# Anti-windup Observer-based Controller Based on Improved Bounded Real Lemma: Active Vibration Control

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Abstract: In this paper, the windup problem is addressed for disturbance rejection control (DRC) based on an unknown input observer (UIO). The proposed combination aims at dealing with actuator input nonlinearities in active vibration control (AVC) to provide a useful tool for systems under unknown environmental stimuli. Accordingly, first, assuming the governing dynamics of the plant to be smooth and differentiable around the equilibrium points, a UIO is designed such that the two transfer functions from disturbance/control input to observation residuals are null. By aiming at rejecting the external perturbations in the plant output and the observation error dynamics, an improved form of the bounded real lemma is employed. Then, based on the Lipschitz condition and considering independent input disturbance signal (mismatch case) and measurement noise, DRC synthesis is formulated in linear matrix equality/inequality framework. To address the saturation of actuators, the developed controller is then augmented with a static anti-windup (AW) compensator in a linear fractional transformation (LFT) form. The proposed combination can be used as a low order yet effective method in model-based AVC of arbitrary mechanical/civil structures.

*Keywords:* bounded real lemma; linear matrix inequality; actuator saturation; disturbance rejection; active vibration control

## 1. INTRODUCTION

In the last two decades, by aiming at high precision process control, various candidates are proposed for estimation of the states of the system (Ciccarella et al., 1993; Raghavan and Hedrick, 1994; Rajamani, 1998, Yaz and Yaz, 2001; Ha and Trinh, 2004; Lu et al., 2006; Mondal et al., 2010; Oveisi and Nestorović, 2016a,b). The structure of the estimator is an essential design decision that determines the dynamics of state estimation error. As a result, the estimation tools such as Kalman filter, high gain observer, and more recent observers developed in linear matrix inequality (LMI) framework may have different behavior when the system is subjected to input/output time-varying disturbances and measurement noise (Darouach et al., 1994; Koenig and Mammar, 2001; Pertew et al., 2005; Koenig, 2006; Xu et al., 2008). In the present research, by aiming at providing a simple yet effective tool for AVC of mechanical/civil structures with actuator saturation, DRC based on a UIO is augmented with static AW compensator. In literature, the mismatch DRC with AW compensator for AVC is not tackled with some exceptions (Zaccarian and Teel, 2000; Teel et al., 2006). The augmented control design is proposed in terms of a two-step convex feasibility problem with linear matrix equality (LME) constraints. Accordingly, first, a UIO is utilized to reconstruct the states of the system under unknown but bounded disturbance and noise signals following a similar approach as Mondal et al. (2010). It is assumed that the nonlinear governing equation of motion for AVC purposes satisfies the Lipschitz condition. Compared to Kalman filter, in which the state and control input matrices in estimator dynamics, are exact replicas of the nominal model, in the proposed method, a less conservative model is selected and then additional constraints are introduced on estimator parameters/structure in terms of some LMEs. In order to keep the feedback control design problem linear, these LME constraints are preliminary solved before moving to DRC synthesis based on quadratic stability lemma. As an advantage, the latter method has a better performance mainly because the transfer function from disturbance to the state observation residuals is set to null. Then, the observed states are utilized to form a feedback signal for coupled DRC design. For this purpose, an extended BRL proposed to reject the disturbance at the plant output.

The control effort may pass through nonlinear functions that are not sufficiently smooth to be considered in the form of Rajamani (1998), such as actuator saturation. These actuator nonlinearities are neglected for observer/controller design and are handled later by the anti-windup bumpless transfer (AWBT) compensator (Kothare and Morari, 1997 & 1999; Hu et al., 2006; Mulder et al., 2009; Tarbouriech and Turner, 2009; Wu and Lin, 2014). In this paper, such nonlinearities will be addressed employing the absolute stability theorem (Mulder et al., 2001). Finally, the main results of the paper as a framework for AVC of civil/mechanical structures is presented and later is implemented on a mechanical vibrating system (Oveisi and Nestorović, 2016c).

In the rest of the paper, $\mathcal{I}$ represents the identity matrix with appropriate dimension, superscript $T$ signifies the transpose of a matrix or a vector, and $\mathfrak{R}$ stands for the set of real numbers. Additionally, $\mathfrak{R}^{n}_{\geq 0}$, $\mathfrak{R}^{n\times n}_{\geq 0}$, and ${}^{+}\mathfrak{R}^{n\times n}_{\geq 0}$ represent time-dependent vectors, matrices, and positive definite matrices with dimensions of $n \times 1$, $n \times n$, $n \times n$ for $t \geq 0$, respectively. Also, $\mathcal{E}(.)$ represents the expected value of a random variable.

