Irradiation Treatment for CircumBinary and Be Disks

1 Overview

This document summarizes the irradiation treatment in the 1D viscous disk code for CircumBinary Disks (CBD) and Be (classical) decretion disks. The thermal balance is governed by

$$Q_{\rm vis} + Q_{\rm irr} = Q_{\rm rad}, \tag{1}$$

where $Q_{\rm vis}$ is viscous heating, $Q_{\rm irr}$ is irradiation heating (per unit area, from both disk faces), and $Q_{\rm rad}$ is radiative cooling. The formulae for $Q_{\rm irr}$ and $Q_{\rm rad}$ differ between optically thick and optically thin regions.

2 Irradiation Heating $Q_{\rm irr}$

2.1 Base Formulation: LOH24 Eq. (16)

The code uses the orbit-averaged irradiation flux from Lee, Okazaki & Hayasaki (2024; LOH24) for a circumbinary disk around a point-like central source. In dimensionless form ($\xi = r/r_{\rm in}$),

$$Q_{\rm irr} = \frac{A_1 L_1}{2\pi r_{\rm in}^2}\,\frac{1}{\xi}\left[(1+Q_{12})\,\frac{dY}{d\xi} - \frac{\beta_1 + Q_{12}\beta_2}{\xi^2}\left(\frac{Y}{\xi} - \frac{1}{2}\frac{dY}{d\xi}\right)\right], \tag{2}$$

where $Y = H/r$ (aspect ratio), $dY/d\xi$ is the flaring gradient, $A_1 = 1 -$ albedo, L_1 is the effective luminosity, and β_1, β_2, Q_{12} depend on the binary mass ratio q and gap parameter $C_{\rm gap}$ (LOH24).

2.2 CBD Case

For CircumBinary Disks:

- $L_1 = L_{\rm acc}/(1+q)$ (or similar), with $L_{\rm acc} = \eta_{\rm acc}\dot{M}c^2$.
- Optional irradiation delay: $L_{\rm irr}(t) = L_{\rm acc}(t-\tau)$ where τ is either explicit (fixed) or viscous ($\tau \propto r_{\rm in}^2/\nu_{\rm in}$).
- Point-source assumption in Eq. (2); the central binary is treated as a single luminosity source.

2.3 Be Disk Case

For Be (classical) decretion disks:

- $L_1 = L_\star$ (stellar luminosity); no accretion-powered irradiation delay.
- Finite stellar size is important: the star subtends a significant solid angle at the inner disk.

2.4 Finite-Size Irradiation Source (Chiang & Goldreich)

When the central source has finite radius $R_\star$, the grazing angle is augmented. Following Chiang & Goldreich (1997) and LOH24:

$$\alpha_{\text{total}} = \alpha_{\text{star}} + \alpha_{\text{flare}}, \tag{3}$$

where

$$\alpha_{\text{star}} = 0.4\,\frac{R_\star}{r}, \tag{4}$$

$$\alpha_{\text{flare}} = \xi\,\frac{dY}{d\xi} = r\,\frac{d(H/r)}{dr}. \tag{5}$$

The irradiation is scaled:

- If $\alpha_{\text{flare}} \geq \alpha_{\text{star}}$: $Q_{\text{irr}} \leftarrow Q_{\text{irr}}^{\text{LOH24}} \times (\alpha_{\text{star}} + \alpha_{\text{flare}})/\alpha_{\text{flare}}$.
- If $\alpha_{\text{flare}} < \alpha_{\text{star}}$: $Q_{\text{irr}} = (\alpha_{\text{star}} + \alpha_{\text{flare}})/\alpha_{\text{star}} \times Q_{\text{irr}}^{\text{flat,star}}$, with $Q_{\text{irr}}^{\text{flat,star}} \propto (R_\star/r_{\text{in}})/\xi^3$.

3 Radiative Cooling Q_{rad}

3.1 Optically Thick Region ($\tau_R \gtrsim 1$)

In the diffusion limit (e.g., Kato, Fukue & Mineshige 2008, Eq. 3.38):

$$Q_{\text{rad}}^{\text{thick}} = \frac{64\,\sigma_{\text{SB}} T^4}{3\,\kappa_R\,\Sigma}, \tag{6}$$

where κ_R is the Rosseland mean opacity, Σ is the surface density, and $\tau_R = \kappa_R \Sigma/2$ (one-side optical depth). The Rosseland mean is appropriate for diffusion through an optically thick medium.

