1 Equations in ρ coordinates

$$0 = \frac{1}{V'}\frac{\partial}{\partial\rho}\left[V'\langle|\nabla\rho|^2\rangle\frac{\partial}{\partial\rho}\Phi\right] + \frac{1}{\epsilon_0}\left(-en_\mathrm{e} + Z_\mathrm{i}en_\mathrm{i} + Z_\mathrm{b}en_\mathrm{b} + Z_\mathrm{b}egn_\mathrm{b}^\mathrm{rp}\right) \tag{1.1}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot\psi_t = V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial}{\partial\rho}\left[\frac{1}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial\rho}\psi_t\right]$$

$$+\frac{\mu_0}{\langle B^\theta\rangle}\left[-en_\mathrm{e}\left(\langle Bu_\mathrm{e\|}\rangle - I\left\langle\frac{u_\mathrm{e\zeta}}{R}\right\rangle\right) + Z_\mathrm{i}en_\mathrm{i}\left(\langle Bu_\mathrm{i\|}\rangle - I\left\langle\frac{u_\mathrm{i\zeta}}{R}\right\rangle\right)\right.$$

$$\left.+Z_\mathrm{b}en_\mathrm{b}\left(\langle Bu_\mathrm{b\|}\rangle - I\left\langle\frac{u_\mathrm{b\zeta}}{R}\right\rangle\right)\right] \tag{1.2}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot\psi = \frac{1}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial\rho}\left[V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial}{\partial\rho}\psi\right]$$

$$-\frac{\mu_0}{\langle R^{-2}\rangle}\left(-en_\mathrm{e}\left\langle\frac{u_\mathrm{e\zeta}}{R}\right\rangle + Z_\mathrm{i}en_\mathrm{i}\left\langle\frac{u_\mathrm{i\zeta}}{R}\right\rangle + Z_\mathrm{b}en_\mathrm{b}\left\langle\frac{u_\mathrm{b\zeta}}{R}\right\rangle\right) \tag{1.3}$$

$$\frac{\partial}{\partial t}\psi = \dot\psi \tag{1.4}$$

$$\frac{\partial}{\partial t}\psi_t = \dot\psi_t \tag{1.5}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_\mathrm{e}) = -\frac{1}{V'}\frac{\partial}{\partial\rho}[V'n_\mathrm{e}(u_\mathrm{e}^\rho - u_g^\rho)] + \nu_\mathrm{ionize}\frac{n_\mathrm{e}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03})$$

$$-\nu_\mathrm{L}(n_\mathrm{e}-n_\mathrm{ediv}) + (1-f_\mathrm{CX})\frac{P_\mathrm{b}}{E_\mathrm{b}} \tag{1.6}$$

$$0 = -(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial\rho}n_\mathrm{e}T_\mathrm{e} + en_\mathrm{e}(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial\rho}\Phi$$

$$-e\frac{\partial\psi}{\partial\rho}In_\mathrm{e}\langle Bu_\mathrm{e\|}\rangle + e\frac{\partial\psi}{\partial\rho}\langle B^2\rangle n_\mathrm{e}\langle Ru_\mathrm{e\zeta}\rangle \tag{1.7}$$

$$m_\mathrm{e}n_\mathrm{e}\frac{\partial}{\partial t}\langle Bu_\mathrm{e\|}\rangle = -\hat\mu_1^\mathrm{e}\langle Bu_\mathrm{e\|}\rangle - \hat\mu_2^\mathrm{e}\langle B\hat q_\mathrm{e\|}\rangle + \hat\mu_1^\mathrm{e}BV_\mathrm{1e} + \hat\mu_2^\mathrm{e}BV_\mathrm{2e}$$

$$+\ell_{11}^\mathrm{ee}\langle Bu_\mathrm{e\|}\rangle + \ell_{11}^\mathrm{ei}\langle Bu_\mathrm{i\|}\rangle + \ell_{11}^\mathrm{eb}\langle Bu_\mathrm{b\|}\rangle - \ell_{12}^\mathrm{ee}\langle B\hat q_\mathrm{e\|}\rangle - \ell_{12}^\mathrm{ei}\langle B\hat q_\mathrm{i\|}\rangle$$

$$-en_\mathrm{e}\langle BE_\|\rangle - \nu_\mathrm{L}m_\mathrm{e}n_\mathrm{e}\langle Bu_\mathrm{e\|}\rangle - \nu_\mathrm{0e}m_\mathrm{e}n_\mathrm{e}\langle Bu_\mathrm{e\|}\rangle$$

$$-\frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|\rangle v_\mathrm{e\zeta}n_\mathrm{e}\langle Bu_\mathrm{e\|}\rangle + \langle|\nabla\rho|\rangle v_\mathrm{e\zeta}\frac{\partial}{\partial\rho}\left(\frac{\langle B^2\rangle}{I}\right)n_\mathrm{e}\langle Bu_\mathrm{e\|}\rangle$$

$$+\frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|^2\rangle\chi_{\mathrm{e}\zeta}n_{\mathrm{e}}\frac{\partial}{\partial\rho}\langle Bu_{\mathrm{e}\|}\rangle-\frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|^2\rangle\chi_{\mathrm{e}\zeta}n_{\mathrm{e}}\frac{\partial}{\partial\rho}\ln\!\left(\frac{\langle B^2\rangle}{I}\right)\langle Bu_{\mathrm{e}\|}\rangle$$

$$-\langle|\nabla\rho|^2\rangle\chi_{\mathrm{e}\zeta}n_{\mathrm{e}}\frac{\partial}{\partial\rho}\ln\!\left(\frac{\langle B^2\rangle}{I}\right)\frac{\partial}{\partial\rho}\langle Bu_{\mathrm{e}\|}\rangle+\langle|\nabla\rho|^2\rangle\chi_{\mathrm{e}\zeta}n_{\mathrm{e}}\left[\frac{\partial}{\partial\rho}\ln\!\left(\frac{\langle B^2\rangle}{I}\right)\right]^2\langle Bu_{\mathrm{e}\|}\rangle \tag{1.8}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'm_{\mathrm{e}}n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle)=-\frac{1}{V'}\frac{\partial}{\partial\rho}V'\left[\langle|\nabla\rho|\rangle v_{\mathrm{e}\zeta}m_{\mathrm{e}}n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle+(u_{\mathrm{e}}^{\rho}-u_{g}^{\rho})m_{\mathrm{e}}n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle\right.$$

$$\left.-\langle|\nabla\rho|^2\rangle\chi_{\mathrm{e}\zeta}m_{\mathrm{e}}n_{\mathrm{e}}\frac{\partial}{\partial\rho}\langle Ru_{\mathrm{e}\zeta}\rangle+\langle\Pi_{\mathrm{e}}^{\mathrm{res}}\rangle\right]$$

$$+\ell_{11}^{\mathrm{ee}}\langle Ru_{\mathrm{e}\zeta}\rangle+\ell_{11}^{\mathrm{ei}}\langle Ru_{\mathrm{i}\zeta}\rangle+\ell_{11}^{\mathrm{eb}}\langle Ru_{\mathrm{b}\zeta}\rangle-\ell_{12}^{\mathrm{ee}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{\mathrm{e}\|}\rangle-\ell_{12}^{\mathrm{ei}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{\mathrm{i}\|}\rangle$$

$$-en_{\mathrm{e}}\langle RE_{\zeta}\rangle-e\frac{\partial\psi}{\partial\rho}n_{\mathrm{e}}u_{\mathrm{e}}^{\rho}$$

$$-\frac{e^2\langle|\nabla\rho|^2\rangle D_{\mathrm{e}}}{T_{\mathrm{e}}}\frac{\partial\psi}{\partial\rho}n_{\mathrm{e}}\left[\frac{\partial\psi}{\partial\rho}\frac{\langle B^2\rangle}{\langle B^2\rangle\langle R^2\rangle-I^2}\langle Ru_{\mathrm{e}\zeta}\rangle-\frac{\partial\psi}{\partial\rho}\frac{I}{\langle B^2\rangle\langle R^2\rangle-I^2}\langle Bu_{\mathrm{e}\|}\rangle\right.$$

$$\left.+\frac{C_{\mathrm{T}}-1}{e}\frac{\partial}{\partial\rho}T_{\mathrm{e}}+\frac{\partial}{\partial\rho}\Phi+\frac{T_{\mathrm{e}}}{e}\frac{\partial\psi}{\partial\rho}C_{\mathrm{p}}\right]-\nu_{\mathrm{L}}m_{\mathrm{e}}n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle-\nu_{0\mathrm{e}}m_{\mathrm{e}}n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle \tag{1.9}$$

$$\frac{3}{2}\frac{1}{V'^{5/3}}\frac{\partial}{\partial t}(V'^{5/3}n_{\mathrm{e}}T_{\mathrm{e}})=-\frac{1}{V'}\frac{\partial}{\partial\rho}V'\left[-\chi_{\mathrm{e}}\langle|\nabla\rho|^2\rangle n_{\mathrm{e}}\frac{\partial}{\partial\rho}T_{\mathrm{e}}+V_{\mathrm{e}}^{\mathrm{hp}}\langle|\nabla\rho|\rangle n_{\mathrm{e}}T_{\mathrm{e}}+\frac{5}{2}n_{\mathrm{e}}T_{\mathrm{e}}(u_{\mathrm{e}}^{\rho}-u_{g}^{\rho})\right]$$

$$-u_{g}^{\rho}\frac{\partial}{\partial\rho}n_{\mathrm{e}}T_{\mathrm{e}}-u_{\mathrm{i}}^{\rho}\frac{\partial}{\partial\rho}n_{\mathrm{i}}T_{\mathrm{i}}-\hat{u}_{\mathrm{i}\theta}\langle\boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\pi}_{\mathrm{i}}\rangle-\frac{3}{2}n_{\mathrm{i}}\frac{T_{\mathrm{e}}-T_{\mathrm{i}}}{\tau_{\mathrm{ie}}}$$

