Rosen-Roback Models

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Rosen-Roback spatial economic models

- In the canonical spatial equilibrium model we have introduced, we did not talk about how wages and population are determined
- We address these considerations by formulating a richer spatial economic model, called "Rosen-Roback model."
  - As in the canonical spatial equilibrium model, the (modified) hedonic approach is justified by the theory for evaluating amenities
  - Named after two classic papers: Rosen (1979 book chapter) and Roback (1982 JPE).
  - Zidar's lecture note is helpful.[1] See also Glaeser (2008 Chapter 3).
- The Rosen-Roback provides a microfoundation for the hedonic approach to evaluate amenities, while taking wages into account.
  - Rosen-Roback is an extension of the canonical spatial equilibrium model.

[1] https://zidar.princeton.edu/sites/g/files/toruqf3371/files/zidar/files/zidar_eco524_s2020_lec2.pdf

Model overview

- There are N locations in this economy. $i = 1, ..., N$.
  - And also an outside location, which gives outside utility $\bar{V}$.
- Markets:
  - Local labor markets. Prices (w_i). Quantity (L_i)
  - Local housing markets, where housing is used both for residence and production. Prices (r_i). Quantity (H_i^R, H_i^P)
  - Tradable goods market. Prices (1). Quantity (Y_i)
- Agents:
  - Workers (continuum, homogeneous)
  - Firms (perfectly competitive, CRS technology)
  - Landlords (They just receives land rents behind the scene)
  - Note: We did not have firms in the canonical spatial model in the previous lecture!
- Indifference conditions
  - Spatial equilibrium condition that equalizes workers' utility across locations
  - Firms are also indifferent in production location (earns zero profit at any location, in equilibrium)

Workers: Indirect utility

- Workers have Cobb-Douglas utility $U = A^R C^\alpha (H^R)^{1-\alpha}$, where
  - A_i^R is the residential amenity
  - C is goods consumption (price is 1)
  - H^R is land consumption (price is r_i)
- Workers in location i maximize this under the budget constraint $C_i + r_i H_i = w_i$:
  - The demand for goods is $C_i = \alpha w_i$
  - The demand for housing is $H_i^P = (1-\alpha)\frac{w_i}{r_i}$
- The indirect utility of living in location i, V_i is written as

$$V_i = \ln w_i - (1-\alpha)\ln r_i + \ln A_i^R + X_w,$$

where X_w is a constant term.

Workers: Indirect utility

- In spatial equilibrium, this equalizes across all locations:

$$\ln w_i - (1-\alpha)\ln r_i + \ln A_i^R + X_w = \ln \bar{V},$$

  for all $i = 1, ..., N$.

- The outside utility $\bar{V}$ is assumed to be exogenously given.
- Rearranging the above spatial equilibrium condition yields the following formula for land prices:

$$\ln r_i = \frac{1}{1-\alpha}\ln w_i + \frac{1}{1-\alpha}\ln A_i^R + X_w,$$

  where X_w again represents a constant number common for all i.

Graphical illustration

- The spatial equilibrium condition is an upward-sloping line in the $\ln r - \ln w$ plane.
- Intuitively, why is this upward-sloping?

Firms: Profit maximization and labor demand

- Firms have the production function $Y = A^P(L)^\beta(H^P)^{1-\beta}$
  - A_i^P is the productivity (capturing natural conditions, infrastructure quality etc)
  - L is labor input (its price is w)
  - H^P is housing used for production
- Firms in location i minimize their production cost $(w_i L_i + r_i H_i^P)$ to achieve a given level of production $(Y_i = \bar{Y})$:
  - Labor input demand: $L_i = \bar{Y}(A_i^P)^{-1}(\beta/(1-\beta))^{1-\beta} w_i^{\beta-1} r_i^{1-\beta}$
  - Housing input demand: $H_i^P = \bar{Y}(A_i^P)^{-1}(\beta/(1-\beta))^{-\beta} w_i^{\beta} r_i^{-\beta}$
- Using this, firms' cost function per one unit good is

$$(A_i^P)^{-1}\left((\beta/(1-\beta))^{1-\beta} + (\beta/(1-\beta))^{-\beta}\right) w_i^\beta r_i^{1-\beta}$$

Zero profit condition and local labor demand

- The goods price is one (numeraire) and firms earn zero profits in equilibrium due to perfect competition.