3.2 Optically Thin Region ($\tau_R \ll 1$)

In the optically thin limit, cooling is dominated by emission. D'Alessio et al. (1998) show that the cooling rate scales with the Planck mean opacity κ_P:

$$\Lambda_d = 4\pi\kappa_P\rho\left(\frac{\sigma_{\text{SB}} T^4}{\pi} - J_d\right), \tag{7}$$

which in the optically thin limit gives $Q_{\text{rad}} \propto \kappa_P \Sigma \sigma_{\text{SB}} T^4$. The implemented form (per unit area, both faces) is

$$Q_{\text{rad}}^{\text{thin}} = 2\,\kappa_P\,\Sigma\,\sigma_{\text{SB}}\,T^4. \tag{8}$$

Planck mean opacity is used because emission weights the spectrum by the Planck function, unlike the Rosseland mean used for radiative diffusion.

3.3 Bridging Between Thick and Thin

A smooth transition is used:

$$Q_{\text{rad}} = \frac{1}{\dfrac{1-w}{Q_{\text{rad}}^{\text{thin}}} + \dfrac{w}{Q_{\text{rad}}^{\text{thick}}}}, \quad w = \frac{\tau_R}{1+\tau_R}. \tag{9}$$

4 Opacity Tables

4.1 Rosseland Mean κ_R

OPAL, Ferguson, AESOPUS, Semenov et al. (2003) tables are used, depending on temperature and density.

4.2 Planck Mean κ_P: Calculation Method

The Planck mean opacity κ_P is required for optically thin radiative cooling (Eq. 8). The calculation switches at a critical temperature:

Temperature $T \le 10^4\,$K: Semenov et al. (2003) Planck mean table.

- File: `data/SemenovPlanckTable.data`
- Grid: $\log_{10}(T/\mathrm{K})$ from 1.0 to 4.0 (i.e., 10 K to 10^4 K), $\log_{10}(\rho/\mathrm{g\,cm^{-3}})$ from -17 to -7
- Values stored as $\log_{10}(\kappa_P/\mathrm{cm^2\,g^{-1}})$
- 2D spline (or polynomial) interpolation in $(\log T, \log \rho)$

The Semenov Planck table covers dust + gas opacity and is appropriate for protoplanetary and cool disk conditions.

Temperature $T > 10^4\,$K: Analytic formula (Kramers free-free + electron scattering). Above 10^4 K, dust is destroyed and the opacity is dominated by free-free absorption and electron scattering. The Planck mean is computed as

$$\kappa_P = \kappa_{\mathrm{ff}} + \kappa_{\mathrm{es}}, \tag{10}$$

where

$$\kappa_{\mathrm{ff}} = 3.68 \times 10^{22}\, g_{\mathrm{ff}}\, (1-Z)(1+X)\, \rho\, T^{-7/2}, \tag{11}$$

$$\kappa_{\mathrm{es}} = 0.2\,(1+X). \tag{12}$$

Implemented parameters: $g_{\mathrm{ff}} = 1$, $X = 0.7$, $Z = 0.02$ (cosmic abundance); $\kappa_{\mathrm{es}} \approx 0.34\,\mathrm{cm^2\,g^{-1}}$. The free-free opacity follows the Kramers approximation; units are CGS (ρ in $\mathrm{g\,cm^{-3}}$, T in K, κ_P in $\mathrm{cm^2\,g^{-1}}$).

Summary:

$T \le 10^4\,$K	Semenov et al. (2003) table interpolation
$T > 10^4\,$K	$\kappa_P = \kappa_{\mathrm{ff}} + \kappa_{\mathrm{es}}$ (Eqs. 11–12)

This choice is motivated by the fact that published Planck mean tables (e.g., Semenov) typically extend only to $T \sim 10^4\,$K, while Be disks and inner CBD regions can reach slightly higher temperatures; the analytic extension ensures physically reasonable cooling in those regimes.

5 Optically Thin Irradiation (Chiang & Goldreich Eq. 12)

Chiang & Goldreich (1997) derive three regimes for the incident stellar flux in optically thin surface layers (Eq. 12a–c):

- Regime (a): optically thin to stellar radiation; flux penetrates and heats the whole column.

