

## 1 Equations in $r$ coordinates

$$0 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \phi + \frac{1}{\epsilon_0} (-en_e + Z_i en_i + Z_b en_b + Z_b e g n_b^{\text{rp}}) \quad (1.1)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\theta = \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r A_\theta + \mu_0 (-en_e u_{e\theta} + Z_i en_i u_{i\theta} + Z_b en_b u_{b\theta}) \quad (1.2)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\phi = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} A_\phi + \mu_0 (-en_e u_{e\phi} + Z_i en_i u_{i\phi} + Z_b en_b u_{b\phi}) \quad (1.3)$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \quad (1.4)$$

$$\frac{\partial}{\partial t} A_\theta = \dot{A}_\theta \quad (1.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e = & -\frac{1}{r} \frac{\partial}{\partial r} r n_e u_{er} + \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \nu_L (n_e - n_{\text{ediv}}) \\ & + (1 - f_{\text{CX}}) \frac{P_b}{E_b} \end{aligned} \quad (1.6)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{er} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{er} n_e u_{er} + \frac{1}{r} u_{e\theta} n_e u_{e\theta} - \frac{\partial}{\partial r} \frac{n_e T_e}{m_e} \\ & - \frac{e}{m_e} n_e E_r - \frac{e}{m_e} B_\phi n_e u_{e\theta} + \frac{e}{m_e} B_\theta n_e u_{e\phi} \end{aligned} \quad (1.7)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{e\theta} = & -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u_{er} n_e u_{e\theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^3 n_e \mu_e \frac{\partial}{\partial r} \frac{u_{e\theta}}{r} - \frac{e}{m_e} n_e E_\theta + \frac{e}{m_e} B_\phi n_e u_{er} \\ & - \nu_{\text{NCe}} n_e u_{e\theta} - \nu_{\text{ei1}} n_e (u_{e\theta} - u_{i\theta}) - \nu_{\text{ei2}} n_e (u_{e\phi} - u_{i\phi}) \\ & - \frac{m_b}{m_e} \nu_{\text{be1}} n_b (u_{e\theta} - u_{b\theta}) - \frac{m_b}{m_e} \nu_{\text{be2}} n_b (u_{e\phi} - u_{b\phi}) \\ & - \frac{e^2 B_\phi^2 D_e}{m_e T_e} n_e \left[ u_{e\theta} - \frac{B_\theta}{B_\phi} u_{e\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \\ & - 2\nu_L n_e u_{e\theta} - \nu_{0e} n_e u_{e\theta} + F_{e\theta}^{\text{neo}} \end{aligned} \quad (1.8)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e u_{e\phi} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{er} n_e u_{e\phi} + \frac{1}{r} \frac{\partial}{\partial r} r n_e \mu_e \frac{\partial}{\partial r} u_{e\phi} - \frac{e}{m_e} n_e E_\phi - \frac{e}{m_e} B_\theta n_e u_{er} \\ & - \nu_{\text{ei3}} n_e (u_{e\phi} - u_{i\phi}) - \nu_{\text{ei2}} n_e (u_{e\theta} - u_{i\theta}) - \frac{m_b}{m_e} \nu_{\text{be2}} n_b (u_{e\theta} - u_{b\theta}) - \frac{m_b}{m_e} \nu_{\text{be3}} n_b (u_{e\phi} - u_{b\phi}) \\ & + \frac{e^2 B_\phi^2 D_e}{m_e T_e} \frac{B_\theta}{B_\phi} n_e \left[ u_{e\theta} - \frac{B_\theta}{B_\phi} u_{e\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \end{aligned}$$

