**Mixed Kalman-Fuzzy Sliding Mode State Observer in Disturbance Rejection Control of a Vibrating Smart Structure**

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**ABSTRACT**

In the controllers that are synthesized on a nominal model of the nonlinear plant, the parametric matched uncertainties and nonlinear/unmodeled dynamics of high order nature can significantly affect the performance of the closed-loop system. In this note, owing to the robust character of the sliding mode observer against modeling perturbations, measurement noise, and unknown disturbances and due to the non-fragile behavior of the Kalman filter against process noise, a mixed Kalman sliding mode state-observer is proposed and later enhanced by the addition of an intelligent fuzzy agent. In light of the proposed technique, the chattering phenomena and the conservative boundary neighboring layer of the high gain sliding mode observer are addressed. Then, a robust active disturbance rejection controller is developed by using static feedback of the estimated states using direct Lyapunov quadratic stability Theorem. The reduced order plant for control design purposes is subjected to some simulated square-integrable disturbances and is assumed to have mismatch uncertainties in system matrices. Finally, the robust performance of the closed-loop scheme with respect to the mentioned perturbation signals and modeling imperfections is tested by implementing the control system on a mechanical vibrating smart cantilever beam.

**Keywords:** Fuzzy system; Nonlinear control; Active disturbance rejection; Kalman Filter; Vibration suppression.

1. **INTRODUCTION**

The uncertainty as the perturbation of the plant model from its real states may lead to unexpected behavior of the closed-loop system. Thus, it is vital to take into account the effects of such perturbations in structural elements during the modeling process [1–3]. Such a view leads to robustness of the predicted response of the system as well as increasing the applicability and reliability of the designed controller. On the other hand, the model-based regulators are developed based on the simplified mathematical replicas of the real plant. These two modeling requirements constrain the control methodologies to derive based on the norm-bounded representations of unmodeled dynamics, parametric uncertainties, and exogenous disturbance. Thus, the robust controller may be considered as the primary candidate that justifies these inquiries [4,5].

Kalman filtering is extensively employed in state estimation. Nevertheless, it is recognized to be unable to handle large modeling uncertainties [6,7]. The sliding mode observer (SMO) is proposed to address this issue both for the dynamical systems with linear and nonlinear behavior under large disturbance signals. Based on the classical sliding mode (SM) technology, a discontinuous switching function is utilized to hold the trajectory of the observation residuals on a manifold. Hence, the observation residuals become independent of the modeling perturbations in estimation error space. A central concern of the SM technique is the sensitivity of the control system to the uncertainties in the reaching phase, during which the error trajectory travels toward the sliding surface [8]. In recent years, some investigations are performed in order to examine the effects of these parametric uncertainties in the robust performance of SMO [9]. A standard design configuration for SMO is based on the equivalent control (EC) theory. Accordingly, for nonlinear dynamical systems subjected to bounded uncertainties, the output observation error together with its high order derivative is employed to construct a sliding manifold [10].

SM controller (SMC) and observer have one common drawback, namely, the chattering phenomenon that excites the high-order dynamics of the system which are mostly neglected in the design procedure. The second issue in the actuating element is the high control activity, non-smooth actuation, and actuation saturation in SMC [11]. An accepted workaround for this problem is modifying the switching function of SM technique to a neighboring boundary function such as saturation function which leads to finite steady-state error [12]. The neighboring layer depth is dependent on the switching gain. Notably, if the system is exposed to large disturbance amplitude and modeling perturbations, the SM system requires a thick boundary layer to account for the chattering problem. Such a behavior prevents the error dynamics from sliding toward the SM manifold, and, therefore, a thick boundary layer is most undesirable in control systems. In contrast, an intelligent element such as a fuzzy set based on linguistic variables may transfer the knowledge of the SM expert to the observer element instead of a passive boundary neighboring function [13]. In this paper, a Mamdani fuzzy mapping is used to create a smooth switching function that can: i) handle the parametric uncertain terms in state, input, and output matrices in state space representation of the plant. ii) treat the nonlinear norm-bounded globally Lipschitz unmodeled dynamics; and iii) mimic the behavior of boundary layer neighboring function without the mentioned drawbacks. For this purpose, a nonlinear multi-input-multi-output (MIMO) system is linearized based on the idea of Rajamani [14], and then, a fuzzy SMO (FSMO) is added to the conventional Kalman filter as a complimentary term in the dynamics of state-estimator. In addition to the asymptotic stability of the estimation error dynamics, the exponential convergence, and the disturbance rejection in estimation error are addressed carefully to provide a robust state observation framework.

