Fitting spatial and spatiotemporal models

FISH 507 – Applied Time Series Analysis

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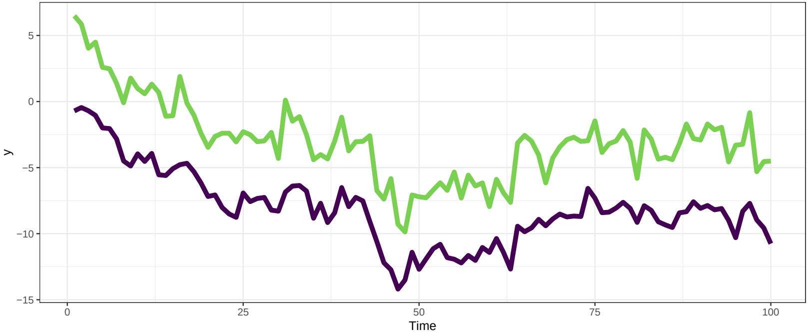
What we've learned so far

- Time series can be useful for identifying structure, improving precision, and accuracy of forecasts
- Modeling multivariate time series
  - e.g. MARSS() function, with each observed time series mapped to a single discrete state
- Using DFA
  - Structure determined by factor loadings

Response generally the same variable (not separate species)

- Making inference about population as a whole involves jointly modeling both time series



Lots of time series -> complicated covariance matrices

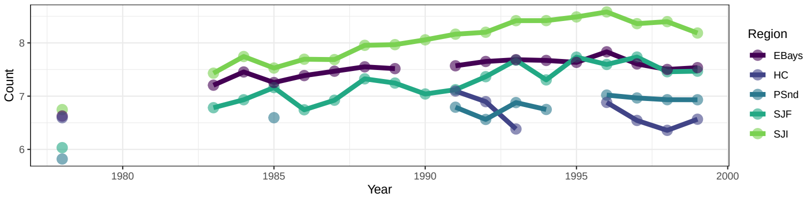
- Last week, in talking about Gaussian process models, we showed the number of covariance matrix parameters $= m * (m + 1)/2$
  - problematic for > 5 or so time series
- MARSS solutions: diagonal matrices or 'equalvarcov'
- DFA runs into same issues with estimating unconstrained R matrix

Potential problems with both the MARSS and DFA approach

- Sites separated by large distances may be grouped together
- Sites close to one another may be found to have very different dynamics

Are there biological mechanisms that may explain this?

- Puget Sound Chinook salmon
  - 21 populations generally cluster into 2-3 groups based on genetics
  - Historically large hatchery programs
- Hood canal harbor seals
  - Visited by killer whales



Motivation of explicitly including spatial structure

- Adjacent sites can be allowed to covary
- Estimated parameters greatly reduced to 2-5

Types of spatial data

Point referenced data (aka geostatistical)

- Typically 2-D, but could be 1-D or 3-D (depth, altitude)
- May be fixed station or random (e.g. trawl surveys)

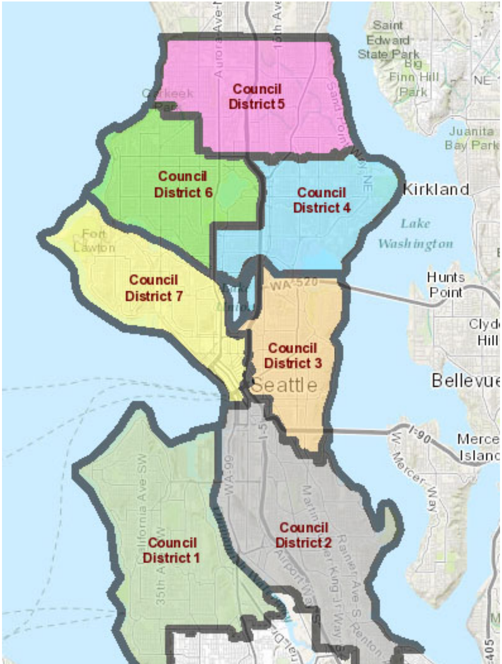
Point pattern data

- Spatially referenced based on outcomes (e.g. presence)
- Inference focused on describing clustering (or not)

Areal data

- Locations occur in blocks
- counties, management zones, etc.

