Canonical Spatial Models and the Hedonic Approach

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Course overview

- I teach theory and empirics in urban and spatial economics.
  - You do not fully understand empirics unless you pay attention to theory
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- Topics (tentative and subject to change). 13 lectures in total.
  - Canonical spatial models and the hedonic approach (1 lectures)
  - Rosen-Roback models (1.5 lectures)
  - Discrete choice models (1.5 lectures)
  - Quantaitive spatial models (3.5 lectures)
  - Empirics of agglomeration effects (1.5 lecture)
  - Sorting in space (1.5 lectures)
  - Housing supply (1.5 lecture)
  - Public policies in space (1-2 lectures, depending on the remaining time)
- Not just students mainly interested in urban and spatial economics, I also welcome those interested in other fields (like labor, public, trade, macro...). My lecture intersects with them.
  - I myself have worked on labor, public, and political topics in a spatial framework.

Course overview

- Evaluation is based on reading assignments and the final report.
  - I require you to submit a summary of some assigned papers (a few times)
  - Final report requires you to present your own research ideas (urban&spatial topics are encouraged, but okay as long as your ideas are tangentially related to urban&spatial economics)
- Homework and final reports should be submitted by email to a.yamagishi@r.hit-u.ac.jp (no hard copy!)
  - After the first lecture, send an email to this address by your university address ("ac.jp" domain) with your name and affiliation
- I have to cancel the in-person class on Sep 30 and Oct 3 due to my overseas business trip.
  - We will set up another date, or make it an online class

Spatial economics and urban economics

There is no definitive way to define urban and spatial economics, but the following is how I define them....

- **Spatial economics** studies the spatial distribution of economic activities, such as people, firms, and land.
  - Why Tokyo attracts so many people and firms?
  - How do location characteristics, such as climate and transportation access, shape the distribution of economic activities and land prices?
  - What is the impact of local policies, such as taxes and public goods, on city's population and welfare?
- **Urban economics** can be considered as a part of spatial economics, but its focus is *within a city*
  - Why Shinjuku attracts more people than Kunitachi?
  - What causes the massive the land price difference between Shinjuku and Kunitachi?
  - What are the economic and welfare impacts of introducing a new train line?

Spatial equilibrium

- **Spatial economic model** is an economic model that includes **location choice**
  - Which city do you live in? City A or City B?
  - Where in a city do you live? City center or suburban area?
- **Spatial equilibrium condition:** people should be optimally choosing their location
- Typically, spatial equilibrium condition implies some form of *indifference condition:* people are indifferent between the chosen location and other locations
  - If utility is not equalized between location A and B, people have an incentive to change their location.

A simplest spatial economic model

- People can choose to live in location X or somewhere else.
- If you live in location X, you enjoy
  - Your wage rate w (exogenously given)
  - Your residential amenities (A), which may include climate, public goods etc.
- But you pay land rents r to live in location X.
  - Assumption: you always consume one unit of land (no endogenous adjustment of living space)
- If you live somewhere else, you get the utility $\bar{u}$
- In the spatial equilibrium, people do not have an incentive to change their location.
- The **spatial equilibrium condition** in this model is

$$w + A - r = \bar{u}$$

- (What happens if this equality does not hold?)

Capitalization in spatial equilibrium

- From the spatial equilibrium condition, we have a formula for the land price:

$$r = w + A - \bar{u}$$

- Land price "capitalizes" the (relative) attractiveness of this location
  - The value of working in location X is w
  - The value of living in location X is A
  - Location X is, relatively speaking, less attractive when other locations are more attractive (higher $\bar{u}$)
- Intuition:
  - In spatial equilibrium, living in location X and elsewhere should give the same utility
  - Land price r must go up to offset such attractiveness of living in location X

Capitalization is a powerful tool for empirical analysis

- Although our spatial equilibrium model is very simple, its empirical implication is quite rich.
- Note that we do not usually observe the values of residential amenities A themselves, which include the attractiveness of various interesting things such as
  - transportation convenience
  - public goods quality
  - value of nice climates
- However, by measuring land prices in data, we can quantify the value of A!

