left(n_{01}+n_{02}+n_{03}\right) + f_{\mathrm{ei}}P_{\mathrm{b}} + P_{\alpha\mathrm{e}} + P_{\mathrm{RFe}} - P_{\mathrm{Br}} \tag{3.10}$$

$$\frac{\partial}{\partial t}n_{\mathrm{i}} = -2\frac{\partial}{\partial s}rn_{\mathrm{i}}u_{\mathrm{ir}} + \frac{\nu_{\mathrm{ionize}}}{Z_{\mathrm{i}}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}\left(n_{01}+n_{02}+n_{03}\right) - \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}}(n_{\mathrm{e}} - n_{\mathrm{ediv}})$$

$$+ \nu_{\mathrm{b}}n_{\mathrm{b}} + \nu_{\mathrm{b}}gn_{\mathrm{b}}^{\mathrm{rp}} - f_{\mathrm{CX}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} + \nu_{\mathrm{bL}}n_{\mathrm{b}} + S_{\mathrm{LC}} - \nu_{\mathrm{OL}}n_{\mathrm{i}} \tag{3.11}$$

$$\frac{\partial}{\partial t}rn_{\mathrm{i}}u_{\mathrm{ir}} = -2r\frac{\partial}{\partial s}u_{\mathrm{ir}}rn_{\mathrm{i}}u_{\mathrm{ir}} + \frac{u_{\mathrm{e}\theta}}{r}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - 2s\frac{\partial}{\partial s}\frac{n_{\mathrm{i}}T_{\mathrm{i}}}{m_{\mathrm{i}}}$$

$$- 2Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}sn_{\mathrm{i}}\frac{\partial}{\partial s}\phi + Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}B_{\phi}rn_{\mathrm{i}}u_{\mathrm{i}\theta} + 2Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}s\frac{\partial A_{\phi}}{\partial s}n_{\mathrm{i}}u_{\mathrm{i}\phi} \tag{3.12}$$

$$\frac{\partial}{\partial t}rn_{\mathrm{i}}u_{\mathrm{i}\theta} = -2\frac{\partial}{\partial s}ru_{\mathrm{ir}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} + \left(4\frac{\partial}{\partial s}s\mu_{\mathrm{i}}\frac{\partial}{\partial s}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - 4\frac{\partial}{\partial s}s\mu_{\mathrm{i}}ru_{\mathrm{i}\theta}\frac{\partial n_{\mathrm{i}}}{\partial s} - 4\frac{\partial}{\partial s}\mu_{\mathrm{i}}rn_{\mathrm{i}}u_{\mathrm{i}\theta}\right)$$

$$- Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}n_{\mathrm{i}}r\dot{A}_{\theta} - Z_{\mathrm{i}}\frac{e}{m_{\mathrm{i}}}B_{\phi}rn_{\mathrm{i}}u_{\mathrm{ir}} - \nu_{\mathrm{NCi}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\nu_{\mathrm{ei1}}rn_{\mathrm{e}}(u_{\mathrm{i}\theta} - u_{\mathrm{e}\theta})$$

$$+ 2\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}s\frac{\partial A_{\phi}}{\partial s}\frac{\nu_{\mathrm{ei2}}}{B_{\theta}}n_{\mathrm{e}}(u_{\mathrm{i}\phi} - u_{\mathrm{e}\phi}) - \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}}\nu_{\mathrm{bi}}rn_{\mathrm{b}}(u_{\mathrm{i}\theta} - u_{\mathrm{b}\theta})$$

$$+ \frac{e^2 B_{\phi}^2 D_{\mathrm{e}}}{m_{\mathrm{i}}T_{\mathrm{e}}}\left[rn_{\mathrm{e}}u_{\mathrm{e}\theta} + 2\frac{\partial A_{\phi}}{\partial s}\frac{s}{B_{\phi}}n_{\mathrm{e}}u_{\mathrm{e}\phi} + sn_{\mathrm{e}}\frac{B}{B_{\phi}}\left\langle\frac{\omega}{m}\right\rangle_{\mathrm{e}} + 2s\frac{1-\alpha_{\mathrm{e}}}{eB_{\phi}}\left(\frac{\partial}{\partial s}n_{\mathrm{e}}T_{\mathrm{e}} - T_{\mathrm{e}}\frac{\partial}{\partial s}n_{\mathrm{e}}\right)\right.$$

