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Introduction to Dynamic Linear Models (DLMs)

1.1 Homework problems

For the homework this week we will use a DLM to examine some of the time-varying properties of the spawner-recruit relationship for Pacific salmon. Much work has been done on this topic, particularly by Randall Peterman and his students and post-docs at Simon Fraser University. To do so, researchers commonly use a Ricker model because of its relatively simple form, such that the number of recruits (offspring) born in year t (R_t) from the number of spawners (parents) (S_t) is

$$R_t = aS_t e^{-bS+v_t}. \tag{1.1}$$

The parameter a determines the maximum reproductive rate in the absence of any density-dependent effects (the slope of the curve at the origin), b is the strength of density dependence, and $v_t \sim N(0,\sigma)$. In practice, the model is typically log-transformed so as to make it linear with respect to the predictor variable S_t, such that

$$\begin{aligned} \log(R_t) &= \log(a) + \log(S_t) - bS_t + v_t \\ \log(R_t) - \log(S_t) &= \log(a) - bS_t + v_t \\ \log(R_t/S_t) &= \log(a) - bS_t + v_t. \end{aligned} \tag{1.2}$$

Substituting $y_t = \log(R_t/S_t)$, $x_t = S_t$, and $\alpha = \log(a)$ yields a simple linear regression model with intercept α and slope b.

Unfortunately, however, residuals from this simple model typically show high-autocorrelation due to common environmental conditions that affect overlaping generations. Therefore, to correct for this and allow for an index of stock productivity that controls for any density-dependent effects, the model may be re-witten as

$$\begin{aligned} \log(R_t/S_t) &= \alpha_t - bS_t + v_t, \\ \alpha_t &= \alpha_{t-1} + w_t, \end{aligned} \tag{1.3}$$

and $w_t \sim N(0,q)$. By treating the brood-year specific productivity as a random walk, we allow it to vary, but in an autocorrelated manner so that consecutive years are not independent from one another.

More recently, interest has grown in using covariates (*e.g.*, sea-surface temperature) to explain the interannual variability in productivity. In that case, we can can write the model as

$$\log(R_t/S_t) = \alpha + \delta_t X_t - bS_t + v_t. \tag{1.4}$$

In this case we are estimating some base-level productivity (α) plus the time-varying effect of some covariate X_t (δ_t).

1.1.1 Spawner-recruit data

The data come from a large public database begun by Ransom Myers many years ago. If you are interested, you can find lots of time series of spawning-stock, recruitment, and harvest for a variety of fishes around the globe. Here is the website:

`http://ram.biology.dal.ca/~myers/about_site.html`

For this exercise, we will use spawner-recruit data for sockeye salmon (*Oncorhynchusnerka*) from the Fraser River in British Columbia. Specifically, the data come from a population in the Chilko River and span the years 1948-1986. In addition, we'll examine the potential effects of the Pacific Decadal Oscillation (PDO) during the salmon's first year in the ocean, which is widely believed to be a "bottleneck" to survival.

Here are the data:

```
# get S-R data; cols are:
# 1: brood yr (brood.yr)
# 2: number of spawners (Sp)
# 3: number of recruits (Rec)
# 4: PDO during first summer at sea (PDO.t2)
# 5: PDO during first winter at sea (PDO.t3)
load("ChilkoSockeye.RData")
```

1.1.2 Questions

Use the information above to do the following:

1. Begin by fitting a reduced form of Equation 1.3 that includes only a time-varying level (α_t) and observation error (v_t). Although you will be modeling productivity as completely independent of their parents, it will provde some insights as to the overall temporal pattern in recruitment. Plot the ts of α_t and note the AICc for this model.

2. Fit the full model specified by Equation 1.3. For this model, obtain the time series of α_t, which is an estimate of the stock productivity in the absence of density-dependent effects. How do these estimates of productivity compare to those from the previous question? Plot the ts of α_t and note the AICc for this model. (*Hint*: If you don't want a parameter to vary with time, what does that say about its process variance?.)

3. Fit the model specified by Equation 1.4 with the summer PDO index as the covariate (`PDO.t2`). What is the mean level of productivity? Plot the ts of δ_t and note the AICc for this model

4. Fit the model specified by Equation 1.4 with the winter PDO index as the covariate (`PDO.t3`). What is the mean level of productivity? Plot the ts of δ_t and note the AICc for this model