## 2. PROBLEM FORMULATION

Consider the multi-input/multi-output nonlinear dynamics of a plant in the form of (1)

$$\begin{aligned}\dot{x}(t) &= f\big(x(t),u(t)\big) + g\big(x(t)\big)d(t) + h\big(x(t)\big)w(t),\\ y(t) &= l\big(x(t),u(t)\big) + w(t),\end{aligned} \tag{1}$$

where, $x(t) \in \Re^{n}_{\geq 0}$, $u(t) \in \Re^{n_u}_{\geq 0}$, $d(t) \in \Re^{n_d}_{\geq 0}$, $w(t) \in \Re^{n_y}_{\geq 0}$, and $y(t) \in \Re^{n_y}_{\geq 0}$ are the state, input, square-integrable disturbance, white Gaussian noise, and output vectors at time $t \geq 0$, respectively. In addition, in order to put the system in the class of nonlinear Lipschitz systems, it is assumed that $f(.,.)\colon \Re^{n}_{\geq 0} \times \Re^{n_u}_{\geq 0} \to \Re^{n}_{\geq 0}$, $g(.)\colon \Re^{n}_{\geq 0} \to \Re^{n_d}_{\geq 0}$, $h(.)\colon \Re^{n}_{\geq 0} \to \Re^{n_y}_{\geq 0}$, and $l(.,.)\colon \Re^{n}_{\geq 0} \times \Re^{n_u}_{\geq 0} \to \Re^{n_y}_{\geq 0}$ are sufficiently smooth and differentiable for UIO/DRC synthesis. Then, the necessary and sufficient conditions on the asymptotic stability of the observer-based control system can be addressed by the eigenvalues of the linearized system (Rajamani, 1998) (see (2)). For causal system in (1), $f(0,0) = 0$, with globally Lipschitz continuous dynamics, (1) can be linearized in the form of (2)

$$\begin{aligned}\dot{x} &= A_P x + B_P u + E_P d + G_P w,\\ y &= C_P x + w,\end{aligned} \tag{2}$$

where $A_P = \partial f(x,u)/\partial x$, $B_P = \partial f(x,u)/\partial u$, $E_P = \partial g(x)/\partial x$, $G_P = \partial h(x)/\partial x$, and $C_P = \partial l(x,u)/\partial x$ assuming that $\partial l(x,u)/\partial u = 0$ to keep the design problem of the observer-based controller convex. In (2) and the rest of the paper, for simplifying the notation, the dependence of the system terms on time is suppressed. Assuming that *i*) $A_P$ is Hurwitz, *ii*) $(A_P, C_P)$ is detectable, *iii*) matrices $C_P$ and $E_P$ are full row and column rank, correspondingly, and *iv*) $C_P E_P$ has the same rank as $E_P$, the dynamics of the UIO mechanism is presented in (3)

$$\begin{aligned}\dot{\mathcal{s}} &= \mathcal{N}\mathcal{s} + \mathcal{L}(y - \hat{y}) + \mathcal{J}u,\\ \hat{x} &= \mathcal{s} - \mathcal{H}y, \qquad \hat{y} = C\hat{x}.\end{aligned} \tag{3}$$

where $\hat{x} \in \Re^{n}_{\geq 0}$, $\hat{y} \in \Re^{n_y}_{\geq 0}$, and $\mathcal{s} \in \Re^{n}_{\geq 0}$ are the observed state, output, and auxiliary observer state vectors, correspondingly.

### 2.1 Preliminary results

*Definition 1.* The state reconstruction system based on UIO in (3) is asymptotically stable if *a*) The Lyapunov stability of the estimator is satisfied and *b*) if the matrices $\mathcal{N} \in \Re^{n\times n}$, $\mathcal{L} \in \Re^{n\times n_y}$, $\mathcal{J} \in \Re^{n\times n_u}$, and $\mathcal{H} \in \Re^{n\times n_y}$ can be calculated such that $\lim_{t\to\infty}\|x - \hat{x}\| = 0$.