$$\left. - 2sn_{\mathrm{e}}\frac{1}{B_{\phi}}\frac{\partial}{\partial s}\phi\right] - 2\nu_{\mathrm{L}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \nu_{\mathrm{0i}}rn_{\mathrm{i}}u_{\mathrm{i}\theta} - \nu_{\mathrm{CX}}rn_{\mathrm{i}}u_{\mathrm{i}\theta}$$

$$+ rS_{\text{LC}\theta} - \nu_{\text{OL}} r n_{\text{i}} u_{\text{i}\theta} + F_{\text{i}\theta}^{\text{neo}} + T_{\text{i}\theta} \tag{3.13}$$

$$\frac{\partial}{\partial t} n_{\text{i}} u_{\text{i}\phi} = -2\frac{\partial}{\partial s} r u_{\text{i}r} n_{\text{i}} u_{\text{i}\phi} + \left(4\frac{\partial}{\partial s} s\mu_{\text{i}} \frac{\partial}{\partial s} n_{\text{i}} u_{\text{i}\phi} - 4\frac{\partial}{\partial s} s\mu_{\text{i}} u_{\text{i}\phi} \frac{\partial n_{\text{i}}}{\partial s} \right) - Z_{\text{i}} \frac{e}{m_{\text{i}}} n_{\text{i}} \dot{A}_{\phi} - 2Z_{\text{i}} \frac{e}{m_{\text{i}}} \frac{\partial A_{\phi}}{\partial s} r n_{\text{i}} u_{\text{i}r}$$

$$- \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei}3} n_{\text{e}} (u_{\text{i}\phi} - u_{\text{e}\phi}) + 2\frac{m_{\text{e}}}{m_{\text{i}}} \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{ei}2}}{B_{\theta}} r n_{\text{e}} (u_{\text{i}\theta} - u_{\text{e}\theta}) - \frac{m_{\text{b}}}{m_{\text{i}}} \nu_{\text{bi}} n_{\text{b}} (u_{\text{i}\phi} - u_{\text{b}\phi})$$

$$+ 2\frac{e^2 B_{\phi} D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} \frac{\partial A_{\phi}}{\partial s} \left[r n_{\text{e}} u_{\text{e}\theta} + s n_{\text{e}} \frac{B}{B_{\phi}} \left\langle \frac{\omega}{m} \right\rangle_{\text{e}} + 2s \frac{1 - \alpha_{\text{e}}}{e B_{\phi}} \left(\frac{\partial}{\partial s} n_{\text{e}} T_{\text{e}} - T_{\text{e}} \frac{\partial}{\partial s} n_{\text{e}} \right) \right.$$

$$\left. -2 s n_{\text{e}} \frac{1}{B_{\phi}} \frac{\partial}{\partial s} \phi \right] + \frac{e^2 B_{\phi}^2 D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} \left(\frac{B_{\theta}}{B_{\phi}} \right)^2 n_{\text{e}} u_{\text{e}\phi} - 2\nu_{\text{L}} n_{\text{i}} u_{\text{i}\phi} - \nu_{0\text{i}} n_{\text{i}} u_{\text{i}\phi} - \nu_{\text{CX}} n_{\text{i}} u_{\text{i}\phi}$$

$$+ S_{\text{LC}\phi} - \nu_{\text{OL}} n_{\text{i}} u_{\text{i}\phi} + T_{\phi,\text{i}} \tag{3.14}$$

$$\frac{\partial}{\partial t} \frac{3}{2} n_{\text{i}} T_{\text{i}} = -5\frac{\partial}{\partial s} r u_{\text{e}r} n_{\text{e}} T_{\text{e}} + \left(4\frac{\partial}{\partial s} s\chi_{\text{e}} \frac{\partial}{\partial s} n_{\text{i}} T_{\text{i}} - 4\frac{\partial}{\partial s} s\chi_{\text{i}} T_{\text{i}} \frac{\partial n_{\text{i}}}{\partial s} \right) + 2 r u_{\text{i}r} \frac{\partial}{\partial s} n_{\text{i}} T_{\text{i}}$$

$$+ Z_{\text{i}} e \frac{E_{\theta}}{r} r n_{\text{i}} u_{\text{i}\theta} + Z_{\text{i}} e E_{\phi} n_{\text{i}} u_{\text{i}\phi} - \frac{3}{2} \nu_{\text{Tei}} n_{\text{e}} (T_{\text{i}} - T_{\text{e}}) + m_{\text{b}} v_{\text{b}} \frac{B_{\theta} u_{\text{i}\theta} + B_{\phi} u_{\text{i}\phi}}{B} \frac{P_{\text{b}\parallel}}{E_{\text{b}}}$$

