Regression versus State-Space

FISH 507 – Applied Time Series Analysis

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Fixed & random effects

Let's go back to Mark's lecture on DFA

$$y_t = \underbrace{\alpha + \beta x_t}_{\text{fixed}} + \underbrace{f_t + e_t}_{\text{random}}$$

Regression (linear or non-linear) with correlated errors gets your the fixed effects while properly taking into account the correlated random effects.

State-space model allows you to model the f_t.

Things to think about

$$y_t = \underbrace{\alpha + \beta x_t}_{\text{fixed}} + \underbrace{f_t + e_t}_{\text{random}}$$

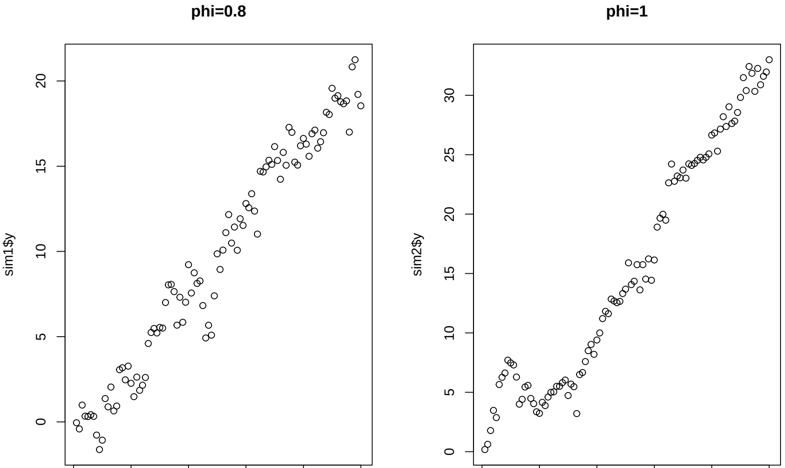
- What if your want f_t, the hidden random walk
- What if you want $E[\alpha + \beta x_t + f_t]$
- What if you want the $E[y_t | y_{1:t-1}]$
- What if we want to forecast y_t?

Let's simulate some data

$$
\begin{gathered}
x_t = t \\
f_t = \phi f_{t-1} + w_t,\ w_t \sim N(0, 1) \\
y_t = \beta x_t + f_t + v_t,\ v_t \sim N(0, \sqrt{0.2})
\end{gathered}
$$

Simulated data

```
set.seed(123)
N <- 100; h <- 100; x <- 1:N
sim1 <- sim.data(N, h=h, phi=0.8)
sim2 <- sim.data(N, h=h, phi=1)
```



Fit with arima(y, xreg=x)

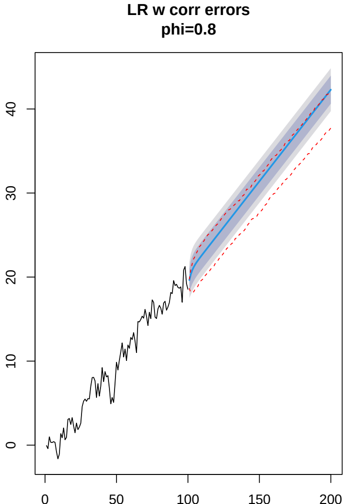
Fits a linear regression with ARMA errors.

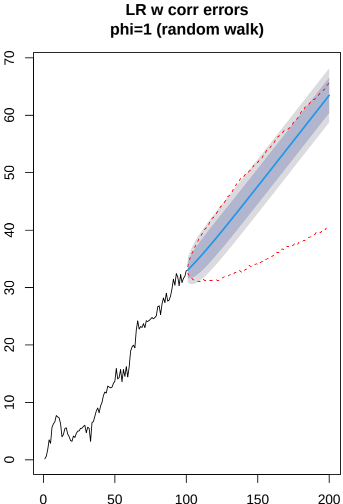
```
library(forecast)
fit <- auto.arima(sim1$y, xreg=x)
fr1 <- forecast(fit, xreg=(N+1):(N+h))
fit <- auto.arima(sim2$y, xreg=x)
fr2 <- forecast(fit, xreg=(N+1):(N+h))
```

Plot forecasts versus true

I created simulations from the true process to get truth.

Plot shows prediction intervals (future y). Red lines are true 80% intervals.

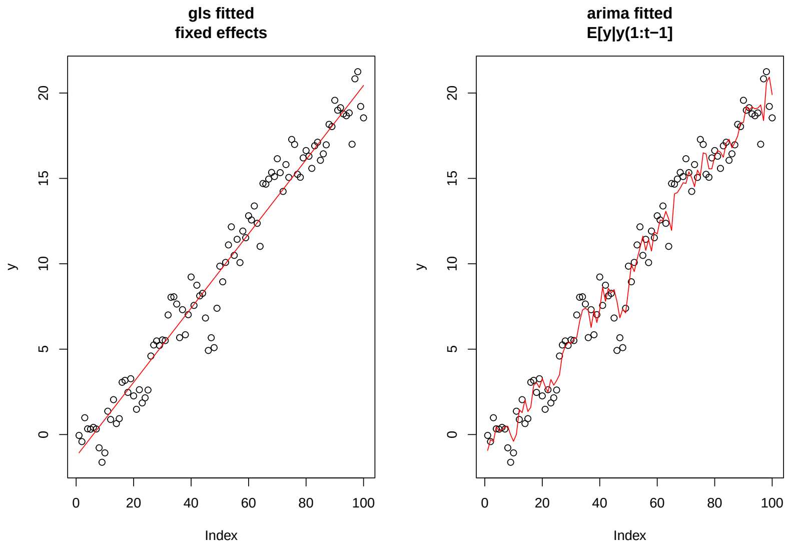




Fitted

You need to be careful to think about what you mean by `fitted()`

- `gls(y~x)` and similar would return $E[\alpha + \beta x_t]$
- `arima(y, xreg=x)` returns $E[\alpha + \beta x_t + e_t | y_{1:t-1}]$
- state-space model would get you $E[\alpha + \beta x_t + f_t | y_{1:N}]$


gls fitted
fixed effects
arima fitted
E[y|y(1:t−1]
y
Index
0
5
10
15
20
0
20
40
60
80
100

Regression w ARMA errors vs ARMAX

These are different models.

Regression w AR1 errors

$$e_t = \phi e_{t-1} + w_t$$
$$y_t = \alpha + \beta x_t + e_t$$

ARMAX: In this case, AR(1)-X

$$y_t = \phi y_{t-1} + \alpha + \beta x_t + w_t$$