Forecasting with ARIMA models

FISH 550 – Applied Time Series Analysis   Download Rmd

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Forecasting with an ARIMA model

The basic idea of forecasting with an ARIMA model to estimate the parameters and forecast forward.

For example, let's say we want to forecast with this ARIMA(2,1,0) model:

$$y_t = \mu + \beta_1 y_{t-1} + \beta_2 y_{t-2} + e_t$$

where $y_t = x_t - x_{t-1}$, the first difference.

`Arima()` would write this model:

$$(y_t - m) = \beta_1(y_{t-1} - m) + \beta_2(y_{t-2} - m) + e_t$$

The relationship between μ and m is $\mu = m(1 - \beta_1 - \beta_2)$.

Let's estimate the β's for this model from the anchovy data.

```
fit <- forecast::Arima(anchovyts, order=c(2,1,0), include.c
coef(fit)
```

```
##        ar1         ar2       drift
## -0.53850433 -0.44732522  0.05367062
```

```
mu <- coef(fit)[3]*(1-coef(fit)[1]-coef(fit)[2])
mu
```

```
##     drift
## 0.1065807
```

So we will forecast with this model:

$$y_t = 0.1065807 - 0.53850433 y_{t-1} - 0.44732522 y_{t-2} + e_t$$

To get our forecast for 1990, we do this

$$(x_{90} - x_{89}) = 0.106 - 0.538(x_{89} - x_{88}) - 0.447(x_{88} - x_{87})$$

Thus

$$x_{90} = x_{89} + 0.106 - 0.538(x_{89} - x_{88}) - 0.447(x_{88} - x_{87})$$

Here is R code to do that:

```
anchovyts[26]+mu+coef(fit)[1]*(anchovyts[26]-anchovyts[25])
  coef(fit)[2]*(anchovyts[25]-anchovyts[24])
```

```
##    drift
## 9.962083
```

Forecasting with forecast()

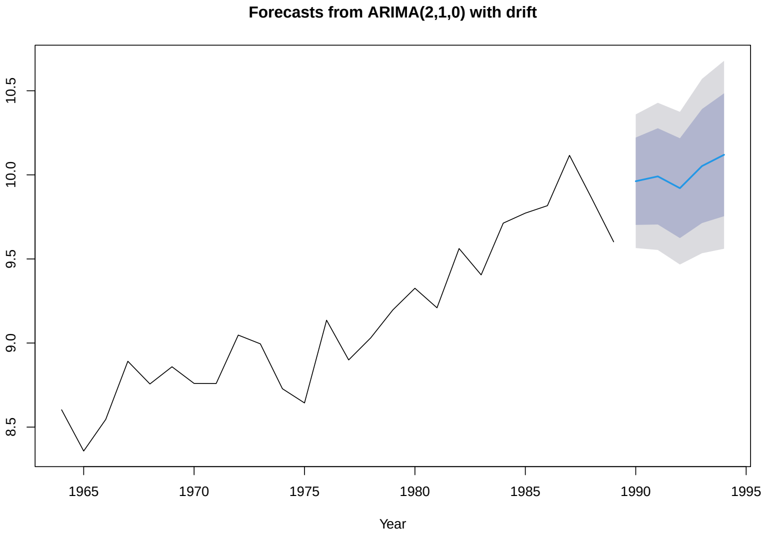
`forecast(fit, h=h)` automates the forecast calculations for us and computes the upper and lower prediction intervals. Prediction intervals include uncertainty in parameter estimates plus the process error uncertainty.

```
fr <- forecast::forecast(fit, h=5)
fr
```

```
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 1990       9.962083 9.702309 10.22186 9.564793 10.35937
## 1991       9.990922 9.704819 10.27703 9.553365 10.42848
## 1992       9.920798 9.623984 10.21761 9.466861 10.37473
## 1993      10.052240 9.713327 10.39115 9.533917 10.57056
## 1994      10.119407 9.754101 10.48471 9.560719 10.67809
```

