Anti-Windup Observer-Based Output Regulation Using Improved Bounded Real Lemma

Abstract

An appropriate estimator is developed for sufficiently smooth and differentiable nonlinear system considering independent input disturbance signal and measurement noise. Based on the Lipschitz condition, the design parameters are formulated in linear matrix equality/inequality framework to guarantee the asymptotic stability. By aiming at rejecting the external perturbations in the plant output and the error dynamics, an improved form of the bounded real lemma is employed to feedback the observed states. To address the saturation of actuators, the developed controller is augmented with a static anti-windup compensator in an LFT form. The proposed method is implemented on the mechanical vibrating structure.

Keywords: bounded real lemma; linear matrix inequality; actuator saturation; observer-based control; active vibration control

1. Introduction

In the last two decades, by aiming at high precision regulation, various candidates are proposed for estimation of the states of the system (Ciccarella *et al*., 1993; Raghavan and Hedrick, 1994; Rajamani, 1998, Yaz and Yaz, 2001; Ha and Trinh, 2004; Lu *et al*., 2006; Mondal *et al*., 2010; Oveisi and Nestorović, 2016). The structure of the estimator and some measure of the system states are two essential factors that are required in the design procedure. Additionally, the estimation tools such as Kalman filter, high gain observer, and more recent observers developed in linear matrix inequality (LMI) framework may have completely different behavior when the system is subjected to input/output time-varying disturbances and measurement noise (Darouach et al., 1994; Koenig and Mammar, 2001; Pertew et al., 2005; Koenig, 2006; Xu, 2008).

In the present work, an alternative approach to Kalman filter is developed based on the unknown input observer that is proposed by Mondal *et al*. (2008; 2010). It is assumed that the nonlinear governing equation of motion satisfies the Lipschitz condition. Compared to Kalman filter, in which the state matrix, as well as the control input in estimator dynamics, are exact replicas of the nominal model, in the proposed method, a less conservative model is initiated and then additional constraints are introduced on estimator parameters in terms of some linear matrix equalities (LMEs). In order to keep the regulation problem in linear form these LMEs are solved before moving to DRC. It is perceived that the latter method has a better performance mainly because the transfer function from disturbance to the estimation residual is set to null. Then, the observed states are utilized to form a feedback signal for coupled DRC synthesis. For this purpose, the bounded real lemma (BRL) is presented as a proposition and then, an extended BRL proposed to reject the disturbance at the plant output quadratically.

Since the control effort may pass through nonlinear functions that are not sufficiently smooth enough to be considered in the form of Rajamani (1998), first, they are neglected for observer/controller design. These control input nonlinearities are handled by anti-windup bumpless transfer (AWBT) compensators (Kothare and Morari, 1997 & 1999; Mulder et al., 2001; Hu et al., 2006; Tarbouriech and Turner, 2009; Wu and Lin, 2014). Then, following the AW compensator synthesis, such nonlinearities will take into account in a separate manner. In this paper, using the absolute stability theorem (Mulder et al., 2001) quadratic stability problem of the saturated controller is addressed. Finally, by aiming at vibration control, the hybrid method is implemented on a mechanical vibrating system (Zaccarian and Teel, 2000; Teel et al., 2006; Oveisi and Nestorović, 2016).

In the rest of the paper, $\mathcal{I}$ represents the identity matrix with appropriate dimension, superscript T signifies the transpose of a matrix or a vector, and $\mathfrak{R}$ stands for the set of real numbers. Additionally, $\mathfrak{R}^{n}_{\geq 0}$, $\mathfrak{R}^{n\times n}_{\geq 0}$, and ${}^{+}\mathfrak{R}^{n\times n}_{\geq 0}$ represent some time dependent vectors, matrices, and positive definite matrices with dimensions of $n\times 1$, $n\times n$, $n\times n$ for $t\geq 0$, respectively. Also, $\mathcal{E}(.)$ represents the expected value of a random variable.

2. Problem formulation

Consider the multi-input/multi-output nonlinear dynamics of a plant in the form of Eq. (1)

$$\dot{x}(t) = f\big(x(t), u(t)\big) + g\big(x(t)\big)d(t) + h\big(x(t)\big)w(t),$$
$$y(t) = l\big(x(t), u(t)\big) + w(t), \tag{1}$$

where, $x(t) \in \Re^{n}_{\geq 0}$, $u(t) \in \Re^{n_u}_{\geq 0}$, $d(t) \in \Re^{n_d}_{\geq 0}$, $w(t) \in \Re^{n_y}_{\geq 0}$, and $y(t) \in \Re^{n_y}_{\geq 0}$ are the state, input, square-integrable disturbance, 2-norm bounded white Gaussian noise, and output vectors at time $t \geq 0$, respectively. In addition, in order to put the system in the class of nonlinear Lipschitz systems, it is assumed that $f(.,.)\colon \Re^{n}_{\geq 0} \times \Re^{n_u}_{\geq 0} \to \Re^{n}_{\geq 0}$, $g(.)\colon \Re^{n}_{\geq 0} \to \Re^{n_d}_{\geq 0}$, $h(.)\colon \Re^{n}_{\geq 0} \to \Re^{n_y}_{\geq 0}$, and $l(.,.)\colon \Re^{n}_{\geq 0} \times \Re^{n_u}_{\geq 0} \to \Re^{n_y}_{\geq 0}$ are sufficiently smooth and differentiable. Then, the necessary and sufficient conditions on the asymptotic stability of the observer-based control system can be addressed by the eigenvalues of the linearized matrices (Rajamani, 1998). For causal system in Eq. (1), $f(0,0) = 0$, with globally Lipschitz continuous dynamics, Eq. (1) can be linearized in the form of Eq. (2)