$$-\dot{\psi}_{t}\langle B^{\theta}\rangle(-en_{\mathrm{e}}\hat{u}_{\mathrm{e}\theta}+Z_{\mathrm{i}}en_{\mathrm{i}}\hat{u}_{\mathrm{i}\theta})+\dot{\psi}\left(-en_{\mathrm{e}}\left\langle\frac{u_{\mathrm{e}\zeta}}{R}\right\rangle+Z_{\mathrm{i}}en_{\mathrm{i}}\left\langle\frac{u_{\mathrm{i}\zeta}}{R}\right\rangle\right)$$

$$-\nu_{\mathrm{L}}T_{\mathrm{e}}(n_{\mathrm{e}}-n_{\mathrm{ediv}})-\frac{3}{2}\nu_{\mathrm{LT}_{\mathrm{e}}}n_{\mathrm{e}}(T_{\mathrm{e}}-T_{\mathrm{ediv}})$$

$$-E_{\mathrm{H}}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{e}}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03})$$

$$+f_{\mathrm{ei}}P_{\mathrm{b}}+P_{\alpha\mathrm{e}}+P_{\mathrm{RFe}}-P_{\mathrm{Br}} \tag{1.10}$$

$$\frac{5}{2}m_{\mathrm{e}}n_{\mathrm{e}}\frac{\partial}{\partial t}\langle B\hat{q}_{\mathrm{e}\|}\rangle=-\hat{\mu}_{2}^{\mathrm{e}}\langle Bu_{\mathrm{e}\|}\rangle-\hat{\mu}_{3}^{\mathrm{e}}\langle B\hat{q}_{\mathrm{e}\|}\rangle+\hat{\mu}_{2}^{\mathrm{e}}BV_{1\mathrm{e}}+\hat{\mu}_{3}^{\mathrm{e}}BV_{2\mathrm{e}}$$

$$-\ell_{21}^{\mathrm{ee}}\langle Bu_{\mathrm{e}\|}\rangle-\ell_{21}^{\mathrm{ei}}\langle Bu_{\mathrm{i}\|}\rangle-\ell_{21}^{\mathrm{eb}}\langle Bu_{\mathrm{b}\|}\rangle+\ell_{22}^{\mathrm{ee}}\langle B\hat{q}_{\mathrm{e}\|}\rangle+\ell_{22}^{\mathrm{ei}}\langle B\hat{q}_{\mathrm{i}\|}\rangle$$

$$-\nu_{\mathrm{L}}m_{\mathrm{e}}n_{\mathrm{e}}\langle B\hat{q}_{\mathrm{e}\|}\rangle-\nu_{0\mathrm{e}}m_{\mathrm{e}}n_{\mathrm{e}}\langle B\hat{q}_{\mathrm{e}\|}\rangle \tag{1.11}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{\mathrm{i}})=-\frac{1}{V'}\frac{\partial}{\partial\rho}[V'n_{\mathrm{i}}(u_{\mathrm{i}}^{\rho}-u_{g}^{\rho})]+\nu_{\mathrm{ionize}}\frac{n_{\mathrm{i}}}{n_{01}+n_{02}+n_{03}}(n_{01}+n_{02}+n_{03})$$

$$-\nu_{\mathrm{L}}(n_{\mathrm{i}}-n_{\mathrm{idiv}})+\nu_{\mathrm{b}}n_{\mathrm{b}}+\nu_{\mathrm{b}}gn_{\mathrm{b}}^{\mathrm{rp}}-f_{\mathrm{CX}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}}+\nu_{\mathrm{bL}}n_{\mathrm{b}}+S_{\mathrm{LC}}-\nu_{\mathrm{OL}}n_{\mathrm{i}} \tag{1.12}$$

2

$$0 = -(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial\rho}n_\mathrm{i}T_\mathrm{i} - Z_\mathrm{i}en_\mathrm{i}(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial\rho}\Phi$$

$$+ Z_\mathrm{i}en_\mathrm{i}\frac{\partial\psi}{\partial\rho}I\langle Bu_{\mathrm{i}\|}\rangle - Z_\mathrm{i}en_\mathrm{i}\frac{\partial\psi}{\partial\rho}\langle B^2\rangle\langle Ru_{\mathrm{i}\zeta}\rangle \tag{1.13}$$

$$m_\mathrm{i}n_\mathrm{i}\frac{\partial}{\partial t}\langle Bu_{\mathrm{i}\|}\rangle = -\hat{\mu}_1^\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle - \hat{\mu}_2^\mathrm{i}\langle Bq_{\mathrm{i}\|}\rangle + \hat{\mu}_1^\mathrm{i}BV_{1\mathrm{i}} + \hat{\mu}_2^\mathrm{i}BV_{2\mathrm{i}}$$

$$+ \ell_{11}^{\mathrm{ie}}\langle Bu_{\mathrm{e}\|}\rangle + \ell_{11}^{\mathrm{ii}}\langle Bu_{\mathrm{i}\|}\rangle + \ell_{11}^{\mathrm{ib}}\langle Bu_{\mathrm{b}\|}\rangle - \ell_{12}^{\mathrm{ie}}\langle B\hat{q}_{\mathrm{e}\|}\rangle - \ell_{12}^{\mathrm{ii}}\langle B\hat{q}_{\mathrm{i}\|}\rangle$$

$$+ Z_\mathrm{i}en_\mathrm{i}\langle BE_\|\rangle - \nu_\mathrm{L}m_\mathrm{i}n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle + \nu_\mathrm{b}m_\mathrm{b}n_\mathrm{b}\langle Bu_{\mathrm{b}\|}\rangle - \nu_{0\mathrm{i}}m_\mathrm{i}n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle$$

$$- \nu_\mathrm{CX}m_\mathrm{i}n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle - \nu_\mathrm{OL}m_\mathrm{i}n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle$$

$$- \frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|\rangle v_{\mathrm{i}\zeta}n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle + \langle|\nabla\rho|\rangle v_{\mathrm{i}\zeta}\frac{\partial}{\partial\rho}\left(\frac{\langle B^2\rangle}{I}\right)n_\mathrm{i}\langle Bu_{\mathrm{i}\|}\rangle$$

$$+ \frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|^2\rangle\chi_{\mathrm{i}\zeta}n_\mathrm{i}\frac{\partial}{\partial\rho}\langle Bu_{\mathrm{i}\|}\rangle - \frac{1}{V'}\frac{\partial}{\partial\rho}V'\langle|\nabla\rho|^2\rangle\chi_{\mathrm{i}\zeta}n_\mathrm{i}\frac{\partial}{\partial\rho}\ln\left(\frac{\langle B^2\rangle}{I}\right)\langle Bu_{\mathrm{i}\|}\rangle$$

$$- \langle|\nabla\rho|^2\rangle\chi_{\mathrm{i}\zeta}n_\mathrm{i}\frac{\partial}{\partial\rho}\ln\left(\frac{\langle B^2\rangle}{I}\right)\frac{\partial}{\partial\rho}\langle Bu_{\mathrm{i}\|}\rangle + \langle|\nabla\rho|^2\rangle\chi_{\mathrm{i}\zeta}n_\mathrm{i}\left[\frac{\partial}{\partial\rho}\ln\left(\frac{\langle B^2\rangle}{I}\right)\right]^2\langle Bu_{\mathrm{i}\|}\rangle \tag{1.14}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'm_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle) = -\frac{1}{V'}\frac{\partial}{\partial\rho}V'\left[\langle|\nabla\rho|\rangle v_{\mathrm{i}\zeta}m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle + (u_\mathrm{i}^\rho - u_g^\rho)m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle\right.$$

$$\left. - \langle|\nabla\rho|^2\rangle\chi_{\mathrm{i}\zeta}m_\mathrm{i}n_\mathrm{i}\frac{\partial}{\partial\rho}\langle Ru_{\mathrm{i}\zeta}\rangle + \langle\Pi_\mathrm{i}^\mathrm{res}\rangle\right]$$

$$+ \ell_{11}^{\mathrm{ie}}\langle Ru_{\mathrm{e}\zeta}\rangle + \ell_{11}^{\mathrm{ii}}\langle Ru_{\mathrm{i}\zeta}\rangle + \ell_{11}^{\mathrm{ib}}\langle Ru_{\mathrm{b}\zeta}\rangle - \ell_{12}^{\mathrm{ie}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{\mathrm{e}\|}\rangle - \ell_{12}^{\mathrm{ii}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{\mathrm{i}\|}\rangle$$

$$+ Z_\mathrm{i}en_\mathrm{i}\langle RE_\zeta\rangle + Z_\mathrm{i}e\frac{\partial\psi}{\partial\rho}n_\mathrm{i}u_\mathrm{i}^\rho$$

$$+ Z_\mathrm{i}\frac{e^2\langle|\nabla\rho|^2\rangle D_\mathrm{e}}{T_\mathrm{e}}\frac{\partial\psi}{\partial\rho}n_\mathrm{i}\left[\frac{\partial\psi}{\partial\rho}\frac{\langle B^2\rangle}{\langle B^2\rangle\langle R^2\rangle - I^2}\langle Ru_{\mathrm{e}\zeta}\rangle - \frac{\partial\psi}{\partial\rho}\frac{I}{\langle B^2\rangle\langle R^2\rangle - I^2}\langle Bu_{\mathrm{e}\|}\rangle\right.$$

$$\left. + \frac{C_\mathrm{T} - 1}{e}\frac{\partial}{\partial\rho}T_\mathrm{e} + \frac{\partial}{\partial\rho}\Phi + \frac{T_\mathrm{e}}{e}\frac{\partial\psi}{\partial\rho}C_\mathrm{p}\right]$$

$$- \nu_\mathrm{L}m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle + \nu_\mathrm{b}m_\mathrm{b}n_\mathrm{b}\langle Ru_{\mathrm{b}\zeta}\rangle - \nu_{0\mathrm{i}}m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle - \nu_\mathrm{CX}m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle$$