- Because the technology is constant-returns-to-scale, the zero profit implies that the unit cost should equal one:

$$(A_i^P)^{-1}\left((\beta/(1-\beta))^{1-\beta}+(\beta/(1-\beta))^{-\beta}\right)w_i^{\beta}r_i^{1-\beta}=1$$

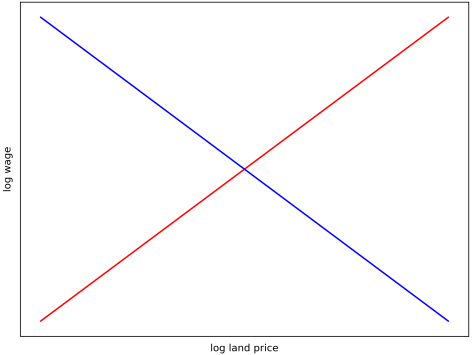
- Rearranging this and ignoring the constant term, we obtain the local labor demand function:

$$\ln w_i=-\frac{1-\beta}{\beta}\ln r_i+\frac{1}{\beta}\ln A_i^P+X_p,$$

where X_p represents a constant term common for all i.

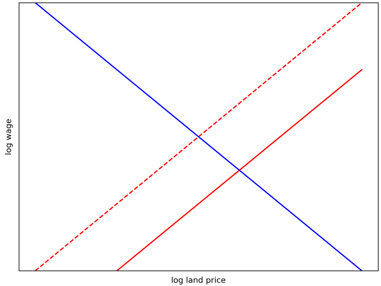
Graphical illustration

- The local labor demand function is downward-sloping in the $\ln r - \ln w$ plane
- Intuitively, why?
- The intersection of the two curves defines the equilibrium (r, w).



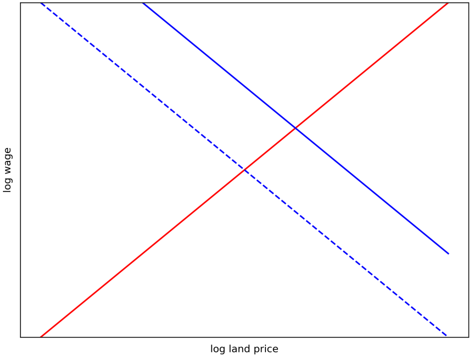
Graphical illustration: Comparative statics

- Suppose that the residential amenity A_i^R improves
  - The red curve shifts downward as workers now accept lower wages
- The equilibrium r increases and w decreases
- When $\beta \simeq 1$ so that production does not use land, the blue line is flat and only r changes
  - This case reduces to a canonical spatial model: wage is fixed and land prices adjust to satisfy the spatial equilibrium



Graphical illustration: Comparative statics

- Suppose that the production amenity A_i^P improves
  - The blue curve shifts upward as firms can pay higher wages
- The equilibrium r and w increase



Inferring the value of amenities for workers

- We have $V_i(w_i(A_i^R), r_i(A_i^R), A_i^R) = \ln w_i - (1-\alpha)\ln r_i + \ln A_i^R + X_w = \ln \bar{V}$ in spatial equilibrium.
- Differentiating this indirect utility with respect to A_i^R, we get the MWTP for amenities

$$
\begin{aligned}
& V_w w' + V_r r' + V_A = 0 \\
\Rightarrow & V_A = -V_w w' - V_r r' = -V_w w' + H_i^R V_w r' \quad (\because \text{Roy's identity}) \\
\Rightarrow & \underbrace{\frac{V_A}{V_w}}_{\text{Marginal WTP}} = \underbrace{H_i^R r'}_{\text{Changes in land cost}} - \underbrace{w'}_{\text{Wage change}} = w(\theta \frac{r'}{r} - \frac{w'}{w})
\end{aligned}
$$

- We can estimate r'/r and w'/w by regressing $\ln r$ and $\ln w$ on residential amenity A_i^R.
  - Wage w and the land share of income $\theta \equiv H_i^R r/w$ are observable in data.
- Integrating this over $[\underline{\alpha_i^R}, \overline{\alpha_i^R}]$, we can evaluate the willingness-to-pay for improving the amenities from $\underline{\alpha_i^R}$ to $\overline{\alpha_i^R}$.
- Note: If $H_i^R = 1$ and $w' = 0$ always holds as in our canonical spatial equilibrium model, the MWTP equals changes in land rents.