$$\dot{x} = A_P x + B_P u + E_P d + G_P w,$$
$$y = C_P x + w, \tag{2}$$

where $A_P = \partial f(x,u)/\partial x$, $B_P = \partial f(x,u)/\partial u$, $E_P = \partial g(x)/\partial x$, $G_P = \partial h(x)/\partial x$, and $C_P = \partial l(x,u)/\partial x$. In Eq. (2) and the rest of the paper, for simplifying the notation, the dependence of the system terms on time is suppressed. Assuming that *i*) A_P is asymptotically stable, *ii*) (A_P, C_P) is detectable, *iii*) matrices C_P and E_P are full row and column rank, correspondingly, and *iv*) $C_P E_P$ has the same rank as E_P, the dynamics of the state estimation mechanism is proposed in Eq. (3).

$$\dot{\mathcal{s}} = \mathcal{N}\mathcal{s} + \mathcal{L}(y - \hat{y}) + \mathcal{J}u,$$
$$\hat{x} = \mathcal{s} - \mathcal{H}y, \tag{3}$$
$$\hat{y} = C\hat{x}.$$

In Eq. (3), $\hat{x} \in \Re^{n}_{\geq 0}$ and $\hat{y} \in \Re^{n_y}_{\geq 0}$ are the estimated state- and output-vectors, correspondingly.

Definition 1. The extended state estimation system in Eq. (3) is asymptotically stable if *a*) The Lyapunov stability of the estimator satisfied and *b*) if the matrices $\mathcal{N} \in \Re^{n\times n}$, $\mathcal{L} \in \Re^{n\times n_y}$, $\mathcal{J} \in \Re^{n\times n_u}$, and $\mathcal{H} \in \Re^{n\times n_y}$ can be calculated such that $\lim_{t\to\infty}\|x - \hat{x}\| = 0$.

The residual signal of the estimation is defined as $e = x - \hat{x}$. Then, using Eqs. (2, 3), the dynamics of state-estimation error can be derived as Eq. (4)

$$\dot{e} = (\mathcal{N} - \mathcal{L}C_P)e + (\mathcal{P}A_P - \mathcal{N}\mathcal{P})x + (\mathcal{P}B_P - \mathcal{J})u + \mathcal{P}E_P d + (\mathcal{P}G_P - \mathcal{N}\mathcal{H} - \mathcal{L})w + H_P v, \tag{4}$$

where $\mathcal{P} = \mathcal{I}_n + \mathcal{H}C_P$ and $v = \dot{w}$. Following (Mondal *et al*., 2010), in order to cancel the direct effects of the states, control input, and input disturbance on the error dynamics it is assumed that

$$\mathcal{P}A_P - \mathcal{N}\mathcal{P} = 0, \tag{5a}$$
$$\mathcal{P}B_P - \mathcal{J} = 0, \tag{5b}$$
$$\mathcal{P}E_P = 0. \tag{5c}$$

Then, employing Eq. (5c), $\mathcal{H} = \mathcal{H}_1 + Y\mathcal{H}_2$, where $\mathcal{H}_1 = -E_P(C_P E_P)^{+}$ and $\mathcal{H}_2 = \mathcal{I}_{n_y} - (C_P E_P)(C_P E_P)^{+}$ with $(C_P E_P)^{+}$ being a generalized inversion. If $n_y = n_d$, there exists a unique solution for the intermediate matrix $Y \in \Re^{n\times n_y}$ that guarantees the satisfaction of Eq. (5c). In such case, Y can be calculated numerically. If $n_d > n_y$, then there is no guarantee on the existence of a solution for Y and it may be calculated by means of Least Square (LS) algorithm. Next, be using Eq. (5c) and after some mathematical manipulations, $\mathcal{P}$ and $\mathcal{J}$ are calculated as $\mathcal{P} = \mathcal{P}_1 + Y\mathcal{P}_2$ and $\mathcal{J} = \{\mathcal{I}_n + [\mathcal{H}_1 + Y\mathcal{H}_2]C_P\}B_P$, where $\mathcal{P}_1 = \mathcal{I}_n - \mathcal{H}_1 C_P$ and $\mathcal{P}_2 = \mathcal{H}_2 C_P$. Finally, by using Eq. (5a) and introducing $\mathcal{K} = \mathcal{N}\mathcal{H}$, the error dynamics can be obtained as $\dot{e} = (\mathcal{P}A_P - \mathcal{K}C_P)e + (\mathcal{P}G_P - \mathcal{K} - \mathcal{L})w + H_P v$. Since, the ultimate goal of the paper is to develop an observer-based DRC, the control law is defined as $u = \mathcal{T}\hat{x}$.