Capitalization is a powerful tool for empirical analysis

- To formalize, suppose there are two locations i and j, sharing the common wage ($w_i = w_j$) and outside utility ($\bar{u}_i = \bar{u}_j$).
- Then, the land price difference $r_i - r_j$ equals the difference in the amenity value $A_i - A_j$
- To measure the value of amenity x of your interest, further suppose that $A_i = \beta x_i + \epsilon_i$
  - x_i may be some specific amenity, such as commuting distance, crime rates, test score of local school etc.
  - ϵ_i is an error term (or "unobserved amenities") for econometricians (us!).
- We can then measure the value of amenity x by the regression.

$$r_i = \beta x_i + \epsilon_i,$$

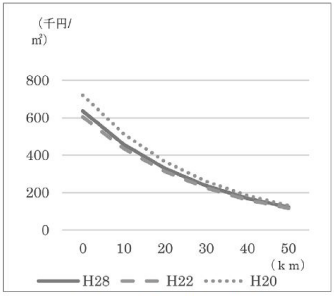
  Using this regression model, we get $r_i - r_j = \beta(x_i - x_j)$, which is the value of increasing the amenity level from x_j to x_i.
- This approach of using land price regressions to evaluate amenities is called a **Hedonic approach.**
  - Moreover, the quantification is *money-metric* and easy to interpret.
  - In practice, we typically apply log transformation for both r_i and x_i in the above regression equation.

Example: The simplest version of monocentric city model

- To fix ideas, suppose there are locations i and j in a city.
- They share the same wage ($w_i = w_j$) because people in both locations commute to the city center and work there.
- The outside utility is also the same ($\bar{u}_i = \bar{u}_j$) because they share the common outside option: living outside the city.
- Location i and j differ in the commuting cost to the city center.
  - Let $A_i = -\beta x_i + \epsilon_i$, where β is the commuting cost per 1km and x_i is distance from the city center.
- Then, the land price difference $r_i - r_j = -\beta(x_i - x_j) + (\epsilon_i - \epsilon_j)$.
  - Land price is lower in a location further away from the city center.
- Therefore, measuring the land price gap between the city center and the suburb corresponds to measuring the magnitude of commuting cost difference.
  - Motivated by this, we can regress land prices on distance from the city center

The rent gradient and the monocentric city model

- Consistent with this prediction, land prices in Tokyo get lower as moving away from city center[1]



（図 4） 地価勾配曲線の推移

- Since everyone commutes to the city center, this model is called "monocentric city model," dating back to Alonso (1964, book)
  - See Brueckner (1987, Handbook of Regional and Urban Economics), Fujita (1989 book), and Yamagishi and Sato (2025) for more discussions on the monocentric city model.

[1] Figure is taken from Ueno (2017) `https://www.jstage.jst.go.jp/article/jares/32/2/32_119/_pdf`

Hedonic approach: applications

- Measurement of commuting cost using land prices is a classic application of hedonic approach.
- But the hedonic approach is not "outdated" at all: The hedonic approach is widely used even in modern empirical papers!
- I cover several empirical examples to illustrate its usefulness in measuring various objects of interest
  - School quality (Black 1999)
  - Severity and persistence of discrimination (Yamagishi and Sato 2025)
  - Optimality of public goods provision (Brueckner 1982; Cellini et al. 2010)
  - Crime risk (Linden and Rockoff 2008)
  - Health risk (Kawaguchi and Yukutake 2017)

Note: Land price vs Housing price

- Note: In the empirical application, we use either housing price data or land price data, depending on which one is available.
  - Japan has maintained good land price data for a long time. while the US and other countries often have richer data in housing prices.
- We can interchangeably use either log housing or land price data in regression, but using housing price data may push all coefficients toward zero when housing supply is more elastic
  - More formally, when housing production is Cobb-Douglas in a static model, we have $\ln(\text{housing price}) = \beta \times \ln(\text{land price}) + \text{constant}$, where $\beta \in (0, 1)$ is the input share of land
- Intuitively, since we can increase floor space by investing in more construction materials to build taller, the distribution of housing prices looks similar to the distribution of land prices but has less variance
- We come back to this point in the "housing supply" lecture