$$- \frac{\nu_{\text{L}}}{Z_{\text{i}}} T_{\text{i}} (n_{\text{e}} - n_{\text{ediv}}) - \frac{3}{2} \nu_{\text{LT}_{\text{i}}} n_{\text{i}} (T_{\text{i}} - T_{\text{idiv}})$$

$$+ \frac{3}{2} \frac{\nu_{\text{ionize}}}{Z_{\text{i}}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03}) - \frac{3}{2} \nu_{\text{CX}} n_{\text{i}} (T_{\text{i}} - T_{01})$$

$$+ (1 - f_{\text{ei}}) P_{\text{b}} + P_{\alpha\text{i}} + P_{\text{RFi}} \tag{3.15}$$

$$\frac{\partial}{\partial t} n_{\text{b}} = \frac{P_{\text{b}}}{E_{\text{b}}} - g\frac{P_{\text{b}\perp}}{E_{\text{b}}} - \nu_{\text{b}} n_{\text{b}} - \nu_{\text{bL}} n_{\text{b}} - \nu_{\text{eff}} g(\sqrt{\delta} n_{\text{b}} - n_{\text{b}}^{\text{rp}}) + 4\frac{\partial}{\partial s} s f_t D_{\text{b}} \frac{\partial}{\partial s} n_{\text{b}} \tag{3.16}$$

$$\frac{\partial}{\partial t} r n_{\text{b}} u_{\text{b}\theta} = -Z_{\text{b}} \frac{e}{m_{\text{b}}} n_{\text{b}} r \dot{A}_{\theta} - \nu_{\text{be}1} r n_{\text{b}} (u_{\text{b}\theta} - u_{\text{e}\theta}) + 2s \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{be}2}}{B_{\theta}} n_{\text{b}} (u_{\text{b}\phi} - u_{\text{e}\phi}) - \nu_{\text{bi}} r n_{\text{b}} (u_{\text{b}\theta} - u_{\text{i}\theta})$$

$$- \nu_{0\text{b}} r n_{\text{b}} u_{\text{b}\theta} - \nu_{\text{CX}} r n_{\text{b}} u_{\text{b}\theta} + r v_{\text{b}\theta} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}} r n_{\text{b}} u_{\text{b}\theta} - \nu_{\text{eff}} g\sqrt{\delta} r n_{\text{b}} u_{\text{b}\theta}$$

$$+ 4\frac{\partial}{\partial s} s f_t D_{\text{b}} \frac{\partial}{\partial s} r n_{\text{b}} u_{\text{b}\theta} - 4\frac{\partial}{\partial s} \left(s f_t D_{\text{b}} \frac{\partial r u_{\text{b}\theta}}{\partial s} \right) n_{\text{b}} \tag{3.17}$$

$$\frac{\partial}{\partial t} n_{\text{b}} u_{\text{b}\phi} = -Z_{\text{b}} \frac{e}{m_{\text{b}}} n_{\text{b}} \dot{A}_{\phi} - \nu_{\text{be}3} n_{\text{b}} (u_{\text{b}\phi} - u_{\text{e}\phi}) + 2\frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{be}2}}{B_{\theta}} n_{\text{b}} (u_{\text{b}\theta} - u_{\text{e}\theta}) - \nu_{\text{bi}} n_{\text{b}} (u_{\text{b}\phi} - u_{\text{i}\phi})$$

$$- \nu_{0\text{b}} n_{\text{b}} u_{\text{b}\phi} - \nu_{\text{CX}} n_{\text{b}} u_{\text{b}\phi} + v_{\text{b}\phi} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}} n_{\text{b}} u_{\text{b}\phi} - \nu_{\text{eff}} g\sqrt{\delta} n_{\text{b}} u_{\text{b}\phi}$$

$$+ 4\frac{\partial}{\partial s} s f_t D_{\text{b}} \frac{\partial}{\partial s} n_{\text{b}} u_{\text{b}\phi} - 4\frac{\partial}{\partial s} \left(s f_t D_{\text{b}} \frac{\partial u_{\text{b}\phi}}{\partial s} \right) n_{\text{b}} \tag{3.18}$$