$$- 2\nu_L n_e u_{e\phi} - \nu_{0e} n_e u_{e\phi} \quad (1.9)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} n_e T_e = & -\frac{1}{r} \frac{\partial}{\partial r} r \left( \frac{5}{2} u_{er} n_e T_e - n_e \chi_e \frac{\partial}{\partial r} T_e \right) + u_{er} \frac{\partial}{\partial r} n_e T_e - e E_\theta n_e u_{e\theta} - e E_\phi n_e u_{e\phi} \\ & - \frac{3}{2} \nu_{Tei} n_e (T_e - T_i) - \nu_L T_e (n_e - n_{\text{ediv}}) - \frac{3}{2} \nu_{LT_e} n_e (T_e - T_{\text{ediv}}) \\ & - E_H \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) + f_{ei} P_b + P_{\alpha e} + P_{\text{RFe}} - P_{\text{Br}} \end{aligned} \quad (1.10)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_i = & -\frac{1}{r} \frac{\partial}{\partial r} r n_i u_{ir} + \frac{\nu_{\text{ionize}}}{Z_i} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \frac{\nu_L}{Z_i} (n_e - n_{\text{ediv}}) \\ & + \nu_b n_b + \nu_b g n_b^{\text{rp}} - f_{\text{CX}} \frac{P_b}{E_b} + \nu_{bL} n_b + S_{\text{LC}} - \nu_{\text{OL}} n_i \end{aligned} \quad (1.11)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_i u_{ir} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{ir} n_i u_{ir} + \frac{1}{r} u_{i\theta} n_i u_{i\theta} - \frac{\partial}{\partial r} \frac{n_i T_i}{m_i} \\ & + Z_i \frac{e}{m_i} n_i E_r + Z_i \frac{e}{m_i} B_\phi n_i u_{i\theta} - Z_i \frac{e}{m_i} B_\theta n_i u_{i\phi} \end{aligned} \quad (1.12)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_i u_{i\theta} = & -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u_{ir} n_i u_{i\theta} + \frac{1}{r^2} \frac{\partial}{\partial r} r^3 n_i \mu_i \frac{\partial}{\partial r} \frac{u_{i\theta}}{r} + Z_i \frac{e}{m_i} n_i E_\theta - Z_i \frac{e}{m_i} B_\phi n_i u_{ir} \\ & - \nu_{\text{NCi}} n_i u_{i\theta} - \frac{m_e}{m_i} \nu_{ei1} n_e (u_{i\theta} - u_{e\theta}) - \frac{m_e}{m_i} \nu_{ei2} n_e (u_{i\phi} - u_{e\phi}) - \frac{m_b}{m_i} \nu_{bi} n_b (u_{i\theta} - u_{b\theta}) \\ & + \frac{e^2 B_\phi^2 D_e}{m_i T_e} n_e \left[ u_{e\theta} - \frac{B_\theta}{B_\phi} u_{e\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \\ & - 2\nu_L n_i u_{i\theta} - \nu_{0i} n_i u_{i\theta} - \nu_{\text{CX}} n_i u_{i\theta} + S_{\text{LC}\theta} - \nu_{\text{OL}} n_i u_{i\theta} + F_{i\theta}^{\text{neo}} + T_{i\theta} \end{aligned} \quad (1.13)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_i u_{i\phi} = & -\frac{1}{r} \frac{\partial}{\partial r} r u_{ir} n_i u_{i\phi} + \frac{1}{r} \frac{\partial}{\partial r} r n_i \mu_i \frac{\partial}{\partial r} u_{i\phi} + Z_i \frac{e}{m_i} n_i E_\phi + Z_i \frac{e}{m_i} B_\theta n_i u_{ir} \\ & - \frac{m_e}{m_i} \nu_{ei3} n_e (u_{i\phi} - u_{e\phi}) - \frac{m_e}{m_i} \nu_{ei2} n_e (u_{i\theta} - u_{e\theta}) - \frac{m_b}{m_i} \nu_{bi} n_b (u_{i\phi} - u_{b\phi}) \\ & - \frac{e^2 B_\phi^2 D_e}{m_i T_e} \frac{B_\theta}{B_\phi} n_e \left[ u_{e\theta} - \frac{B_\theta}{B_\phi} u_{e\phi} + \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e r + \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial r} T_e + \frac{E_r}{B_\phi} \right] \\ & - 2\nu_L n_i u_{i\phi} - \nu_{0i} n_i u_{i\phi} - \nu_{\text{CX}} n_i u_{i\phi} + S_{\text{LC}\phi} - \nu_{\text{OL}} n_i u_{i\phi} + T_{i\phi} \end{aligned} \quad (1.14)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} n_i T_i = & -\frac{1}{r} \frac{\partial}{\partial r} r \left( \frac{5}{2} u_{ir} n_i T_i - n_i \chi_i \frac{\partial}{\partial r} T_i \right) + u_{ir} \frac{\partial}{\partial r} n_i T_i + Z_i e E_\theta n_i u_{i\theta} + Z_i e E_\phi n_i u_{i\phi} \\ & - \frac{3}{2} \nu_{Tei} n_e (T_i - T_e) + m_b \nu_b \frac{B_\theta u_{i\theta} + B_\phi u_{i\phi}}{B} \frac{P_{b\parallel}}{E_b} - \frac{\nu_L}{Z_i} T_i (n_e - n_{\text{ediv}}) \\ & - \frac{3}{2} \nu_{LT_i} n_i (T_i - T_{\text{idiv}}) + \frac{3}{2} \frac{\nu_{\text{ionize}}}{Z_i} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} T_{01} + n_{02} T_{02} + n_{03} T_{03}) \end{aligned}$$