Areal data: Seattle Council Districts



Computationally convenient approaches

CAR (conditionally autoregressive models)

- Better suited for Bayesian methods
- Goal of both is to write the distribution of a single prediction analytically in terms of the joint (y1, y2)

SAR (simultaneous autregressive models)

- Better suited for ML methods
- Simultaneously model distribution of predicted values

'Autoregressive' in the sense of spatial dependency / correlation between locations

CAR models (Besag 1991)

$$Y_i = BX_i + \phi_i + \varepsilon_i$$

X_i are predictors (regression) $\phi_i i$ spatial component, (aka markov random field) ε_i residual error term

- Create spatial adjacency matrix W, based on neighbors, e.g.
- W(i,j) = 1 if neighbors, 0 otherwise
- W often row-normalized (rows sum to 1)
- Diagonal elements W(i,i) are 0

Adjacency matrix example

CAR models

In matrix form,

$$\phi \sim N\left(0, (I - \rho W)^{-1}\widetilde{D}\right)$$
$$\widetilde{D}_{ii} = \sigma_i$$

- Implemented in ‘spdep’, ‘CARBayes’, etc

CAR models

- Each element has conditional distribution dependent on others,

$$\phi_i \mid \phi_j, \sim \mathrm{N}\left(\sum_{j=1}^{n} W_{ij}\phi_j, \sigma^2\right) \text{ for } j \neq i$$

SAR models

- Simultaneous autoregressive model

$$\phi \sim N\left(0, (I - \rho W)^{-1}\widetilde{D}(I - \rho W)^{-1}\right)$$
$$\widetilde{D}_{ii} = \sigma_i$$

- Remember that the CAR was

$$\phi \sim N\left(0, (I - \rho W)^{-1}\widetilde{D}\right)$$
$$\widetilde{D}_{ii} = \sigma_i$$

Example adjacency matrix

- $I - \rho W$
- Example W matrix,

```
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    1    0    1
## [3,]    0    1    0
```

Example adjacency matrix

- $(I - 0.3W)$, $\rho = 0.3$

```
##       [,1] [,2] [,3]
## [1,]  1.0 -0.3  0.0
## [2,] -0.3  1.0 -0.3
## [3,]  0.0 -0.3  1.0
```

Example adjacency matrix

- $(I - 0.3W)\widetilde{D}$
- Let's assume same variance ~ 0.1
- D $=$ diag(0.1,3)

```
##       [,1]  [,2]  [,3]
## [1,]  0.10 -0.03  0.00
## [2,] -0.03  0.10 -0.03
## [3,]  0.00 -0.03  0.10
```

Commonalities of both approaches

- Adjacency matrix W can also instead be modified to include distance
- Models spatial dependency as a function of single parameter ρ
- Models don't include time dimension in spatial field
  - One field estimated for all time steps
  - How could we do this?

brms() includes both approaches

```
?brms::sar
?brms::car
```

General challenge with CAR and SAR models

Wall (2004) "A close look at the spatial structure implied by the CAR and SAR models".

- Note: not a 1-to-1 mapping of distance and correlation

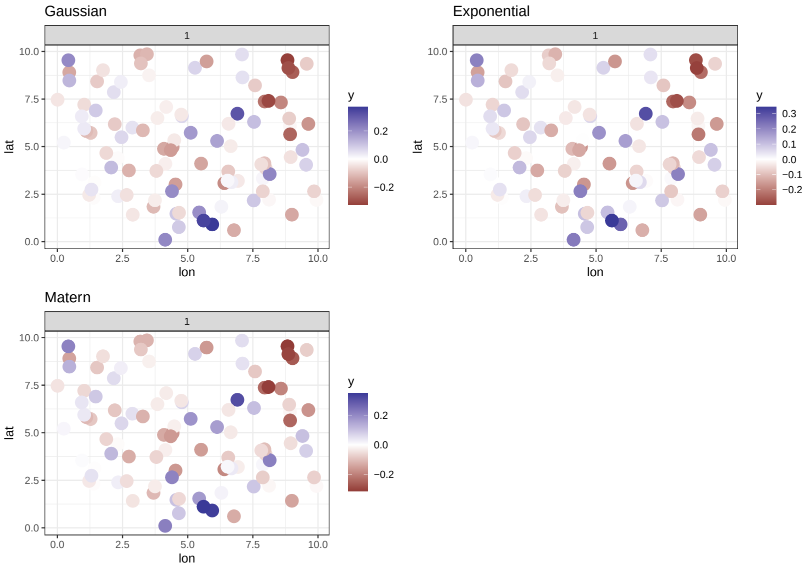
Alternative to CAR & SAR

- model elements of Q as functions
- Create matrix D, as pairwise distances
- This can be 1-D, or any dimension
  - We'll use Euclidian distances for 2-D

