Differentiation and Integration

Introduction to Derivative

In mathematics.

- The rate of change of a function with respect to a variable is known as derivative.

Derivative Vs. Differentiation

- Differentiation is a technique for calculating a derivative, which is the rate of change of a function's output y in relation to the change of a variable x.

Representation of Differentials

- Differentials are represented as *d*x, *d*y, *d*t, and so on, where *d*x represents a small change in x, *d*y represents a small change in y, and *d*t is a small change in t. When comparing changes in related quantities where y is the function of x.

http://www.differencebetween.net/science/mathematics-statistics/difference-between-differential-and-derivative/

Representation of derivative

→ "The derivative of" can be written as $\boxed{\frac{d}{dx}}$

For Example:

$\frac{d}{dx}\cos(x)$ and $\cos(x)'$

they both represents "the derivative of cos(x)"

⇒ **What is the derivative of cos(x).**

It can be written as

$\boxed{\frac{d}{dx} = \cos(x) = -\sin(x)}$ or $\boxed{\cos(x)' = -\sin(x)}$

Techniques of differentiation

Technique	**Function**	**Derivative**
Multiplication by constant	cf	cf’
Power Rule	x^n	nx^{n-1}
Sum Rule	f + g	f’ + g’
Difference Rule	f - g	f’ – g’
Product Rule	fg	f g’ + f’ g
Quotient Rule	f/g	f’ g – g’ fg^2
Reciprocal Rule	1/f	$-f'/f^2$
Chain Rule(as "Composition of Functions")	f º g	(f’ º g) × g’
Chain Rule (using ’)	f(g(x))	f’(g(x))g’(x)
Chain Rule (using d/dx)	Dy/dx = (dy/du)(du/dx)	

https://www.mathsisfun.com/calculus/derivatives-rules.html

Techniques of differentiation Cont.

Power Rule: $\frac{d}{dx} x^4$

Power rule = $\frac{d}{dx} x^n = n x^{n-1}$.

So, $\frac{d}{dx} x^4$

$\Rightarrow 4 x^{4-1}$

$\Rightarrow 4 x^3$.

The derivative of x^4 is $4x^3$.

Multiplication by Constant:

What is $4x^6$?

$\Rightarrow \frac{d}{dx} 4x^6 = (4)(6) x^{6-1}$

$\frac{d}{dx.} = 24 x^5$.

Techniques of differentiation Cont.

Sum Rule:

What is derivative of $5x^2 + 10x^3$?

From the Sum rule $f+g = f' + g'$

$\frac{d}{dx} 5x^2 = 5(2)x^{2-1} = 10x$

$\frac{d}{dx} 10x^3 = 10(3)x^{3-1} = 30x^2$

the derivative of $5x^2 + 10x^3 = 10x + 30x^2$

Difference Rule:

- The derivative of $f-g = f' - g'$

What is the derivative of $3f^3 - 4f^3$

→ $\frac{d}{dx} 3f^3 = (3)(3)f^{3-1} = 9f^2$

→ $\frac{d}{dx} 4f^3 = (4)(3)f^{3-1} = 12f^2$

the derivative of $3f^3 - 4f^3$ is $9f^2 - 12f^2$

Techniques of differentiation Cont.

Product Rule:

$$\frac{d}{dx} \sin(x) \cos(x)$$

According to product rule:

the derivative of $\cancel{\sin(x) \cos(x)} = fg = fg' + f'g$

$f = \sin(x)$, $g = \cos(x)$

$$\frac{d}{dx} \cos(x) = -\sin x$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

So the derivative will be

$$\Rightarrow \sin(x)[-\sin(x)] + \cos(x)\cos(x)$$

$$= -\sin^2(x) + \cos^2(x)$$

$$= \cos^2(x) - \sin^2(x)$$

Quotient Rule:

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$$

What is the derivative of $\frac{\sin(x)}{x}$

$$\left[\frac{\sin(x)}{x}\right]' = \frac{x[\cos(x)] - [\sin x](1)}{x^2}$$

$$\boxed{\left[\frac{\sin(x)}{x}\right]' = \frac{x\cos(x) - \sin x}{x^2}}$$

Techniques of differentiation Cont.

Resiprocal Rule:

$$\frac{d}{dx}\frac{1}{x^2}$$

$$\Rightarrow \frac{d}{dx} x^{-2} \Rightarrow -2x^{-2-1}$$

$$= \frac{-2}{x^3}$$

The derivative of $\frac{1}{x^2} = \frac{-2}{x^3}$.