Black (1999 QJE)

- How can we measure school quality?
  - School has various effects on children (e.g., test score, non-cognitive skills, friend network...), and it is hard to measure directly.
- Suppose that A_i includes school quality in school district i.
  - $A_i = s_i + X_i$, where s_i is school quality and X_i is the other location characteristics of district i.
- Then, the land price difference $r_i - r_j$ equals the difference in school quality $s_i - s_j$, *if* $X_i = X_j$ *holds so that school districts i and j are homogeneous except for school quality*
  - To be precise, we also need $w_i = w_j$ and $\bar{u}_i = \bar{u}_j$.
- How can we ensure $X_i = X_j$?

Border design

- Black (1999) adopts **border design** to focus on similar land that differs only in school quality
  - Right across the border of school districts, land plots should have similar characteristics so that $X_i = X_j$. But school quality differs.
  - This is a small area, so that $w_i = w_j$ and $\bar{u}_i = \bar{u}_j$ also seem reasonable.
- To implement the border design in a regression, focus on samples around the border and regress housing prices on school-district fixed effects.

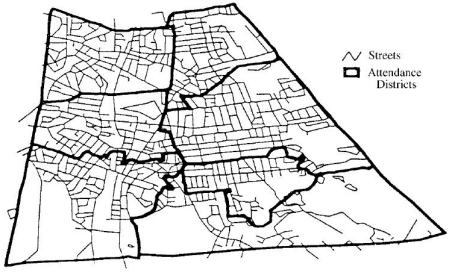


FIGURE I
Example of Data Collection for One City: Melrose Streets, and Attendance District Boundaries

Border design

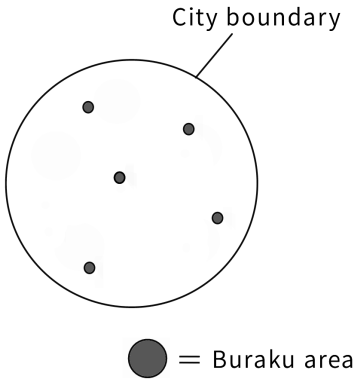
- Black: In suburban Boston, people are willing to pay 0.5% more housing prices for 1% increase in test scores.
  - See Kuroda (2022 JRS) and Nakazawa and Inukai (2025, wp) for a Japanese situation.
- Takeaway: In evaluating a particular amenity (school quality in this study) using land prices, you should make sure to compare "similar land plots" that differ only in this particular amenity.
- Border design is one plausible and commonly-used strategy to ensure this.
  - Not just controlling for *observable* location characteristics, border design allows us to control for *unobservable* location characteristics as long as they do not jump at the border.
- I will next introduce my paper as a more recent example of this strategy.

Yamagishi and Sato (2025 REStat)

- Discrimination often persists in society, even after *de jure* discrimination against a minority group was abolished,
- It has been difficult to quantify persistence of discrimination against a minority group:
  - Data availability
  - The abashment of discriminatory rules is often relatively recent
  - No random variation in minority group affiliation
- We analyze *buraku discrimination* in Japan, in which one faces higher risk of being identified as former outcaste (buraku) if they live in historical neighborhoods of the outcaste group.
- This paper provides quantitative evidence on persistence of discrimination, combining 100 years of granular land price data, a simple spatial economic model, and a border design

