Fitting spatial and spatiotemporal models - continued

FISH 507 – Applied Time Series Analysis

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4 Mar 2021

Where we left off

- Many approaches we've talked about were forms of spatial regression

$$Y_i = \mathbf{b}X_i + \phi_i + \varepsilon_i$$

- ϕ is a vector of random effects describing a spatial field / process
- Is this a state space model?

Gaussian predictive process model

- Instead of estimating giant covariance matrix describing ϕ,
- Estimate smaller random effects at a subset of points ϕ^*
- Use some covariance kernel, and distances between points describing ϕ and ϕ^* to project estimates
- Can be done in ML or Bayesian framework, Latimer et al. 2009

Method 3: glmmfields

- ‘glmmfields’ implements these models using Stan backend
- Easy to use formula syntax like gam() or spBayes()
- Allows spatial field to be modeled with Multivariate-T field, as alternative to Multivariate-Normal to better capture extremes
- Also includes lots of non-normal families for observation model

glmmfields

Let's start with a simple model. This includes only 6 knots (probably way too few) and 100 iterations (too few!) for run time.

- Time set to NULL here to fit a model with constant spatial field

```
m <- glmmfields(Feb ~ 0, time = NULL,
 lat = "Latitude", lon = "Longitude", data = d,
 nknots = 6, iter = 100, chains = 2, seed = 1)
```

glmmfields

Ok, now let's change the covariance function to matern, and estimate separate spatial fields by year.

- The 'estimate_ar' argument is important for determining whether the field is an AR(1) process or independent by year

```
m <- glmmfields(Feb ~ 0, time = "Water.Year",
 lat = "Latitude", lon = "Longitude", covariance="matern",
  data = d, nknots = 6, iter = 100, chains = 2, seed = 1,
  estimate_ar=FALSE)
```

glmmfields

Finally let's include an example with modeling the spatial field as a Multivariate-t distribution

- We do this with the 'estimate_df' argument

```
m <- glmmfields(Feb ~ 0, time = "Water.Year",
 lat = "Latitude", lon = "Longitude", covariance="matern",
  data = d, nknots = 6, iter = 100, chains = 2, seed = 1,
  estimate_AR=FALSE, estimate_df= TRUE)
```

Method 4: INLA

GP spatial models like glmmfields are extremely powerful

- May get overwhelmed by number of points
- Approaches to incorporate knots using sparse covariance matrices
- Integrated Nested Laplace Approximation - not on CRAN

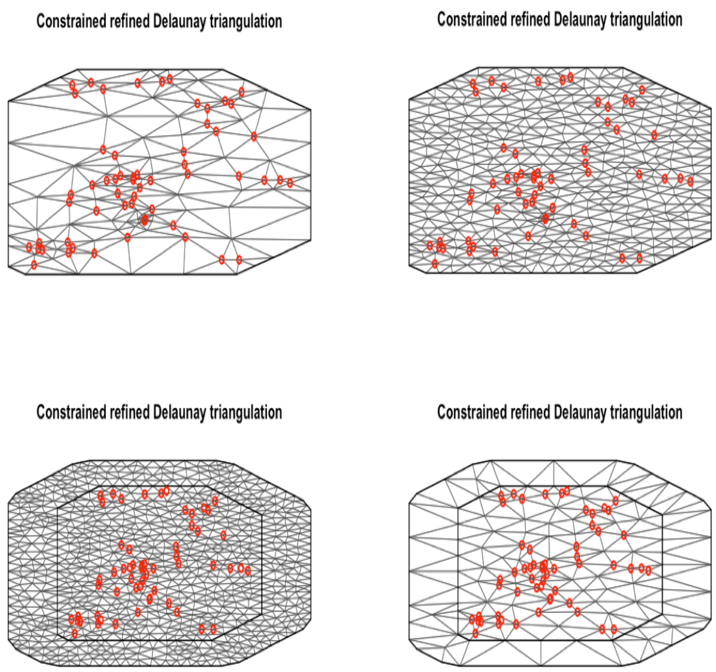
Motivation of INLA

- Some problems contain simple spatial structure
  - e.g. the harbor seal data in MARSS() with only 5 – 7 time series
- Others are much more complex
  - WA SWE data
  - Fisheries survey data (1000s of points)
- Including time-varying spatial fields becomes very computationally difficult
- Doing all of the above in a Bayesian setting can be prohibitive, but we can use Laplace approximation