$$-\frac{3}{2}\nu_{\text{CX}}n_{\text{i}}(T_{\text{i}} - T_{01}) + (1 - f_{\text{ei}})P_{\text{b}} + P_{\alpha\text{i}} + P_{\text{RFi}} \quad (1.15)$$

$$\frac{\partial}{\partial t}n_{\text{b}} = \frac{P_{\text{b}}}{E_{\text{b}}} - g\frac{P_{\text{b}\perp}}{E_{\text{b}}} - \nu_{\text{b}}n_{\text{b}} - \nu_{\text{bL}}n_{\text{b}} - \nu_{\text{eff}}g(\sqrt{\delta}n_{\text{b}} - n_{\text{b}}^{\text{rp}}) + \frac{1}{r}\frac{\partial}{\partial r}rf_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \quad (1.16)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{\text{b}}u_{\text{b}\theta} &= Z_{\text{b}}\frac{e}{m_{\text{b}}}n_{\text{b}}E_{\theta} - \nu_{\text{be1}}n_{\text{b}}(u_{\text{b}\theta} - u_{\text{e}\theta}) - \nu_{\text{be2}}n_{\text{b}}(u_{\text{b}\phi} - u_{\text{e}\phi}) - \nu_{\text{bi}}n_{\text{b}}(u_{\text{b}\theta} - u_{\text{i}\theta}) \\ &\quad - \nu_{0\text{b}}n_{\text{b}}u_{\text{b}\theta} - \nu_{\text{CX}}n_{\text{b}}u_{\text{b}\theta} + v_{\text{b}\theta}\frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}}n_{\text{b}}u_{\text{b}\theta} - \nu_{\text{eff}}g\sqrt{\delta}n_{\text{b}}u_{\text{b}\theta} \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}ru_{\text{b}\theta}f_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \end{aligned} \quad (1.17)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{\text{b}}u_{\text{b}\phi} &= Z_{\text{b}}\frac{e}{m_{\text{b}}}n_{\text{b}}E_{\phi} - \nu_{\text{be3}}n_{\text{b}}(u_{\text{b}\phi} - u_{\text{e}\phi}) - \nu_{\text{be2}}n_{\text{b}}(u_{\text{b}\theta} - u_{\text{e}\theta}) - \nu_{\text{bi}}n_{\text{b}}(u_{\text{b}\phi} - u_{\text{i}\phi}) \\ &\quad - \nu_{0\text{b}}n_{\text{b}}u_{\text{b}\phi} - \nu_{\text{CX}}n_{\text{b}}u_{\text{b}\phi} + v_{\text{b}\phi}\frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}}n_{\text{b}}u_{\text{b}\phi} - \nu_{\text{eff}}g\sqrt{\delta}n_{\text{b}}u_{\text{b}\phi} \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}ru_{\text{b}\phi}f_{\text{t}}D_{\text{b}}\frac{\partial}{\partial r}n_{\text{b}} \end{aligned} \quad (1.18)$$

$$\begin{aligned} \frac{\partial}{\partial t}n_{01} &= \frac{1}{r}\frac{\partial}{\partial r}rD_{01}\frac{\partial}{\partial r}n_{01} - \frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}}n_{01} - \nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01} + n_{02}}n_{01} \\ &\quad + \gamma_0\frac{\nu_{\text{L}}}{Z_{\text{i}}}(n_{\text{e}} - n_{\text{ediv}}) + \frac{1}{r}\frac{\partial}{\partial r}r\Gamma^{\text{puff}}\Big|_{\text{b}} \end{aligned} \quad (1.19)$$

$$\frac{\partial}{\partial t}n_{02} = \frac{1}{r}\frac{\partial}{\partial r}rD_{02}\frac{\partial}{\partial r}n_{02} - \frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}}n_{02} + \nu_{\text{CX}}\frac{n_{\text{i}}}{n_{01} + n_{02}}n_{01} \quad (1.20)$$

$$\frac{\partial}{\partial t}n_{03} = \frac{1}{r}\frac{\partial}{\partial r}rD_{03}\frac{\partial}{\partial r}n_{03} - \frac{1}{Z_{\text{i}}}\nu_{\text{ionize}}\frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}}n_{03} + f_{\text{CX}}\frac{P_{\text{b}}}{E_{\text{b}}} \quad (1.21)$$

$$\frac{\partial}{\partial t}n_{\text{b}}^{\text{rp}} = \frac{P_{\text{b}\perp}}{E_{\text{b}}} + \nu_{\text{eff}}(\sqrt{\delta}n_{\text{b}} - n_{\text{b}}^{\text{rp}}) - \nu_{\text{b}}n_{\text{b}}^{\text{rp}} - \frac{1}{r}\frac{\partial}{\partial r}\left(ru_{\text{b}}^{\text{rp}}n_{\text{b}}^{\text{rp}} - rD_{\text{b}}^{\text{rp}}\frac{\partial}{\partial r}n_{\text{b}}^{\text{rp}}\right) \quad (1.22)$$

