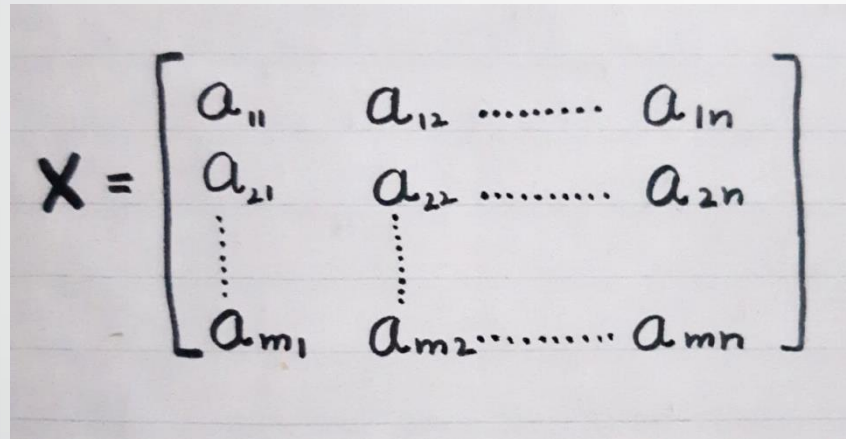




# Matrices and their Properties

# Matrix

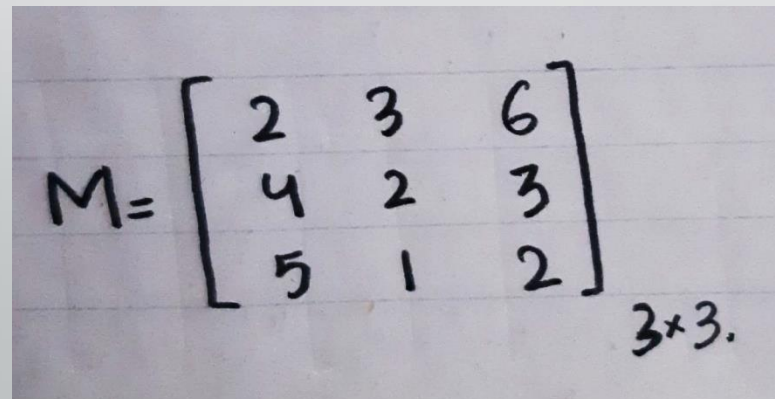
- A matrix is a two dimensional array of numbers.
- Matrixes are usually represented by an uppercase bold letter, such as **X**.



A handwritten representation of a general matrix  $X$  enclosed in large square brackets. The matrix has  $m$  rows and  $n$  columns. The elements are labeled  $a_{ij}$ , where  $i$  is the row index and  $j$  is the column index. The first row contains  $a_{11}, a_{12}, \dots, a_{1n}$ . The second row contains  $a_{21}, a_{22}, \dots, a_{2n}$ . Vertical dots between the second and  $m$ th rows indicate the continuation of the pattern. The last row contains  $a_{m1}, a_{m2}, \dots, a_{mn}$ .

$$X = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

- $M$  is a matrix of dimensions  $3 \times 3$



A handwritten representation of a specific  $3 \times 3$  matrix  $M$  enclosed in large square brackets. The matrix contains the following values:

$$M = \begin{bmatrix} 2 & 3 & 6 \\ 4 & 2 & 3 \\ 5 & 1 & 2 \end{bmatrix}$$

Below the matrix, the dimensions  $3 \times 3$  are written.

# Diagonal Matrix and Identity Matrix

## Diagonal Matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}_{3 \times 3}$$

## Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Matrix multiplication

$$X = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 0 & 2 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 6 \\ 2 & 7 \\ 3 & 8 \end{bmatrix}$$