Territorial stigma and discrimination risk

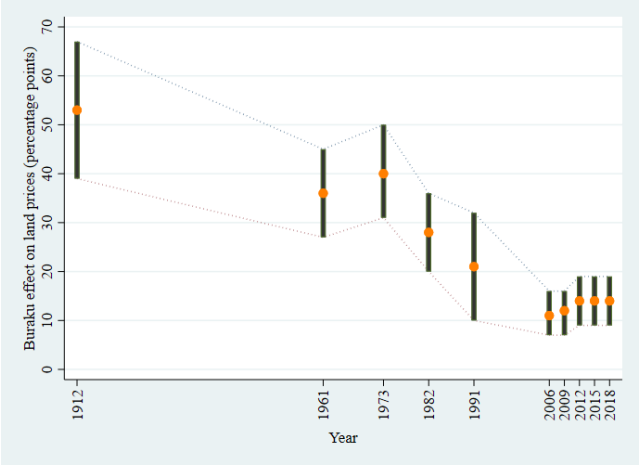
- The former outcaste (*buraku*) has little visible distinction from the majority, in terms of ethnicity, language etc.
  - Discrimination stems from the occupation in the pre-modern period.
  - *"Japan's Invisible Race"*
- Living in the buraku areas still carries a stigma.
- Under this situation, **living in a buraku area may signal the group affiliation and increase the risk of experiencing discrimination**
  - Discrimination risk is considered as a part of (negative) amenities A_i of buraku neighborhood i.



100 years of the buraku land price penalty

Apply the border design to 100 years of land price data of Kyoto city, the land price discount of buraku areas has substantially declined but it still persists

→ **novel quantitative evidence on severity and persistence of buraku discrimination**



Brueckner (1982 JPUBE)

- Local governments may decide the level of public goods provision, such as public schools and firefighting
  - More public goods mean higher local tax rates, so there should be some "optimal" level of local public goods provision
- Brueckner's key claim: the optimal level of local public goods maximizes land prices

Public goods optimality and land prices

- To see this point, let t be the spending level for public goods (i.e., the local tax)
- Since more taxes are required to provide more public goods, the amenities $A(t)$ are increasing in the tax rate t.
- The utility in this region is written as

$$(w - t) + A(t) - r,$$

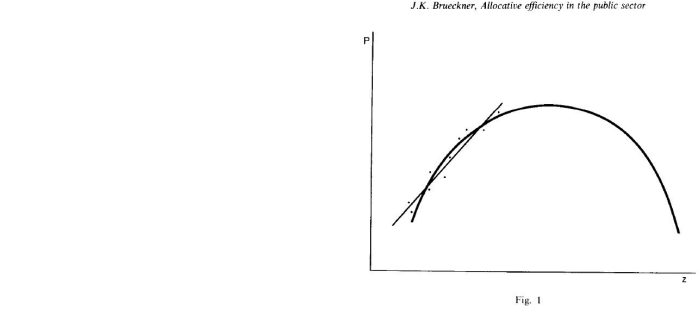
  which is maximized at t^* that satisfies the first-order condition $A'(t^*) = 1$.
- From the spatial equilibrium condition, the land price is written as

$$r = (w - t) + A(t) - \bar{u},$$

  which is also maximized at t^*!
- Intuition: When the public good is optimally provided, people are willing to pay the highest land prices to live in this location.

Empirical implication

- Taking t on horizontal axis and r on vertical axis, we would get a U-shaped relationship (see Figure)
  - This happens if $A(t)$ is U-shaped, which seems natural.
- Regress r on t. If it is positively sloped as in the Figure, the public goods are under-provided.
  - Zero slope around the optimum. Negative slope if over-provided.
- Brueckner: both education and non-education spending had zero slope in Massachusetts communities $\rightarrow$ optimal provision



Remark on the generality of Brueckner's approach

- Note that Brueckner's approach is more general than the case of public goods provision with local taxes.
- The optimality of *any* policy that has inverse U-shape impact on utility can be evaluated using Brueckner's approach
  - Broad applicability: I have already written two papers that use Brueckner's approach!
- Yamagishi (2021 RSUE): evaluating the desirability of minimum wages based on Brueckner's approach
  - Minimum wages are likely to improve workers' welfare when they are low and the disemployment effect is small
  - However, as they rise, the disemployment effect may get larger and start damaging workers.
  - Using Japanese quasi-experiment to identify the impact of minimum wages on housing rents of low-quality apartments.
- Goto and Yamagishi (2024 wp) also invoke Brueckner's approach for evaluating the public-sector wage cut in Japan.