$$- \nu_\mathrm{OL}m_\mathrm{i}n_\mathrm{i}\langle Ru_{\mathrm{i}\zeta}\rangle \tag{1.15}$$

$$\frac{3}{2}\frac{1}{V'^{5/3}}\frac{\partial}{\partial t}(V'^{5/3}n_\mathrm{i}T_\mathrm{i}) = -\frac{1}{V'}\frac{\partial}{\partial\rho}V'\left[-\chi_\mathrm{i}\langle|\nabla\rho|^2\rangle n_\mathrm{i}\frac{\partial}{\partial\rho}T_\mathrm{i} + V_\mathrm{i}^\mathrm{hp}\langle|\nabla\rho|\rangle n_\mathrm{i}T_\mathrm{i} + \frac{5}{2}n_\mathrm{i}T_\mathrm{i}(u_\mathrm{i}^\rho - u_g^\rho)\right]$$

$$+ (u_\mathrm{i}^\rho - u_g^\rho)\frac{\partial}{\partial\rho}n_\mathrm{i}T_\mathrm{i} + \hat{u}_{\mathrm{i}\theta}\langle\boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\pi}_\mathrm{i}\rangle + \frac{3}{2}n_\mathrm{i}\frac{T_\mathrm{e} - T_\mathrm{i}}{\tau_\mathrm{ie}}$$

$$- \nu_\mathrm{L}T_\mathrm{i}(n_\mathrm{i} - n_\mathrm{idiv}) - \frac{3}{2}\nu_{\mathrm{L}T_\mathrm{i}}n_\mathrm{i}(T_\mathrm{i} - T_\mathrm{idiv})$$

$$+ \frac{3}{2}\nu_{\text{ionize}} \frac{n_{\text{i}}}{n_{01} + n_{02} + n_{03}} (n_{01}T_{01} + n_{02}T_{02} + n_{03}T_{03})$$

$$- \frac{3}{2}\nu_{\text{CX}}n_{\text{i}}(T_{\text{i}} - T_{01}) + (1 - f_{\text{ei}})P_{\text{b}} + m_{\text{b}}\frac{P_{\text{b}}}{E_{\text{b}}}\langle Bu_{\text{b}\|}\rangle\frac{\langle Bu_{\text{i}\|}\rangle}{\langle B^2\rangle} + P_{\alpha\text{i}} + P_{\text{RFi}} \tag{1.16}$$

$$\frac{5}{2}m_{\text{i}}n_{\text{i}}\frac{\partial}{\partial t}\langle B\hat{q}_{\text{i}\|}\rangle = -\hat{\mu}_2^{\text{i}}\langle Bu_{\text{i}\|}\rangle - \hat{\mu}_3^{\text{i}}\langle B\hat{q}_{\text{i}\|}\rangle + \hat{\mu}_2^{\text{i}}BV_{1\text{i}} + \hat{\mu}_3^{\text{i}}BV_{2\text{i}}$$

$$- \ell_{21}^{\text{ie}}\langle Bu_{\text{e}\|}\rangle - \ell_{21}^{\text{ii}}\langle Bu_{\text{i}\|}\rangle - \ell_{21}^{\text{ib}}\langle Bu_{\text{b}\|}\rangle + \ell_{22}^{\text{ie}}\langle B\hat{q}_{\text{e}\|}\rangle + \ell_{22}^{\text{ii}}\langle B\hat{q}_{\text{i}\|}\rangle$$

$$- \nu_{\text{L}}m_{\text{i}}n_{\text{i}}\langle B\hat{q}_{\text{i}\|}\rangle - \nu_{0\text{i}}m_{\text{i}}n_{\text{i}}\langle B\hat{q}_{\text{i}\|}\rangle \tag{1.17}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{\text{b}}) = -\frac{1}{V'}\frac{\partial}{\partial \rho}[V'n_{\text{b}}(u_{\text{b}}^{\rho} - u_g^{\rho})] + \frac{P_{\text{b}}}{E_{\text{b}}} - g\frac{P_{\text{b}\perp}}{E_{\text{b}}} - \nu_{\text{b}}n_{\text{b}} - \nu_{\text{bL}}n_{\text{b}} - \nu_{\text{eff}}g(\sqrt{\delta}n_{\text{b}} - n_{\text{b}}^{\text{rp}})$$

$$+ \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle|\nabla\rho|^2\rangle f_t D_{\text{b}}\frac{\partial}{\partial \rho}n_{\text{b}} \tag{1.18}$$

$$0 = -(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial \rho}n_{\text{b}}T_{\text{b}} - Z_{\text{b}}en_{\text{b}}(\langle B^2\rangle\langle R^2\rangle - I^2)\frac{\partial}{\partial \rho}\Phi$$

$$+ Z_{\text{b}}en_{\text{b}}\frac{\partial \psi}{\partial \rho}I\langle Bu_{\text{b}\|}\rangle - Z_{\text{b}}en_{\text{b}}\frac{\partial \psi}{\partial \rho}\langle B^2\rangle\langle Ru_{\text{b}\zeta}\rangle \tag{1.19}$$

$$m_{\text{b}}\frac{\partial}{\partial t}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle = -(\hat{\mu}_1^{\text{b}} - \ell_{11}^{\text{bb}})\langle Bu_{\text{b}\|}\rangle + \sum_{a\neq\text{b}}\ell_{11}^{\text{ba}}\langle Bu_{a\|}\rangle + \hat{\mu}_1^{\text{b}}BV_{1\text{b}} + Z_{\text{b}}en_{\text{b}}\langle BE_{\|}^A\rangle$$

$$- \nu_{\text{bL}}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle - \nu_{\text{b}}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle - \nu_{0\text{b}}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle - \nu_{\text{CX}}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle + \langle BS_{\text{m}\|\text{b}}\rangle$$

$$- \nu_{\text{eff}}g\sqrt{\delta}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\|}\rangle + \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle|\nabla\rho|^2\rangle m_{\text{b}}\langle Bu_{\text{b}\|}\rangle f_t D_{\text{b}}\frac{\partial}{\partial \rho}n_{\text{b}} \tag{1.20}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'm_{\text{b}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle) = -\frac{1}{V'}\frac{\partial}{\partial \rho}[V'(u_{\text{b}}^{\rho} - u_g^{\rho})m_{\text{b}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle]$$

$$+ \ell_{11}^{\text{be}}\langle Ru_{\text{e}\zeta}\rangle + \ell_{11}^{\text{bi}}\langle Ru_{\text{i}\zeta}\rangle + \ell_{11}^{\text{bb}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle$$

$$+ Z_{\text{b}}en_{\text{b}}\dot{\psi} + Z_{\text{b}}e\frac{\partial \psi}{\partial \rho}n_{\text{b}}u_{\text{b}}^{\rho} - \nu_{\text{bL}}m_{\text{b}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle - \nu_{\text{b}}m_{\text{b}}n_{\text{b}}\langle Bu_{\text{b}\zeta}\rangle$$

$$- \nu_{0\text{i}}m_{\text{b}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle - \nu_{\text{CX}}m_{\text{b}}n_{\text{b}}\langle Ru_{\text{b}\zeta}\rangle + \frac{I}{\langle B^2\rangle}\langle BS_{\text{m}\|\text{b}}\rangle \tag{1.21}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{01}) = \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle|\nabla\rho|^2\rangle D_{01}\frac{\partial}{\partial \rho}n_{01} - \nu_{\text{ionize}}\frac{n_{\text{i}}}{n_{01} + n_{02} + n_{03}}n_{01}$$

$$- \nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01} + n_{02}}n_{01} + \gamma_0\nu_{\text{L}}(n_{\text{i}} - n_{\text{idiv}}) + \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle\mathbf{\Gamma}^{\text{puff}}\cdot\nabla\rho\rangle\Big|_{\text{b}} \tag{1.22}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{02}) = \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle|\nabla\rho|^2\rangle D_{02}\frac{\partial}{\partial \rho}n_{02} - \nu_{\text{ionize}}\frac{n_{\text{i}}}{n_{01} + n_{02} + n_{03}}n_{02} + \nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01} + n_{02}}n_{01} \tag{1.23}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{03}) = \frac{1}{V'}\frac{\partial}{\partial \rho}V'\langle|\nabla\rho|^2\rangle D_{03}\frac{\partial}{\partial \rho}n_{03} - \nu_{\text{ionize}}\frac{n_{\text{i}}}{n_{01}+n_{02}+n_{03}}n_{03} + f_{\text{CX}}\frac{P_{\text{b}}}{E_{\text{b}}} \tag{1.24}$$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_{\text{b}}^{\text{rp}}) = \frac{P_{\text{b}\perp}}{E_{\text{b}}} + \nu_{\text{eff}}(\sqrt{\delta}n_{\text{b}} - n_{\text{b}}^{\text{rp}}) - \nu_{\text{b}}n_{\text{b}}^{\text{rp}} - \frac{1}{V'}\frac{\partial}{\partial \rho}V'\left(\langle|\nabla\rho|\rangle u_{\text{b}}^{\text{rp}}n_{\text{b}}^{\text{rp}} - \langle|\nabla\rho|^2\rangle D_{\text{b}}^{\text{rp}}\frac{\partial}{\partial \rho}n_{\text{b}}^{\text{rp}}\right) \tag{1.25}$$