The observation residual is defined as $e = x - \hat{x}$. Then, using (2) and (3), the dynamics of state-estimation error can be derived as $\dot{e} = (\mathcal{N} - \mathcal{L}C_P)e + (\mathcal{P}A_P - \mathcal{N}\mathcal{P})x + (\mathcal{P}B_P - \mathcal{J})u + \mathcal{P}E_P d + (\mathcal{P}G_P - \mathcal{N}\mathcal{H} - \mathcal{L})w + H_P v$ with $\mathcal{P} = \mathcal{I}_n + \mathcal{H}C_P$ and $v = \dot{w}$. Following (Mondal et al., 2010), in order to cancel the direct effects of the states, control input, and input disturbance on the error dynamics it is assumed that: (a) $\mathcal{P}A_P - \mathcal{N}\mathcal{P} = 0$, (b) $\mathcal{P}B_P - \mathcal{J} = 0$, and (c) $\mathcal{P}E_P = 0$. Then, employing (c), we acquire $\mathcal{H} = \mathcal{H}_1 + Y\mathcal{H}_2$, where $\mathcal{H}_1 = -E_P(C_P E_P)^+$ and $\mathcal{H}_2 = \mathcal{I}_{n_y} - (C_P E_P)(C_P E_P)^+$ with $(C_P E_P)^+$ being a generalized inversion. If $n_y = n_d$, there exists a unique solution for the intermediate matrix $Y \in \Re^{n\times n_y}$ that guarantees the satisfaction of (c). If $n_d > n_y$, then a solution for $Y$ may be calculated by means of Least Square (LS) algorithm. Next, by using (c) and after some mathematical manipulations, $\mathcal{P}$ and $\mathcal{J}$ are calculated as $\mathcal{P} = \mathcal{P}_1 + Y\mathcal{P}_2$ and $\mathcal{J} = \{\mathcal{I}_n + [\mathcal{H}_1 + Y\mathcal{H}_2]C_P\}B_P$, where $\mathcal{P}_1 = \mathcal{I}_n - \mathcal{H}_1 C_P$ and $\mathcal{P}_2 = \mathcal{H}_2 C_P$. Finally, by using (a) and introducing $\mathcal{K} = \mathcal{N}\mathcal{H}$, the state estimation error dynamics can be obtained as $\dot{e} = (\mathcal{P}A_P - \mathcal{K}C_P)e + (\mathcal{P}G_P - \mathcal{K} - \mathcal{L})w + H_P v$. Since, the ultimate goal of the paper is to develop an observer-based DRC, the control law is defined as $u = \mathcal{T}\hat{x}$.

*Proposition 1.* (Improved BRL) *System $\dot{q} = Aq + B_w w + B_d d$, and $y = Cq + w$ is asymptotically stable and the quadratic stability of the transfer function from the input disturbance (d) and measurement noise (w) to the system output is guaranteed if there exist symmetric positive definite matrices $P$, $S_1$, and $S_2$ and positive scalars $\gamma_d$ and $\gamma_w$ such that the following LMI is satisfied*

$$Q = \begin{bmatrix} \mathcal{G} & S_1 - A^T S_2^T & (P - S_1)B_w + C^T & (P - S_1)B_d \\ & S_2 + S_2^T & -S_2 B_w & -S_2 B_d \\ & & (1 - \gamma_w^2 \mathcal{I}_{n_w}) & 0 \\ * & & & -\gamma_d^2 \mathcal{I}_{n_d} \end{bmatrix} < 0, \tag{4}$$

*where $\mathcal{G} = A^T P + PA + C^T C - S_1 A - A^T S_1^T$ and $*$ and 0 represent the transpose of all upper triangular elements of $Q$ and zero matrix with appropriate dimension, respectively.*

Proof of Proposition 1 is trivial using Lyapunov stability theorem based on the Lyapunov function $V_1(q,\dot{q}) = q^T P q + \int_0^t \big(y^T y - \gamma_w^2 w^T w - \gamma_d^2 d^T d\big)d\tau + \int_0^t [(q^T S_1 + \dot{q}^T S_2)(\dot{q} - Aq - B_w w - B_d d)]d\tau$.

### 2.2 DRC based on UIO

*Theorem 1. System (2) together with UIO mechanism (3) is asymptotically stable in terms of Definition 1 and rejects the input disturbance and measurement noise in output quadratically if there exist symmetric positive definite matrices $\mathcal{X} \in \Re^{n\times n}$ and $\mathcal{Q}_2 \in \Re^{n\times n}$, matrices $\hat{\mathcal{T}} \in \Re^{n_u\times n}$, $\hat{\mathcal{K}} \in \Re^{n\times n_y}$, and $\hat{\mathcal{L}} \in \Re^{n\times n_y}$, positive scalars $\gamma_d$, $\gamma_w$, and $\gamma_v$, and user-defined positive weighting scalars $\alpha$ and $\beta$ such that LMI condition (5a) is satisfied*

$$\Pi^* = \begin{bmatrix} \mathcal{G}_{11} & 0 & 2G_P & 2E_P & 0 & \mathcal{X}C_P^T \\ & \mathcal{G}_{22} & \mathcal{G}_{23} & 0 & \mathcal{Q}_2\mathcal{H} & 0 \\ & & -\gamma_w^2 \mathcal{I}_{n_y} & 0 & 0 & \mathcal{I}_{n_y} \\ & & & -\gamma_d^2 \mathcal{I}_{n_d} & 0 & 0 \\ & & & & -\gamma_v^2 \mathcal{I}_{n_y} & 0 \\ * & & & & & -\alpha \mathcal{I}_{n_y} \end{bmatrix} < 0, \tag{5a}$$