## 2 Relations Between Fields and Potentials

$$E_r = -\frac{\partial}{\partial r}\phi \quad (2.1)$$

$$E_{\theta} = -\frac{\partial}{\partial t}A_{\theta} = -\dot{A}_{\theta} \quad (2.2)$$

$$E_{\phi} = -\frac{\partial}{\partial t}A_{\phi} = -\dot{A}_{\phi} \quad (2.3)$$

$$B_\theta = -\frac{\partial}{\partial r} A_\phi \quad (2.4)$$

$$B_\phi = \frac{1}{r} \frac{\partial}{\partial r} r A_\theta \quad (2.5)$$

### 3 Equations in $s(=r^2)$ coordinates

$$0 = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} \phi + \frac{1}{\epsilon_0} (-e n_e + Z_i e n_i + Z_b e n_b + Z_b e g n_b^{\text{rp}}) \quad (3.1)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} r \dot{A}_\theta = \left( 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} r A_\theta - 4 \frac{\partial}{\partial s} r A_\theta \right) + \mu_0 (-e r n_e u_{e\theta} + Z_i e r n_i u_{i\theta} + Z_b e r n_b u_{b\theta}) \quad (3.2)$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \dot{A}_\phi = 4 \frac{\partial}{\partial s} s \frac{\partial}{\partial s} A_\phi + \mu_0 (-e n_e u_{e\phi} + Z_i e n_i u_{i\phi} + Z_b e n_b u_{b\phi}) \quad (3.3)$$

$$\frac{\partial}{\partial t} A_\phi = \dot{A}_\phi \quad (3.4)$$

$$\frac{\partial}{\partial t} r A_\theta = r \dot{A}_\theta \quad (3.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} n_e &= -2 \frac{\partial}{\partial s} r n_e u_{er} + \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \nu_L (n_e - n_{\text{ediv}}) \\ &+ (1 - f_{\text{CX}}) \frac{P_b}{E_b} \end{aligned} \quad (3.6)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_e u_{er} &= -2r \frac{\partial}{\partial s} u_{er} r n_e u_{er} + \frac{u_{e\theta}}{r} r n_e u_{e\theta} - 2s \frac{\partial}{\partial s} \frac{n_e T_e}{m_e} \\ &+ 2 \frac{e}{m_e} s n_e \frac{\partial}{\partial s} \phi - \frac{e}{m_e} B_\phi r n_e u_{e\theta} - 2 \frac{e}{m_e} s \frac{\partial A_\phi}{\partial s} n_e u_{e\phi} \end{aligned} \quad (3.7)$$

$$\begin{aligned} \frac{\partial}{\partial t} r n_e u_{e\theta} &= -2 \frac{\partial}{\partial s} r u_{er} r n_e u_{e\theta} + \left( 4 \frac{\partial}{\partial s} s \mu_e \frac{\partial}{\partial s} r n_e u_{e\theta} - 4 \frac{\partial}{\partial s} s \mu_e r u_{e\theta} \frac{\partial n_e}{\partial s} - 4 \frac{\partial}{\partial s} \mu_e r n_e u_{e\theta} \right) \\ &+ \frac{e}{m_e} n_e r \dot{A}_\theta + \frac{e}{m_e} B_\phi r n_e u_{er} - \nu_{\text{NCe}} r n_e u_{e\theta} - \nu_{\text{eil}} r n_e (u_{e\theta} - u_{i\theta}) + 2s \frac{\partial A_\phi}{\partial s} \frac{\nu_{\text{ei2}}}{B_\theta} n_e (u_{e\phi} - u_{i\phi}) \\ &- \frac{m_b}{m_e} \nu_{\text{be1}} r n_b (u_{e\theta} - u_{b\theta}) + 2 \frac{m_b}{m_e} s \frac{\partial A_\phi}{\partial s} \frac{\nu_{\text{be2}}}{B_\theta} n_e (u_{e\phi} - u_{b\phi}) \\ &- \frac{e^2 B_\phi^2 D_e}{m_e T_e} n_e \left[ r u_{e\theta} + 2 \frac{\partial A_\phi}{\partial s} \frac{s}{B_\phi} u_{e\phi} + s \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2s \frac{1 - \alpha_e}{e B_\phi} \frac{\partial}{\partial s} T_e - 2s \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] \\ &- 2\nu_L r n_e u_{e\theta} - \nu_{0e} r n_e u_{e\theta} + r F_{e\theta}^{\text{neo}} \end{aligned} \quad (3.8)$$