2 Additional important relationship

$$B_p = \frac{|\nabla\rho|}{R}\frac{\partial\psi}{\partial\rho}, \quad \langle B_p^2\rangle = \left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\left(\frac{\partial\psi}{\partial V}\right)^2 \tag{2.1}$$

$$B_t = \frac{I}{R} = \frac{1}{R}\frac{4\pi^2}{\langle R^{-2}\rangle}\frac{\partial\psi_t}{\partial V}, \quad \langle B_t^2\rangle = \frac{16\pi^4}{\langle R^{-2}\rangle}\left(\frac{\partial\psi_t}{\partial V}\right)^2 \tag{2.2}$$

$$\langle B^2\rangle = \langle B_t^2\rangle + \langle B_p^2\rangle = \frac{16\pi^4}{\langle R^{-2}\rangle}\left(\frac{\partial\psi_t}{\partial V}\right)^2 + \left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\left(\frac{\partial\psi}{\partial V}\right)^2 \tag{2.3}$$

$$\langle B^\theta\rangle = \frac{4\pi^2}{V'}\frac{\partial\psi}{\partial\rho} = 4\pi^2\frac{\partial\psi}{\partial V} = \frac{I\langle R^{-2}\rangle}{q} = 2\pi\left(\oint\frac{dl_p}{B_p}\right)^{-1} \tag{2.4}$$

$$\hat{q}^2 \equiv \frac{I^2}{2\langle B_p^2\rangle}\left(\left\langle\frac{1}{R^2}\right\rangle - \frac{1}{\langle R^2\rangle}\right) \tag{2.5}$$

$$\langle B^2\rangle\langle R^2\rangle - I^2 = \langle R^2\rangle\left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\left(\frac{\partial\psi}{\partial V}\right)^2(1+2\hat{q}^2) \tag{2.6}$$

$$\langle B\hat{q}_{s\|}\rangle \equiv \frac{2\langle Bq_{s\|}\rangle}{5p_s} \tag{2.7}$$

$$\hat{u}_{s\theta} = \frac{\langle Bu_{s\|}\rangle - BV_{1s}}{\langle B^2\rangle} \tag{2.8}$$

$$\left\langle\frac{u_{s\zeta}}{R}\right\rangle = \frac{1}{1+2\hat{q}^2}\frac{1}{\langle R^2\rangle}\langle Ru_{s\zeta}\rangle + \frac{2\hat{q}^2}{1+2\hat{q}^2}\frac{1}{I}\langle Bu_{s\|}\rangle \tag{2.9}$$

$$= \frac{\langle Ru_{s\zeta}\rangle}{\langle R^2\rangle} + \frac{I}{\langle B^2\rangle}\left(\left\langle\frac{1}{R^2}\right\rangle - \frac{1}{\langle R^2\rangle}\right)(\langle Bu_{s\|}\rangle - BV_{1s})$$

$$= \frac{\langle Ru_{s\zeta}\rangle}{\langle R^2\rangle} + I\left(\left\langle\frac{1}{R^2}\right\rangle - \frac{1}{\langle R^2\rangle}\right)\hat{u}_{s\theta}$$

$$BV_{1s} = -I\left(\frac{1}{e_s n_s}\frac{\partial n_s T_s}{\partial\psi} + \frac{\partial\Phi}{\partial\psi}\right), \quad BV_{2s} = -\frac{I}{e_s}\frac{\partial T_s}{\partial\psi} \tag{2.10}$$

$$u^\rho \equiv \langle\boldsymbol{u}\cdot\nabla\rho\rangle, \quad u^V = \langle\boldsymbol{u}\cdot\nabla V\rangle \tag{2.11}$$

$$u_g^\rho = -\frac{4\pi^2}{IV'\langle R^{-2}\rangle}\dot\psi_t, \quad u_g^V = -\frac{4\pi^2}{I\langle R^{-2}\rangle}\dot\psi_t = -\frac{V'}{2\rho\psi_{ta}}\dot\psi_t \tag{2.12}$$

$$\frac{1}{V'}\frac{\partial V'}{\partial t} = \frac{\partial}{\partial V}\langle \boldsymbol{u}_g \cdot \nabla V\rangle = \frac{\partial u_g^V}{\partial V} \tag{2.13}$$

$$\langle E_r\rangle = -\langle|\nabla\rho|^2\rangle\frac{\partial r}{\partial\rho}\frac{\partial}{\partial\rho}\Phi \tag{2.14}$$

$$\langle BE_\parallel\rangle = I\langle R^{-2}\rangle\left(-\frac{\dot\psi_t}{q} + \dot\psi\right) \tag{2.15}$$

$$RE_\zeta = \dot\psi = \frac{V_{\text{loop}}}{2\pi} \tag{2.16}$$

$$\langle \boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\pi}_{\text{i}}\rangle = \hat\mu_1^{\text{i}}(\langle Bu_{\text{i}\parallel}\rangle - BV_{1\text{i}}) + \hat\mu_2^{\text{i}}\left(\frac{2\langle Bq_{\text{i}\parallel}\rangle}{5p_{\text{i}}} - BV_{2\text{i}}\right) \tag{2.17}$$

$$\hat\ell_{ij}^{ab} \equiv \frac{\ell_{ij}^{ab}}{m_a n_a}, \quad \hat\ell_{ij}^{\text{bb}} \equiv \frac{\ell_{ij}^{\text{bb}}}{m_{\text{e}} n_{\text{e}}} \tag{2.18}$$

$$\Gamma^{\text{puff}} \simeq \frac{\langle \boldsymbol{\Gamma}^{\text{puff}}\cdot\nabla\rho\rangle}{\langle|\nabla\rho|\rangle} \tag{2.19}$$

3   Equations in the coordinates suitable for implementation

$$0 = \frac{\partial}{\partial V}\left[\langle|\nabla V|^2\rangle\frac{\partial}{\partial V}\Phi\right] + \frac{1}{\epsilon_0}\left(-en_{\text{e}} + Z_{\text{i}}en_{\text{i}} + Z_{\text{b}}en_{\text{b}} + Z_{\text{b}}egn_{\text{b}}^{\text{rp}}\right) \tag{3.1}$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\dot\psi_t = \frac{\partial}{\partial V}\left[\left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\frac{1}{\langle R^{-2}\rangle}\frac{\partial}{\partial V}\psi_t\right] - \frac{1}{\langle R^{-2}\rangle}\frac{\partial}{\partial V}\left[\left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\right]\frac{\partial}{\partial V}\psi_t$$

$$+ \frac{\mu_0}{\langle B^\theta\rangle}\left[-e\left(n_{\text{e}}\langle Bu_{\text{e}\parallel}\rangle - In_{\text{e}}\left\langle\frac{u_{\text{e}\zeta}}{R}\right\rangle\right) + Z_{\text{i}}e\left(n_{\text{i}}\langle Bu_{\text{i}\parallel}\rangle - In_{\text{i}}\left\langle\frac{u_{\text{i}\zeta}}{R}\right\rangle\right)\right.$$

$$\left. + Z_{\text{b}}e\left(n_{\text{b}}\langle Bu_{\text{b}\parallel}\rangle - In_{\text{b}}\left\langle\frac{u_{\text{b}\zeta}}{R}\right\rangle\right)\right] \tag{3.2}$$

$$\frac{1}{c^2}\left\langle\frac{1}{R^2}\right\rangle\frac{\partial}{\partial t}\dot\psi = \frac{\partial}{\partial V}\left[\left\langle\frac{|\nabla V|^2}{R^2}\right\rangle\frac{\partial}{\partial V}\psi\right]$$

$$- \mu_0\left(-en_{\text{e}}\left\langle\frac{u_{\text{e}\zeta}}{R}\right\rangle + Z_{\text{i}}en_{\text{i}}\left\langle\frac{u_{\text{i}\zeta}}{R}\right\rangle + Z_{\text{b}}en_{\text{b}}\left\langle\frac{u_{\text{b}\zeta}}{R}\right\rangle\right) \tag{3.3}$$

$$\frac{\partial}{\partial t}\psi = \dot\psi \tag{3.4}$$

$$\frac{\partial}{\partial t}\psi_t = \dot{\psi}_t \tag{3.5}$$

$$\frac{\partial}{\partial t}n_{\rm e} = -\frac{\partial}{\partial V}n_{\rm e}u_{\rm e}^V + u_g^V\frac{\partial}{\partial V}n_{\rm e} + \nu_{\rm ionize}\frac{n_{\rm e}}{n_{01}+n_{02}+n_{03}}\left(n_{01}+n_{02}+n_{03}\right)$$
$$-\nu_{\rm L}(n_{\rm e}-n_{\rm ediv}) + (1-f_{\rm CX})\frac{P_{\rm b}}{E_{\rm b}} \tag{3.6}$$

$$0 = -\left(\langle B^2\rangle\langle R^2\rangle - I^2\right)\frac{1}{m_{\rm e}}\frac{\partial}{\partial V}n_{\rm e}T_{\rm e} + \frac{e}{m_{\rm e}}n_{\rm e}\left(\langle B^2\rangle\langle R^2\rangle - I^2\right)\frac{\partial}{\partial V}\Phi$$
$$-\frac{e}{m_{\rm e}}\frac{\partial\psi}{\partial V}In_{\rm e}\langle Bu_{\rm e\|}\rangle + \frac{e}{m_{\rm e}}\frac{\partial\psi}{\partial V}\langle B^2\rangle n_{\rm e}\langle Ru_{\rm e\zeta}\rangle \tag{3.7}$$

