Heterogeneity and Sorting

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Heterogeneous workers and sorting

- When workers are heterogeneous, **geographical sorting** might happen
  - People of similar type tend to locate in the same location
  - "Type" can be defined in many ways: education, income, race, age etc....
  - "Birds of the feather flock together"
- Common definition of sorting: compared to the national average share of type X people, some locations have the substantially larger share than the national average and other locations have the lower share
  - This situation is also called segregation
- I first briefly discuss classic theories of sorting
  - Tiebout (1956 JPE)
  - Schelling (1969 AER; 1971 Journal of Mathematical Sociology)
  - Cutler, Glaeser, Vigdor (1999, JPE)
- I then introduce a simple model of sorting to discuss sources of residential sorting
  - Following Diamond and Gaubert (2022 Annual Review of Economics, Section 3)
- I also discuss empirical evidence of sorting along the way

Tiebout (1956 JPE)

- Lays out the so-called "Tiebout hypothesis"
- Suppose there are two types of workers: A and B
  - Worker A enjoys public school quality
  - Worker B do not enjoy public school quality
- There are two locations: a and b. Each location decides on school quality while imposing local taxes
  - Workers can choose between these two locations
- Tiebout hypothesis: The equilibrium outcome involves sorting:
  - Every type A worker lives in location a, and location a provides the best school quality for worker type A.
  - Every type B worker lives in location b, and location a provides the best school quality for worker type B.
- This equilibrium outcome is *efficient*
  - Workers *"vote with their feet"* about school quality
  - Called "Tiebout Sorting"

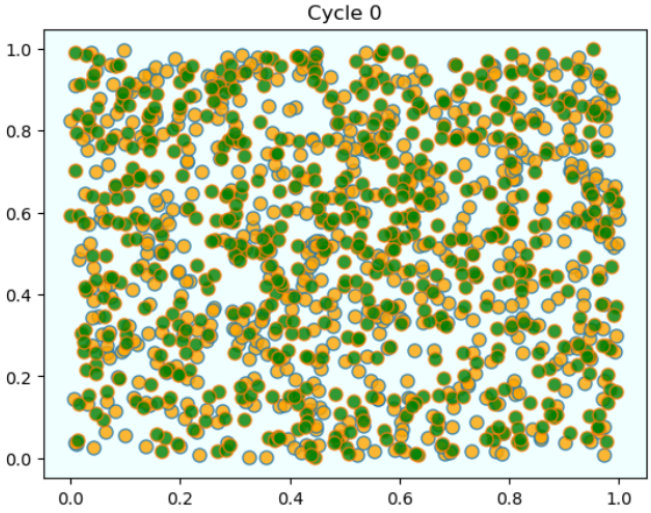
Schelling (1969 AER; 1971 JMS)

- Analyzes the cause of sorting by race
  - Such sorting is often called "segregation."
- A simple model demonstrating that even a small preferences for living together with the same race leads to extreme sorting by race
- A worker with race x is *happy* if more than 5 of their 10 nearest neighbors are of the same race
  - They are happy to live in a mixed neighborhood, but they have some preferences for the same race.
- If they are unhappy, they reallocate to a random location in a city
- The analysis is based on a simulation
  - QuantEcon provides a simple simulation code in Python[1]

[1] `https://intro.quantecon.org/schelling.html`

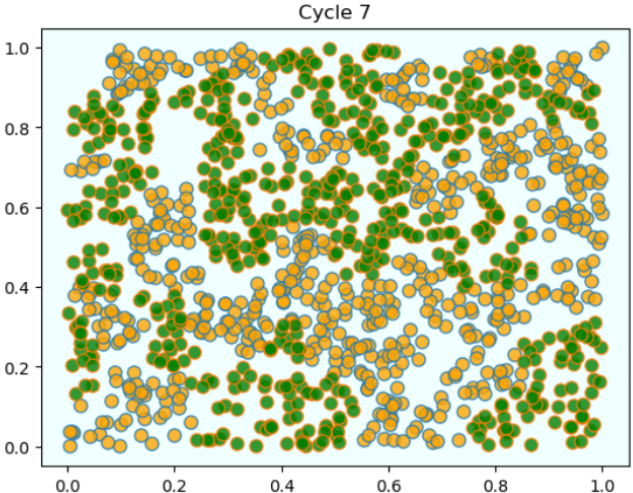
Schelling (1969 AER; 1971 JMS)

- Start with a perfectly mixed situation



Schelling (1969 AER; 1971 JMS)