Proposition 1. (BRL) (Ahmadizadeh *et al*., 2014) *System $\dot{q} = Aq + Bu$, and $y = Cx$ is asymptotically stable if there exists a symmetric positive definite matrix P and a positive scalar γ such that the following LMI is satisfied*

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma^2 \mathcal{I}_{n_u} & 0 \\ C & 0 & -\mathcal{I}_{n_y} \end{bmatrix} < 0, \tag{6}$$

Proposition 2. (Improved BRL) *System $\dot{q} = Aq + B_w w + B_d d$, and $y = Cq + w$ is asymptotically stable and guarantees the quadratic stability of the transfer function from the input disturbance (d) and measurement noise (w) to the system output if there exist symmetric positive definite matrices P, S_1, and S_2 and positive scalars γ_d and γ_w such that the following LMI is satisfied*

$$\mathcal{Q} = \begin{bmatrix} \mathcal{G} & S_1 - A^T S_2^T & (P - S_1)B_w + C^T & (P - S_1)B_d \\ & S_2 + S_2^T & -S_2 B_w & -S_2 B_d \\ & & (1 - \gamma_w^2 \mathcal{I}_{n_w}) & 0 \\ * & & & -\gamma_d^2 \mathcal{I}_{n_d} \end{bmatrix} < 0, \tag{7}$$

Where $\mathcal{G} = A^T P + PA + C^T C - S_1 A - A^T S_1^T$ and $*$ represents the transpose of all upper triangular elements of $\mathcal{Q}$.

Proof. Selecting the Lyapunov function as $V_1(q,\dot{q}) = q^T P q + \int_0^t (y^T y - \gamma_w^2 w^T w - \gamma_d^2 d^T d) d\tau + \int_0^t [(q^T S_1 + \dot{q}^T S_2)(\dot{q} - Aq - B_w w - B_d d)] d\tau$ and then, derivation of the Lyapunov function with respect to time results in $\dot{V}_1(q,\dot{q}) = q_h^T \mathcal{Q} q_h < 0$ in which $q_h^T = [q^T \quad \dot{q}^T \quad w^T \quad d^T]$. This completes the proof.

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Theorem 1. *System (2) together with the state estimation mechanism (3) is asymptotically stable by means of Definition 1 and rejects the input disturbance and measurement noise in output quadratically if there exist symmetric positive definite matrices $\mathcal{X} \in \Re^{n\times n}$ and $\mathcal{Q}_2 \in \Re^{n\times n}$, matrices $\hat{\mathcal{T}} \in \Re^{n_u\times n}$, $\hat{\mathcal{K}} \in \Re^{n\times n_y}$, and $\hat{\mathcal{L}} \in \Re^{n\times n_y}$, positive scalars γ_d, γ_w, and γ_v, and user-defined positive weighting scalars α and β such that LMI condition (8) is satisfied*

$$\Pi = \begin{bmatrix} \mathcal{G}_{11} & 0 & 2G_P & 2E_P & 0 & \mathcal{X}C_P^T \\ & \mathcal{G}_{22} & \mathcal{G}_{23} & 0 & \mathcal{Q}_2\mathcal{H} & 0 \\ & & -\gamma_w^2\mathcal{I}_{n_y} & 0 & 0 & \mathcal{I}_{n_y} \\ & & & -\gamma_d^2\mathcal{I}_{n_d} & 0 & 0 \\ & & & & -\gamma_v^2\mathcal{I}_{n_y} & 0 \\ * & & & & & -\alpha\mathcal{I}_{n_y} \end{bmatrix} < 0, \tag{8a}$$

where,

$$\begin{gathered} \mathcal{G}_{11} = 2\left(B_P\hat{\mathcal{T}} + \hat{\mathcal{T}}^T B_P^T\right) + 2(\mathcal{X}A_P^T + A_P\mathcal{X}), \\ \mathcal{G}_{22} = A_P^T\mathcal{P}^T\mathcal{Q}_2 + \mathcal{Q}_2\mathcal{P}A_P - \hat{\mathcal{K}}C_P - C_P^T\hat{\mathcal{K}}^T + \beta\mathcal{I}_n, \\ \mathcal{G}_{23} = \mathcal{Q}_2\mathcal{P}G_P - \hat{\mathcal{K}} - \hat{\mathcal{L}}. \end{gathered} \tag{8b}$$

Then, the controller gain $\mathcal{T} = \hat{\mathcal{T}}\mathcal{X}$, and observer gains $\mathcal{K} = \mathcal{Q}_2^{-1}\hat{\mathcal{K}}$ and $\mathcal{L} = \mathcal{Q}_2^{-1}\hat{\mathcal{L}}$ can be calculated.

Proof. The proof is a direct use of Proposition 2 and Eq. (7) on the Lyapunov function $V_2(x,\dot{x},e) = x^T\mathcal{Q}_1 x + e^T\mathcal{Q}_2 e + \int_0^t(\alpha y^T y + \beta e^T e - \gamma_w^2 w^T w - \gamma_d^2 d^T d - \gamma_v^2 v^T v)d\tau + \int_0^t[(x^T S_1 + \dot{x}^T S_2)(\dot{x} - A_P x - B_P u - E_P d - G_P w)]d\tau$ to obtain $\dot{V}_2(x,\dot{x},e) = q_h^T \Pi q_h$, in which $[x^T \quad e^T \quad w^T \quad d^T \quad v^T]$. Then, setting $S_1 = -\mathcal{Q}_1$ and $S_2 = 0_n$ and applying the congruence transformation $\mathrm{diag}\left(\mathcal{Q}_1^{-1}, \mathcal{I}_n, \mathcal{I}_{n_y}, \mathcal{I}_{n_d}, \mathcal{I}_{n_y}\right)$ on Π and defining $\mathcal{X} = \mathcal{Q}_1^{-1}$. This completes the proof.