Inferring the value of amenities for firms

- In equilibrium, the unit cost function $c(w_i(A_i^P), r_i(A_i^P), A_i^P) = (A_i^P)^{-1}\left((\beta/(1-\beta))^{1-\beta} + (\beta/(1-\beta))^{-\beta}\right) w_i^{\beta} r_i^{1-\beta} = 1$ holds.
- Differentiating this unit cost function, we get

$$
\begin{aligned}
& c_w w' + c_r r' + c_A = 0 \\
\Rightarrow & c_A = (-c_w w' - c_r r') = (-\frac{P_i}{Y_i} w' - \frac{H_i^P}{Y_i} r') \quad (\because \text{Shephard's Lemma}) \\
\Rightarrow & c_A = (-\Theta_w \frac{w'}{w} - \Theta_r \frac{r'}{r}),
\end{aligned}
$$

  where Θ_w and Θ_r are the cost share of labor and land, respectively (these are observable in data).
- As the above discussion is about the per-unit cost saving, the total value of amenities for firms is $-Y_i c_A$

Inferring the total value of amenities

- Now consider the case in which residential and production functions are common.
  - Let $A_i^R = A_i^P = A_i$ and we marginally increase A_i.
- Now we should combine the amenity value for workers and firms.
- The sum of workers' and firms' MWTP is

$$P_i \frac{V_A}{V_w} - Y_i c_A = P_i H_i^R r' - P_i w' + P_i w' + H_i^P r' = \bar{H} r',$$

where P_i is the population of location i.

- Thus, the increase in aggregate land value corresponds to the total value of amenities.
  - Workers and firms are perfectly mobile, but land is perfectly immobile $\rightarrow$ land bears 100% incidence.
  - Our canonical spatial equilibrium model is a spatial case, in which the total value of amenities for workers = the increase in aggregate land value.

Aside: Population and housing/land market

- Our argument so far did not require us to derive equilibrium population level
  - This is because the level of population does not affect the decisions of each worker and firm in our specification
  - The two indifference conditions about the prices (spatial equilibrium condition and zero profit condition) are enough to pin down (w, r)
- To determine the population level of location i, P_i, we need a model of housing/land market
- To illustrate, suppose that inverse housing supply function $r_i = \eta P_i^{\gamma}$, where γ is housing supply elasticity
  - This constant-elasticity supply function is log-linear local approximation of any housing supply function and widely adopted in the empirical literature (e.g., Saiz 2010; Kline and Moretti 2014)
- We have already derived equilibrium r_i, and equilibrium population is given by $P_i = (\frac{r_i}{\eta})^{1/\gamma}$

Aside: Population and housing/land market

- Alternatively we can assume that location i has $\bar{H}_i$ amount of housing and it is inelastically supplied.
  - You can think that people directly consume land
  - Or, housing supply is fixed in the short-run as it is a durable stock (Glaeser and Gyourko 2005 JPE).
- The demand for housing per capita by workers is $(1-\alpha)\frac{w_i}{r_i}$, and the total demand is $P_i(1-\alpha)\frac{w_i}{r_i}$.
  - Therefore, in equilibrium, firms use $H_i^P = \bar{H}_i - P_i(1-\alpha)\frac{w_i}{r_i}$.
  - Since firms employ P_i workers in equilibrium, the equilibrium production is $Y_i = A_i^P (P_i)^\beta (\bar{H}_i - P_i(1-\alpha)\frac{w_i}{r_i})^{1-\beta}$
- The demand for housing by firms is $Y_i (A_i^P)^{-1} (\beta/(1-\beta))^{-\beta} w_i^\beta r_i^{-\beta}$. This equals $H_i^P = \bar{H}_i - P_i(1-\alpha)\frac{w_i}{r_i}$ in equilibrium.
- Thus, the equilibrium P_i is implicitly defined as a solution to

$$P_i^\beta (\bar{H}_i - P_i(1-\alpha)\frac{w_i}{r_i})^{-\beta} (\beta/(1-\beta))^{-\beta} w_i^\beta r_i^{-\beta} = 1$$

Applications

- I now talk about three applications of the Rosen-Roback model
  - Air pollution (Bayer, Keohane, Timmins 2009 JEEM)
  - Air pollution and information disclosure (Gao, Song, Timmins 2023 JDE)
  - City shape (Harari 2020 AER)
- As we have seen, the Rosen-Roback model takes into account wage changes in evaluating amenities
- As such, it is suitable for comparing different cities. Both applications belong to such cross-city case.
  - If we focus on a small area (e.g., the border design case), then wages are likely to be the same.
  - In such cases, the "canonical spatial equilibrium model" we discussed in the previous lecture often suffices.