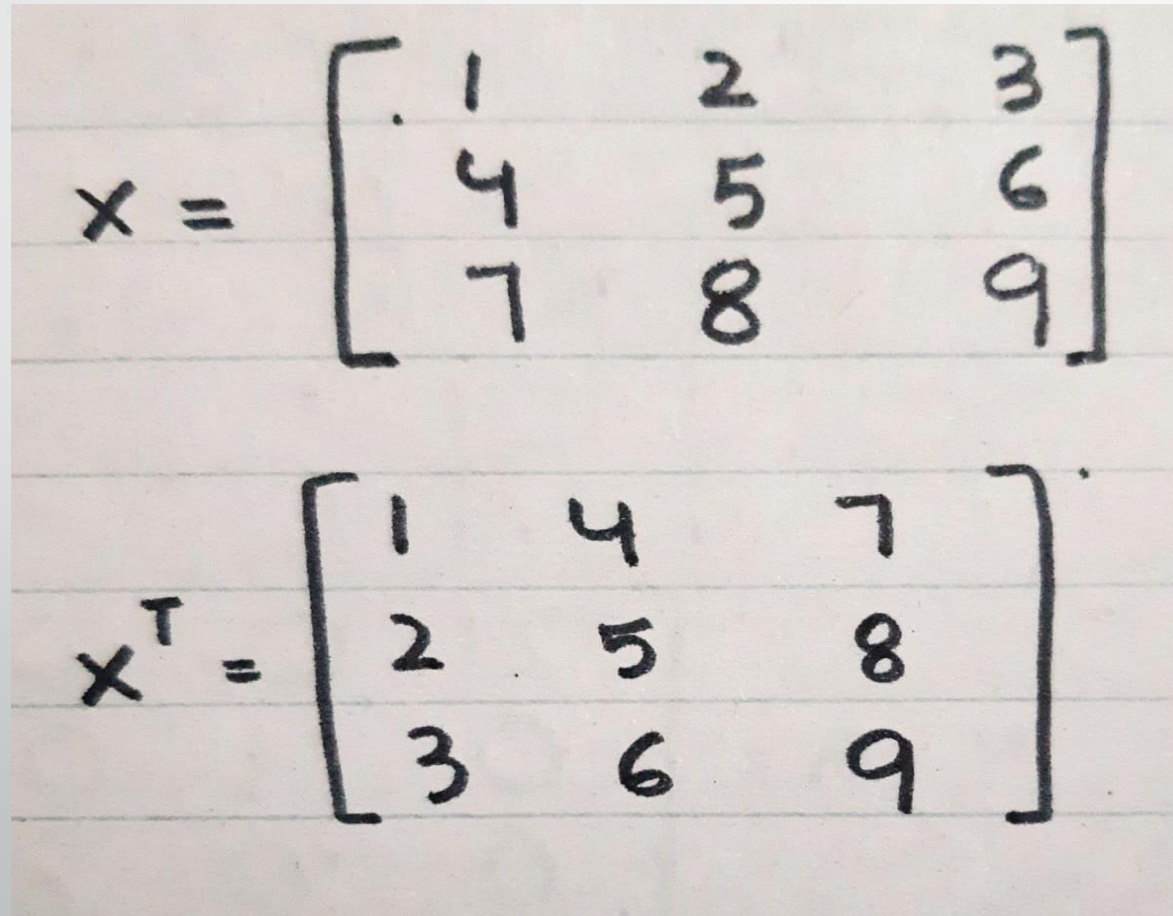
$$Z = XY = \begin{bmatrix} 2 \times 1 + 4 \times 2 + 6 \times 3 & 2 \times 6 + 4 \times 7 + 6 \times 8 \\ 8 \times 1 + 0 \times 2 + 2 \times 3 & 8 \times 6 + 0 \times 7 + 2 \times 8 \end{bmatrix}$$

$$Z = \begin{bmatrix} 2 + 8 + 18 & 12 + 28 + 48 \\ 8 + 0 + 6 & 48 + 0 + 16 \end{bmatrix}$$

$$Z = \begin{bmatrix} 28 & 88 \\ 14 & 64 \end{bmatrix}$$

# Transpose of a Matrix

- The result of swapping the rows and columns of a matrix **X** is the transpose of that matrix.
- When we take the transpose, element (i, j) goes to position (j, i).



The image shows two handwritten matrices on lined paper. The first matrix is labeled  $X =$  and is a 3x3 matrix with elements 1, 2, 3 in the first row; 4, 5, 6 in the second row; and 7, 8, 9 in the third row. The second matrix is labeled  $X^T =$  and is a 3x3 matrix with elements 1, 4, 7 in the first row; 2, 5, 8 in the second row; and 3, 6, 9 in the third row. This illustrates the process of transposing a matrix, where rows become columns and columns become rows.

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$
$$X^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$



# Inverse of a Matrix

$$AA^{-1} = I = A^{-1}A.$$

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}.$$

$$A \cdot A^{-1} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -3/2+3/2 \\ 4-4 & -2+3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$