Regression Analysis

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Agenda

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  - Method of Least Square

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  - Limitation of this approach

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  - The percentage of SSE reduction $= r^2 * 100 = 33.33\%$

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- The percentage of SSE reduction from the base model if we use the regression model instead.
- r^2 is the percentage of the variation explained by the model

Example from 10.66

The strength of concrete pipes over time

```
library(gdata)
```

```
d = read.xls('http://www.typ-stats.com/xdatasets/PIPELOAD.X
head(d)
```

```
##    LOAD AGE
## 1 11450  20
## 2 10420  20
## 3 11142  20
## 4 10840  25
## 5 11170  25
## 6 10540  25
```

Example from 10.66

```r
mymodel = lm(LOAD~AGE, data=d)
summary(mymodel)
```

```
Residual standard error: 460.7 on 7 degrees of freedom
Multiple R-squared:  0.7311,  Adjusted R-squared:  0.6927
F-statistic: 19.03 on 1 and 7 DF,  p-value: 0.003305
```

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- What if it's much lower? like 30% .

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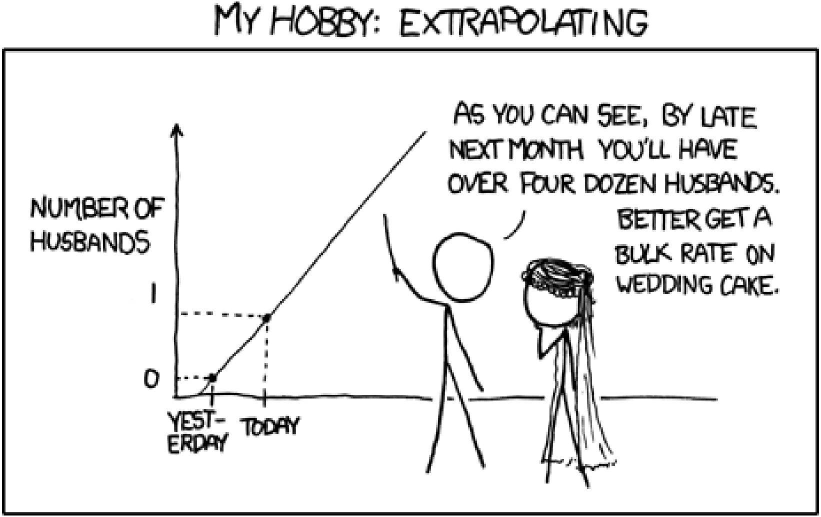
- Interpret r^2
- Determine whether β_1 is significant. Interpret the significance.

Limitation of regression analysis



Limitation of regression analysis

- One outlier can really mess up your regression model. β_0, β_1, r^2 won't be accurate anymore.

```
d = read.xls('http://www.typ-stats.com/xdatasets/PIPELOAD.X
d[10,] = c(20000, 30) #<--- this will screw up the data
library(ggplot2)
ggplot(mapping=aes(x=AGE,y=LOAD),data=d) + geom_point() +
  geom_smooth(method='lm')
```