INLA's approximation: SNOTEL data

- INLA meshes describe resolution of surface to estimate spatial process

`INLA::meshbuilder`



INLA's approximation: SNOTEL data

- How many points fall on vertices?
- Is the boundary area large enough?
- Choosing this must be done very carefully!

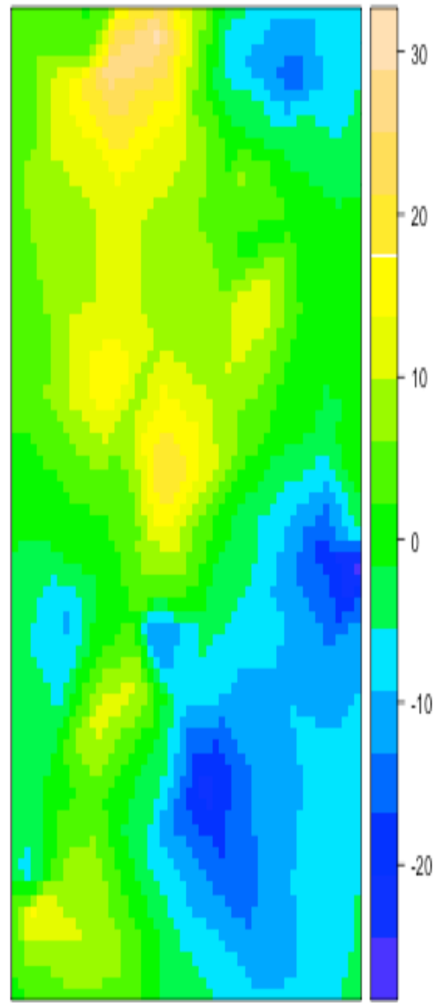
Estimation done via maximum likelihood

- Estimates seems similar to those from gls() and spBayes()
- Year included as numeric here (not significant)
- Alternatively, we can include year in spatial field
- Year can also be included as factor