The active disturbance rejection control (ADRC) is extensively researched in the last decades [15,16]. ADRC may be interpreted as a frequent alteration of the classical feedback control theory such as PID controller which was further elaborated in [15]. In this paper, as the second contribution, an ADRC is proposed based on the feedback of the observed states (mixed Kalman-FSMO). Thereupon, an appropriate constraint is adjured on the transfer function from disturbance to the desired output for the class of systems with parametric uncertainties in all system matrices. Most of the mentioned ADRC schemes are limited to some abstract plant dynamics. In contrast, an adaptive fuzzy integral SMC (FISMC) is developed by Wai et al. to control the position of an electrical servo drive [17]. An adaptive FSMO for a class of nonlinear MIMO systems with application in the modular and reconfigurable robot system is proposed in [18]. Mechanical vibrating systems based on the concept of smart structures are also categorized as the potential case studies for observer-based ADRC [19,20]. In this paper, the control method is realized on a smart vibrating clamped-free beam to evaluate the disturbance rejection, state estimation, and chattering prevention performance.

In the rest of the paper, represents the identity matrix with appropriate dimension, shows the norm for systems, superscript denotes the transpose of a matrix or a vector, and stands for the set of real numbers. Additionally, , , and represent some time dependent vectors, matrices, and positive definite matrices with dimension of , , for , respectively. Moreover, represents the well-known scalar sign function and represents the block diagonal matrix of square sub-matrices each of which has the dimension of . states for Transpose of all previous Terms e.g., for all matrices with appropriate dimensions. Also, represents the expected value of a random variable.

1. **PROBLEM FORMULATION**

Consider the nonlinear dynamics of a MIMO plant

|  |  |
| --- | --- |
|  | (1) |

where, ,,, and are the state, input, square-integrable disturbance, 2-norm bounded white Gaussian noise, and output vectors at time , respectively. For the sake of tractability of the problem, it is assumed that and . In addition, in order to put the system in the class of nonlinear Lipschitz systems, it is assumed that , , , and are sufficiently smooth and differentiable. Then, the necessary and sufficient conditions on the asymptotic stability of the observer-based control system can be addressed by the eigenvalues of the linearized matrices [14]. Unstructured unmodeled dynamics of the system are parameterized in as a Lipschitz nonlinearity with the positive Lipschitz constant , in which is the estimation of the states. For causal system (1), with globally Lipschitz continuous dynamics, (1) can be linearized in the form of Eq. (2)

|  |  |
| --- | --- |
|  | (2) |

where,

|  |  |
| --- | --- |
|  | (3) |

Additionally, , , and are the time-dependent perturbation terms that are associated with parametric uncertainty of modeling. In Eq. (2) and the rest of the paper, for simplifying the notation, the dependence of the system terms on time is suppressed. The perturbation terms are decomposed in form of , where and are known matrices with appropriate dimensions and are the time-dependent unknown matrices that satisfy .

**Remark 1.** In Eq. (2), mathematically, can be converted to or included in the form of unmodeled dynamics () and the main purpose of this decomposition is the available uncertainty quantification methods in the structural dynamical modeling [21,22].