- Regime (b): intermediate; partial absorption.
- Regime (c): optically thick to stellar radiation; absorption at surface (standard treatment).

The current code applies the same $Q_{\rm irr}$ profile (LOH24 + finite-star correction) to both optically thick and thin cells. This corresponds to a single effective regime (c-like for thick, and an approximation for thin). For outer disk regions where $\tau_R < 1$ but stellar irradiation is still absorbed, the adopted $Q_{\rm irr}$ remains a reasonable approximation; true limb-to-limb optically thin irradiation would require a regime-dependent treatment per Chiang & Goldreich Eq. 12a–c.

6 Summary by Case

	CBD	Be Disk
$L_{\rm irr}$	$L_{\rm acc}$ (optionally delayed)	$L_\star$
Finite star	Optional ($\alpha_{\rm star}$)	Recommended ($\alpha_{\rm star}$)
$Q_{\rm irr}$	LOH24 Eq. 16 + Eq. (3)	Same
Optically thick $Q_{\rm rad}$	Eq. (6), κ_R	Same
Optically thin $Q_{\rm rad}$	Eq. (8), κ_P	Same

Table 1: Irradiation and cooling treatments by disk type.

7 Implementation Guide

This section provides code file paths and input parameters for readers who wish to locate or modify the implementation.

7.1 Code Files

File	Role
`irradiation_mod.f90`	$Q_{\rm irr}$ (LOH24 Eq. 16), finite-star correction, $L_{\rm irr}$ from accretion or $L_\star$, irradiation delay buffer
`disk_thermal_mod.f90`	Thermal balance: `heating_cooling_cell`, $Q_{\rm vis}$, $Q_{\rm rad}$ (thick/thin bridging), κ_R, κ_P usage
`opacity_table_mod.f90`	`get_opacity_Planck_rhoT`, Semenov Planck table, Kramers+$\kappa_{\rm es}$ for $T > 10^4$ K
`disk_energy_mod.f90`	Thermal solver, iteration over $T_{\rm mid}$ and $Q_{\rm irr}$
`output_mod.f90`	Disk structure output, $Q_{\rm rad}$ for diagnostics
`mod_global.f90`	Global flags: `use_irradiation`, `use_be_decretion`, etc.
`setup.f90`	Namelist input (`/disk_mode/`)

7.2 Key Input Parameters

Namelist `/disk_mode/` (in `ad1d.in` or equivalent):

Parameter	Default	Description
`use_irradiation`	`.false.`	Enable irradiation heating
`use_be_decretion`	`.false.`	Be disk mode; $L_{\rm irr} = L_\star$, no delay
`use_irradiation_delay`	`.false.`	Delay $L_{\rm irr}$ by τ (CBD accretion)
`use_finite_irradiation_source`	`.false.`	Add $\alpha_{\rm star}$ (Eq. 4)
`tau_irr_lag_mode`	`'explicit'`	`'explicit'`: fixed τ; `'viscous'`: $\tau \propto r_{\rm in}^2/\nu_{\rm in}$
`tau_irr_lag_nd`	0	`explicit`: delay in units of t_0; `viscous`: prefactor

Related parameters (in `/star_params/`, `/scale_params/`, or elsewhere): `R_star`, `L_star`, `q` (mass ratio), `f_edd_cap` (cap $L_{\rm acc}/L_{\rm Edd}$). Albedo is hardcoded in `irradiation_mod.f90` (`albedo = 0.9`).

8 References

1. Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368
   "Spectral Energy Distributions of T Tauri Stars with Passive Circumstellar Disks"

2. D'Alessio, P., Canto, J., Calvet, N., & Lizano, S. 1998, ApJ, 500, 411
   "Accretion Disks around Young Objects. I."

3. Kato, S., Fukue, J., & Mineshige, S. 2008, *Black-Hole Accretion Disks*
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4. Lee, Y., Okazaki, A. T., & Hayasaki, K. 2024, ApJ, 975, 65 (LOH24)
   "Circumbinary disk spectra irradiated by two central accretion disks in a binary black hole system"

5. Semenov, D., Henning, T., Helling, C., Ilgner, M., & Sedlmayr, E. 2003, A&A, 410, 611
   "Rosseland and Planck mean opacities for protoplanetary discs"