$$\frac{\partial}{\partial t} n_{01} = 4\frac{\partial}{\partial s} s D_{01} \frac{\partial}{\partial s} n_{01} - \frac{1}{Z_{\text{i}}} \nu_{\text{ionize}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} n_{01} - \nu_{\text{CX}} \frac{n_{\text{i}}}{n_{01} + n_{02}} n_{01}$$

$$+ \gamma_0 \frac{\nu_{\mathrm{L}}}{Z_{\mathrm{i}}} \left(n_{\mathrm{e}} - n_{\mathrm{ediv}} \right) + 2 \frac{\partial}{\partial s} r \Gamma^{\mathrm{puff}} \Big|_{\mathrm{b}} \tag{3.19}$$

$$\frac{\partial}{\partial t} n_{02} = 4 \frac{\partial}{\partial s} s D_{02} \frac{\partial}{\partial s} n_{02} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{02} + \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01} \tag{3.20}$$

$$\frac{\partial}{\partial t} n_{03} = 4 \frac{\partial}{\partial s} s D_{03} \frac{\partial}{\partial s} n_{03} - \frac{1}{Z_{\mathrm{i}}} \nu_{\mathrm{ionize}} \frac{n_{\mathrm{e}}}{n_{01} + n_{02} + n_{03}} n_{03} + f_{\mathrm{CX}} \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} \tag{3.21}$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}}^{\mathrm{rp}} = \frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} + \nu_{\mathrm{eff}} (\sqrt{\delta} n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}}) - \nu_{\mathrm{b}} n_{\mathrm{b}}^{\mathrm{rp}} - 2 \frac{\partial}{\partial s} r u_{\mathrm{b}}^{\mathrm{rp}} n_{\mathrm{b}}^{\mathrm{rp}} + 4 \frac{\partial}{\partial s} s D_{\mathrm{b}}^{\mathrm{rp}} \frac{\partial}{\partial s} n_{\mathrm{b}}^{\mathrm{rp}} \tag{3.22}$$

4 Boundary Conditions

num.	name	variable	center	edge
1	LQm1	ϕ	N	0
2	LQm2	$r\dot{A}_\theta$	0	$2s_b B_{\mathrm{V}}$
3	LQm3	$\dot{A}_\phi$	N	$-2b B_{\theta\mathrm{b}}$
4	LQm4	A_ϕ	$\times$	$\times$
5	LQm5	rA_θ	$\times$	$\times$
6	LQe1	n_{e}	N	N($\times$)
7	LQe2	$rn_{\mathrm{e}}u_{\mathrm{e}r}$	0	$\times$(0)
8	LQe3	$rn_{\mathrm{e}}u_{\mathrm{e}\theta}$	0	0
9	LQe4	$n_{\mathrm{e}}u_{\mathrm{e}\phi}$	N	0
10	LQe5	$n_{\mathrm{e}}T_{\mathrm{e}}$	N	N
11	LQi1	n_{i}	N	N($\times$)
12	LQi2	$rn_{\mathrm{i}}u_{\mathrm{i}r}$	0	$\times$(0)
13	LQi3	$rn_{\mathrm{i}}u_{\mathrm{i}\theta}$	0	0
14	LQi4	$n_{\mathrm{i}}u_{\mathrm{i}\phi}$	N	0
15	LQi5	$n_{\mathrm{i}}T_{\mathrm{i}}$	N	N
16	LQb1	n_{b}	$\times$(N)	$\times$(N)
17	LQb3	$rn_{\mathrm{b}}u_{\mathrm{b}\theta}$	$\times$(0)	$\times$(0)
18	LQb4	$n_{\mathrm{b}}u_{\mathrm{b}\phi}$	$\times$(N)	$\times$(0)
19	LQn1	n_{01}	N	$0(2bS_{\mathrm{gas}})$
20	LQn2	n_{02}	N	0
20	LQn3	n_{03}	N	0
21	LQr1	$n_{\mathrm{b}}^{\mathrm{rp}}$	$\times$(N)	$\times$(N)

0: Dirichlet condition ($u = 0$)

N: Neumann condition ($u' = 0$)

value: Neumann condition ($u' = $ value)

$\times$: no condition

Excess BCs are imposed on the equations LQe1 and LQi1, from the aspect of the order of their derivative. However, these BCs must be required to keep quasi-neutrality.