Bayer, Keohane, Timmins (2009, JEEM)

- How can we evaluate air quality (PM10)?
- Based on our formula, we evaluate how wages and land prices respond to PM10:

$$\underbrace{\frac{V_A}{V_w}}_{\text{Marginal WTP}} = \underbrace{H_i^R r'}_{\text{Changes in land cost}} - \underbrace{w'}_{\text{Wage change}}$$

- Bayer et al. estimate the following specification (in their notation, Y is income and ρ is housing cost):

$$\ln \Delta Y_j = \gamma_{PM,Y} \Delta \ln PM_j + \Delta \mathbf{Z}_j' \boldsymbol{\beta}_Z + \gamma_{R,Y} R_j + u_j^Y,$$

$$\ln \Delta \rho_j = \gamma_{PM,\rho} \Delta \ln PM_j + \Delta \mathbf{Z}_j' \boldsymbol{\beta}_Z + \gamma_{R,\rho} R_j + u_j^\rho.$$

Impact of air quality on income and housing cost

- PM10 may be endogenous to economic variables (income, housing price)
- To deal with this, Bayer et al took two measures:
  - first-differencing by measuring all variables at two periods
  - IV strategy that focuses in air pollution by distant plants
- In OLS, air pollution decreases housing prices and income
  - Decline of income is inconsistent with Rosen-Roback model's theoretical prediction when A_i^R decreases
- After using the IV, the effect on income becomes near zero.

Table 3
Results from conventional wage-hedonic regressions.

Dependent variable	OLS		IV	
	(1)	(2)	(3)	(4)
$\Delta \ln \rho$	-0.232^{**} (0.097)	-0.292^{***} (0.098)	-0.497^{***} (0.179)	-0.634^{***} (0.185)
$\Delta \ln Y$	-0.073^{***} (0.022)	-0.074^{***} (0.023)	-0.035 (0.041)	-0.006 (0.043)
MSA covariates	No	Yes	No	Yes
Regional dummies	Yes	Yes	Yes	Yes

Notes: This table presents results from conventional wage-hedonic regressions. The cells contain the coefficients on $\Delta \ln(\text{PM10})$ pertaining to housing services (ρ) and income (Y) with respect to increases in air pollution. Columns (1) and (2) present results from OLS regressions; columns (3) and (4) present results using estimated PM10 from sources farther than 80 km as an instrument. Standard errors are in parentheses; * denotes significance at 10%; ** at 5%; *** at 1%.

Marginal willingness-to-pay for air quality

- The resulting MWTP: People are willing to pay $55 per year to avoid one unit increase of PM10 (column 2).
- Bayer et al. compare this result with the estimates from a different method (discrete choice approach)
  - We revisit this paper when discussing the discrete choice approach

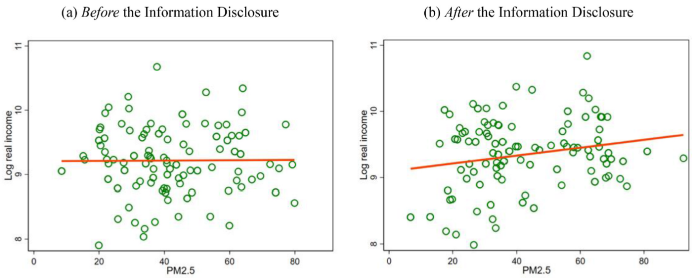
Measure	Hedonics		Residential sorting			
	OLS	IV	OLS	IV		
	Full specification (1)	Full specification (2)	Full specification (3)	Full specification (4)	No covariates (5)	No control for population (6)
WTP Elasticity	0.06	0.13	0.16	0.34	0.38	0.42
MWTP ($)	25.40	55.20	69.10	148.70	164.72	184.89