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Using Theorem 1, the extended observer-based controller dynamics can be transformed in Eq. (9)

$$\begin{gathered} \dot{x}_K = A_K x_K + B_K y, \\ u = C_K x_K + D_K y. \end{gathered} \tag{9}$$

where $A_K = \mathcal{N} - \mathcal{J}\mathcal{T} - \mathcal{L}C$, $B_K = \mathcal{L} + \mathcal{L}C\mathcal{H} - \mathcal{J}\mathcal{T}\mathcal{H}$, $C_K = \mathcal{T}$, and $D_K = -\mathcal{T}\mathcal{H}$.

Based on Kothare *et al*. (1994), the closed-loop system is configured in axiomatic compensation framework for the regulation problem presented in Fig. 1a. In this figure, the augmented controller is parameterized by system matrices $(A_K, [B_K \quad \mathcal{I}_n \quad 0_{n\times n_u}], C_K, [D_K \quad 0_{n_u\times n} \quad \mathcal{I}_{n_u}])$, also, N, ξ, and $\Lambda^T = [\Lambda_1^T \quad \Lambda_2^T]$ represents actuation nonlinearity (saturation function in this paper), AW compensation signal, and a static gain matrix for AW compensation, respectively. The augmented controller's configuration guarantees the independent effect of the AW compensator on controller states and output (Mulder *et al*., 2001).

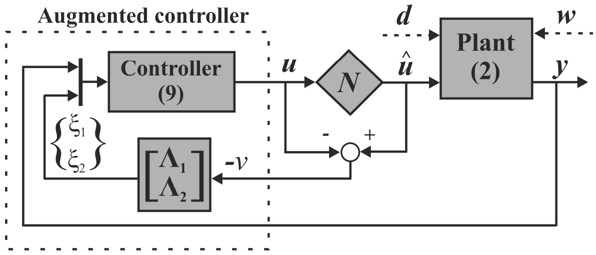


Fig. 1 The AW observer-based closed-loop system interconnection

Marcopoli and Philips (1996) and Mulder *et al*. (1999) presented an apparent convexity in controller synthesis for plants with the saturated actuators. Their observation led to recasting the AW synthesis in LMI framework by transforming the system in Fig. 1a to LFT form of Fig. 2. In this figure, all of the exogenous inputs are included in ϑ and $\Delta = \mathcal{I}_n - N$. As a natural claim on the AW system, when $\nu = 0$, the controller recovers from Fig. 1 to linear observed state-feedback controller obtained from Theorem 1. The dynamic equation governing **PLANT** in Fig. 2 can be formulated as (Mulder *et al*., 2009)

$$\begin{aligned}\dot{\sigma} &= A_\sigma \sigma + \left(B_\nu - B_\xi \Lambda\right)\nu + B_\vartheta \vartheta,\\ u &= C_u \sigma + \left(D_{u\nu} - D_{u\xi}\Lambda\right)\nu + D_{u\vartheta}\vartheta,\\ y &= C_y \sigma + \left(D_{y\nu} - D_{y\xi}\Lambda\right)\nu + D_{y\vartheta}\vartheta,\end{aligned} \tag{10a}$$

$$\begin{aligned} u(s) &= [P_{u1}(s) \quad P_{u2}(s) \quad P_{u3}(s)][\nu^T \quad \vartheta^T \quad \xi^T]^T,\\ y(s) &= [P_{y1}(s) \quad P_{y2}(s) \quad P_{y3}(s)][\nu^T \quad \vartheta^T \quad \xi^T]^T,\\ -\nu &= [-\mathcal{I} \quad 0 \quad 0][\nu^T \quad \vartheta^T \quad \xi^T]^T,\end{aligned} \tag{10b}$$

where,

$$\begin{aligned} P_{u1}(s) &= -\mathbf{P}_u (C_K(s\mathcal{I} - A_K)^{-1}B_K + D_K)\mathbf{P}_y,\\ P_{u2}(s) &= \mathbf{P}_u (C_K(s\mathcal{I} - A_K)^{-1}B_K + D_K)(C_P(s\mathcal{I} - A_P)^{-1}[E_P \quad G_P]),\\ P_{u3}(s) &= \mathbf{P}_u (C_K(s\mathcal{I} - A_K)^{-1}B_K + D_K)[C_K(s\mathcal{I} - A_K)^{-1} \quad \mathcal{I}],\\ \mathbf{P}_y &= C_P(s\mathcal{I} - A_P)^{-1}B_P,\\ \mathbf{P}_u &= \left\{\mathcal{I} - \left[(C_K(s\mathcal{I} - A_K)^{-1}B_K + D_K)\left((C_P(s\mathcal{I} - A_P)^{-1}B_P)\right)\right]\right\}^{-1},\\ P_{y1}(s) &= \mathbf{P}_y P_{u1}(s) - \mathbf{P}_y,\\ P_{y2}(s) &= \mathbf{P}_y P_{u2}(s) - C_P(s\mathcal{I} - A_P)^{-1}[E_P \quad G_P],\\ P_{y3}(s) &= \mathbf{P}_y P_{u3}(s),\end{aligned} \tag{10c}$$

Additionally, A_σ, B_ν, B_ξ, B_ϑ, C_u, C_y, $D_{u\nu}$, $D_{y\nu}$, $D_{u\xi}$, $D_{y\xi}$, $D_{u\vartheta}$, $D_{y\vartheta}$ are matrices with appropriate dimension that can be obtained from minimal state-space realization of Eq. (10b). By reducing Fig. 1a in to LFT form in Fig1b, the objective of static AW compensator is to design Λ.