Cellini, Ferreira, Rothstein (2010 QJE)

- In implementing Brueckner's optimality test, "regressing r on t" using OLS may suffer from endogeneity:
  - For example, parents caring about education quality (omitted variable) may increase t, but they also create good neighborhoods and increase r
  - If such a variable exists, locations with different t are no longer comparable as they have different characteristics other than t.
- Cellini et al. look at the case of California, in which increasing investment in school requires bond issues but the bond issue requires voting
- The area with 49% vote share and the area with the 51% vote share are likely very similar, but only the latter increase public spending
  - Regression discontinuity approach in public investment in schools.

Discontinuous jump in education investment

- The public expenditure on education investment indeed jumps discontinuously at 50% vote share.
- Data on the year before the elcction is shown as a "placebo test"
  - If election results matter, then we should see no discontinuity in public spending before the election

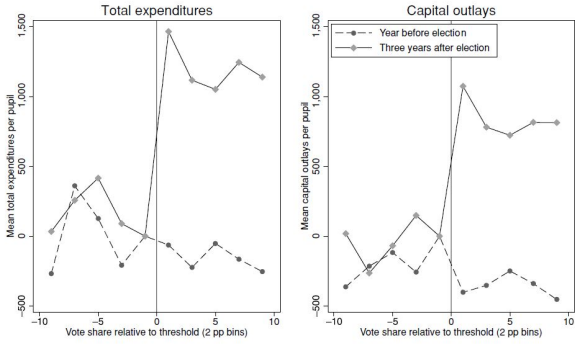
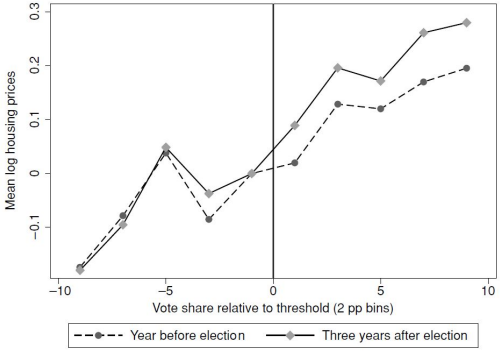


FIGURE II
Total Spending and Capital Outlays per Pupil, by Vote Share, One Year before and Three Years after Election

Discontinuous jump in housing prices

- Housing prices also increase discontinuously
- According to Brueckner's (1982) model, this suggests underinvestment in education
- Key takeaway: comparing locations with similar characteristics seems important
  - For achieving such "apples-to-apples" comparison, this paper exploits discontinuous jumps in spending with respect to vote share.



Linden and Rockoff (2008 AER)

- How costly is local criminal risk?
- Linden and Rockoff: combining location information disclosure of sex offenders and the hedonic approach.
- Since sex offernders tend to commit sex crime again, their new move-in increases the crime risk
- This high crime risk, included in A as a negative amenity, should capitalize into land prices.

Location information of sex offenders

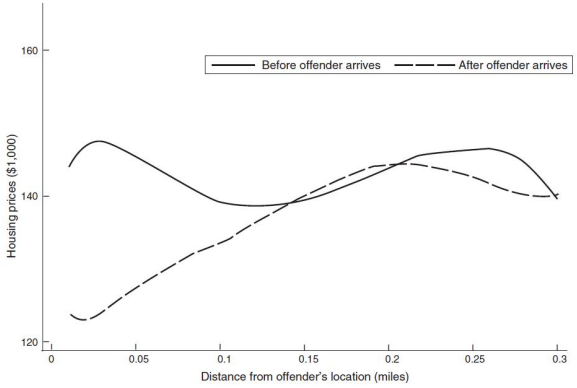
- North Carolina discloses information about the location of sex offenders (see Figure).