In this paper, the novel methods in uncertainty quantification are considered in the control development as one inspiration point to realize the gain variations in a plant in a practical manner. The interested reader may refer to [23]. Henceforth, the authors are active in developing a new mechanism for uncertainty quantification in mechanical structures with complex geometries that can be used as an alternative to conventional methods [21]. Uncertainty quantification regarding the unmodeled dynamics of high order nature is classically dealt with as a lumped stable bounded time-varying functions. In terms of controller synthesis, such a view leads to conservative results in controller design and closed-loop performance. However, analytical modeling of simpler geometries under large vibration amplitudes hands the structure of uncertainty. Next, a combination of parametric identification methods and practical approaches that provide time-dependent responses of the nonlinear system make an effective tool available for creating global models. The importance of considering such a parametric uncertainty depends on the DRC’s application. In view of that matter, if the disturbance is active in the high-frequency range, then the high-order dynamics of the plant are more efficient in the response of the system [24]. In the application of vibration control, where the active methods are useful at frequencies up to 1 kHz, the introduction of unmodeled dynamics is crucial for addressing nonlinear damping models and nonlinear vibration [25]. In contrast, the sensitivity of the controller/observer to the parametric uncertainties remain as the key constraint in robust control analysis due to the uncertainty propagation phenomenon [26]. The bounded uncertainty of disturbance/noise input matrices ( and ) are neglected in this paper since they may be considered as an additional source of disturbance. Additionally, the bound of the Euclidean norm of the disturbance is assumed to be , i.e. there exists a positive real value satisfying . By assuming the system to be stabilizable and detectable, the dynamics of the Kalman-SMO-based (KSMO) state-estimation mechanism is considered in the form of Eq. (4)

|  |  |
| --- | --- |
|  | (4) |

in which, and are the estimated state- and output-vectors, correspondingly. The stability of the nominal observer is guaranteed by unbiased Kalman filter algorithm with Kalman gain based on the assumptions for and . The residual signal of the estimation is defined as . In Eq. (4), the -term with SMO gain () is assigned for compensating the effects of the disturbance, noise, and unknown bounded uncertainties of various nature. Hence, the hybrid KSMO system embraces Kalman filter’s term along with the discontinuous element with separate design mechanisms. Using Eqs. (2-4), the dynamics of state-observation residuals can be derived as

|  |  |
| --- | --- |
|  | (5) |

Considering the control law for regulation problem as , in which is static observed-state feedback gain, the dynamics of the states-estimation error takes the form of Eq. (6)

|  |  |
| --- | --- |
|  | (6) |

Next, the necessary and sufficient conditions for quadratic stability of the estimation system as well as the conditions on performance index are defined. As a result, the disturbance is rejected in residual signal by introducing an appropriate objective on control system regarding control gain and SMO gain .

1. **MAIN RESULTS**
   1. **Robust Stability and Performance of KFSMO**

The primary objective of this section is to formulate the quadratic stability of the state-estimation in Eq. (4) in terms of a convex feasibility/optimization problem. The outcome is the designed gain, , which accounts for the parametric uncertainties, unmodeled dynamics, and disturbance. It is assumed that for the nominal system, is a strictly Hurwitz matrix.

**Lemma 1.** *The hybrid state estimator in Eq. (4) has a quadratic stable residual dynamics and satisfies the disturbance rejection on estimator regarding the -norm bounded constraint on the transfer function , in which is a strictly positive real value if there exist positive symmetric matrix , matrices and , and such that simultaneously and the following matrix inequality is satisfied*

|  |  |
| --- | --- |
|  | (7a) |

*where,*

|  |  |
| --- | --- |
|  | (7b) |

*in which,* *represents zero matrix with appropriate dimension. Then, the SMO gain and the controller gain are calculated.*