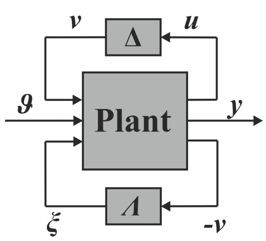


Fig. 2 Standard AW LFT configuration

Theorem 2. ($\mathfrak{L}_2$ gain criterion) [Teel and Kapoor, 1997; Mulder *et al*., 2001] *The AW compensator in Eq. (10) is $\mathfrak{L}_2$ gain stabilizable, if there exists symmetric positive definite matrix $\mathfrak{Q}$ with appropriate dimension, positive definite block diagonal matrix $\mathfrak{M} \in \mathfrak{R}^{n_u \times n_n}$, matrix $\mathfrak{X}$ and positive scalars Γ, $\mathfrak{g}$, and δ_1 such that the LMI equation (11) is satisfied*

$$\Theta=\begin{bmatrix}\mathfrak{Q}A^T+A\mathfrak{Q} & B_\vartheta & \Theta_{13} & \mathfrak{Q}C_y^T & 0\\ & -\Gamma\mathcal{I} & D_{uv}^T & D_{yv}^T & 0\\ & & \Theta_{33} & \mathfrak{M}D_{yv}^T-\mathfrak{X}^TD_{y\xi}^T & \mathfrak{M}\\ & & & -\mathrm{g}^{-1}\mathcal{I} & 0\\ & * & & & -\delta_1\mathcal{I}\end{bmatrix}<0, \tag{11a}$$

with

$$\begin{aligned}\Theta_{13}&=B_\nu\mathfrak{M}-B_\xi\mathfrak{X}+\mathfrak{Q}C_u^T,\\ \Theta_{33}&=-2\,\mathfrak{M}+D_{uv}\mathfrak{M}+\mathfrak{M}D_{uv}^T-D_{u\xi}\mathfrak{X}-\mathfrak{X}^TD_{u\xi}^T.\end{aligned} \tag{11b}$$

Then, $\Lambda=\mathfrak{X}\mathfrak{M}^{-1}$.

3. Numerical example

3.1. Modeling of open loop plant

In this section, the observer-based AW compensator is implemented on a smart structure which is oscillating under the effect of a bounded disturbance signal. The objective of AWDRC is to attenuate the vibration of the structure at the system output. Additionally, if the saturation appears in the actuation elements, the AW compensator accounts for the matter to keep the stability of the system and if $\nu=0$, the controller should recover from wind-up state. The plant configuration, as shown in Fig. 3, consists of a clamped-free aluminum beam with Young's modulus 70 Gpa and density 2.7 g/cm³ and two piezo-patches ($30\times50\times0.5$ mm) bounded on one side of the structure. The piezo-actuators (DuraActTM P-876.A15) are activated in operating voltage range of [-150 150] V with electrical capacitance of 45 nF, and blocking force of 775 N.

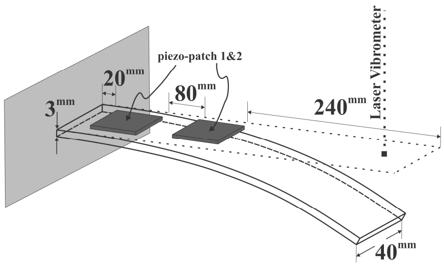


Fig. 3 Geometry of the smart beam

Next, first, the controller ($u=\mathcal{T}\hat{x}$) and observer system (Eq. 3) are designed based on the results in Theorem 1. The closed-loop system configuration is presented in Fig. 4. The piezolaminated beam plant has three independent inputs: the control inputs which act on the two actuator piezo-patches and the exogenous signal which excites the structure through an independent disturbance channel.

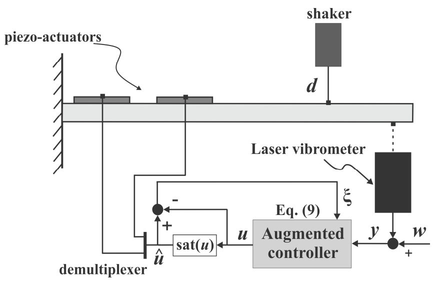


Fig. 4 Schematic interconnection of the plant and extended observer-based AW-controller