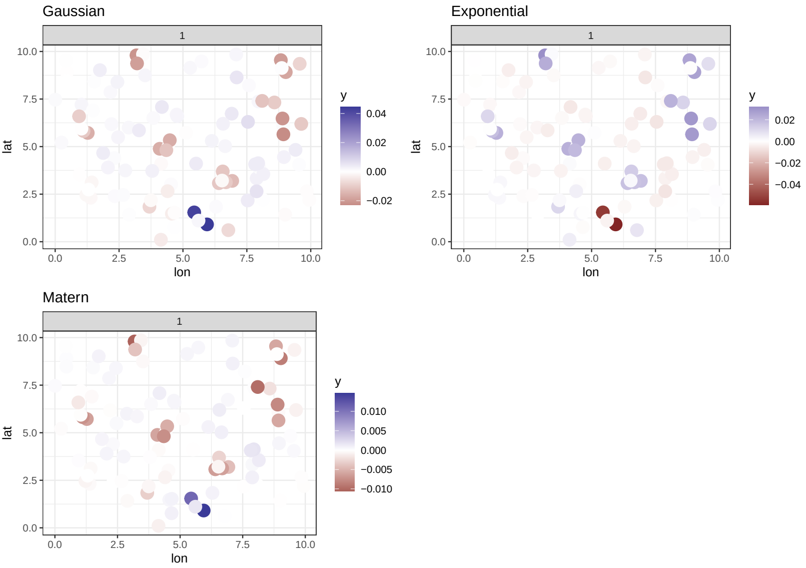
Squared exponential covariance

Squared exponential covariance

Which covariance kernel to choose?



Which covariance kernel to choose?



Considerations for time series models

Should spatial dependency be included? If so, how to model it?

- Constant
- Time varying
- Autoregressive
- Random walk
- Independent variation

Model-based geostatistical approaches

1. Generalized least squares
2. Bayesian methods in spBayes, glmmfields
3. INLA models
4. Spatial GAMs
5. TMB (VAST, sdmTMB)

Method 1: using gls()

- Generalized least squares function
  - similar syntax to lm, glm, etc.
- Flexible correlation structures
  - corExp()
  - corGaus()
  - corLin()
  - corSpher()
- Allows irregularly spaced data / NAs
  - unlike Arima(), auto.arima(), etc.

WA Snotel sites

Modeling WA Snotel data

We'll use Snow Water Equivalent (SWE) data in Washington state

70 SNOTEL sites

- we'll focus only on Cascades

1981-2013

Initially start using just the February SWE data

1518 data points (only 29 missing values!)

Use AIC to evaluate different correlation models

```
mod.exp = gls(Feb ~ elev,
  correlation = corExp(form=~lat+lon,nugget=T),
  data = y[which(is.na(y$Feb)==F & y$Water.Year==2013),])
AIC(mod.exp) = 431.097

mod.gaus = gls(Feb ~ elev,
  correlation = corGaus(form=~lat+lon,nugget=T),
  data = y[which(is.na(y$Feb)==F & y$Water.Year==2013),])
AIC(mod.gaus) = 433.485
```

Diagnostics: fitting variograms

```
var.exp <- Variogram(mod.exp, form =~ lat+lon)
plot(var.exp,main="Exponential",ylim=c(0,1))

var.gaus <- Variogram(mod.gaus, form =~ lat+lon)
plot(var.gaus,main="Gaussian",ylim=c(0,1))
```

Exponential variogram

$$\text{Semivariance} = 0.5 \quad Var(x_1, x_2)$$
$$\text{Semivariance} = \text{Sill - } Cov(x_1, x_2)$$

Gaussian variogram

Extensions of these models

corExp and corGaus spatial structure useful for wide variety of models / R packages