*Proof.* Consider the following Lyapunov function ()

|  |  |
| --- | --- |
|  | (8) |

Then, by derivation of the Lyapunov function with respect to time, one can obtain

|  |  |
| --- | --- |
|  | (9) |

Next, by using the assumptions on the structure of the perturbation matrices , , and and applying Lemma A.1, in Appendix A, on uncertain terms of Eq. (9) except for , hands the following inequalities

|  |  |
| --- | --- |
| \_lities can be statedov function with respect to time, one can obtainection on estimator in terms of the | (10) |

Furthermore, by using the globally Lipschitz assumption on the unmodeled dynamics and employing Lemma A.2 in Appendix A, it is obvious that . By defining the augmented state vector as and using Eq. (10), can be transformed into

|  |  |
| --- | --- |
|  | (11a) |

where,

|  |  |
| --- | --- |
|  | (11b) |

in which,

|  |  |
| --- | --- |
|  | (11c) |

Finally, by applying the Congruence Transformation on Eq. (11b), defining and , and the successive use of the Schur complement Lemma, LMI condition Eq. (7) can be obtained and this completes the proof.

Consequently, by guaranteeing the LMI condition as well as the condition on noise channel which can be expressed as in a more conservative form, it is proven that the estimation error converges to zero and the disturbance is rejected in the state estimation error dynamics. Since the latter condition is time-dependent and there exists no information on the noise signal, the dynamics of the estimation system is enhanced by introducing a Fuzzy system instead of the discontinuous term. Such a change reduces the conservatism on the KSMO system. Accordingly, an intelligent Fuzzy framework is employed to address the well-known chattering problem of the observer by revising its dynamic as shown in Eq. (12).

|  |  |
| --- | --- |
|  | (12) |

where is the observer gain associated with switching function . In Eq. (12), the switching function is constructed based on the system of *if-then* rules with two inputs of estimation error and its derivation w.r.t. time and the crisp output based on fuzzy logic of the expert. The generated rules are supposed to mimic the behavior of function. The fuzzy system realizes the nonlinear mapping between the linguistic input variables and the crisp output as . In this paper the input and output membership functions are assumed to be symmetric Gaussian [27]. The fuzzy sets for inputs and outputs of are labeled as P, Z, N, S, M, and B which stand for Positive, Zero, Negative, Small, Medium, and Big, respectively. By normalizing both of the input linguistic variables in the range of [-1, 1], the membership function of input/output variables are presented in Fig. 1. Following the fuzzy rule table in [28], Table 1 is created based on 49 *if-then* rules.

Table 1. Fuzzy rules

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | NB | NM | NS | Z | PS | PM | PB |
|  |  |  |  |  |  |  |  |  |
| NB |  | NB | NB | NB | NB | NM | NS | Z |
| NM |  | NB | NB | NB | NM | NS | Z | PS |
| NS |  | NB | NB | NM | NS | Z | PS | PM |
| Z |  | NB | NM | NS | Z | PS | PM | PB |
| PS |  | NM | NS | Z | PS | PM | PB | PB |
| PM |  | NS | Z | PS | PM | PB | PB | PB |
| PB |  | Z | PS | PM | PB | PB | PB | PB |

The nonlinear mapping from the input linguistic variables to the output of is presented in Fig. 2.

|  |  |
| --- | --- |
|  |  |
| **Figure 1.** Membership functions for input/output fuzzy sets. | **Figure 2.** Fuzzy surface representing the nonlinear mapping between and and . |

**Corollary 1.** *The KFSMO of Eq. (12) has a quadratic stable state observation residual dynamics and rejects the disturbance on the state estimator in terms of the -norm bounded constraint on transfer function , if there exist positive symmetric matrix , matrices and , and positive such that the following matrix inequality is satisfied*

|  |  |
| --- | --- |
|  | (13a) |

*where,*

|  |  |
| --- | --- |
|  | (13b) |

*Then, the FSMO observer gain and the controller gain are calculated.*

*Proof.* By using the same Lyapunov function as in Eq. (8) moreover, following the same steps as in Lemma 1, but, this time, defining the augmented vector as and using the Congruence Transformation and Schur complement Lemma, LMI (13a) can be obtained.