$$\begin{aligned}
\frac{\partial}{\partial t} n_e u_{e\phi} = & -2 \frac{\partial}{\partial s} r u_{er} n_e u_{e\phi} + \left( 4 \frac{\partial}{\partial s} s \mu_e \frac{\partial}{\partial s} n_e u_{e\phi} - 4 \frac{\partial}{\partial s} s \mu_e u_{e\phi} \frac{\partial n_e}{\partial s} \right) \\
& + \frac{e}{m_e} n_e \dot{A}_\phi + 2 \frac{e}{m_e} \frac{\partial A_\phi}{\partial s} r n_e u_{er} - \nu_{ei3} n_e (u_{e\phi} - u_{i\phi}) + 2 \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} r n_e (u_{e\theta} - u_{i\theta}) \\
& + 2 \frac{m_b}{m_e} \frac{\partial A_\phi}{\partial s} \frac{\nu_{be2}}{B_\theta} r n_e (u_{e\theta} - u_{i\theta}) - \frac{m_b}{m_e} \nu_{be3} n_b (u_{e\phi} - u_{b\phi}) \\
& - 2 \frac{e^2 B_\phi D_e}{m_e T_e} \frac{\partial A_\phi}{\partial s} \left[ r n_e u_{e\theta} + s n_e \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2s \frac{1 - \alpha_e}{e B_\phi} \left( \frac{\partial}{\partial s} n_e T_e - T_e \frac{\partial}{\partial s} n_e \right) \right. \\
& \left. - 2s n_e \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] - \frac{e^2 B_\phi^2 D_e}{m_e T_e} \left( \frac{B_\theta}{B_\phi} \right)^2 n_e u_{e\phi} - 2\nu_L n_e u_{e\phi} - \nu_{0e} n_e u_{e\phi}
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\frac{\partial}{\partial t} \frac{3}{2} n_e T_e = & -5 \frac{\partial}{\partial s} r u_{er} n_e T_e + \left( 4 \frac{\partial}{\partial s} s \chi_e \frac{\partial}{\partial s} n_e T_e - 4 \frac{\partial}{\partial s} s \chi_e T_e \frac{\partial n_e}{\partial s} \right) + 2 r u_{er} \frac{\partial}{\partial s} n_e T_e \\
& - e \frac{E_\theta}{r} r n_e u_{e\theta} - e E_\phi n_e u_{e\phi} - \frac{3}{2} \nu_{Tei} n_e (T_e - T_i) \\
& - \nu_L T_e (n_e - n_{ediv}) - \frac{3}{2} \nu_{LT_e} n_e (T_e - T_{ediv}) \\
& - E_H \nu_{ionize} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) + f_{ei} P_b + P_{ae} + P_{RFe} - P_{Br}
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
\frac{\partial}{\partial t} n_i = & -2 \frac{\partial}{\partial s} r n_i u_{ir} + \frac{\nu_{ionize}}{Z_i} \frac{n_e}{n_{01} + n_{02} + n_{03}} (n_{01} + n_{02} + n_{03}) - \frac{\nu_L}{Z_i} (n_e - n_{ediv}) \\
& + \nu_b n_b + \nu_b g n_b^{rp} - f_{CX} \frac{P_b}{E_b} + \nu_{bL} n_b + S_{LC} - \nu_{OL} n_i
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\frac{\partial}{\partial t} r n_i u_{ir} = & -2r \frac{\partial}{\partial s} u_{ir} r n_i u_{ir} + \frac{u_{e\theta}}{r} r n_i u_{i\theta} - 2s \frac{\partial}{\partial s} \frac{n_i T_i}{m_i} \\
& - 2Z_i \frac{e}{m_i} s n_i \frac{\partial}{\partial s} \phi + Z_i \frac{e}{m_i} B_\phi r n_i u_{i\theta} + 2Z_i \frac{e}{m_i} s \frac{\partial A_\phi}{\partial s} n_i u_{i\phi}
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\frac{\partial}{\partial t} r n_i u_{i\theta} = & -2 \frac{\partial}{\partial s} r u_{ir} r n_i u_{i\theta} + \left( 4 \frac{\partial}{\partial s} s \mu_i \frac{\partial}{\partial s} r n_i u_{i\theta} - 4 \frac{\partial}{\partial s} s \mu_i r u_{i\theta} \frac{\partial n_i}{\partial s} - 4 \frac{\partial}{\partial s} \mu_i r n_i u_{i\theta} \right) \\
& - Z_i \frac{e}{m_i} n_i r \dot{A}_\theta - Z_i \frac{e}{m_i} B_\phi r n_i u_{ir} - \nu_{NCi} r n_i u_{i\theta} - \frac{m_e}{m_i} \nu_{ei1} r n_e (u_{i\theta} - u_{e\theta}) \\
& + 2 \frac{m_e}{m_i} s \frac{\partial A_\phi}{\partial s} \frac{\nu_{ei2}}{B_\theta} n_e (u_{i\phi} - u_{e\phi}) - \frac{m_b}{m_i} \nu_{bi} r n_b (u_{i\theta} - u_{b\theta}) \\
& + \frac{e^2 B_\phi^2 D_e}{m_i T_e} \left[ r n_e u_{e\theta} + 2 \frac{\partial A_\phi}{\partial s} \frac{s}{B_\phi} n_e u_{e\phi} + s n_e \frac{B}{B_\phi} \left\langle \frac{\omega}{m} \right\rangle_e + 2s \frac{1 - \alpha_e}{e B_\phi} \left( \frac{\partial}{\partial s} n_e T_e - T_e \frac{\partial}{\partial s} n_e \right) \right. \\
& \left. - 2s n_e \frac{1}{B_\phi} \frac{\partial}{\partial s} \phi \right] - 2\nu_L r n_i u_{i\theta} - \nu_{0i} r n_i u_{i\theta} - \nu_{CX} r n_i u_{i\theta}
\end{aligned}$$