$$n_{\rm e}\frac{\partial}{\partial t}\langle Bu_{\rm e\|}\rangle = -\frac{1}{m_{\rm e}}\hat{\mu}_1^{\rm e}\langle Bu_{\rm e\|}\rangle - \frac{1}{m_{\rm e}}\hat{\mu}_2^{\rm e}\langle B\hat{q}_{\rm e\|}\rangle + \frac{1}{em_{\rm e}}\hat{\mu}_1^{\rm e}I\frac{\partial V}{\partial\psi}\frac{1}{n_{\rm e}}\frac{\partial}{\partial V}n_{\rm e}T_{\rm e} - \frac{1}{m_{\rm e}}\hat{\mu}_1^{\rm e}I\frac{\partial V}{\partial\psi}\frac{\partial}{\partial V}\Phi$$
$$+\frac{1}{em_{\rm e}}\hat{\mu}_2^{\rm e}I\frac{\partial V}{\partial\psi}\frac{1}{n_{\rm e}}\frac{\partial}{\partial V}n_{\rm e}T_{\rm e} - \frac{1}{em_{\rm e}}\hat{\mu}_2^{\rm e}I\frac{\partial V}{\partial\psi}\frac{T_{\rm e}}{n_{\rm e}}\frac{\partial}{\partial V}n_{\rm e}$$
$$+\hat{\ell}_{11}^{\rm ee}n_{\rm e}\langle Bu_{\rm e\|}\rangle + \hat{\ell}_{11}^{\rm ei}n_{\rm e}\langle Bu_{\rm i\|}\rangle + \hat{\ell}_{11}^{\rm eb}\frac{n_{\rm e}}{n_{\rm b}}n_{\rm b}\langle Bu_{\rm b\|}\rangle - \hat{\ell}_{12}^{\rm ee}n_{\rm e}\langle B\hat{q}_{\rm e\|}\rangle - \hat{\ell}_{12}^{\rm ei}n_{\rm e}\langle B\hat{q}_{\rm i\|}\rangle$$
$$-e\frac{n_{\rm e}}{m_{\rm e}}I\langle R^{-2}\rangle\left(-\frac{\dot{\psi}_t}{q}+\dot{\psi}\right) - \nu_{\rm L}n_{\rm e}\langle Bu_{\rm e\|}\rangle - \nu_{0\rm e}n_{\rm e}\langle Bu_{\rm e\|}\rangle$$
$$-\frac{\partial}{\partial V}\langle|\nabla V|\rangle v_{\rm e\zeta}n_{\rm e}\langle Bu_{\rm e\|}\rangle + \langle|\nabla V|\rangle v_{\rm e\zeta}\frac{\partial}{\partial V}\ln\left(\frac{\langle B^2\rangle}{I}\right)n_{\rm e}\langle Bu_{\rm e\|}\rangle$$
$$+\frac{\partial}{\partial V}\langle|\nabla V|^2\rangle\chi_{\rm e\zeta}n_{\rm e}\frac{\partial}{\partial V}\langle Bu_{\rm e\|}\rangle - \frac{\partial}{\partial V}\langle|\nabla V|^2\rangle\chi_{\rm e\zeta}n_{\rm e}\frac{\partial}{\partial V}\ln\left(\frac{\langle B^2\rangle}{I}\right)\langle Bu_{\rm e\|}\rangle$$
$$-\langle|\nabla V|^2\rangle\chi_{\rm e\zeta}n_{\rm e}\frac{\partial}{\partial V}\ln\left(\frac{\langle B^2\rangle}{I}\right)\frac{\partial}{\partial V}\langle Bu_{\rm e\|}\rangle + \langle|\nabla V|^2\rangle\chi_{\rm e\zeta}n_{\rm e}\left[\frac{\partial}{\partial V}\ln\left(\frac{\langle B^2\rangle}{I}\right)\right]^2\langle Bu_{\rm e\|}\rangle \tag{3.8}$$

$$\frac{\partial}{\partial t}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle = -(u_{\rm e}^V - u_g^V)\frac{\partial}{\partial V}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle - \frac{\partial u_{\rm e}^V}{\partial V}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle - \frac{\partial}{\partial V}[\langle|\nabla V|\rangle v_{\rm e\zeta}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle]$$
$$+\frac{\partial}{\partial V}\langle|\nabla V|^2\rangle\chi_{\rm e\zeta}\frac{\partial}{\partial V}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle - \frac{\partial}{\partial V}\langle|\nabla V|^2\rangle\chi_{\rm e\zeta}\langle Ru_{\rm e\zeta}\rangle\frac{\partial}{\partial V}n_{\rm e} - \frac{1}{m_{\rm e}}\frac{\partial}{\partial V}\langle\Pi_{\rm e}^{\rm res}\rangle$$
$$+\hat{\ell}_{11}^{\rm ee}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle + \hat{\ell}_{11}^{\rm ei}\frac{n_{\rm e}}{n_{\rm i}}n_{\rm i}\langle Ru_{\rm i\zeta}\rangle + \hat{\ell}_{11}^{\rm eb}\frac{n_{\rm e}}{n_{\rm b}}n_{\rm b}\langle Ru_{\rm b\zeta}\rangle - \hat{\ell}_{12}^{\rm ee}\frac{I}{\langle B^2\rangle}n_{\rm e}\langle B\hat{q}_{\rm e\|}\rangle - \hat{\ell}_{12}^{\rm ei}\frac{I}{\langle B^2\rangle}n_{\rm e}\langle B\hat{q}_{\rm i\|}\rangle$$
$$-e\frac{n_{\rm e}}{m_{\rm e}}\dot{\psi} - e\frac{1}{m_{\rm e}}\frac{\partial\psi}{\partial V}n_{\rm e}u_{\rm e}^V$$
$$-\frac{e^2\langle|\nabla V|^2\rangle D_{\rm e}}{m_{\rm e}T_{\rm e}}\frac{\partial\psi}{\partial V}\left[\frac{\partial\psi}{\partial V}\frac{\langle B^2\rangle}{\langle B^2\rangle\langle R^2\rangle - I^2}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle - \frac{\partial\psi}{\partial V}\frac{I}{\langle B^2\rangle\langle R^2\rangle - I^2}n_{\rm e}\langle Bu_{\rm e\|}\rangle\right.$$
$$\left.+\frac{C_{\rm T}-1}{e}\left(\frac{\partial}{\partial V}n_{\rm e}T_{\rm e} - T_{\rm e}\frac{\partial}{\partial V}n_{\rm e}\right) + n_{\rm e}\frac{\partial}{\partial V}\Phi + \frac{T_{\rm e}}{e}\frac{\partial\psi}{\partial V}C_{\rm p}n_{\rm e}\right] - \nu_{\rm L}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle - \nu_{0\rm e}n_{\rm e}\langle Ru_{\rm e\zeta}\rangle \tag{3.9}$$

$$\frac{3}{2}\frac{\partial}{\partial t}n_\mathrm{e}T_\mathrm{e} = -\frac{5}{2}(u_\mathrm{e}^V - u_g^V)\frac{\partial}{\partial V}n_\mathrm{e}T_\mathrm{e} - \frac{5}{2}\frac{\partial u_\mathrm{e}^V}{\partial V}n_\mathrm{e}T_\mathrm{e} - \frac{\partial}{\partial V}[V_\mathrm{e}^\mathrm{hp}\langle|\nabla V|\rangle n_\mathrm{e}T_\mathrm{e}]$$

$$+ \frac{\partial}{\partial V}\chi_\mathrm{e}\langle|\nabla V|^2\rangle\frac{\partial}{\partial V}n_\mathrm{e}T_\mathrm{e} - \frac{\partial}{\partial V}\chi_\mathrm{e}\langle|\nabla V|^2\rangle T_\mathrm{e}\frac{\partial}{\partial V}n_\mathrm{e}$$

$$- u_g^V\frac{\partial}{\partial V}n_\mathrm{e}T_\mathrm{e} - u_\mathrm{i}^V\frac{\partial}{\partial V}n_\mathrm{i}T_\mathrm{i} - \hat{u}_{\mathrm{i}\theta}\langle\boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\pi}_\mathrm{i}\rangle - \frac{3}{2}\nu_\mathrm{Tei}\left(n_\mathrm{e}T_\mathrm{e} - \frac{n_\mathrm{e}}{n_\mathrm{i}}n_\mathrm{i}T_\mathrm{i}\right)$$

$$- 4\pi^2\frac{\partial\psi}{\partial V}\dot{\psi}_t(-en_\mathrm{e}\hat{u}_{\mathrm{e}\theta} + Z_\mathrm{i}en_\mathrm{i}\hat{u}_{\mathrm{i}\theta}) + \dot{\psi}\left(-en_\mathrm{e}\left\langle\frac{u_{\mathrm{e}\zeta}}{R}\right\rangle + Z_\mathrm{i}en_\mathrm{i}\left\langle\frac{u_{\mathrm{i}\zeta}}{R}\right\rangle\right)$$

$$- \nu_\mathrm{L}(n_\mathrm{e}T_\mathrm{e} - n_\mathrm{ediv}T_\mathrm{e}) - \frac{3}{2}\nu_{\mathrm{L}T_\mathrm{e}}(n_\mathrm{e}T_\mathrm{e} - n_\mathrm{e}T_\mathrm{ediv}) - E_\mathrm{H}\nu_\mathrm{ionize}\frac{n_\mathrm{e}}{n_{01} + n_{02} + n_{03}}(n_{01} + n_{02} + n_{03})$$

$$+ f_\mathrm{ei}P_\mathrm{b} + P_{\alpha\mathrm{e}} + P_\mathrm{RFe} - P_\mathrm{Br} \tag{3.10}$$

$$\frac{5}{2}n_\mathrm{e}\frac{\partial}{\partial t}\langle B\hat{q}_{\mathrm{e}\parallel}\rangle = -\frac{1}{m_\mathrm{e}}\hat{\mu}_2^\mathrm{e}\langle Bu_{\mathrm{e}\parallel}\rangle - \frac{1}{m_\mathrm{e}}\hat{\mu}_3^\mathrm{e}\langle B\hat{q}_{\mathrm{e}\parallel}\rangle + \frac{1}{em_\mathrm{e}}\hat{\mu}_2^\mathrm{e}I\frac{\partial V}{\partial\psi}\frac{1}{n_\mathrm{e}}\frac{\partial}{\partial V}n_\mathrm{e}T_\mathrm{e} - \frac{1}{m_\mathrm{e}}\hat{\mu}_2^\mathrm{e}I\frac{\partial V}{\partial\psi}\frac{\partial}{\partial V}\Phi$$

$$+ \frac{1}{em_\mathrm{e}}\hat{\mu}_3^\mathrm{e}I\frac{\partial V}{\partial\psi}\frac{1}{n_\mathrm{e}}\frac{\partial}{\partial V}n_\mathrm{e}T_\mathrm{e} - \frac{1}{em_\mathrm{e}}\hat{\mu}_3^\mathrm{e}I\frac{\partial V}{\partial\psi}\frac{T_\mathrm{e}}{n_\mathrm{e}}\frac{\partial}{\partial V}n_\mathrm{e}$$