*where,*

$$\begin{aligned}\mathcal{G}_{11} &= 2\left(B_P\hat{\mathcal{T}} + \hat{\mathcal{T}}^T B_P^T\right) + 2\left(\mathcal{X}A_P^T + A_P\mathcal{X}\right),\\ \mathcal{G}_{22} &= A_P^T\mathcal{P}^T\mathcal{Q}_2 + \mathcal{Q}_2\mathcal{P}A_P - \hat{\mathcal{K}}C_P - C_P^T\hat{\mathcal{K}}^T + \beta\mathcal{I}_n,\\ \mathcal{G}_{23} &= \mathcal{Q}_2\mathcal{P}G_P - \hat{\mathcal{K}} - \hat{\mathcal{L}}.\end{aligned} \tag{5b}$$

*Then, the controller gain $\mathcal{T} = \hat{\mathcal{T}}\mathcal{X}$, and observer gains $\mathcal{K} = \mathcal{Q}_2^{-1}\hat{\mathcal{K}}$ and $\mathcal{L} = \mathcal{Q}_2^{-1}\hat{\mathcal{L}}$ can be calculated.*

**Proof**. The proof is a direct use of Proposition 1 on the Lyapunov function $V_2(x,\dot{x},e) = x^T\mathcal{Q}_1 x + e^T\mathcal{Q}_2 e + \int_0^t\left(\alpha y^T y + \beta e^T e - \gamma_w^2 w^T w - \gamma_d^2 d^T d - \gamma_v^2 v^T v\right)d\tau + \int_0^t\left[\left(x^T S_1 + \dot{x}^T S_2\right)\left(\dot{x} - A_P x - B_P u - E_P d - G_P w\right)\right]d\tau$ to obtain $\dot{V}_2(x,\dot{x},e) = q_h^T\Pi q_h$, in which $q_h^T = \begin{bmatrix}x^T & e^T & w^T & d^T & v^T\end{bmatrix}$. Then, setting $S_1 = -\mathcal{Q}_1$ and $S_2 = 0$ and applying the congruence transformation $\operatorname{diag}\left(\mathcal{Q}_1^{-1}, \mathcal{I}_n, \mathcal{I}_{n_y}, \mathcal{I}_{n_d}, \mathcal{I}_{n_y}\right)$ on $\Pi$ and defining $\mathcal{X} = \mathcal{Q}_1^{-1}$, $\Pi^*$ can be obtained. This completes the proof.

■

Using Theorem 1, the extended observer-based controller dynamics can be transformed in form of (6)

$$\begin{aligned}\dot{x}_K &= A_K x_K + B_K y,\\ u &= C_K x_K + D_K y.\end{aligned} \tag{6}$$

where $A_K = \mathcal{N} - \mathcal{J}\mathcal{T} - \mathcal{L}C$, $B_K = \mathcal{L} + \mathcal{L}C\mathcal{H} - \mathcal{J}\mathcal{T}\mathcal{H}$, $C_K = \mathcal{T}$, and $D_K = -\mathcal{T}\mathcal{H}$.

### *2.3 Static AW compensator*

In order to address non-smooth nonlinearities of the actuator, based on Kothare et al. (1994), the closed-loop system is configured in axiomatic compensation framework for the regulation problem presented in Fig. 1. In this figure, the augmented controller is parameterized by system matrices $(A_K, [B_K \quad \mathcal{I}_n \quad 0_{n\times n_u}], C_K, [D_K \quad 0_{n_u\times n} \quad \mathcal{I}_{n_u}])$. Also, $N$, $\xi$, and $\Lambda^T = \begin{bmatrix}\Lambda_1^T & \Lambda_2^T\end{bmatrix}$ represents actuation nonlinearity (saturation function in this paper), AW compensation signal, and a static gain matrix for AW compensation, respectively. The augmented controller's configuration guarantees the independent effect of the AW compensator on controller states and output (Mulder et al., 2001).