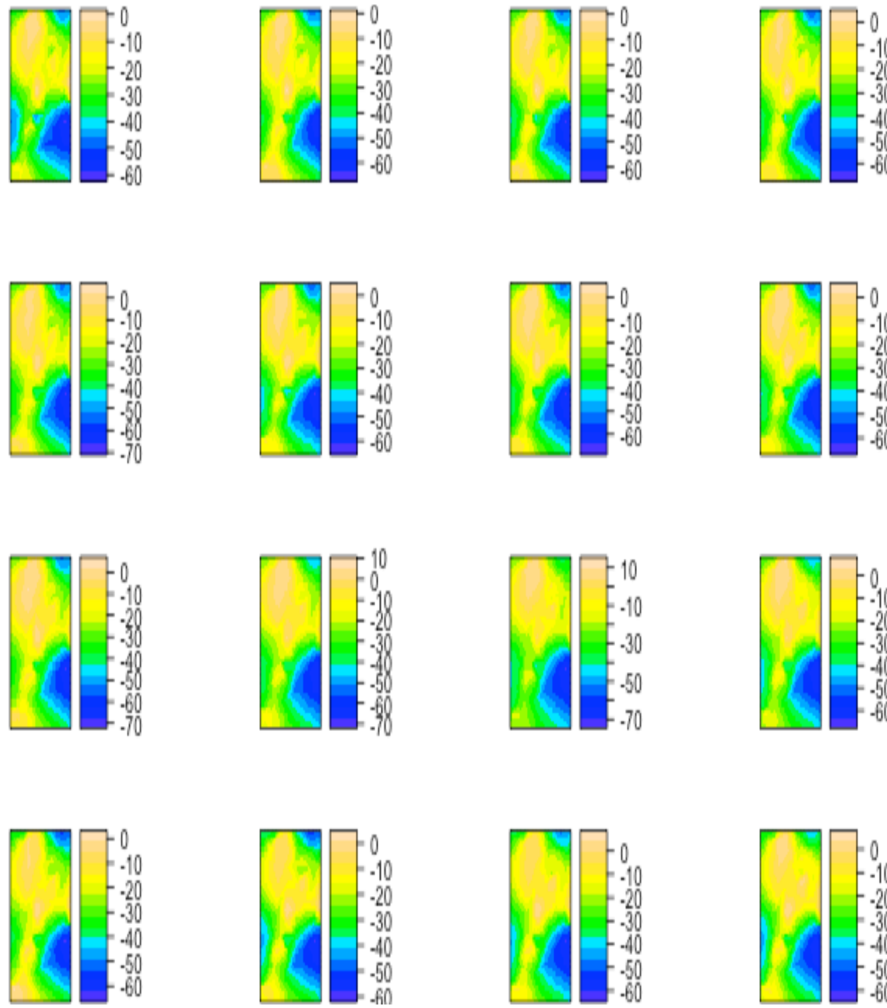
```
> res2pos$summary.fixed
                  mean           sd    0.025quant      0.5quant  0.975quant         mode          kld
Intercept -14.576549539 2.788559e+01 -69.319288831 -14.579598755 40.13298938 -14.583354523 2.924022e-16
elev        0.010464086 8.931255e-04   0.008740241   0.010453876  0.01224473   0.010432282 6.838504e-13
year       -0.004199109 1.409759e-02  -0.031884876  -0.004197175  0.02345057  -0.004192103 5.540561e-16
> 
```

  - splines, AR coefficients also possible for time-varying effects

Projecting INLA estimates to surface



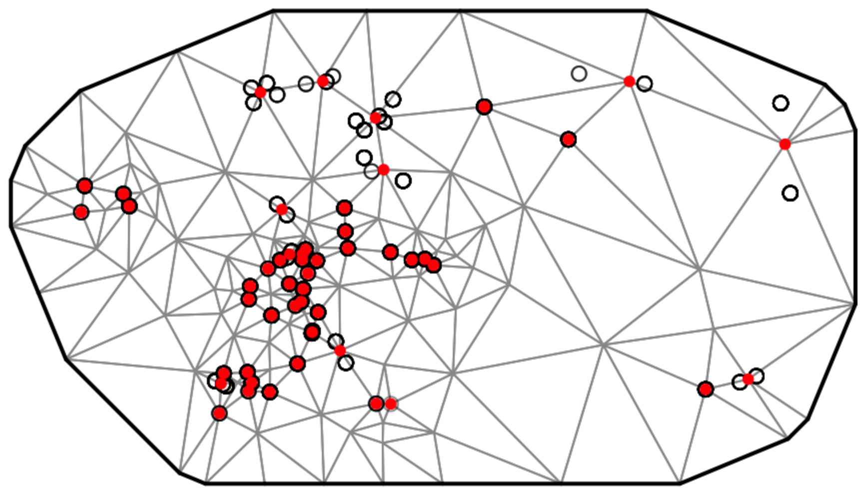
Spatial SWE fields by year



Implementation in sdmTMB

- Start with the built in make_mesh function

```
snow_spde <- make_mesh(d, c("Longitude", "Latitude"), n_knots=50)
plot(snow_spde)
```