Gao, Song, Timmins (2023 JDE)

- Do people really know amenity values in making migration decisions?
- China's information disclosure about air quality (PM2.5) started since 2012.
- If people update their belief about air quality, the information disclosure might lower the "perceived" residential amenities

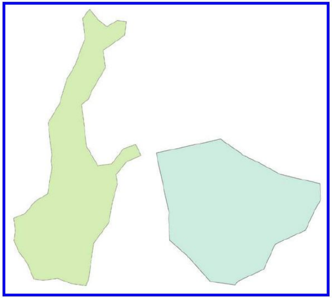
PM2.5 information disclosure and real income

- After the information disclosure of PM2.5, the positive correlation between PM2.5 and real income emerged
  - Positive correlation appears after instrumenting the PM2.5 for wind directions and distant plants
- Consistent with the prediction of the Rosen-Roback model when (perceived) residential amenity A_i^R gets worse due to the information disclosure
- The WTP for avoiding PM2.5 exposure almost doubled



Harari (2020 AER)

- Bad city shape affect within-city transportation access, which in turn may affect quality of life and economic growth.
- Regress wages and housing rents on the "compactness" measure of the city
  - Compactness is regarded as amenities that affect both quality of life and productivity (A_i^R and A_i^P)
  - But some economic shocks may affect both wages/rents and city compactness

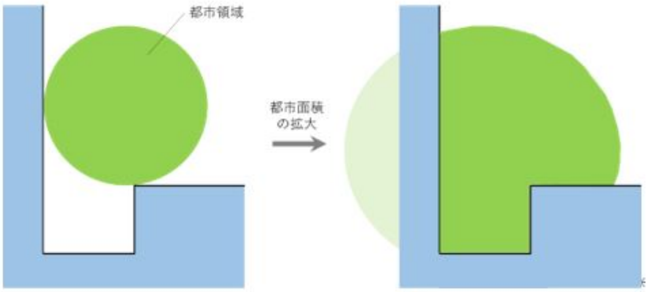


Shape metric	Kolkata	Normalized	Bangalore	Normalized
Disconnection, km	20.4	1.2	16	0.94
Remoteness, km	14.8	0.87	11.8	0.69
Spin, km^2	287.1	0.99	9.4	0.55
Range, km	65.2	3.83	44.5	2.62

Figure 1. Shape Metrics: An Example

Geography and city compactness

- Instrumental variable: City compactness implied by the natural geography
- Idea: As cities develop, it may hit undeveloped areas (e.g., seas) and it reduces compactness of the city
  - Using mechanically-predicted city growth and topographical information, it is orthogonal to local economic shocks



Source of Figure: Yamagishi (2022 Keizai Seminar)

Effects of bad city shape

- Reducing compactness (i.e., urban sprawl) increases wages and decreases rents
  - Note that this case is consistent with the decline in A_i^R (while holding A_i^P fixed) in page 10
- This implies that reducing compactness of the city reduces residential amenities, given our formula

$$\underbrace{\frac{V_A}{V_w}}_{\text{Marginal WTP}} = \underbrace{H_i^R r'}_{\text{Changes in land cost}} - \underbrace{w'}_{\text{Wage change}}$$

- However, the impact on productivity is near zero. Intuitively, the increase in wages and the decrease in rents offset with each other in the formula:

$$c_A = (-\Theta_w \frac{w'}{w} - \Theta_r \frac{r'}{r}),$$

Taking stock

- Rosen-Roback model extends the canonical spatial economic model by endogenizing wages and population
- We have seen applications of Rosen-Roback to evaluate various amenities
- The Rosen-Roback model still includes the exogenous "outside utility," and it is partial-equilibrium analysis in this sense
  - Let's say we look at Tokyo and Osaka. When Tokyo's amenities change, don't we think that it affects the utility level of Osaka?
  - We address this issue in discussing Quantitative Spatial Model (QSM) later.
- Note that the Rosen-Roback model does not use individual-level location choice data
  - You can implement this approach once you have city-level data on wages, rents, and amenities
  - But it ignores various individual heterogeneity (e.g., mobility costs from hometown)
- We next discuss how to evaluate amenities by exploiting individual-level location choice data
  - "Discrete choice approach" for amenity evaluation