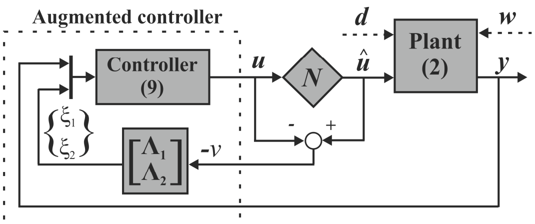


Fig. 1. The AW observer-based closed-loop system interconnection

Marcopoli and Philips (1996) and Mulder et al. (1999) presented an apparent convexity in controller synthesis for plants with the saturated actuators. Their observation led to recasting the AW synthesis in the LMI framework by transforming the system in Fig. 1 to LFT form of Fig. 2. In this figure, all of the exogenous inputs are included in $\vartheta$ and $\Delta = \mathcal{I}_n - N$. As a natural claim on the AW system, when $\nu = 0$, the controller recovers from Fig. 1 to linear observed state-feedback controller obtained from Theorem 1. The dynamic equation governing **PLANT** in Fig. 2 can be formulated as (7)

$$\begin{aligned}\dot{\sigma} &= A_\sigma\sigma + \left(B_\nu - B_\xi\Lambda\right)\nu + B_\vartheta\vartheta,\\ u &= C_u\sigma + \left(D_{u\nu} - D_{u\xi}\Lambda\right)\nu + D_{u\vartheta}\vartheta,\\ y &= C_y\sigma + \left(D_{y\nu} - D_{y\xi}\Lambda\right)\nu + D_{y\vartheta}\vartheta,\end{aligned} \tag{7}$$

where $A_\sigma$, $B_\nu$, $B_\xi$, $B_\vartheta$, $C_u$, $C_y$, $D_{u\nu}$, $D_{y\nu}$, $D_{u\xi}$, $D_{y\xi}$, $D_{u\vartheta}$, $D_{y\vartheta}$ are matrices with appropriate dimension obtained from the standard LFT. By reducing Fig. 1 in to LFT form in Fig. 2, the objective of static AW compensator is to design $\Lambda$.

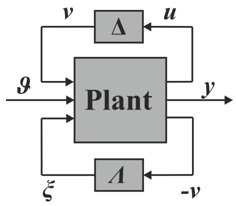


Fig. 2. Standard AW LFT configuration

*Theorem 2.* ($\mathfrak{L}_2$ gain criterion) [Teel and Kapoor, 1997; Mulder et al., 2001] *The AW compensator in (7) is $\mathfrak{L}_2$ gain stabilizable, if there exists symmetric positive definite matrix $\mathfrak{Q}$ with appropriate dimension, positive definite block diagonal matrix $\mathfrak{M} \in \mathfrak{R}^{n_u\times n_n}$, matrix $\mathfrak{X}$ and positive scalars $\Gamma$, $\mathrm{g}$, and $\delta_1$ such that the LMI (8) is satisfied*

$$\Theta = \begin{bmatrix}\mathfrak{Q}A^T + A\mathfrak{Q} & B_\vartheta & \Theta_{13} & \mathfrak{Q}C_y^T & 0\\ & -\Gamma\mathcal{I} & D_{u\nu}^T & D_{y\nu}^T & 0\\ & & \Theta_{33} & \mathfrak{M}D_{y\nu}^T - \mathfrak{X}^T D_{y\xi}^T & \mathfrak{M}\\ & & & -\mathrm{g}^{-1}\mathcal{I} & 0\\ & * & & & -\delta_1\mathcal{I}\end{bmatrix} < 0, \tag{8a}$$

*with*

$$\begin{aligned}\Theta_{13} &= B_\nu\mathfrak{M} - B_\xi\mathfrak{X} + \mathfrak{Q}C_u^T,\\ \Theta_{33} &= -2\,\mathfrak{M} + D_{u\nu}\mathfrak{M} + \mathfrak{M}D_{u\nu}^T - D_{u\xi}\mathfrak{X} - \mathfrak{X}^T D_{u\xi}^T.\end{aligned} \tag{8b}$$

*Then, $\Lambda = \mathfrak{X}\mathfrak{M}^{-1}$.*

## 3. AVC BASED ON UIO/DRC

In this section, the unknown input observer-based DRC augmented with AW compensator is implemented on a smart structure which is oscillating under the effect of a bounded mismatch disturbance signal. The objective of AWDRC is to attenuate the vibration of the structure at the system output. Additionally, if the saturation appears in the actuation elements, the AW compensator accounts for the matter to keep the stability of the system and if $\nu = 0$, the controller should recover from windup state. The plant configuration, as shown in Fig. 3, consists of a clamped-free aluminum beam with Young's modulus 70 Gpa and density 2.7 g/cm³ and two piezo-patches ($30 \times 50 \times 0.5$ mm) bounded on one side of the structure. The piezo-actuators are activated in operating voltage range of [-150 150] V with electrical capacitance of 45 nF.