It should be mentioned that the torsional modes are neglected in the modeling of the beam since only the bending vibration suppression is analyzed in this paper. The dynamics of the actuation is obtained using finite element analysis in coupled electro-mechanical domain. By assuming $\mathbb{M}$, $\mathbb{C}$, and $\mathbb{K}$ as mass, damping, and stiffness matrices in the framework of FEM, the governing ordinary differential equation (ODE) of motion in matrix form can be written as $\mathbb{M}\ddot{q} + \mathbb{C}\dot{q} + \mathbb{K}q = \mathbb{F}$ with q and $\mathbb{F}$ being the nodal states of displacement and electric potential/applied external inputs, respectively (Oveisi and Nestorović, 2014). For control purposes, the structure of control effort is assumed to be $\mathbb{F} = \mathcal{B}u(t)$, where, $\mathcal{B}$ represents the position of generalized control effort in FE structure. The output of the plant based on the nodal displacements and velocities is expressed as $y = C_{0q}q + C_{0v}\dot{q}$ with C_{0q} and C_{0v} being the modal output displacement and velocity matrices, respectively. The natural frequencies (w_i) of the coupled structure as well as the mode shapes (φ_i) for $i = 1 \dots n$ are obtained by employing the classical harmonic solution ($q = \varphi e^{iwt}$) and solving the obtained algebraic equation. Then, by applying the transformation $q = \boldsymbol{\varphi} q_m$, the nodal model of the plant can be transformed to the generalized modal displacement model, in which, $\boldsymbol{\varphi}$ is the matrix of eigen vectors. Finally, by using the proportional damping model, $\mathbb{C} = a\mathbb{M} + b\mathbb{K}$, orthogonality of mode shapes, and assuming the state vector as $x_{q_m} = [\boldsymbol{w} q_m \quad \dot{q}_m]^T$, with $\boldsymbol{w} = \mathrm{diag}(w_i)$, the governing equation of motion can be presented in state space form of Eq. (2). The interpretation of the state, input, and output matrices based on the FE approach can be presented as Eq. (12) (Marinkovic *et al.*, 2009)

$$\begin{gathered} A_P = \begin{bmatrix} \mathbf{0} & \boldsymbol{w} \\ -\boldsymbol{w} & 2\mathcal{Z}\boldsymbol{w} \end{bmatrix}, \\ B_P = [\mathbf{0} \quad B_m^T]^T, \\ C_P = [C_{mq} \quad C_{mv}], \end{gathered} \tag{12}$$

where $\mathcal{Z} = \mathrm{diag}(\zeta_i)$ with ζ_i being the damping ratio of ith mode shape and $B_m = \boldsymbol{\varphi}^T\mathcal{B}$, $C_{mq} = C_{0q}\boldsymbol{\varphi}$, and $C_{mv} = C_{0v}\boldsymbol{\varphi}$.

3.2. Controller and Observer design

In this section, the numerical results of the employment of the proposed controller are presented on the piezo-laminated beam shown in Fig. 3. For design purposes, a reduced order system is obtained by considering two bending mode shapes of the smart structure. The two fundamental natural frequencies of the system are 87.3 and 480 rad/s and their associated damping ratio is assumed to be 1×10^{-2}. The nominal system is a transformed in state space model with the system matrices shown in Eq. (13).

$$\begin{gathered} A_P = \begin{bmatrix} 0 & 87.27 & 0 & 0 \\ -87.27 & -1.74 & 0 & 0 \\ 0 & 0 & 0 & 480.07 \\ 0 & 0 & -480.07 & -9.6 \end{bmatrix}, \\ [B_P \quad E_P] = \begin{bmatrix} 0 & 4.35 & 0 & 19.27 \\ 0 & 2.79 & 0 & -4.23 \end{bmatrix}^T, \\ C_P = [0 \quad -5.1 \quad 0 \quad 4.03]. \end{gathered} \tag{13}$$

For examination of the enactment of the closed-loop system, an observer-based controller is designed using Theorem 1. Accordingly, the controller and observer gains are obtained as

$$\begin{gathered} \mathcal{N} = \begin{bmatrix} 2.28E-4 & 221.44 & -9.91E-4 & -105.88 \\ -37.08 & -2.8603 & -217.91 & -2.686 \\ 1.09E-4 & -16.06 & -4.72E-4 & 492.75 \\ -46.98 & -2.258 & -276.1 & -4.481 \end{bmatrix}, \\ \mathcal{L} = [-25.754 \quad 0.4098 \quad 3.0631 \quad 0.2549]^T, \\ \mathcal{J} = \begin{bmatrix} 2.84E-5 & 10.596 & 1.35E-5 & 13.426 \\ -1.6E-5 & -0.733 & -7.63E-6 & -0.928 \end{bmatrix}^T, \\ \mathcal{H} = [5.13E-7 \quad 0.1127 \quad 2.44E-7 \quad -0.1055]^T, \\ \mathcal{T} = \begin{bmatrix} 12.308 & -263.269 & -3.71 & 207.852 \\ 36.038 & -4381.2 & -68.391 & 3456.8 \end{bmatrix}. \end{gathered} \tag{14a}$$

The results in Eq. (14a) are obtained based on the intermediate parameter in Eq. (14b)

$$\begin{gathered} Y = [5.236E-7 \quad 0.1127 \quad 2.494E-7 \quad -0.1055]^T, \\ \alpha = \beta = 1. \end{gathered} \tag{14b}$$

Next, by transforming the results in Eq. (14) into Eq. (9) and then (10), and finally utilizing Theorem 2, the AW static compensation gain is calculated in Eq. (15)

$$\Lambda = \begin{bmatrix} 0.0044 & -0.0528 & 0.0215 & -0.0669 & -0.9667 & 0.5518 \\ 0.0024 & -0.093 & 0.017 & -0.1179 & 0.0583 & -0.0302 \end{bmatrix}^T, \qquad (15)$$
$$\mathfrak{g} = 10.$$

By implementing the control system based Eqs. (14, 15) on the plant with system matrices as in Eq. (13), the prospect of the operational vibration attenuation performance is assessed. For this purpose, first, the disturbance channel is activated by the realization of a chirp signal. The frequency of the interference signal is swept from zero to 90 Hz within a nine seconds time window in three cases of the AWDRC, saturated DRC, and uncontrolled systems. Investigations are carried out in the time domain by means of the structure in Fig. 3 with the sampling frequency of 10 kHz. All of the control systems, as well as the plant, are modeled on SIMULINK platform with standard fixed step explicit ODE5 solver (Dormand-Prince method)[1]. The response of the system for the closed-loop and open loop cases is shown in Fig. 5a in the time domain based on the velocity measurement signal.