- But things quickly converge to extreme sorting by race
- Here, everyone is "happy" and people no longer move



Cutler, Glaeser, Vigdor (1999 JPE)

- Tiebout and Schelling implicitly assume that people do not actively exclude certain types of people
  - Discrimination may prevent some people from living in certain locations, which naturally leads to a sorted outcome
- Cutler, Glaeser, Vigdor considers the possibility of "collective exclusion"
  - Whites may exclude blacks by discrimination in the housing market, violence, bullying etc.
- If blacks are discriminated against, they may pay higher housing prices in equilibrium than whites
  - Intuitively, the blacks are willing to avoid pay money for moving away from whites and avoiding discrimination
- In the mid-twintieth century, blacks clustered in certain locations ("ghettos") and indeed paid more housing costs than whites $\rightarrow$ consistent with collective exclusion
- By 1990, whites now pay more for housing
  - Consistent with "white flight": whites pay money to avoid living with blacks
- See Boustan (2013 wp) for more discussions

Is sorting good or bad?

- Often, sorting (or segregation) is seen as a "bad" phenomenon
  - A more mixed situation is often considered as "desirable"
  - For instance, the segregation of the blacks in certain area ("Ghettos") is considered as a bad situation in light of racial equity
- But this is not obvious at all!
- Both Tiebout and Schelling suggest that sorting may be good in terms of *everyone's* welfare
  - Different people have difference preferences, and sorting is a way to respect such heterogeneous preferences
- Reasons to still dislike sorting
  - Maybe we should not judge the social welfare based on discriminatory preferences
    - Whites may dislike blacks and Asians. But should we satisfy such preferences?
  - We care regional inequality itself, above and beyond the individual-level inequality
    - See Gaubert, Kline, Vergara, Yagan (2025 AER)
  - There could be negative externality associated with sorting
    - For instance, sorting of the rich and the poor may lead to bad performances of children (e.g., Chetty et al. 2016 AER; 2022 Nature).

A model of skill sorting: Diamond and Gaubert (2022 ARE)

- There are two types of workers: Skilled ($\theta = S$) and Unskilled ($\theta = U$).
- Similar to Redding (2016 JIE), a worker of type θ has Cobb-Douglas utility:

$$u_i^\theta(\omega) = A_i^\theta (c_i^\theta)^{1-\alpha^\theta} (h_i^\theta)^{\alpha^\theta} \epsilon_i^\theta(\omega)$$

  - A_i^θ is the amenity, which can be type-specific.
  - c_i is numeraire goods consumption and h_i is the housing consumption
  - α^θ is the spending share of land for housing.
    - $\alpha^U > \alpha^S$ to respect the data that the poor spend more share of their income on housing.
  - $\epsilon_i^\theta(\omega)$ is the Frechet idiosyncratic taste shock with the dispersion parameter κ^θ.
- Maximizing this under the budget constraint $c_i + r_i h_i = w_i^\theta$, the indirect utility is

$$v_i^\theta(\omega) = \frac{A_i^\theta w_i^\theta}{r_i^{\alpha^\theta}} \epsilon_i^\theta(\omega)$$

Location choice probability and welfare

- Due to the Frechet idiosyncratic shock, the location choice probability is

$$\lambda_i^\theta = \frac{L_i^\theta}{\bar{L}^\theta} = \frac{\left(\frac{A_i^\theta w_i^\theta}{r_i^{\kappa^\theta}}\right)^{\kappa^\theta}}{\sum_j \left(\frac{A_j^\theta w_j^\theta}{r_j^{\kappa^\theta}}\right)^{\kappa^\theta}}$$

- Note that the migration elasticity κ^θ is different across types
- The expected welfare of type θ:

$$W^\theta = \Gamma\left(\frac{\kappa^\theta - 1}{\kappa^\theta}\right)\left[\sum_k \left(\frac{A_j^\theta w_j^\theta}{r_j^{\kappa^\theta}}\right)^{\kappa^\theta}\right]^{1/\kappa^\theta}$$

Production

- Production function of the free-trade numeraire goods is of the CES type:

$$Y_i = \left[(z_i^U)(L_i^U)^{\frac{\rho-1}{\rho}} + (z_i^S)(L_i^S)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

- z_i^U and z_i^S reflect local productivity (can be exogenous or endogenous)
- By solving firms' cost minimization problem, the relative labor demand satisfies the following:

$$\ln\left(\frac{L_i^S}{L_i^U}\right) = \ln\left(\frac{z_i^S}{z_i^U}\right) - \rho \ln\left(\frac{w_i^S}{w_i^U}\right)$$