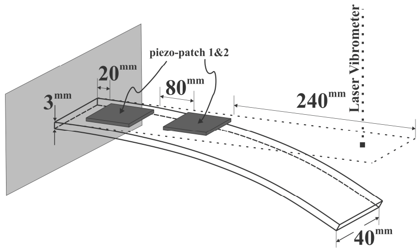


Fig. 3. Geometry of the smart beam

The closed-loop system configuration is presented in Fig. 4. The piezolaminated beam plant has three independent inputs: the control inputs which act on the two actuator piezo-patches and the exogenous signal which excites the structure through an independent disturbance channel.

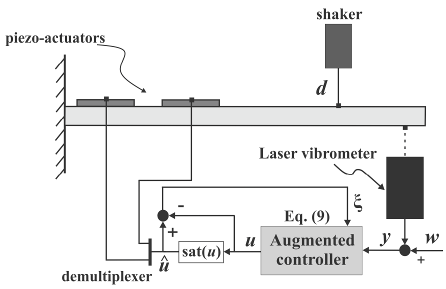


Fig. 4. Schematic interconnection of the plant and UIO-based AWDRC

The dynamics of the actuation is obtained using finite element method (FEM) in coupled electro-mechanical domain. By assuming $\mathbb{M}$, $\mathbb{C}$, and $\mathbb{K}$ as mass, damping, and stiffness matrices in the framework of FEM, the governing ordinary differential equation (ODE) of motion in matrix form can be written as $\mathbb{M}\ddot{q} + \mathbb{C}\dot{q} + \mathbb{K}q = \mathbb{F}$ with $q$ and $\mathbb{F}$ being the nodal states of displacement and electric potential/applied external inputs, respectively. This representation can be transformed into state-space representation of (2). Interested reader may refer to Marinkovic et al., (2009) and Oveisi and Nestorović, (2014).

*3.1 Numerical simulation*

In this subsection, the numerical results of the employment of the proposed controller are presented on the piezo-laminated beam shown in Fig. 3. For design purposes, a reduced order system is obtained by considering two bending mode shapes of the smart structure. The two fundamental natural frequencies of the system are 87.3 and 480 rad/s and their associated damping ratio is assumed to be $1\times 10^{-2}$. The nominal system is a transformed in state space model with the system matrices shown in (9).

$$\begin{aligned}
A_P &= \begin{bmatrix} 0 & 87.27 & 0 & 0 \\ -87.27 & -1.74 & 0 & 0 \\ 0 & 0 & 0 & 480.07 \\ 0 & 0 & -480.07 & -9.6 \end{bmatrix}, \\
[B_P \quad E_P] &= \begin{bmatrix} 0 & 4.35 & 0 & 19.27 \\ 0 & 2.79 & 0 & -4.23 \\ 0 & -5.42 & 0 & 5.08 \end{bmatrix}^T, \\
C_P &= [0 \quad -5.1 \quad 0 \quad 4.03].
\end{aligned} \tag{9}$$

First, the controller ($u = \mathcal{T}\hat{x}$) and observer system (3) are designed based on the results in Theorem 1. Accordingly, using Scilab, the controller and observer gains are obtained as

$$\begin{aligned}
&\mathcal{N} \\
&= \begin{bmatrix} 2.28E-4 & 221.44 & -9.91E-4 & -105.88 \\ -37.08 & -2.8603 & -217.91 & -2.686 \\ 1.09E-4 & -16.06 & -4.72E-4 & 492.75 \\ -46.98 & -2.258 & -276.1 & -4.481 \end{bmatrix}, \\
&\mathcal{L} = [-25.754 \quad 0.4098 \quad 3.0631 \quad 0.2549]^T, \\
&\mathcal{J} \\
&= \begin{bmatrix} 2.84E-5 & 10.596 & 1.35E-5 & 13.426 \\ -1.6E-5 & -0.733 & -7.63E-6 & -0.928 \end{bmatrix}^T, \\
&\mathcal{H} = [5.13E-7 \quad 0.1127 \quad 2.44E-7 \quad -0.1055]^T, \\
&\mathcal{T} = \begin{bmatrix} 12.308 & -263.269 & -3.71 & 207.852 \\ 36.038 & -4381.2 & -68.391 & 3456.8 \end{bmatrix}.
\end{aligned} \tag{10}$$