Estimation in INLA

- Can be slow for very large models

Implementation in sdmTMB

- This would fit a model with static spatial field

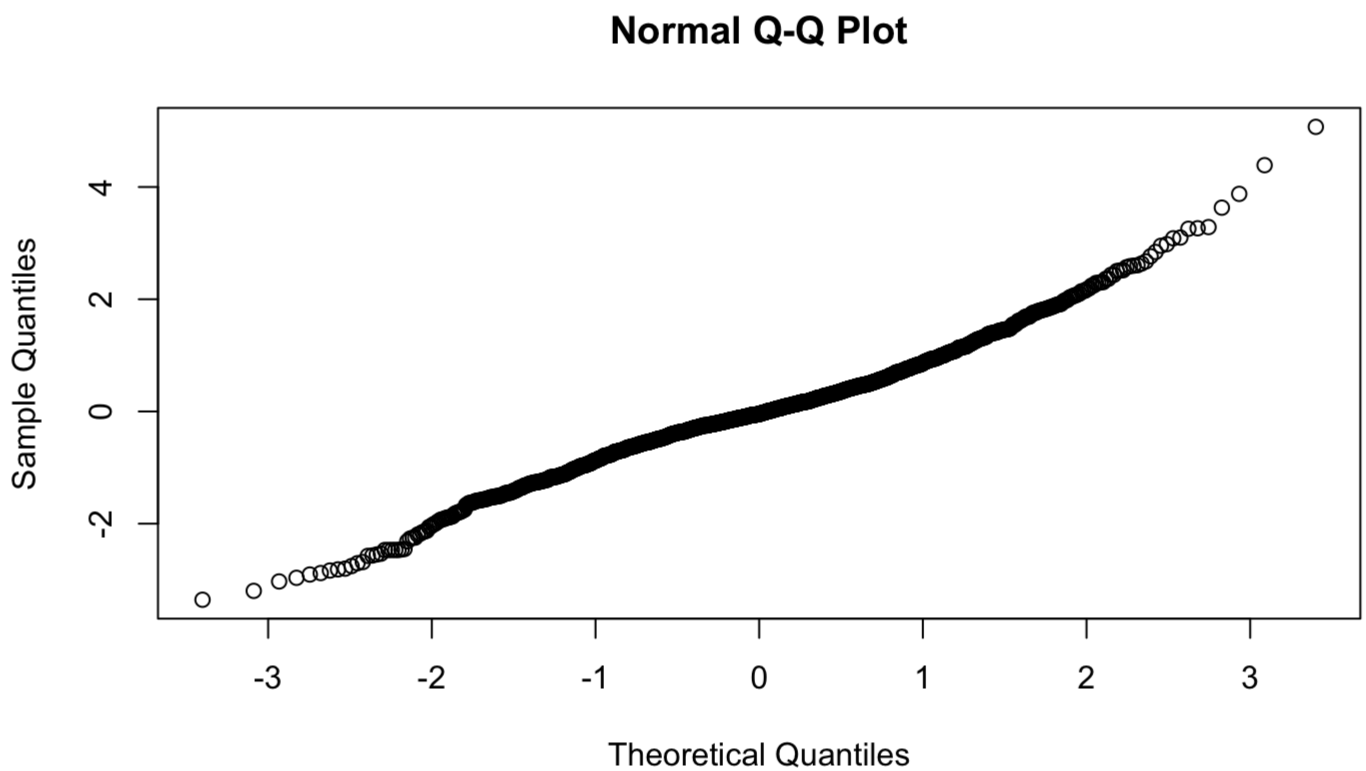
```
fit <- sdmTMB(Feb ~ 1, mesh = snow_spde, data=d)
```