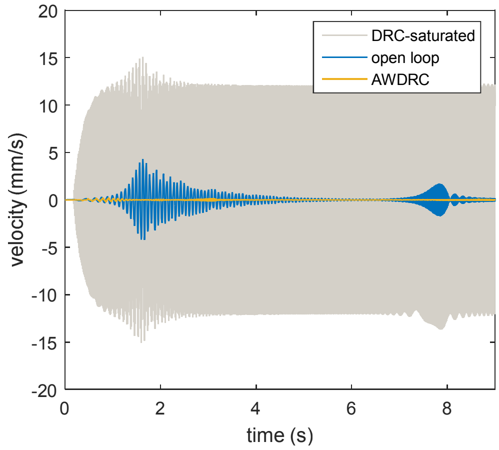


Fig. 5a Comparison of measured outputs: instability of closed-loop in saturated case

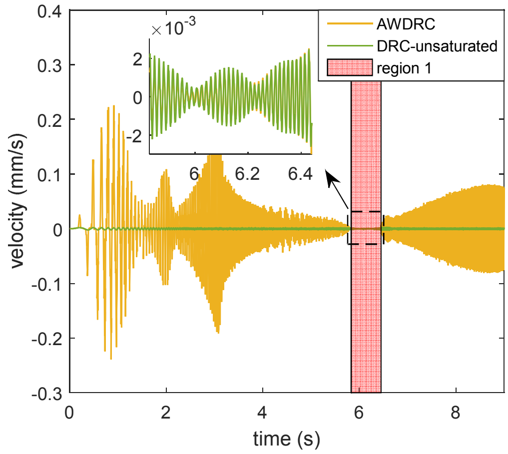
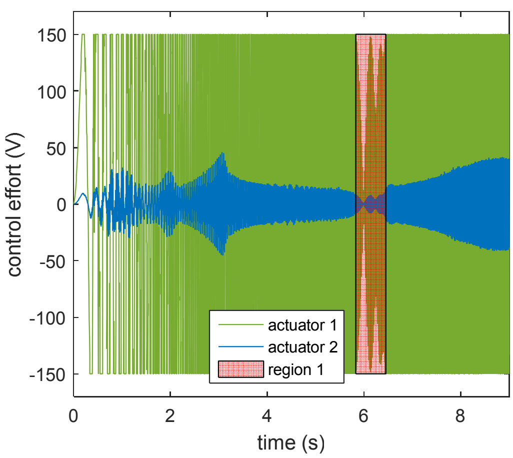
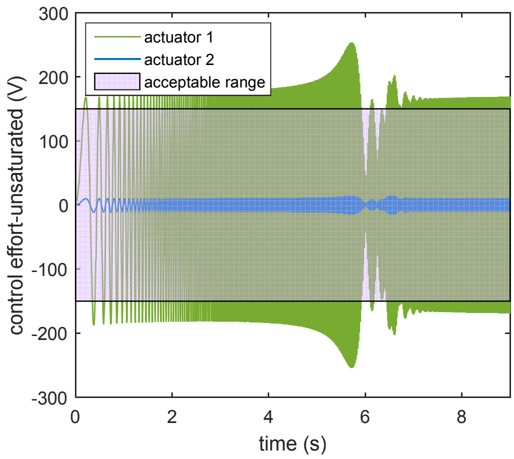


Fig. 5b Comparison of measured outputs in time domain: AW performance reduction

It can be seen in Fig. 5a that the control system based on Theorem 1 is unable to deal with actuation nonlinearity and although the AWDRC lost performance compared to the unsaturated system, it suppressed the vibration magnitude within the considered frequency range. Moreover, the hybrid compensator based on Theorems 1 and 2 is asymptotically stable that can recover from the windup state. This can be seen by comparing the control effort in Fig. 6a with the system output Fig. 5b at region 1 in which the performance of the AW compensated controller reaches to linear case. Additionally, Fig. 6b represents the control effort for the unsaturated system.





[1] Authors are willing to share the script for designing the controller/observer as an .m file as well as the SIMULINK model with the interested reader.

