Matrices and their Properties

Matrix

- A matrix is a two dimensional array of numbers.
- Matrixes are usually represented by an uppercase bold letter, such as **X**.

$$\mathbf{X} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- M is a matrix of dimensions 3×3

$$M = \begin{bmatrix} 2 & 3 & 6 \\ 4 & 2 & 3 \\ 5 & 1 & 2 \end{bmatrix}_{3\times 3.}$$

Diagonal Matrix and Identity Matrix

Diagonal Matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}_{3\times 3}$$

Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix multiplication

$$X = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 0 & 2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & 6 \\ 2 & 7 \\ 3 & 8 \end{bmatrix}$$

$$Z = XY = \begin{bmatrix} 2\times 1 + 4\times 2 + 6\times 3 & 2\times 6 + 4\times 7 + 6\times 8 \\ 8\times 1 + 0\times 2 + 2\times 3 & 8\times 6 + 0\times 7 + 2\times 8 \end{bmatrix}$$

$$Z = \begin{bmatrix} 2+8+18 & 12+28+48 \\ 8+0+6 & 48+0+16 \end{bmatrix}$$

$$Z = \begin{bmatrix} 28 & 88 \\ 14 & 64 \end{bmatrix}$$

Transpose of a Matrix

- The result of swapping the rows and columns of a matrix **X** is the transpose of that matrix.
- When we take the transpose, element (i, j) goes to position (j, i).

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Inverse of a Matrix

$$AA^{-1} = I = A^{-1}A.$$

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}.$$

$$A \cdot A^{-1} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -3/2 + 3/2 \\ 4-4 & -2+3 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$