- Versus this one, where the field is independent by year

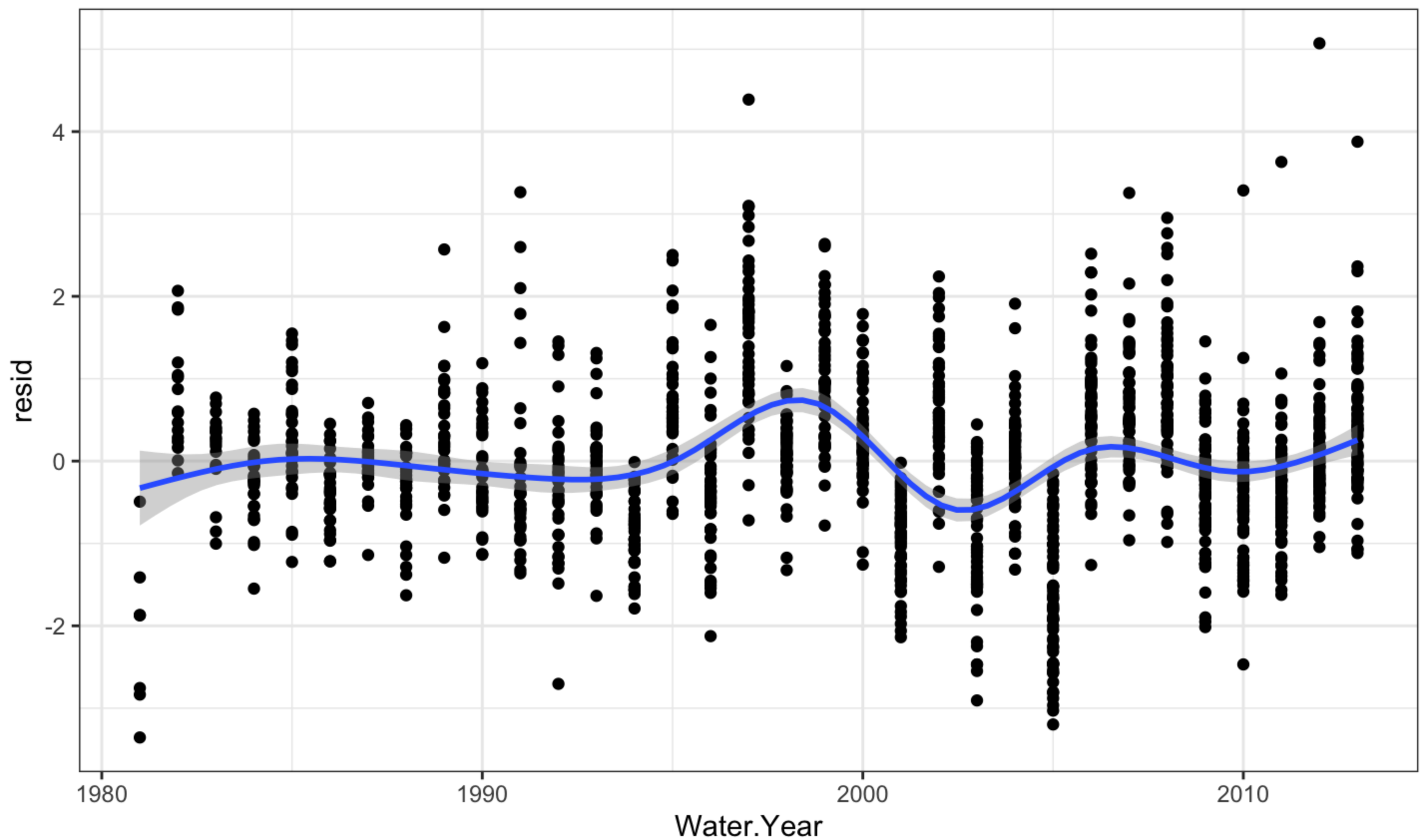
```
fit <- sdmTMB(Feb ~ 1, mesh = snow_spde, time="Water.Year", data=d)
```

Diagnostics: residuals for a given time slice

```
fit <- sdmTMB(Feb ~ 1, mesh = snow_spde, data=d)
qqnorm(residuals(fit))
```



Diagnostics: residuals over time: are they stationary?



INLA & TMB references:

Lindgren 2013

Lindgren & Rue 2015

INLA book

Kristensen et al. 2016