Plotting our forecasts

```
plot(fr, xlab="Year")
```



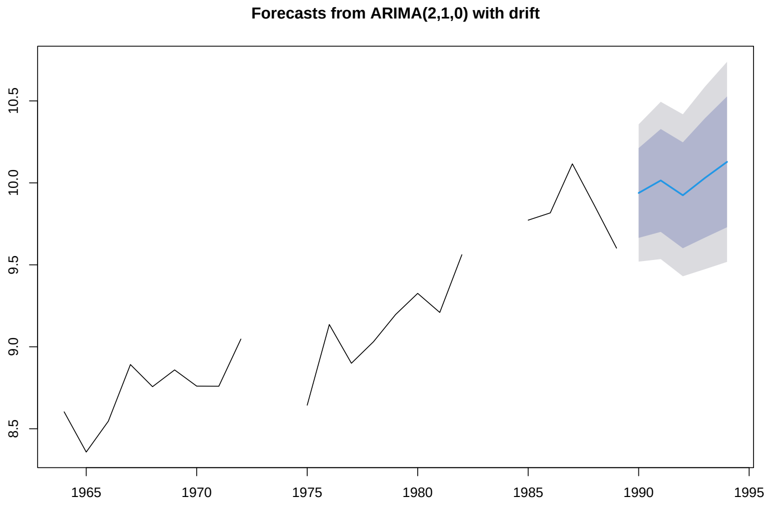
Missing values

Missing values are allowed for `forecast::Arima()`. We can produce forecasts with the same code.

```
anchovy.miss <- anchovyts
anchovy.miss[10:11] <- NA
anchovy.miss[20:21] <- NA
fit <- forecast::Arima(anchovy.miss, order=c(2,1,0), incluc
fr <- forecast::forecast(fit, h=5)
fr
```

```
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 1990       9.938269 9.664479 10.21206 9.519543 10.35700
## 1991      10.014686 9.700961 10.32841 9.534885 10.49449
## 1992       9.924208 9.601147 10.24727 9.430129 10.41829
## 1993      10.029988 9.666069 10.39391 9.473421 10.58656
## 1994      10.128066 9.729066 10.52707 9.517848 10.73828
```

```
plot(fr)
```



Forecasts from ARIMA(2,1,0) with drift
8.5
9.0
9.5
10.0
10.5
1965
1970
1975
1980
1985
1990
1995

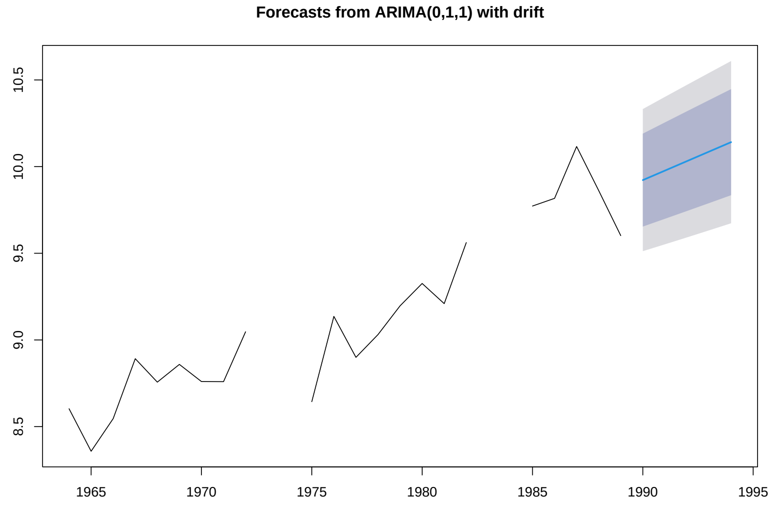
Using auto.arima()

We can let forecast to select the ARIMA model:

```
anchovy.miss <- anchovyts
anchovy.miss[10:11] <- NA
anchovy.miss[20:21] <- NA
fit <- forecast::auto.arima(anchovy.miss)
fit
```

```
## Series: anchovy.miss
## ARIMA(0,1,1) with drift
##
## Coefficients:
##           ma1   drift
##       -0.7240  0.0548
## s.e.   0.2283  0.0125
##
## sigma^2 = 0.04355:  log likelihood = 3.52
## AIC=-1.03   AICc=0.11   BIC=2.63
```

```r
fr <- forecast::forecast(fit, h=5)
plot(fr)
```