Linear/non-linear mixed effect models

- lme() / nlme() in nlme package

Generalized linear mixed models

- glmmPQL() in MASS package

Generalized additive mixed models

- gamm() in mgcv package

Method 2: GAMs

Previous approaches modeled errors (or random effects) as correlated

- Spatial GAMs generally model mean as spatially correlated

Example with SNOTEL data

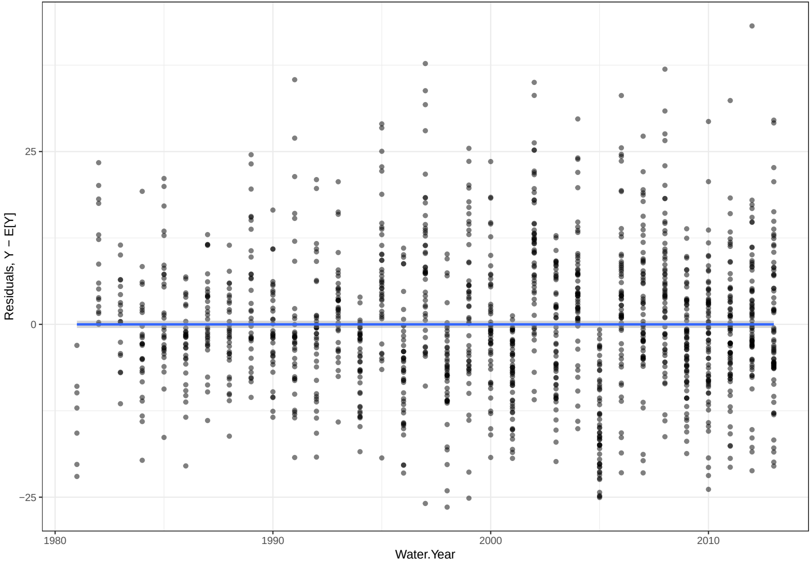
First a simple GAM, with latitude and longtide.

- Note we're not transforming to UTM, but probably should
- Note we're not including tensor smooths te(), but probably should

```
d = dplyr::filter(d, !is.na(Water.Year), !is.na(Feb))
mod = gam(Feb ~ s(Water.Year) +
    s(Longitude, Latitude), data=d)
d$resid = resid(mod)
```

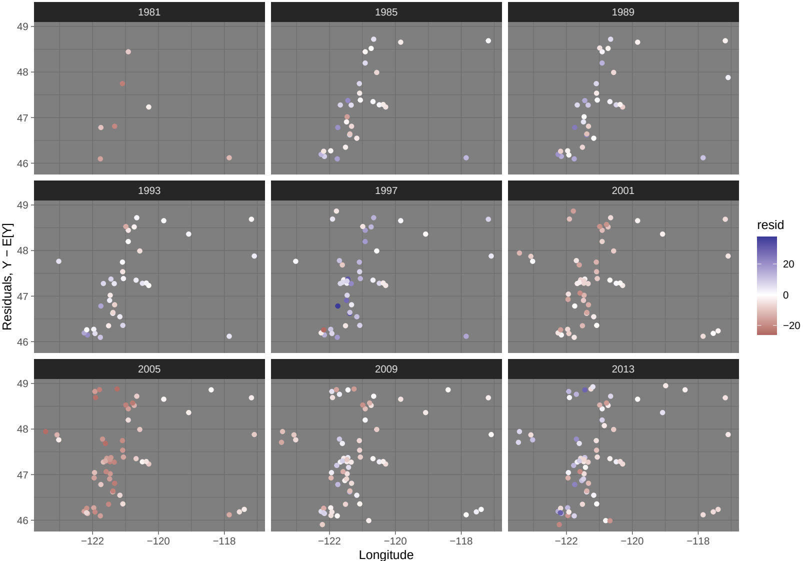
Ok, let's look at the residuals

First, residuals through time



Now residuals spatially

- We'll use a subset of years here



Two ways to include spatio-temporal interactions

- What do these interactions mean in the context of our SNOTEL data?

1. First, spatial field may vary by year

```
mod2 = gam(Feb ~ s(Water.Year) +
    s(Longitude, Latitude, by=as.factor(Water.Year)), data=
d$resid = resid(mod2)
```

Two ways to include spatio-temporal interactions

2. Spatial effect may be continuous and we can include space-time interaction

- Why ti() instead of te() ? ti() only is the interaction where te() includes the main effects

```
mod2 = gam(Feb ~ s(Longitude, Latitude) + s(Water.Year) +
    ti(Longitude, Latitude,Water.Year, d=c(2,1)), data=d)
d$resid = resid(mod2)
```

GAM extensions and resources

Quickly evolving features for spatio-temporal models

- Random effects gamm
- Interface with inla ginla
- Bayesian estimation bam

Lots of detailed examples and resources

*e.g. ESA workshop by Simpson/Pederson/Ross/Miller

https://noamross.github.io/mgcv-esa-workshop/

Potential limitations of spatial GAMs

- Hard to specify covariance structure explicitly
- Hard to add constraints, like making spatial field be an AR(1) process
- Motivates more custom approaches

Bayesian: spBayes

Slightly more complicated syntax:

Specify:

- Priors on parameters
- Tuning parameters for Metropolis sampling (jumping variance)
- Starting / initial values
- Covariance structure ("exponential", "gaussian", "matern", etc)
- Number of MCMC iterations / burn-in, etc.