Fig. 6a control effort: AW-compensated observer-based controller

Fig. 6b control effort: observer-based controller (Theorem 1)

Next, the residual of the estimation of the system output is shown in Figs. 7 for three cases: AWDRC, unsaturated DRC, and estimation error based on Kalman filter. For this purpose, first, for the nominal deterministic model (Eq. 2), the well-known Kalman filter is calculated by finding the solution of the algebraic Riccati equation (ARE): $L_K = \left(\mathfrak{P}C_P^T + \bar{\mathfrak{N}}\right)\mathcal{R}^{-1}$. In this equation, $\bar{\mathfrak{N}} = G\mathfrak{N}$ with $\mathfrak{N} = \mathcal{E}(wv^T)$, $\mathcal{R} = \mathcal{E}(vv^T)$, and $\mathfrak{P}$ is the steady-state error covariance $(\lim_{t\to\infty} \mathcal{E}(\{x-\hat{x}\}\{x-\hat{x}\}^T))$. Consequently, the obtained L_K is presented in Eq. (16).

$$L_K = [4.1441 \quad -1715.8 \quad 28.884 \quad 1604.5]^T. \tag{16}$$

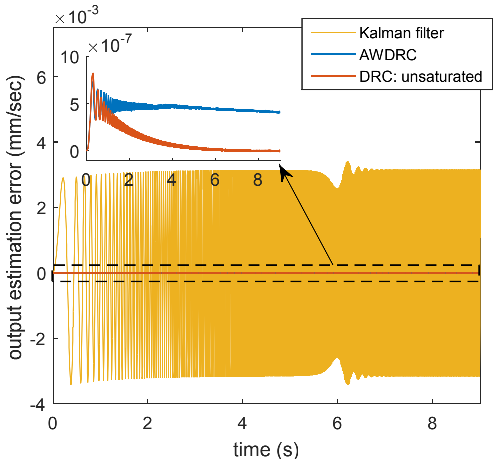


Fig. 7. Estimation error of the closed-loop system: comparison between AWDRC and unsaturated DRC

It can be seen that the proposed observer method has a better performance in state estimation than the well-known Kalman observer. Finally, in order to examine the disturbance rejection performance of the proposed method in Theorem 1, it is compared to the Linear Quadratic Gaussian (LQG) controller in combination with AW compensator in Theorem 2. The comparison of the vibration suppression quality is presented in Fig. 8. It can be seen that the DRC based on Theorem 1 has a better performance compared to AW-compensated LQG controller. The control effort for both of the closed-loop systems, AWDRC, and AWLQG, under the external disturbance, is depicted in Fig. 9a and 9b, respectively.

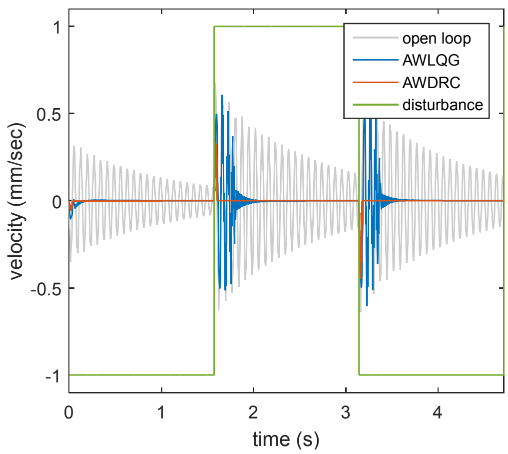


Fig. 8 Comparison of measured outputs: instability of closed-loop in saturated case

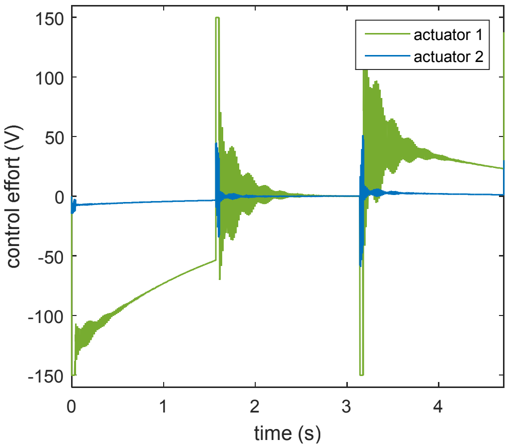


Fig. 9a Control effort: AWDRC

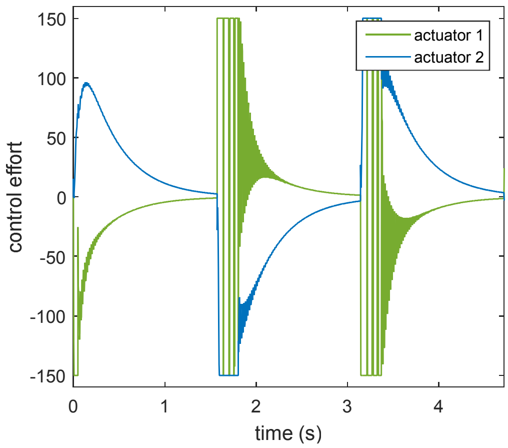


Fig. 9b Control effort: AWLQG

Conclusion

In this paper, the trend in unknown input observation is used to estimate the states of the system in a less conservative agenda. For this purpose, a set of LMEs is defined on to account for multiple unknown matrices in dynamics of the observer. Then, the remaining parameters are acquired as a solution of coupled asymptotic stability problem in terms of improved BRL and stability of the observation residuals. The solution of the observer-based controller is transformed in the framework of static AWBT compensation. For this purpose, the absolute stability of the coupled system is guaranteed by solving and additional LMI. Both of the proposed observer and controller are compared with standard Kalman Filter and LQG regulator, respectively, on a mechanical vibrating plant. It has been observed that the proposed observer-based method has a better performance in vibration attenuation. Additionally, for nonlinear systems with multiple equilibrium points, the proposed method can be easily generalized by employing the Takagi-Sugeno fuzzy global approximation (Tanaka and Wang, 2001).

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