FIGURE 1. AN OFFENDER AREA AND SURROUNDING NEIGHBORHOODS

Note: X marks the center of the offender's exact location. The surrounding circle marks all parcels within one-quarter mile. Neighborhoods are distinguished by shades of gray. Parcels within a neighborhood are usually, but not necessarily, contiguous.

Sex offenders and housing price decline

- Substantial house price decline near the sex offenders, after the arrival of sex offenders
- Assuming something like "land price decline = increased crime risk $\times$ money-metric utility cost of victimization", the cost of sex crime amounts to 1.2 million US dollars
  - Substantially larger than conventional methods based on surveys or jury decisions



Kawaguchi and Yukutake (2017 JUE)

- The 2011 Tohoku earthquake and tsunami caused a nuclear accident at Fukushima Daiichi power plant, resulting in radioactive contamination around the plant.
- People were afraid of its risk for health
- How can we quantify the cost of radioactive contamination?
  - Use the hedonic approach, supposing that A includes health risk by living in this location.

Geography of radioactive contamination

- The contamination is concentrated around the power plant, mostly within 30km.
- Due to the wind direct as of the accident, the spread of contamination is concentrated in the northwest of the plant.
- Kawaguchi and Yukutake regress land prices on the measure of radioactive density to quantify the health risk

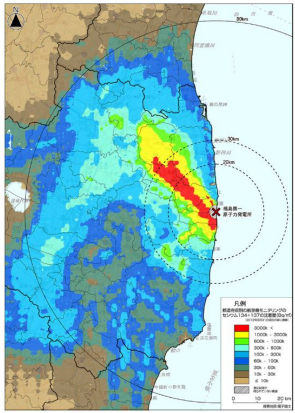
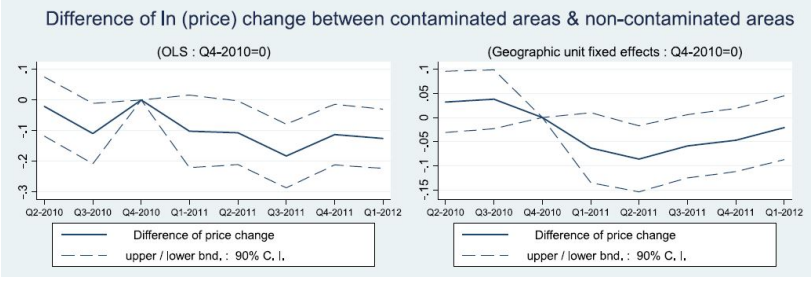


Fig. 1. The density of cesium-134 and cesium-137, as of May 31, 2012 Source: Results of the airborne survey conducted by the Ministry of Education, Culture, Sports, Science and Technology

Land price decline and radioactive contamination

- After the accident, the land prices of contaminated areas declined relative to non-contaminated areas
- The total damage caused by the Fukushima accident is estimated to be around 1.5 to 3.0 trillion Yen, or 0.13% to 0.25% of Japan's total land value
- However, not just the actual health risk, this may include some "over-shooting" as people initially over-estimate the risk of nuclear accidents
  - *Even in the US,* housing prices near a nuclear power plant declined for a year after the Fukushima accident (Tanaka and Zabel 2018 JEEM).



Taking stock

- Even a simplest spatial equilibrium model has rich empirical implications
  - We can measure the value of various residential amenities, even when they are not directly observable
  - School quality, discrimination, optimality of public goods provision, crime risk, health risk....
  - You can come up with what you want to measure and write a paper using the capitalization approach.
- However, the simplest model is unsatisfactory in some senses:
  - Endogenous response of wages is ignored
  - We have focused on land prices, but don't we want to talk also about population distribution?
  - What exactly is the "outside region" and the associated outside utility?
- We next introduce the Rosen-Roback model: a general-equilibrium spatial economic model with a local labor market.
  - The Rosen-Roback relaxes the first two limitations
  - We further relax the last limitation when discussing a quantitative spatial model