$$+ rS_{\text{LC}\theta} - \nu_{\text{OL}} \textcolor{red}{r}n_{\text{i}}u_{\text{i}\theta} + F_{\text{i}\theta}^{\text{neo}} + T_{\text{i}\theta} \quad (3.13)$$

$$\begin{aligned} \frac{\partial}{\partial t} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} = & -2 \frac{\partial}{\partial s} r u_{\text{ir}} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} + \left( 4 \frac{\partial}{\partial s} s \mu_{\text{i}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} - 4 \frac{\partial}{\partial s} s \mu_{\text{i}} u_{\text{i}\phi} \frac{\partial \textcolor{red}{n}_{\text{i}}}{\partial s} \right) - Z_{\text{i}} \frac{e}{m_{\text{i}}} n_{\text{i}} \dot{\textcolor{red}{A}}_{\phi} - 2 Z_{\text{i}} \frac{e}{m_{\text{i}}} \frac{\partial A_{\phi}}{\partial s} \textcolor{blue}{r}n_{\text{i}}u_{\text{ir}} \\ & - \frac{m_{\text{e}}}{m_{\text{i}}} \nu_{\text{ei}3} \textcolor{red}{n}_{\text{e}}(u_{\text{i}\phi} - u_{\text{e}\phi}) + 2 \frac{m_{\text{e}}}{m_{\text{i}}} \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{ei}2}}{B_{\theta}} \textcolor{red}{r}n_{\text{e}}(u_{\text{i}\theta} - u_{\text{e}\theta}) - \frac{m_{\text{b}}}{m_{\text{i}}} \nu_{\text{bi}} n_{\text{b}}(u_{\text{i}\phi} - u_{\text{b}\phi}) \\ & + 2 \frac{e^2 B_{\phi} D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} \frac{\partial A_{\phi}}{\partial s} \left[ \textcolor{red}{r}n_{\text{e}}u_{\text{e}\theta} + s \textcolor{red}{n}_{\text{e}} \frac{B}{B_{\phi}} \left\langle \frac{\omega}{m} \right\rangle_{\text{e}} + 2s \frac{1 - \alpha_{\text{e}}}{e B_{\phi}} \left( \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{e}} T_{\text{e}} - T_{\text{e}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{e}} \right) \right. \\ & \left. - 2s \textcolor{red}{n}_{\text{e}} \frac{1}{B_{\phi}} \frac{\partial}{\partial s} \phi \right] + \frac{e^2 B_{\phi}^2 D_{\text{e}}}{m_{\text{i}} T_{\text{e}}} \left( \frac{B_{\theta}}{B_{\phi}} \right)^2 \textcolor{red}{n}_{\text{e}}u_{\text{e}\phi} - 2\nu_{\text{L}} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} - \nu_{0\text{i}} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} - \nu_{\text{CX}} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} \\ & + S_{\text{LC}\phi} - \nu_{\text{OL}} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} + T_{\phi,\text{i}} \end{aligned} \quad (3.14)$$