$$- \hat{\ell}_{21}^\mathrm{ee}n_\mathrm{e}\langle Bu_{\mathrm{e}\parallel}\rangle - \hat{\ell}_{21}^\mathrm{ei}n_\mathrm{e}\langle Bu_{\mathrm{i}\parallel}\rangle - \hat{\ell}_{21}^\mathrm{eb}\frac{n_\mathrm{e}}{n_\mathrm{b}}n_\mathrm{b}\langle Bu_{\mathrm{b}\parallel}\rangle + \hat{\ell}_{22}^\mathrm{ee}n_\mathrm{e}\langle B\hat{q}_{\mathrm{e}\parallel}\rangle + \hat{\ell}_{22}^\mathrm{ei}n_\mathrm{e}\langle B\hat{q}_{\mathrm{i}\parallel}\rangle$$

$$- \nu_\mathrm{L}n_\mathrm{e}\langle B\hat{q}_{\mathrm{e}\parallel}\rangle - \nu_{0\mathrm{e}}n_\mathrm{e}\langle B\hat{q}_{\mathrm{e}\parallel}\rangle \tag{3.11}$$

$$n_\mathrm{e}\left\langle\frac{u_{\mathrm{e}\zeta}}{R}\right\rangle = \frac{1}{\langle R^2\rangle}n_\mathrm{e}\langle Ru_{\mathrm{e}\zeta}\rangle - \frac{2\hat{q}^2}{1+2\hat{q}^2}\frac{1}{\langle R^2\rangle}n_\mathrm{e}\langle Ru_{\mathrm{e}\zeta}\rangle + \frac{2\hat{q}^2}{1+2\hat{q}^2}\frac{1}{I}n_\mathrm{e}\langle Bu_{\mathrm{e}\parallel}\rangle \tag{3.12}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} = -\frac{\partial}{\partial V} n_{\mathrm{i}} u_{\mathrm{i}}^{V} + u_{g}^{V} \frac{\partial}{\partial V} n_{\mathrm{i}} + \nu_{\mathrm{ionize}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02} + n_{03}} \left(n_{01} + n_{02} + n_{03} \right) - \nu_{\mathrm{L}}(n_{\mathrm{i}} - n_{\mathrm{idiv}})$$

$$+ \nu_{\mathrm{b}} n_{\mathrm{b}} + \nu_{\mathrm{b}} g n_{\mathrm{b}}^{\mathrm{rp}} - f_{\mathrm{CX}} \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} + \nu_{\mathrm{bL}} n_{\mathrm{b}} + S_{\mathrm{LC}} - \nu_{\mathrm{OL}} n_{\mathrm{i}} \tag{3.13}$$

$$0 = -\left(\langle B^2 \rangle \langle R^2 \rangle - I^2 \right) \frac{1}{m_{\mathrm{i}}} \frac{\partial}{\partial V} n_{\mathrm{i}} T_{\mathrm{i}} - Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} \left(\langle B^2 \rangle \langle R^2 \rangle - I^2 \right) \frac{\partial}{\partial V} \Phi$$

$$+ Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} \frac{\partial \psi}{\partial V} I n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle - Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} \frac{\partial \psi}{\partial V} \langle B^2 \rangle n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle \tag{3.14}$$

$$n_{\mathrm{i}} \frac{\partial}{\partial t} \langle B u_{\mathrm{i}\|} \rangle = -\frac{1}{m_{\mathrm{i}}} \hat{\mu}_1^{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle - \frac{1}{m_{\mathrm{i}}} \hat{\mu}_2^{\mathrm{i}} \langle B \hat{q}_{\mathrm{i}\|} \rangle - \frac{1}{Z_{\mathrm{i}} e m_{\mathrm{i}}} \hat{\mu}_1^{\mathrm{i}} I \frac{\partial V}{\partial \psi} \frac{1}{n_{\mathrm{i}}} \frac{\partial}{\partial V} n_{\mathrm{i}} T_{\mathrm{i}} - \frac{1}{m_{\mathrm{i}}} \hat{\mu}_1^{\mathrm{i}} I \frac{\partial V}{\partial \psi} \frac{\partial}{\partial V} \Phi$$

$$- \frac{1}{Z_{\mathrm{i}} e m_{\mathrm{i}}} \hat{\mu}_2^{\mathrm{i}} I \frac{\partial V}{\partial \psi} \frac{1}{n_{\mathrm{i}}} \frac{\partial}{\partial V} n_{\mathrm{i}} T_{\mathrm{i}} + \frac{1}{Z_{\mathrm{i}} e m_{\mathrm{i}}} \hat{\mu}_2^{\mathrm{i}} I \frac{\partial V}{\partial \psi} \frac{T_{\mathrm{i}}}{n_{\mathrm{i}}} \frac{\partial}{\partial V} n_{\mathrm{i}}$$

$$+ \hat{\ell}_{11}^{\mathrm{ie}} n_{\mathrm{i}} \langle B u_{\mathrm{e}\|} \rangle + \hat{\ell}_{11}^{\mathrm{ii}} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle + \hat{\ell}_{11}^{\mathrm{ib}} \frac{n_{\mathrm{i}}}{n_{\mathrm{b}}} n_{\mathrm{b}} \langle B u_{\mathrm{b}\|} \rangle - \hat{\ell}_{12}^{\mathrm{ie}} n_{\mathrm{i}} \langle B \hat{q}_{\mathrm{e}\|} \rangle - \hat{\ell}_{12}^{\mathrm{ii}} n_{\mathrm{i}} \langle B \hat{q}_{\mathrm{i}\|} \rangle$$

$$+ Z_{\mathrm{i}} e \frac{n_{\mathrm{i}}}{m_{\mathrm{i}}} I \langle R^{-2} \rangle \left(-\frac{\dot{\psi}_t}{q} + \dot{\psi} \right)$$

$$- \nu_{\mathrm{L}} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle + \nu_{\mathrm{b}} \frac{m_{\mathrm{b}}}{m_{\mathrm{i}}} n_{\mathrm{b}} \langle B u_{\mathrm{b}\|} \rangle - \nu_{0\mathrm{i}} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle - \nu_{\mathrm{CX}} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle - \nu_{\mathrm{OL}} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle$$

$$- \frac{\partial}{\partial V} \langle |\nabla V| \rangle v_{\mathrm{i}\zeta} n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle + \langle |\nabla V| \rangle v_{\mathrm{i}\zeta} \frac{\partial}{\partial V} \ln\left(\frac{\langle B^2 \rangle}{I} \right) n_{\mathrm{i}} \langle B u_{\mathrm{i}\|} \rangle$$

$$+ \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} n_{\mathrm{i}} \frac{\partial}{\partial V} \langle B u_{\mathrm{i}\|} \rangle - \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} n_{\mathrm{i}} \frac{\partial}{\partial V} \ln\left(\frac{\langle B^2 \rangle}{I} \right) \langle B u_{\mathrm{i}\|} \rangle$$

$$- \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} n_{\mathrm{i}} \frac{\partial}{\partial V} \ln\left(\frac{\langle B^2 \rangle}{I} \right) \frac{\partial}{\partial V} \langle B u_{\mathrm{i}\|} \rangle + \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} n_{\mathrm{i}} \left[\frac{\partial}{\partial V} \ln\left(\frac{\langle B^2 \rangle}{I} \right) \right]^2 \langle B u_{\mathrm{i}\|} \rangle \tag{3.15}$$

$$\frac{\partial}{\partial t} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle = -(u_{\mathrm{i}}^{V} - u_{g}^{V}) \frac{\partial}{\partial V} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle - \frac{\partial u_{\mathrm{i}}^{V}}{\partial V} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle - \frac{\partial}{\partial V} \langle |\nabla V| \rangle v_{\mathrm{i}\zeta} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle$$

$$+ \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} \frac{\partial}{\partial V} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle - \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle \chi_{\mathrm{i}\zeta} \langle R u_{\mathrm{i}\zeta} \rangle \frac{\partial}{\partial V} n_{\mathrm{i}} - \frac{1}{m_{\mathrm{i}}} \frac{\partial}{\partial V} \langle \Pi_{\mathrm{i}}^{\mathrm{res}} \rangle$$

$$+ \hat{\ell}_{11}^{\mathrm{ie}} \frac{n_{\mathrm{i}}}{n_{\mathrm{e}}} n_{\mathrm{e}} \langle R u_{\mathrm{e}\zeta} \rangle + \hat{\ell}_{11}^{\mathrm{ii}} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle + \hat{\ell}_{11}^{\mathrm{ib}} \frac{n_{\mathrm{i}}}{n_{\mathrm{b}}} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle - \hat{\ell}_{12}^{\mathrm{ie}} \frac{I}{\langle B^2 \rangle} n_{\mathrm{i}} \langle B \hat{q}_{\mathrm{e}\|} \rangle - \hat{\ell}_{12}^{\mathrm{ii}} \frac{I}{\langle B^2 \rangle} n_{\mathrm{i}} \langle B \hat{q}_{\mathrm{i}\|} \rangle$$

$$+ Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} n_{\mathrm{i}} \dot{\psi} + Z_{\mathrm{i}} \frac{e}{m_{\mathrm{i}}} \frac{\partial \psi}{\partial V} n_{\mathrm{i}} u_{\mathrm{i}}^{V}$$

$$+ Z_{\mathrm{i}} \frac{e^2 \langle |\nabla V|^2 \rangle D_{\mathrm{e}}}{m_{\mathrm{i}} T_{\mathrm{e}}} \frac{\partial \psi}{\partial V} \left[\frac{\partial \psi}{\partial V} \frac{\langle B^2 \rangle}{\langle B^2 \rangle \langle R^2 \rangle - I^2} \frac{n_{\mathrm{i}}}{n_{\mathrm{e}}} n_{\mathrm{e}} \langle R u_{\mathrm{e}\zeta} \rangle - \frac{\partial \psi}{\partial V} \frac{I}{\langle B^2 \rangle \langle R^2 \rangle - I^2} n_{\mathrm{i}} \langle B u_{\mathrm{e}\|} \rangle \right.$$