The results in (10) are calculated based on the intermediate parameter in (11) assuming $\alpha = \beta = 1$.

$$\begin{aligned}
&Y \\
&= [5.236E-7 \quad 0.1127 \quad 2.494E-7 \quad -0.1055]^T.
\end{aligned} \tag{11}$$

Next, utilizing Theorem 2, the AW static compensation gain is calculated in (12) for $\mathfrak{g} = 10$.

$$\begin{aligned}
&\Lambda \times 10^3 \\
&= \begin{bmatrix} 4.4 & -52.8 & 21.5 & -66.9 & -966.7 & 551.8 \\ 2.4 & -93 & 17 & -117.9 & 58.3 & -30.2 \end{bmatrix}^T.
\end{aligned} \tag{12}$$

For the sake of performance evaluation, first, the disturbance channel is activated by the realization of a chirp signal. The frequency of the disturbance signal is swept from zero to 90 Hz within a nine seconds time window in three cases of the AWDRC, saturated DRC, and uncontrolled systems. The responses of the systems are shown in Fig. 5 (first subplot) in the time domain based on the velocity measurement signal.

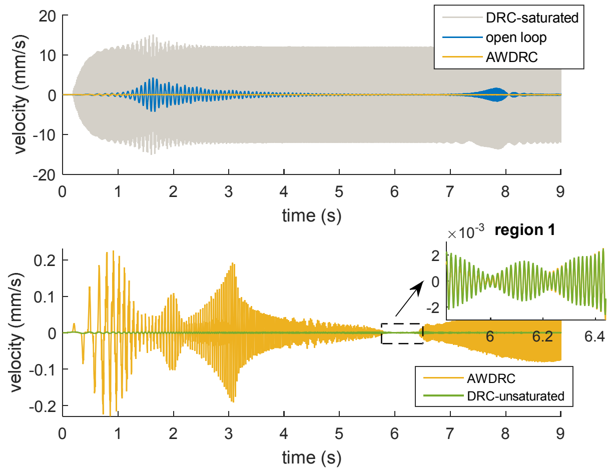


Fig. 5. Comparison of the performance of AVC system

It can be seen in the first subplot of Fig. 5 that the control system based on Theorem 1 is unable to deal with actuation nonlinearity. Although the AWDRC is losing performance compared to the unsaturated system, it suppressed the vibration magnitude within the considered frequency range. Moreover, the AWDRC based on the proposed two-step convex feasibility problem (Theorems 1 and 2) can recover from the windup state which can be seen by comparing the control effort in Fig. 6 (first subplot) with the system output

Fig. 5 (second subplot) at region 1 in which the performance of the AW compensated controller and linear case are almost identical. Additionally, the second subplot of Fig. 6 represents the control effort for the unsaturated system.

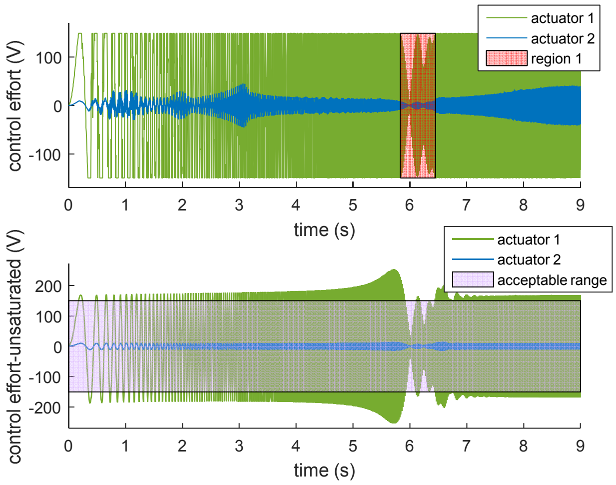


Fig. 6. control effort: AW-compensated obsever-based controller

Next, the UIO residual of system output is shown in Fig. 7 for three cases: AWDRC, unsaturated DRC, and estimation error based on Kalman filter. For this purpose, first, for the nominal model (2), the Kalman filter is designed by finding the solution of the algebraic Riccati equation (ARE): $L_K = (\mathfrak{P}C_P^T + \bar{\mathfrak{N}})\mathcal{R}^{-1}$. In this equation, $\bar{\mathfrak{N}} = G\mathfrak{N}$ with $\mathfrak{N} = \mathcal{E}(wv^T)$, $\mathcal{R} = \mathcal{E}(vv^T)$, and $\mathfrak{P}$ is the steady-state error covariance $(\lim_{t\to\infty} \mathcal{E}(\{x-\hat{x}\}\{x-\hat{x}\}^T))$. $L_K$ is obtained as $L_K = [4.1441 \quad -1715.8 \quad 28.884 \quad 1604.5]^T$.

