Irradiation Treatment Revised

1 Opacity Treatment

In this work, we distinguish between the Rosseland mean opacity (κ_R) and the Planck mean opacity (κ_P), which play different physical roles in radiative transfer.

1.1 Planck Mean Opacity (κ_P)

The Planck mean opacity is used in the optically thin cooling limit and therefore must represent *true absorption* processes only. It is defined as

$$\kappa_P = \frac{\int \kappa_\nu B_\nu, d\nu}{\int B_\nu, d\nu}, \tag{1}$$

and thus includes only processes that can emit radiation.

In this work, we adopt:

- For $T \leq 10^4$ K: Semenov et al. (2003) opacity tables (dust + gas absorption)
- For $T > 10^4$ K: free-free opacity (κ_{ff})

Important: electron scattering opacity (κ_{es}) is **not included** in κ_P, since Thomson scattering does not contribute to radiative cooling in the absence of Comptonization.

1.2 Rosseland Mean Opacity (κ_R)

The Rosseland mean opacity governs radiative diffusion in optically thick regions and is defined as

$$\frac{1}{\kappa_R} = \frac{\int (1/\kappa_\nu)(\partial B_\nu/\partial T), d\nu}{\int (\partial B_\nu/\partial T), d\nu}. \tag{2}$$

In this work, κ_R is taken directly from opacity tables (OP + AESOPUS + Semenov 2003). These tables are assumed to represent the dominant absorption processes.

We do not explicitly add electron scattering opacity to κ_R, since in the present simulations the Rosseland mean opacity is already sufficiently large ($\kappa_R \gg 0.34, \mathrm{cm}^2, \mathrm{g}^{-1}$) in the high-temperature regions, and scattering effects are subdominant.

2 Radiative Cooling

We model radiative cooling using a bridging formula between optically thin and optically thick limits.

2.1 Optical Depths

We define two optical depths:

$$\tau_R = \frac{1}{2}\kappa_R\Sigma, \tag{3}$$

$$\tau_P = \frac{1}{2}\kappa_P\Sigma. \tag{4}$$

Here, τ_R governs radiative diffusion, while τ_P governs local emission/absorption.

2.2 Limiting Cases

The optically thick cooling rate is given by

$$Q_{\text{thick}} = \frac{64\sigma T^4}{3\kappa_R\Sigma}, \tag{5}$$

and the optically thin cooling rate is

$$Q_{\text{thin}} = 2\kappa_P\Sigma\sigma T^4. \tag{6}$$

2.3 Bridging Formula

To smoothly connect these limits, we adopt a Hubeny-type bridging formula:

$$Q^- = \frac{16\sigma T^4}{3\tau_R + 2/\tau_P}. \tag{7}$$

This expression reproduces the correct asymptotic behavior in both limits and provides improved numerical stability across opacity transitions.

2.4 Reference for Bridging Formula

The above bridging form is motivated by radiative transfer treatments in accretion disks (e.g., Hubeny 1990, ApJ, 351, 632) and is commonly used in vertically averaged disk models.

3 Remarks

The present model does not resolve the vertical disk structure. Therefore, the opacity treatment should be regarded as an effective approximation. In particular:

- The separation of κ_P and κ_R ensures physical consistency between emission and diffusion processes.
- The neglect of electron scattering in κ_P avoids artificial enhancement of cooling in optically thin regions.
- The omission of explicit scattering in κ_R is justified for the current parameter regime but may need revision in fully ionized, low-opacity conditions.

4 Future Work

A more detailed treatment including vertical structure and frequency-dependent radiative transfer would be required for higher accuracy, especially in regimes where scattering becomes important.