Amenities and housing

- Amenities can be exogenous or endogenous:

$$A_i^{\theta} = A^{\theta}(\bar{A}_i, L_i^U, L_i^S)$$

- Housing supply is the upward-sloping curve:

$$H_i = \bar{H} r_i^{\eta_i},$$

where η_i represents the local housing supply elasticity

Sources of sorting

- Let Δ_{ij} denote the difference between locations i and j: $\Delta_{ij}x = x_i - x_j$
- $\Delta_{ij} \ln(\frac{L^S}{L^U}) \neq 0$ represents sorting
  - The share of the high-skilled is higher in one location than in another location
- By the tedious rearrangement of the location choice probability expression, we get the following decomposition formula of the sources of sorting

$$\Delta_{ij} \ln\left(\frac{L^S}{L^U}\right) = \underbrace{\frac{\tilde{\kappa}^S}{\rho}\Delta_{ij} \ln\left(\frac{z^S}{z^U}\right)}_{\text{Productivity}} + \underbrace{\tilde{\kappa}^S \Delta_{ij} \ln\left(\frac{A^S}{A^U}\right)}_{\text{Amenities}}$$

$$+ \underbrace{\tilde{\kappa}^S(\alpha^U - \alpha^S)\Delta_{ij} \ln r}_{\text{Housing cost}} + \underbrace{\frac{\tilde{\kappa}^S}{\kappa^U}(1 - \frac{\kappa^U}{\kappa^S})\Delta_{ij} \ln L^U}_{\text{Heterogeneous migration elasticities}}$$

Sources of sorting 1: Productivity

$$\underbrace{\frac{\tilde{\kappa}^S}{\rho}\Delta_{ij}\ln\left(\frac{z^S}{z^U}\right)}_{\text{Productivity}}$$

- When the comparative productivity of the skilled $\left(\frac{z^S}{z^U}\right)$ is higher in location i than j, then location i has higher share of the skilled workers
  - Note that the improvement of the skilled productivity in *all* locations does not induce the sorting
- Empirical evidence: agglomeration forces in productivity work stronger for the higher-skilled workers (Baum-Snow, et al. 2018 AEJ Applied)
  - So larger cities tend to have higher $\frac{z^S}{z^U} \rightarrow$ sorting of the skilled into larger cities

Sources of sorting 2: Amenities

- To fix ideas, suppose that A_i^θ is written as the combination of common amenity level and type-specific preferences:

$$A^\theta = A_i^{\gamma_A^\theta}$$

- Then, the amenity term is rewritten as

$$\underbrace{\tilde{\kappa}^S \Delta_{ij} \ln\left(\frac{A^S}{A^U}\right)}_{\text{Amenities}} = \tilde{\kappa}^S \Delta_{ij} (\gamma_A^S - \gamma_A^U) \Delta_{ij} A.$$

  That is, if preferences for amenities are stronger for the skilled, the higher amenity level induces sorting of the skilled (and vice versa)

- Empirical evidence:
  - Albouy et al (2016 JAERE): college-educated households are willing to pay more for good weather
  - Diamond (2016 AER): Sorting of the skilled induces the provision of amenities that the skilled like (e.g., museums, shopping environments), and it amplifies sorting.

Sources of sorting 3: Housing costs

$$\underbrace{\tilde{\kappa}^S(\alpha^U - \alpha^S)\Delta_{ij} \ln r}_{\text{Housing cost}}$$

- Since the poorer people spend more share of their income on housing, higher housing costs tend to induce the sorting of the skilled.
  - $\alpha^U - \alpha^S > 0$
- Empirical evidence:
  - Rosenthal (2014 AER): The poorer indeed spend more share of their income for housing.
  - Gyourko, Mayer, Sinai (2013 AEJ Policy): Cities with inelastic land supply experienced the appreciation of housing costs, and it displaced low-income households.