$$\begin{aligned} \frac{\partial}{\partial t} \frac{3}{2} \textcolor{red}{n}_{\text{i}}T_{\text{i}} = & -5 \frac{\partial}{\partial s} r u_{\text{er}} \textcolor{red}{n}_{\text{e}}T_{\text{e}} + \left( 4 \frac{\partial}{\partial s} s \chi_{\text{e}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{i}}T_{\text{i}} - 4 \frac{\partial}{\partial s} s \chi_{\text{i}} T_{\text{i}} \frac{\partial \textcolor{red}{n}_{\text{i}}}{\partial s} \right) + 2 r u_{\text{ir}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{i}}T_{\text{i}} \\ & + Z_{\text{i}} e \frac{E_{\theta}}{r} \textcolor{red}{r}n_{\text{i}}u_{\text{i}\theta} + Z_{\text{i}} e E_{\phi} \textcolor{red}{n}_{\text{i}}u_{\text{i}\phi} - \frac{3}{2} \nu_{\text{Tei}} \textcolor{red}{n}_{\text{e}}(T_{\text{i}} - T_{\text{e}}) + m_{\text{b}} v_{\text{b}} \frac{B_{\theta} u_{\text{i}\theta} + B_{\phi} u_{\text{i}\phi}}{B} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} \\ & - \frac{\nu_{\text{L}}}{Z_{\text{i}}} T_{\text{i}}(n_{\text{e}} - n_{\text{ediv}}) - \frac{3}{2} \nu_{\text{LTi}} \textcolor{red}{n}_{\text{i}}(T_{\text{i}} - T_{\text{idiv}}) \\ & + \frac{3}{2} \frac{\nu_{\text{ionize}}}{Z_{\text{i}}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} (\textcolor{red}{n}_{01} T_{01} + \textcolor{red}{n}_{02} T_{02} + \textcolor{red}{n}_{03} T_{03}) - \frac{3}{2} \nu_{\text{CX}} \textcolor{red}{n}_{\text{i}}(T_{\text{i}} - T_{01}) \\ & + (1 - f_{\text{ei}}) P_{\text{b}} + P_{\alpha\text{i}} + P_{\text{RFi}} \end{aligned} \quad (3.15)$$

$$\frac{\partial}{\partial t} \textcolor{red}{n}_{\text{b}} = \frac{P_{\text{b}}}{E_{\text{b}}} - g \frac{P_{\text{b}\perp}}{E_{\text{b}}} - \nu_{\text{b}} \textcolor{red}{n}_{\text{b}} - \nu_{\text{bL}} \textcolor{red}{n}_{\text{b}} - \nu_{\text{eff}} g (\sqrt{\delta} \textcolor{red}{n}_{\text{b}} - \textcolor{red}{n}_{\text{b}}^{\text{rp}}) + 4 \frac{\partial}{\partial s} s f_{\text{t}} D_{\text{b}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{b}} \quad (3.16)$$

$$\begin{aligned} \frac{\partial}{\partial t} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} = & -Z_{\text{b}} \frac{e}{m_{\text{b}}} n_{\text{b}} \dot{\textcolor{red}{A}}_{\theta} - \nu_{\text{be}1} \textcolor{red}{r}n_{\text{b}}(u_{\text{b}\theta} - u_{\text{e}\theta}) + 2s \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{be}2}}{B_{\theta}} \textcolor{red}{n}_{\text{b}}(u_{\text{b}\phi} - u_{\text{e}\phi}) - \nu_{\text{bi}} \textcolor{red}{r}n_{\text{b}}(u_{\text{b}\theta} - u_{\text{i}\theta}) \\ & - \nu_{0\text{b}} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} - \nu_{\text{CX}} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} + r v_{\text{b}\theta} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} - \nu_{\text{eff}} g \sqrt{\delta} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} \\ & + 4 \frac{\partial}{\partial s} s f_{\text{t}} D_{\text{b}} \frac{\partial}{\partial s} \textcolor{red}{r}n_{\text{b}}u_{\text{b}\theta} - 4 \frac{\partial}{\partial s} \left( s f_{\text{t}} D_{\text{b}} \frac{\partial r u_{\text{b}\theta}}{\partial s} \right) \textcolor{red}{n}_{\text{b}} \end{aligned} \quad (3.17)$$

$$\begin{aligned} \frac{\partial}{\partial t} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} = & -Z_{\text{b}} \frac{e}{m_{\text{b}}} n_{\text{b}} \dot{\textcolor{red}{A}}_{\phi} - \nu_{\text{be}3} \textcolor{red}{n}_{\text{b}}(u_{\text{b}\phi} - u_{\text{e}\phi}) + 2 \frac{\partial A_{\phi}}{\partial s} \frac{\nu_{\text{be}2}}{B_{\theta}} \textcolor{red}{n}_{\text{b}}(u_{\text{b}\theta} - u_{\text{e}\theta}) - \nu_{\text{bi}} \textcolor{red}{n}_{\text{b}}(u_{\text{b}\phi} - u_{\text{i}\phi}) \\ & - \nu_{0\text{b}} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} - \nu_{\text{CX}} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} + v_{\text{b}\phi} \frac{P_{\text{b}\parallel}}{E_{\text{b}}} - \nu_{\text{bL}} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} - \nu_{\text{eff}} g \sqrt{\delta} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} \\ & + 4 \frac{\partial}{\partial s} s f_{\text{t}} D_{\text{b}} \frac{\partial}{\partial s} \textcolor{red}{n}_{\text{b}}u_{\text{b}\phi} - 4 \frac{\partial}{\partial s} \left( s f_{\text{t}} D_{\text{b}} \frac{\partial u_{\text{b}\phi}}{\partial s} \right) \textcolor{red}{n}_{\text{b}} \end{aligned} \quad (3.18)$$