$$\left. + \frac{C_{\mathrm{T}} - 1}{e} \frac{n_{\mathrm{i}}}{n_{\mathrm{e}}} \left(\frac{\partial}{\partial V} n_{\mathrm{e}} T_{\mathrm{e}} - T_{\mathrm{e}} \frac{\partial}{\partial V} n_{\mathrm{e}} \right) + n_{\mathrm{i}} \frac{\partial}{\partial V} \Phi + \frac{T_{\mathrm{e}}}{e} \frac{\partial \psi}{\partial V} C_{\mathrm{p}} n_{\mathrm{i}} \right]$$

$$-\nu_{\mathrm{L}}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle + \nu_{\mathrm{b}}\frac{m_{\mathrm{b}}}{m_{\mathrm{i}}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\zeta}\rangle - \nu_{0\mathrm{i}}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle - \nu_{\mathrm{CX}}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle - \nu_{\mathrm{OL}}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle \tag{3.16}$$

$$\frac{3}{2}\frac{\partial}{\partial t}n_{\mathrm{i}}T_{\mathrm{i}} = -\frac{5}{2}(u_{\mathrm{i}}^{V}-u_{g}^{V})\frac{\partial}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} - \frac{5}{2}\frac{\partial u_{\mathrm{i}}^{V}}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} - \frac{\partial}{\partial V}V_{\mathrm{i}}^{\mathrm{hp}}\langle|\nabla V|\rangle n_{\mathrm{i}}T_{\mathrm{i}}$$

$$+ \frac{\partial}{\partial V}\chi_{\mathrm{i}}\langle|\nabla V|^{2}\rangle\frac{\partial}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} - \frac{\partial}{\partial V}\chi_{\mathrm{i}}\langle|\nabla V|^{2}\rangle T_{\mathrm{i}}\frac{\partial}{\partial V}n_{\mathrm{i}}$$

$$+ (u_{\mathrm{i}}^{V}-u_{g}^{V})\frac{\partial}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} + \hat{u}_{\mathrm{i}\theta}\langle\boldsymbol{B}\cdot\nabla\cdot\overset{\leftrightarrow}{\pi}_{\mathrm{i}}\rangle - \frac{3}{2}\nu_{\mathrm{Tei}}\left(\frac{n_{\mathrm{e}}}{n_{\mathrm{i}}}n_{\mathrm{i}}T_{\mathrm{i}} - n_{\mathrm{e}}T_{\mathrm{e}}\right)$$

$$- \nu_{\mathrm{L}}\left(n_{\mathrm{i}}T_{\mathrm{i}} - n_{\mathrm{idiv}}T_{\mathrm{i}}\right) - \frac{3}{2}\nu_{\mathrm{L}T_{\mathrm{i}}}(n_{\mathrm{i}}T_{\mathrm{i}} - n_{\mathrm{i}}T_{\mathrm{idiv}})$$

$$+ \frac{3}{2}\nu_{\mathrm{ionize}}\frac{n_{\mathrm{i}}}{n_{01}+n_{02}+n_{03}}\left(n_{01}T_{01} + n_{02}T_{02} + n_{03}T_{03}\right)$$

$$- \frac{3}{2}\nu_{\mathrm{CX}}(n_{\mathrm{i}}T_{\mathrm{i}} - n_{\mathrm{i}}T_{01}) + (1-f_{\mathrm{ei}})P_{\mathrm{b}} + m_{\mathrm{b}}\frac{1}{n_{\mathrm{b}}}\frac{P_{\mathrm{b}}}{E_{\mathrm{b}}}\frac{\langle Bu_{\mathrm{i}\|}\rangle}{\langle B^{2}\rangle}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle + P_{\alpha\mathrm{i}} + P_{\mathrm{RFi}} \tag{3.17}$$

$$\frac{5}{2}n_{\mathrm{i}}\frac{\partial}{\partial t}\langle B\hat{q}_{\mathrm{i}\|}\rangle = -\frac{1}{m_{\mathrm{i}}}\hat{\mu}_{2}^{\mathrm{i}}\langle Bu_{\mathrm{i}\|}\rangle - \frac{1}{m_{\mathrm{i}}}\hat{\mu}_{3}^{\mathrm{i}}\langle B\hat{q}_{\mathrm{i}\|}\rangle - \frac{1}{Z_{\mathrm{i}}em_{\mathrm{i}}}\hat{\mu}_{2}^{\mathrm{i}}I\frac{\partial V}{\partial\psi}\frac{1}{n_{\mathrm{i}}}\frac{\partial}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} - \frac{1}{m_{\mathrm{i}}}\hat{\mu}_{2}^{\mathrm{i}}I\frac{\partial V}{\partial\psi}\frac{\partial}{\partial V}\Phi$$

$$- \frac{1}{Z_{\mathrm{i}}em_{\mathrm{i}}}\hat{\mu}_{3}^{\mathrm{i}}I\frac{\partial V}{\partial\psi}\frac{1}{n_{\mathrm{i}}}\frac{\partial}{\partial V}n_{\mathrm{i}}T_{\mathrm{i}} + \frac{1}{Z_{\mathrm{i}}em_{\mathrm{i}}}\hat{\mu}_{3}^{\mathrm{i}}I\frac{\partial V}{\partial\psi}\frac{T_{\mathrm{i}}}{n_{\mathrm{i}}}\frac{\partial}{\partial V}n_{\mathrm{i}}$$

$$- \tilde{\ell}_{21}^{\mathrm{ie}}n_{\mathrm{i}}\langle Bu_{\mathrm{e}\|}\rangle - \tilde{\ell}_{21}^{\mathrm{ii}}n_{\mathrm{i}}\langle Bu_{\mathrm{i}\|}\rangle - \tilde{\ell}_{21}^{\mathrm{ib}}\frac{n_{\mathrm{i}}}{n_{\mathrm{b}}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle + \tilde{\ell}_{22}^{\mathrm{ie}}n_{\mathrm{i}}\langle B\hat{q}_{\mathrm{e}\|}\rangle + \tilde{\ell}_{22}^{\mathrm{ii}}n_{\mathrm{i}}\langle B\hat{q}_{\mathrm{i}\|}\rangle$$

$$- \nu_{\mathrm{L}}n_{\mathrm{i}}\langle B\hat{q}_{\mathrm{i}\|}\rangle - \nu_{0\mathrm{i}}n_{\mathrm{i}}\langle B\hat{q}_{\mathrm{i}\|}\rangle \tag{3.18}$$

$$n_{\mathrm{i}}\left\langle\frac{u_{\mathrm{i}\zeta}}{R}\right\rangle = \frac{1}{\langle R^{2}\rangle}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle - \frac{2\hat{q}^{2}}{1+2\hat{q}^{2}}\frac{1}{\langle R^{2}\rangle}n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle + \frac{2\hat{q}^{2}}{1+2\hat{q}^{2}}\frac{1}{I}n_{\mathrm{i}}\langle Bu_{\mathrm{i}\|}\rangle \tag{3.19}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}} = -\frac{\partial}{\partial V}n_{\mathrm{b}}u_{\mathrm{b}}^{V} + u_{g}^{V}\frac{\partial}{\partial V}n_{\mathrm{b}} + \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} - g\frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} - \nu_{\mathrm{b}}n_{\mathrm{b}} - \nu_{\mathrm{bL}}n_{\mathrm{b}} - \nu_{\mathrm{eff}}g(\sqrt{\delta}n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}})$$

$$+ \frac{\partial}{\partial V}\langle|\nabla V|^{2}\rangle f_{t}D_{\mathrm{b}}\frac{\partial}{\partial V}n_{\mathrm{b}} \tag{3.20}$$

$$0 = -\left(\langle B^{2}\rangle\langle R^{2}\rangle - I^{2}\right)\frac{1}{m_{\mathrm{b}}}T_{\mathrm{b}}\frac{\partial}{\partial V}n_{\mathrm{b}} - \left(\langle B^{2}\rangle\langle R^{2}\rangle - I^{2}\right)\frac{1}{m_{\mathrm{b}}}\frac{\partial T_{\mathrm{b}}}{\partial V}n_{\mathrm{b}}$$

$$- Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}n_{\mathrm{b}}\left(\langle B^{2}\rangle\langle R^{2}\rangle - I^{2}\right)\frac{\partial}{\partial V}\Phi + Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}\frac{\partial\psi}{\partial V}In_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle - Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}\frac{\partial\psi}{\partial V}\langle B^{2}\rangle n_{\mathrm{b}}\langle Ru_{\mathrm{b}\zeta}\rangle \tag{3.21}$$

$$\frac{\partial}{\partial t}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle = -\frac{1}{m_{\mathrm{b}}n_{\mathrm{b}}}\hat{\mu}_{1}^{\mathrm{b}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle - \frac{1}{Z_{\mathrm{b}}em_{\mathrm{b}}}\hat{\mu}_{1}^{\mathrm{b}}I\frac{\partial V}{\partial\psi}\frac{T_{\mathrm{b}}}{n_{\mathrm{b}}}\frac{\partial}{\partial V}n_{\mathrm{b}} - \frac{1}{Z_{\mathrm{b}}em_{\mathrm{b}}}\hat{\mu}_{1}^{\mathrm{b}}I\frac{\partial V}{\partial\psi}\frac{\partial T_{\mathrm{b}}}{\partial V} - \frac{1}{m_{\mathrm{b}}}\hat{\mu}_{1}^{\mathrm{b}}I\frac{\partial V}{\partial\psi}\frac{\partial}{\partial V}\Phi$$

$$+ \tilde{\ell}_{11}^{\mathrm{bb}}\frac{m_{\mathrm{e}}n_{\mathrm{e}}}{m_{\mathrm{b}}n_{\mathrm{b}}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle + \sum_{a\neq\mathrm{b}}\tilde{\ell}_{11}^{\mathrm{ba}}\frac{m_{\mathrm{e}}n_{\mathrm{e}}}{m_{\mathrm{b}}}\langle Bu_{a\|}\rangle$$