Example with SNOTEL data

```r
# This syntax is dependent on model parameters. See vignett
priors <- list("beta.Norm"=list(rep(0,p),
  diag(1000,p)), "phi.Unif"=c(3/1, 3/0.1),
  "sigma.sq.IG"=c(2, 2), "tau.sq.IG"=c(2, 0.1))

# Phi is spatial scale parameter, sigma.sq is spatial vari
# tau.sq = residual
starting <- list("phi"=3/0.5, "sigma.sq"=50, "tau.sq"=1)

# variance of normal proposals for Metropolis algorithm
tuning <- list("phi"=0.1, "sigma.sq"=0.1, "tau.sq"=0.1)

m.1 <- spLM(y~X-1, coords=cords,
  n.samples=10000,
  cov.model = "exponential", priors=priors,
  tuning=tuning, starting=starting)
```

Coefficients need to be extracted

```
##recover beta and spatial random effects
burn.in <- 5000
m.1 <- spRecover(m.1, start=burn.in, verbose=FALSE)
```

Standard MCMC diagnostics

Output is of class ‘mcmc’

Method 3: glmmfields

- New R package that implements these models using Stan and Gaussian predictive process models
- Easy to use formula syntax like gam() or spBayes()
- Allows spatial field to be modeled with Multivariate-T field, as alternative to Multivariate-Normal to better capture extremes
- Also includes lots of non-normal responses for observation model

glmmfields

Let's start with a simple model. This includes only 6 knots (probably way too few) and 100 iterations (too few!) for run time.

- Time set to NULL here to fit a model with constant spatial field

```
m <- glmmfields(Feb ~ 0, time = NULL,
 lat = "Latitude", lon = "Longitude", data = d,
 nknots = 6, iter = 100, chains = 2, seed = 1)
```

glmmfields

Ok, now let's change the covariance function to matern, and estimate separate spatial fields by year.

- The 'estimate_ar' argument is important for determining whether the field is an AR(1) process or independent by year

```
m <- glmmfields(Feb ~ 0, time = "Water.Year",
 lat = "Latitude", lon = "Longitude", covariance="matern",
  data = d, nknots = 6, iter = 100, chains = 2, seed = 1,
  estimate_ar=FALSE)
```

glmmfields

Finally let's include an example with modeling the spatial field as a Multivariate-t distribution

- We do this with the 'estimate_df' argument

```
m <- glmmfields(Feb ~ 0, time = "Water.Year",
 lat = "Latitude", lon = "Longitude", covariance="matern",
  data = d, nknots = 6, iter = 100, chains = 2, seed = 1,
  estimate_AR=FALSE, estimate_df= TRUE)
```

Method 4: INLA

GP spatial models are extremely powerful

- May get overwhelmed by number of points
- Approaches to incorporate knots using sparse covariance matrices
- Integrated Nested Laplace Approximation - not on CRAN

Motivation of INLA

Some problems contain simple spatial structure e.g. the harbor seal data in MARSS() with only 5 – 7 time series

Others are much more complex

- WA SWE data
- Fisheries survey data (1000s of points)

Including time-varying spatial fields becomes very computationally difficult

Doing all of the above in a Bayesian setting can be prohibitive, but we can use Laplace approximation

Varying spatial fields example: eulachon-shrimp bycatch

INLA's approximation: SNOTEL data

How many points fall on vertices? Is the boundary area large enough? Choosing this must be done very carefully!

```
INLA::meshbuilder
```

Estimation done via maximum likelihood

- Estimates seems similar to those from gls() and spBayes()
- Year included as numeric here (not significant)
- Alternatively, we can include year in spatial field
- Year can also be included as factor

Projecting INLA estimates to surface

Spatial SWE fields by year

Implementation in sdmTMB

Modeling intervention effects with spatial models

How do we model change points / break points in the mean in regression, ARIMA models, or GAMs?

```
lm(Feb ~ Water.Year)
gam(Feb ~ s(Water.Year))
auto.arima(Feb, xreg=Water.Year)
```

Modeling intervention effects with spatial models

How do we model change points / break points in the mean in regression, ARIMA models, or GAMs?

```
lm(Feb ~ Water.Year)
gam(Feb ~ s(Water.Year))
auto.arima(Feb, xreg=Water.Year)
```

- Use indicator functions before / after breakpoint

Example: modeling spatiotemporal responses of fish to oil spills

Ward et al. 2018

2 approaches for looking at effects:

- indicator / change point associated with the spill
- Or post-hoc approach to look at changes in spatial variability post-spill
  - why: lagged effects, non-stationary spatial effects

Example: modeling spatiotemporal responses of fish to oil spills

Example: modeling spatiotemporal responses of fish to oil spills