$$\frac{\partial}{\partial t} \textcolor{red}{n}_{01} = 4 \frac{\partial}{\partial s} s D_{01} \frac{\partial}{\partial s} \textcolor{red}{n}_{01} - \frac{1}{Z_{\text{i}}} \nu_{\text{ionize}} \frac{n_{\text{e}}}{n_{01} + n_{02} + n_{03}} \textcolor{red}{n}_{01} - \nu_{\text{CX}} \frac{n_{\text{i}}}{n_{01} + n_{02}} \textcolor{red}{n}_{01}$$

$$+ \gamma_0 \frac{\nu_L}{Z_i} (n_e - n_{\text{ediv}}) + 2 \frac{\partial}{\partial s} r \Gamma^{\text{puff}} \Big|_{\text{b}} \quad (3.19)$$

$$\frac{\partial}{\partial t} n_{02} = 4 \frac{\partial}{\partial s} s D_{02} \frac{\partial}{\partial s} n_{02} - \frac{1}{Z_i} \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} n_{02} + \nu_{\text{CX}} \frac{n_i}{n_{01} + n_{02}} n_{01} \quad (3.20)$$

$$\frac{\partial}{\partial t} n_{03} = 4 \frac{\partial}{\partial s} s D_{03} \frac{\partial}{\partial s} n_{03} - \frac{1}{Z_i} \nu_{\text{ionize}} \frac{n_e}{n_{01} + n_{02} + n_{03}} n_{03} + f_{\text{CX}} \frac{P_b}{E_b} \quad (3.21)$$

$$\frac{\partial}{\partial t} n_b^{\text{rp}} = \frac{P_{b\perp}}{E_b} + \nu_{\text{eff}} (\sqrt{\delta} n_b - n_b^{\text{rp}}) - \nu_b n_b^{\text{rp}} - 2 \frac{\partial}{\partial s} r u_b^{\text{rp}} n_b^{\text{rp}} + 4 \frac{\partial}{\partial s} s D_b^{\text{rp}} \frac{\partial}{\partial s} n_b^{\text{rp}} \quad (3.22)$$

## 4 Boundary Conditions

num.	name	variable	center	edge
1	LQm1	$\phi$	N	0
2	LQm2	$r \dot{A}_\theta$	0	$2s_b B_V$
3	LQm3	$\dot{A}_\phi$	N	$-2b B_{\theta b}$
4	LQm4	$A_\phi$	$\times$	$\times$
5	LQm5	$r A_\theta$	$\times$	$\times$
6	LQe1	$n_e$	N	N( $\times$ )
7	LQe2	$r n_e u_{er}$	0	$\times(0)$
8	LQe3	$r n_e u_{e\theta}$	0	0
9	LQe4	$n_e u_{e\phi}$	N	0
10	LQe5	$n_e T_e$	N	N
11	LQi1	$n_i$	N	N( $\times$ )
12	LQi2	$r n_i u_{ir}$	0	$\times(0)$
13	LQi3	$r n_i u_{i\theta}$	0	0
14	LQi4	$n_i u_{i\phi}$	N	0
15	LQi5	$n_i T_i$	N	N
16	LQb1	$n_b$	$\times(\text{N})$	$\times(\text{N})$
17	LQb3	$r n_b u_{b\theta}$	$\times(0)$	$\times(0)$
18	LQb4	$n_b u_{b\phi}$	$\times(\text{N})$	$\times(0)$
19	LQn1	$n_{01}$	N	$\times(2b S_{\text{gas}})$
20	LQn2	$n_{02}$	N	0
21	LQn3	$n_{03}$	N	0
22	LQr1	$n_b^{\text{rp}}$	$\times(\text{N})$	$\times(\text{N})$

0: Dirichlet condition ( $u = 0$ )

N: Neumann condition ( $u' = 0$ )

value: Neumann condition ( $u' = \text{value}$ )

$\times$ : no condition

Excess BCs are imposed on the equations LQe1 and LQi1, from the aspect of the order of their derivative. However, these BCs must be required to keep quasi-neutrality.