Sources of sorting 4: Heterogeneous migration elasticity

$$\underbrace{\frac{\tilde{\kappa}^S}{\kappa^U}\left(1-\frac{\kappa^U}{\kappa^S}\right)\Delta_{ij}\ln L^U}_{\text{Heterogeneous migration elasticities}}$$

- A bit hard to establish formally, but the coefficient $\frac{\tilde{\kappa}^S}{\kappa^U}\left(1-\frac{\kappa^U}{\kappa^S}\right)$ roughly captures the migration elasticity of the skilled relative to the unskilled
  - Can be rewritten as $\frac{\rho}{\kappa^S+\rho}(\kappa^S/\kappa^U-1)$
- Intuitively, $\Delta_{ij}\ln L^U$ represents how attractive location i is relative to j
  - Measured by how popular location i is among the unskilled.
- If the location i offers higher utility $\Delta_{ij}\ln L^U > 0$ and the skilled are more responsive to this positive utility difference, then location i has more skilled workforce
- Empirical evidence
  - The mobility of the high-skilled is higher (e.g., Diamond 2016; Kaplan and Schulhofer-Wohl 2017 IER)

Welfare implications in a sorting model

- How can we evaluate the welfare of workers in a sorting model?
- Let $\hat{x}$ be the change in the variable x over time. Then, the change in welfare of type θ is expressed as

$$\hat{W}^\theta = \frac{W^\theta_{t_2}}{W^\theta_{t_1}} = \left[\sum_k \left(\frac{A^\theta_{jt_2} w^\theta_{jt_2}}{r^{\kappa^\theta}_{jt_2}}\right)^{\kappa^\theta} \left(W^\theta_{t_1}\right)^{-\kappa^\theta}\right]^{1/\kappa^\theta}$$

$$= \left[\sum_k \frac{\left(\frac{A^\theta_{jt_2} w^\theta_{jt_2}}{r^{\kappa^\theta}_{jt_2}}\right)^{\kappa^\theta}}{\left(\frac{A^\theta_{jt_1} w^\theta_{jt_1}}{r^{\kappa^\theta}_{jt_1}}\right)^{\kappa^\theta}} \left(\frac{A^\theta_{jt_1} w^\theta_{jt_1}}{r^{\kappa^\theta}_{jt_1}}\right)^{\kappa^\theta} \left(W^\theta_{t_1}\right)^{-\kappa^\theta}\right]^{1/\kappa^\theta} = \left[\sum_k \left(\hat{V}^\theta_k\right)^{\kappa^\theta} \left(\lambda^\theta_{kt_1}\right)\right]^{1/\kappa^\theta},$$

where $V^\theta_k \equiv \frac{A^\theta_k w^\theta_k}{r^{\kappa^\theta}_k}$ and $\lambda^\theta_{kt_1}$ is the choice probability of location k in period t_1.

Welfare implications in a sorting model

- In particular, change in the skilled-unskilled relative welfare, a measure of inequality, can be written as follows

$$\frac{\hat{W}^S}{\hat{W}^U} = \frac{\left[\sum_k \left(\hat{V}_k^S\right)^{\kappa^S} \lambda_{kt_1}^{\theta}\right]^{1/\kappa^S}}{\left[\sum_k \left(\hat{V}_k^U\right)^{\kappa^U} \lambda_{kt_1}^{\theta}\right]^{1/\kappa^U}}$$

- Using this formula, we can understand how sorting, which accompanies changes in A, w, r affects inequality
- Results (see Diamond and Gaubert 2022 Section 4.1.2 and Diamond 2016 AER):
  - Wages induced inequality over the last 40 years of the US
  - Taking into account housing cost reduces inequality
  - Considering amenities magnifies inequality
- Caution: You should be careful about whether the utility function you use is adequate for measuring welfare (see slide page 7).

Some examples of sorting studies

- To fix ideas, I note a few examples (my favorites!) that include sorting
- Glaeser, Kahn, Rappaport (2008 JUE)
  - In standard monocentric city models, the rich are often predicted to live in the suburbs because they appreciate more housing space.
  - After arguing that this prediction does not seem to hold empirically
  - Instead, they argue that public transportation that is evaluated by the poor explains why the poor choose to live in the city center
  - See Tabuchi (2019 JUE) for a related analysis on the situation of Tokyo
- Heblich, Trew, Zylberberg (2021 JPE)
  - The poor sort into the east side of the city in the UK
  - Historically, these areas experienced severe pollution due to wind direction, and the poor chose to live there because they care less about pollution
  - Such sorting pattern continues even today, despite that pollution no longer exists
- Bayer, Ferreira, McMillan (2007 JPE)
  - Sorting across the school district borders. We have already seen this paper in the discrete choice lecture!

Taking stock

- Sorting of heterogeneous people into different locations
- We have first seen some classic studies about the mechanism behind sorting
- We have then seen a simple spatial model with heterogeneous types of workers
  - Implies a "decomposition formula" for sorting
- Discussed the four sources of sorting:
  - Productivity
  - Amenities
  - Housing costs
  - Heterogeneous migration elasticity