* 1. **ADRC based on KFSMO**

In this section, a second objective is introduced to address the ADRC on the desired output. Before moving to the main results, it should be noted that LMIs (7a) and (13a) are nonstrict ones which can be converted to strict feasible LMIs through eradicating the existing LME constraints and then reducing the resulted LMIs by eliminating any constant nullspace (page 19 of [29]). Moreover, nonstrict LMI constraints do not reduce the generality of the method since satisfying strict inequality constraints in applied optimization interfaces e.g. YAMIP are infeasible.

**Bounded Real Lemma (BRL).** [30]*For the following LTI system in the state-space form*

|  |  |
| --- | --- |
|  | (14) |

*in which is the objective function signal, for a prescribed strictly positive and based on the performance index that is defined as , if and only if there exist a symmetric positive definite matrix known as Lyapunov function matrix such that the LMI condition (15) is satisfied*

|  |  |
| --- | --- |
|  | (15) |

Based on BRL, equivalent Theorem 1 is developed to guarantee the asymptotic stability of the system based on the KFSMO (Eq. (13)):

**Theorem 1.** *The uncertain system (2) with observer system (12) satisfies the -norm bounded objective functions and if there exist symmetric positive definite matrices and , matrix , positive constants , and such that the following LMI/LME is satisfied*

|  |  |
| --- | --- |
|  | (16a) |

*where,*

|  |  |
| --- | --- |
|  | (16b) |

*Then, the FSMO observer gain and the controller gain are calculated.*

*Proof.* The equivalent form of BRL for Eq. (2) can be obtained by assuming the Lyapunov function as where is the desired output. Then, by derivation of the Lyapunov function w.r.t. time and using Lemmas A.1 and A.2, the following inequalities can be stated

|  |  |
| --- | --- |
|  | (17) |

Considering with , and using the Congruence Transformation based on the block diagonal matrix , can be obtained as

|  |  |
| --- | --- |
|  | (18a) |

where

|  |  |
| --- | --- |
|  | (18b) |

Finally, by defining and repetitive employment of Schur complement Lemma on all of the terms with , , , and multipliers second LMI of Eq. (16) is obtained. The first LMI is the direct use of the results from Corollary 1. It should be mentioned that for output disturbance rejection an additional LME should be defined as This completes the proof.

**Remark 2.** LME is the coupling term between the feasibility problem of the robust state estimation and robust ADRC.

**Remark 3.** By defining and replacing the associated terms and in LMI Eq. (16a) by and , respectively, the maximum disturbance attenuation rate can be achieved by solving the following convex optimization problem:

**Corollary 2.** *For the uncertain system (2) with observer system (12), the maximum disturbance attenuation based on the -norm bounded objective functions and can be achieved if there exist symmetric positive definite matrices and , matrix , positive constants , and such that the following convex optimization problem has a solution*

|  |  |
| --- | --- |
|  | (19) |

*subject to LMI/LME system (16a) and (16b), in which, and are user-defined values expressing the weight of the ADRC performance objective compared to robust KFSMO objective.*

**Remark 4.** If an exponential convergence with the desired rate such as , is prescribed by control engineer, then two additional constraints on the derivation of Lyapunov functions w.r.t. time should be added. Accordingly, in Corollary 1 and Theorem 1 the in equalities [31].

1. **NUMERICAL EXAMPLE**
   1. **Modeling of the Open Loop Plant**

In this section, by aiming at evaluating the performance of the proposed observer-based control system, the ADRC scheme is implemented on a mechanical vibrating plant. The objective of ADRC system is to suppress the oscillation of the beam at the plant output. The structure of the system, as presented in Fig. 3, consists of a clamped-free aluminum beam with Young’s modulus 70 Gpa and density 2.7 g/cm³ and two piezo-patches bounded on one side of the structure. The controller and observer systems are calculated based on the results in Theorem 1. The closed-loop system configuration is presented in Fig. 4. The piezolaminated beam plant has three inputs: the control inputs which act on the two actuator piezo-patches and the exogenous signal which excites the structure through an independent disturbance channel.