$$+ Z_{\mathrm{b}}\frac{e}{m_{\mathrm{b}}}n_{\mathrm{b}}I\langle R^{-2}\rangle\left(-\frac{\dot{\psi}_{t}}{q} + \dot{\psi}\right) - \nu_{\mathrm{bL}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle - \nu_{\mathrm{b}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle - \nu_{0\mathrm{b}}n_{\mathrm{b}}\langle Bu_{\mathrm{b}\|}\rangle$$

$$- \nu_{\mathrm{CX}} n_{\mathrm{b}} \langle B u_{\mathrm{b}\|} \rangle + \frac{1}{m_{\mathrm{b}}} \langle B S_{\mathrm{m}\|\mathrm{b}} \rangle - \nu_{\mathrm{eff}} g \sqrt{\delta} n_{\mathrm{b}} \langle B u_{\mathrm{b}\|} \rangle + \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle \langle B u_{\mathrm{b}\|} \rangle f_t D_{\mathrm{b}} \frac{\partial}{\partial V} n_{\mathrm{b}} \quad (3.22)$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle = u_g^V \frac{\partial}{\partial V} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle - \frac{\partial}{\partial V} \langle R u_{\mathrm{b}\zeta} \rangle n_{\mathrm{b}} u_{\mathrm{b}}^V$$

$$+ \frac{1}{m_{\mathrm{b}}} \ell_{11}^{\mathrm{be}} \frac{1}{n_{\mathrm{e}}} n_{\mathrm{e}} \langle R u_{\mathrm{e}\zeta} \rangle + \frac{1}{m_{\mathrm{b}}} \ell_{11}^{\mathrm{bi}} \frac{1}{n_{\mathrm{i}}} n_{\mathrm{i}} \langle R u_{\mathrm{i}\zeta} \rangle + \frac{1}{m_{\mathrm{b}}} \ell_{11}^{\mathrm{bb}} \frac{1}{n_{\mathrm{b}}} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle$$

$$+ Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} n_{\mathrm{b}} \dot{\psi} + Z_{\mathrm{b}} \frac{e}{m_{\mathrm{b}}} \frac{\partial \psi}{\partial V} n_{\mathrm{b}} u_{\mathrm{b}}^V$$

$$- \nu_{\mathrm{bL}} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle - \nu_{\mathrm{b}} n_{\mathrm{b}} \langle B u_{\mathrm{b}\zeta} \rangle - \nu_{0\mathrm{i}} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle - \nu_{\mathrm{CX}} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle + \frac{1}{m_{\mathrm{b}}} \frac{I}{\langle B^2 \rangle} \langle B S_{\mathrm{m}\|\mathrm{b}} \rangle \quad (3.23)$$

$$n_{\mathrm{b}} \left\langle \frac{u_{\mathrm{b}\zeta}}{R} \right\rangle = \frac{1}{\langle R^2 \rangle} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle - \frac{2\hat{q}^2}{1 + 2\hat{q}^2} \frac{1}{\langle R^2 \rangle} n_{\mathrm{b}} \langle R u_{\mathrm{b}\zeta} \rangle + \frac{2\hat{q}^2}{1 + 2\hat{q}^2} \frac{1}{I} n_{\mathrm{b}} \langle B u_{\mathrm{b}\|} \rangle \quad (3.24)$$

$$\frac{\partial}{\partial t} n_{01} = -\frac{\partial u_g^V}{\partial V} n_{01} + \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle D_{01} \frac{\partial}{\partial V} n_{01} - \nu_{\mathrm{ionize}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02} + n_{03}} n_{01}$$

$$- \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01} + \gamma_0 \nu_{\mathrm{L}} (n_{\mathrm{i}} - n_{\mathrm{idiv}}) + \frac{\partial}{\partial V} V' \Gamma^{\mathrm{puff}} \bigg|_{\mathrm{b}} \quad (3.25)$$

$$\frac{\partial}{\partial t} n_{02} = -\frac{\partial u_g^V}{\partial V} n_{02} + \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle D_{02} \frac{\partial}{\partial V} n_{02} - \nu_{\mathrm{ionize}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02} + n_{03}} n_{02} + \nu_{\mathrm{CX}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02}} n_{01} \quad (3.26)$$

$$\frac{\partial}{\partial t} n_{03} = -\frac{\partial u_g^V}{\partial V} n_{03} + \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle D_{03} \frac{\partial}{\partial V} n_{03} - \nu_{\mathrm{ionize}} \frac{n_{\mathrm{i}}}{n_{01} + n_{02} + n_{03}} n_{03} + f_{\mathrm{CX}} \frac{P_{\mathrm{b}}}{E_{\mathrm{b}}} \quad (3.27)$$

$$\frac{\partial}{\partial t} n_{\mathrm{b}}^{\mathrm{rp}} = -\frac{\partial u_g^V}{\partial V} n_{\mathrm{b}}^{\mathrm{rp}} \frac{P_{\mathrm{b}\perp}}{E_{\mathrm{b}}} + \nu_{\mathrm{eff}} (\sqrt{\delta} n_{\mathrm{b}} - n_{\mathrm{b}}^{\mathrm{rp}}) - \nu_{\mathrm{b}} n_{\mathrm{b}}^{\mathrm{rp}}$$

$$- \frac{\partial}{\partial V} \langle |\nabla V| \rangle u_{\mathrm{b}}^{\mathrm{rp}} n_{\mathrm{b}}^{\mathrm{rp}} + \frac{\partial}{\partial V} \langle |\nabla V|^2 \rangle D_{\mathrm{b}}^{\mathrm{rp}} \frac{\partial}{\partial V} n_{\mathrm{b}}^{\mathrm{rp}} \quad (3.28)$$

4 Boundary Conditions

num.	name	variable	highest order derivative	center	edge
1	LQm1	Φ	2	N	0
2	LQm2	$\dot\psi_t$	2	0	$\langle|\nabla V|^2/R^2\rangle|_{V_b} I_{\mathrm{vac}}/(4\pi^2)$
3	LQm3	$\dot\psi$	2	N	$2\pi\mu_0 I_{\mathrm{p}}$
4	LQm4	ψ	0	×	×
5	LQm5	ψ_t/μ_0	0	×	×
6	LQe1	n_{e}	2	N	N
7	LQe2	$n_{\mathrm{e}}u_{\mathrm{e}}^V$	1	0	×
8	LQe3	$\langle Bu_{\mathrm{e}\parallel}\rangle$	2(0)	N(×)	N(×)
9	LQe4	$n_{\mathrm{e}}\langle Ru_{\mathrm{e}\zeta}\rangle$	2	N	N
10	LQe5	$n_{\mathrm{e}}T_{\mathrm{e}}$	2	N	N
11	LQe6	$\langle B\hat q_{\mathrm{e}\parallel}\rangle$	0	×	×
12	LQe7	$n_{\mathrm{e}}\langle u_{\mathrm{e}\zeta}/R\rangle$	0	×	×
13	LQi1	n_{i}	2	N	N
14	LQi2	$n_{\mathrm{i}}u_{\mathrm{i}}^V$	1	0	×
15	LQi3	$\langle Bu_{\mathrm{i}\parallel}\rangle$	2(0)	N(×)	N(×)
16	LQi4	$n_{\mathrm{i}}\langle Ru_{\mathrm{i}\zeta}\rangle$	2	N	N
17	LQi5	$n_{\mathrm{i}}T_{\mathrm{i}}$	2	N	N
18	LQi6	$\langle B\hat q_{\mathrm{i}\parallel}\rangle$	0	×	×
19	LQi7	$n_{\mathrm{i}}\langle u_{\mathrm{i}\zeta}/R\rangle$	0	×	×
20	LQb1	n_{b}	0(2)	×(N)	×(N)
21	LQb2	$n_{\mathrm{b}}u_{\mathrm{b}}^V$	1	0	×
22	LQb3	$n_{\mathrm{b}}\langle Bu_{\mathrm{b}\parallel}\rangle$	0	×	×
23	LQb4	$n_{\mathrm{b}}\langle Ru_{\mathrm{b}\zeta}\rangle$	0	×	×
24	LQb7	$n_{\mathrm{b}}\langle u_{\mathrm{b}\zeta}/R\rangle$	0	×	×
25	LQn1	n_{01}	2	N	$\langle|\nabla V|\rangle S_{\mathrm{gas}}|_b$
26	LQn2	n_{02}	2	N	0
27	LQn3	n_{03}	2	N	0
28	LQr1	$n_{\mathrm{b}}^{\mathrm{rp}}$	0(2)	×(N)	×(N)

0: Dirichlet condition ($u = 0$)

N: Neumann condition ($u' = 0$)

value: Neumann condition ($u' = $ value)

×: no condition

NOTE: The dependent variable of LQm4 turns to be ψ/μ_0 when NB is applied. Subsequently, the boundary condition of LQm3 is also changed to $2\pi I_{\mathrm{p}}$.

NOTE: In the table the column labeled by the highest order derivative means the order of the highest order derivative of a dependent variable, not an equation, in the corresponding row. Please note that the term not integrated by parts is never counted. For example, the first row shows the highest order of derivatives is equal to 2 for Φ because the LQm1 equation has the diffusion term having the second derivative of Φ with respect to V. In addition, the LQe2 and LQi2 equations also have the first derivative of Φ, which is not a dependent

variable of these equations. As a whole, the highest order derivative of Φ in this equation system is 2 that comes from the LQm1 equation. When we look at n_{e}, we find that the highest order derivative of n_{e} is 2 shown in the LQe5 equation, even though no derivative appears in the LQe1 equation. The Neumann conditions for n_{e} come from the LQe5 equation. When it comes to $n_{\mathrm{e}}u_{\mathrm{e}}^{V}$, the highest derivative is found as order 1 in the LQe1 equation, but this term is not integrated by parts. It means that any boundary condition is not naturally imposed on $n_{\mathrm{e}}u_{\mathrm{e}}^{V}$. The Dirichlet boundary condition should be imposed at axis.