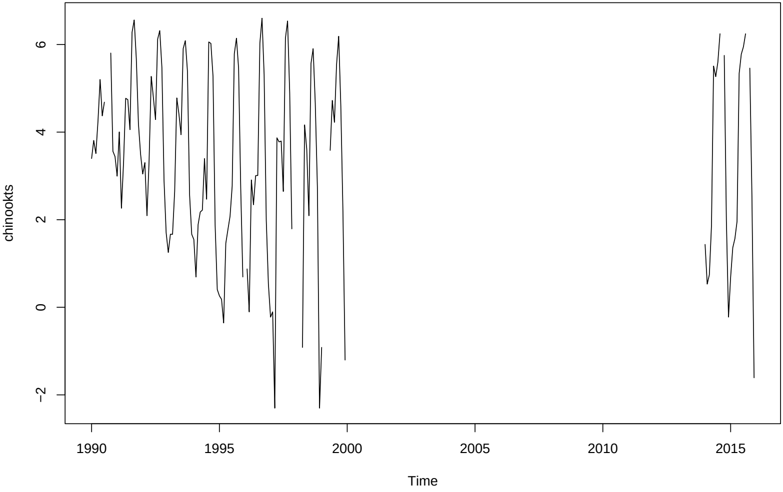
Forecasting with a Seasonal model

Load the chinook salmon data

```
load("chinook.RData")
chinookts <- ts(chinook$log.metric.tons, start=c(1990,1),
                frequency=12)
```

Plot seasonal data

```
plot(chinookts)
```



Seasonal ARIMA model

Seasonally differenced data, e.g. chinook data January 1990 - chinook data January 1989.

$$z_t = x_t - x_{t+s} - m$$

Basic structure of a seasonal AR model

z_t = AR(p) + AR(season) + AR(p+season)

e.g. z_t = AR(1) + AR(12) + AR(1+12)

Example AR(1) non-seasonal part + AR(1) seasonal part

$$z_t = \phi_1 z_{t-1} + \Phi_1 z_{t-12} - \phi_1 \Phi_1 z_{t-13}$$

Notation

ARIMA (p,d,q)(ps,ds,qs)S

ARIMA (1,0,0)(1,1,0)[12]

auto.arima() for seasonal ts

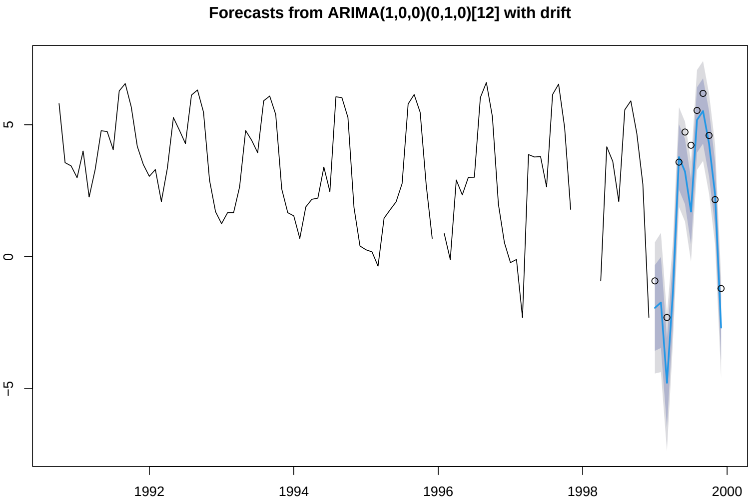
auto.arima() will recognize that our data has season and fit a seasonal ARIMA model to our data by default. We will define the training data up to 1998 and use 1999 as the test data.

```
traindat <- window(chinookts, c(1990,10), c(1998,12))
testdat <- window(chinookts, c(1999,1), c(1999,12))
fit <- forecast::auto.arima(traindat)
fit
```

```
## Series: traindat
## ARIMA(1,0,0)(0,1,0)[12] with drift
##
## Coefficients:
##          ar1    drift
##       0.3676  -0.0320
## s.e.  0.1335   0.0127
##
## sigma^2 = 0.8053:  log likelihood = -107.37
## AIC=220.73   AICc=221.02   BIC=228.13
```

Forecast using seasonal model

```
fr <- forecast::forecast(fit, h=12)
plot(fr)
points(testdat)
```



Missing values

Missing values are ok when fitting a seasonal ARIMA model