It should be mentioned that the torsional modes are neglected in the modeling of the beam since only the bending vibration suppression is analyzed in this paper. The dynamics of the actuation is obtained by means of finite element analysis (FEA) in the coupled electro-mechanical domain. Although it is assumed that the displacements are small to satisfy linear piezo-elasticity, in the case of large deformation (geometrical nonlinearities), the unmodeled nonlinear dynamics of the system can be addressed in the form of in Eq. (2) [32]. By assuming , , and as mass, damping, and stiffness matrices in the framework of FEM, the governing ordinary differential equation (ODE) of motion in matrix form can be written as with and being the nodal states of displacement and electric potential and applied external inputs, respectively. The structure of the control effort is assumed to be , where, depends on the position of generalized control effort in FE structure. The output of the plant based on the nodal displacements and velocities is expressed as with and being the modal output displacement and velocity matrices, respectively. The natural frequencies () of the coupled structure as well as the mode shapes () for is obtained by employing the classical harmonic solution () and solving the obtained algebraic equation. Then, by applying the transformation , the nodal model of the plant can be transformed to the generalized modal displacement model, in which, is the matrix of eigen vectors. Finally, by using the proportional damping model, , orthogonality of mode shapes, and assuming the state vector as , with , the governing equation of motion can be presented in state space form of Eq. (2). The interpretation of the state, input, and output matrices based on the FEA can be presented as Eq. (20)

|  |  |
| --- | --- |
|  | (20) |

where with being the damping ratio of th mode shape and , , and **.**

|  |  |
| --- | --- |
|  |  |
| **Figure 3.** Geometry of the smart beam. | **Figure 4.** Schematic interconnection of the plant and KFSMO-based controller. |

**Remark 5.** In the state space representation of Eq. (20), the parametric uncertainty terms of Eq. (2) are not available, and their quantification is highly dependent on the operating frequency range of disturbance which acts on the structure. If the interference is active at high oscillation rates, then the effect of unmodeled/nonlinear dynamics in the nominal plant model is more important. In this frequency range, after efficient modeling of dynamics of low-frequency nature, the bound of the unknown additive uncertainty function in plant model can be obtained following the experimental method in [24,26]. In contrast, if the frequency of the disturbance is not higher than the highest natural frequency of the modeled reduced order system, stochastic finite element method (SFEM) is an appropriate method for calculating the known matrices ( and ) in mismatch uncertainty of and , respectively [33].

* 1. **Controller and Observer Design**

In this section, the numerical results of the employment of the proposed controller are presented on the piezolaminated beam shown in Fig. 3. For design purposes, a reduced order system is obtained by considering three bending mode shapes of the smart structure. The three fundamental natural frequencies of the system are 96.7, 467.84, and 542.95 rad/sec and their associated damping ratios are calculated as 1.25, 5.02, and 3.79, respectively. The remaining higher order modes are reflected in the form of the unstructured uncertainty of a norm-bounded time-dependent term. The nominal system is transformed in state space representation with the system matrices shown in Eq. (21), in which maximum ten percent parametric variations are assumed in the form of, , , and .

|  |  |
| --- | --- |
|  | (21a) |

with

|  |  |
| --- | --- |
|  | (21b) |

For examination of the robust enactment of the closed-loop system which includes parametric uncertainty of Eqs. (21a) and (21b) and , an observer-based controller is designed. For this purpose, first, for the nominal deterministic model, the well-known Kalman filter is designed by solving the algebraic Riccati equation (ARE) of . In this equation, with , , and is the steady-state error covariance (). Accordingly, the obtained is presented in Eq. (22).