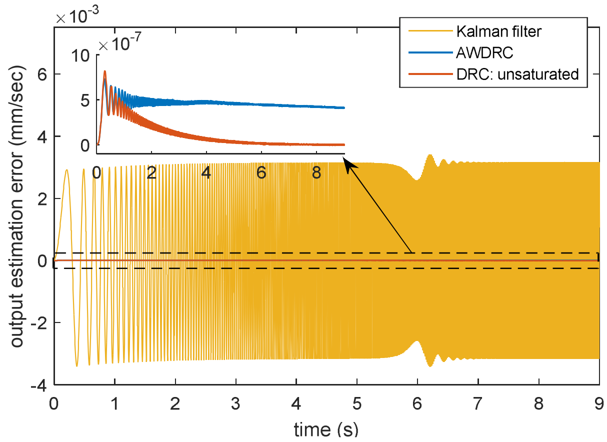


Fig. 7. Estimation error of the closed-loop system: comparison between AWDRC and unsaturated DRC

It can be seen that the UIO has a better performance in output observation than the Kalman observer in the presence of unknown mismatch disturbance. Finally, in order to examine the disturbance rejection performance of the proposed method in Theorem 1, it is compared to the Linear Quadratic Gaussian (LQG) controller in combination with AW compensator in Theorem 2. The comparison of the vibration suppression quality is presented in Fig. 8. The DRC based on Theorem 1 has a better performance compared to AW-compensated LQG controller. The control effort for both of the closed-loop systems: AWDRC and AWLQG, under the external disturbance, is depicted in Fig. 9.

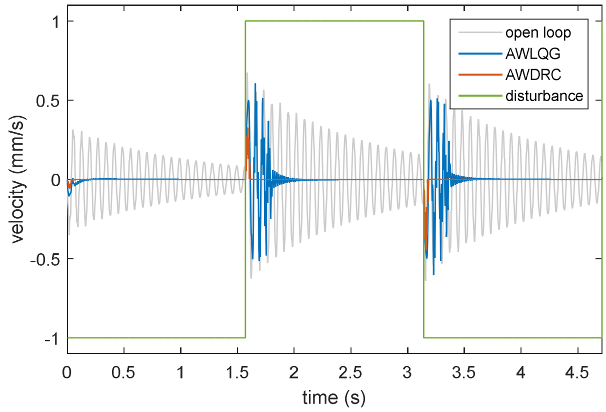


Fig. 8. Comparison of measured outputs: performance of two closed-loop in saturated case

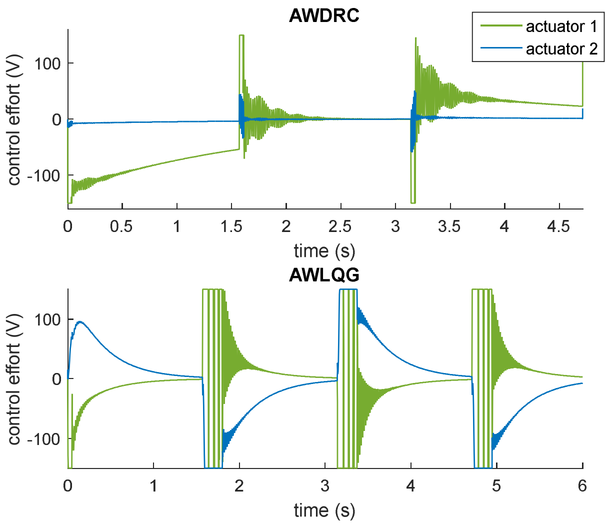


Fig. 9. Control effort: AWDRC and AWLQG

## 4. CONCLUSION AND FURTHER WORK

In this paper, the trend in unknown input observation is used to reconstruct the states of the system in a less conservative agenda. For this purpose, a set of LMEs is defined to account for multiple unknown matrices in dynamics of the observer. Then, the remaining parameters are acquired as a solution of coupled asymptotic stability problem in terms of improved BRL and stability of the observation residuals. The solution of the observer-based controller is transformed into the framework of static AW compensation. For this purpose, the absolute stability of the coupled system is guaranteed by solving an additional LMI. Both of the proposed observer and controller are compared with standard Kalman Filter and LQG regulator, respectively, on a mechanical vibrating plant. It has been observed that the proposed observer-based method has a better performance in vibration attenuation. Additionally, for nonlinear systems with multiple equilibrium points, the proposed method can be generalized by employing the Takagi-Sugeno fuzzy global approximation method.

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