|  |  |
| --- | --- |
|  | (22) |

Then, using the results from Theorem 1 (Eq. (16)), the controller gain and FSMO gain are obtained as

|  |  |
| --- | --- |
|  | (23) |

By implementing the control system based on the calculated gain in Eq. (23) and two observer gains and on the plant with system matrices as in Eqs. (21a,b), the prospect of the operational vibration attenuation performance is assessed. The optimal solution for LMI/LME system of Eq. (16) is acquired by using Scilab. Next, the disturbance channel is activated by realization of a chirp signal. The frequency of the disturbance signal is swept from zero to 100 Hz within a twenty second time window in two cases of the controlled and uncontrolled systems. Investigations are carried out on the structure in Fig. 3 in time-domain with the sampling frequency of 1 kHz. Both of the robust controller and KFSMO systems are modeled on SIMULINK platform with standard fixed step explicit ODE5 solver (Dormand-Prince method). The response of the system for the close-loop and open loop cases is shown in Fig. 5 in time-domain based on the velocity measurement signal.



**Figure 5.** Comparison of measured outputs in time domain.

It can be seen that the control system based on the uncertain plant, suppressed the vibration magnitude within the considered frequency range. However, the quality of the ADRC using KFSMO (Eq. (12)) is better than the one based on KSMO by a small margin. Additionally, the applied control signals that are implemented through piezo-modules are depicted in Fig. 6.

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| --- | --- |
|  |  |
| **Figure 6a.** Control signal based on Kalman SMO. | **Figure 6b.** Control signal based on Kalman-Fuzzy SMO. |

Fig. 6a shows that the control efforts that are actuated by each of the piezo-transducer based on KSMO have a chattering behavior in the region indicated inside the box. However, the controller based on the KFSMO had a smooth response without any sudden jump, and it is limited to [-50 50] V. Next, the residual of the estimation of the system output is shown in Figs. 7 for both of the controllers based on KSMO and KFSMO.



**Figure 7.** Estimation error of the closed-loop system: comparison between KSMO and KFSMO.

The comparison of the switching function () in Eq. (12) and in dynamic equation of KSMO of Eq. (4) in the forced vibration is presented in Fig. 8a. Figures 8a indicates a stable switching function from the initial time for closed-loop system based on the proposed observer-based system. The chattering behavior of the control effort for KSMO-based controller is due to the sudden jumps that appear in the switching function. However, the smooth nonlinear mapping of the intelligent fuzzy system (see Fig. 4) mimics the behavior of the SMO in a jump-less manner. This can be observed in the Fig. 8b, which presents the mapping of the input linguistic variables to the output variable by means of the rule Table 1.

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| **Figure 8a.** Switching functions in forced vibration control. | **Figure 8b.** The nonlinear mapping of the fuzzy component. |

**CONCLUSION**

In this note, Kalman filter is mixed with the sliding mode observer in order to specifically address the unmodeled dynamics of nonlinear nature in plant and the modeling uncertainties of the state, input and output matrices. In order to solve the chattering issue of KSMO and removing the constraint on the time-dependent term in Lyapunov function, a fuzzy system is assigned to the observer system to imitate the behavior of the switching function with employing the knowledge of the expert. Then, the proposed state estimation scheme is used in ADRC problem based on the bounded real Lemma. Given ADRC, the uncertain terms of modeling in system matrices are taken into account and as a result, the stability of the closed-loop system, as well as the conditions for quadratic convergence, are derived. The solution of the problem is transformed in terms of a convex optimization problem subjected to some LMI/LME constraints.

**APPENDIX A**

**Lemma A.1** [29] *For real matrices and symmetric matrix , the following first statement can be guaranteed if and only if the second one holds for a positive scalar and ,*

|  |  |
| --- | --- |
|  | (A1) |

**Lemma A.2.** [34] *For two arbitrary vectors with appropriate dimension such as the following inequality is valid for*

|  |  |
| --- | --- |
|  | (A2) |

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