Chain Rule:

$$\frac{d}{dx}\cos(x^3)$$

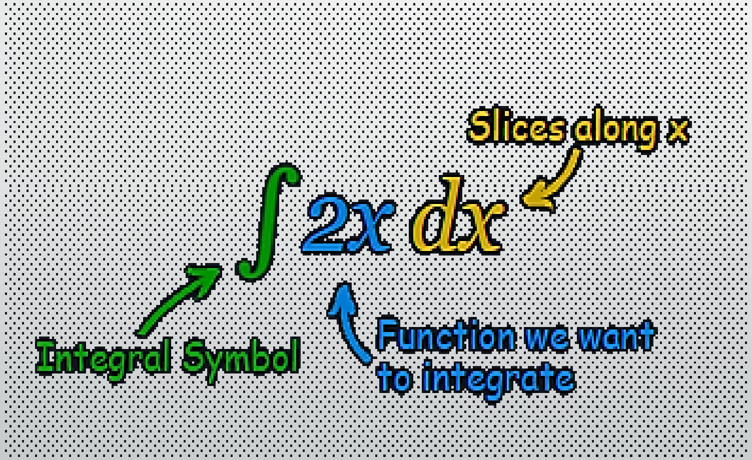
$$\frac{d}{dx}\cos(x^3) = -\sin(x^3)\ (3x)$$

$$\frac{d}{dx}\cos(x^3) = -3x\sin(x^3).$$

The derivative of $\cos(x^3)$ is $-3x\sin(x^3)$.

Introduction to Integration

- Integration is a method of finding the total by combining sections.
- It can be used to find areas, volumes, central points and many useful things
- The process of finding an Integral is the inverse of finding a Derivative.
- The symbol for "Integral" is a stylish "S“ (for "Sum", the idea of summing sections)



Techniques of integration

Rules	Function	Integral
Multiplication by constant	$\int cf(x)\,dx$	$c\int f(x)\,dx$
Power Rule ($n\neq -1$)	$\int x^n\,dx$	$x^{n+1}n+1 + C$
Sum Rule	$\int (f + g)\,dx$	$\int f\,dx + \int g\,dx$
Difference Rule	$\int (f - g)\,dx$	$\int f\,dx - \int g\,dx$
Integration by Parts	$\int u\,v\,dx = u\int v\,dx - \int u'\,(\int v\,dx)\,dx$	
Substitution Rule	Will discuss later	

Techniques of integration Cont.

Power Rule:

What is $\int x^4\,dx$?

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C.$$

$$\int x^4\,dx = \frac{x^{4+1}}{4+1} + C$$

$$\int x^4\,dx = \frac{x^5}{5} + C.$$

Multiplication By Constant:

What is integration of $3x^3$?

$$\Rightarrow \int 3x^3\,dx = 3\int x^3\,dx.$$

$$\Rightarrow 3\cdot\frac{x^4}{4} + C$$

$$\Rightarrow \frac{3}{4}x^4 + C.$$

Techniques of integration Cont.

Sum Rule:

$$\int (\sec^2 x + 2x + x)\, dx \,?$$

$$\int (\sec^2 x + 2x + x)\, dx = \int \sec^2 x + \int 2x + \int x .$$

$$\int \sec^2 x = \tan x + C$$

$$2\int x = 2\,\frac{x^2}{2} = x^2 + C$$

$$\int x = \frac{x^2}{2} + C.$$

$$\int (\sec^2 x + 2x + x)\, dx = \tan x + x^2 + \frac{x^2}{2} + C$$

Techniques of integration Cont.

Integration By Parts:

$$\int uv \, dx = u\int v \, dx - \int \left[u' \left(\int v \, dx\right)\right] dx$$

→ $\int x \, \mathrm{Sin}(x).$

$$= x\int \mathrm{Sin}(x) \, dx - \int \left[x' \left(\int \mathrm{Sin}(x) \, dx\right)\right] dx$$

$$= (x)(-\cos(x)) - \int \left[(1)(-\cos(x))\right] dx$$

$$= -x\cos(x) - \int -\cos(x) \, dx$$

$$= -x\cos(x) - (-\mathrm{Sin}(x)) + C$$

$$= -x\cos(x) + \mathrm{Sin}\, x + C$$

$$= \mathrm{Sin}\, x - x\cos(x) + C.$$

Techniques of integration Cont.

Substitution Rule:

$$\int f(g(x))\ g'(x)\ dx.$$

$$\int \sin(x^2)\ 2x\ dx.$$

$\Rightarrow \quad u = x^2$

$\Rightarrow \quad \frac{du}{dx} = 2x \quad , \quad du = 2dx.$

$\Rightarrow \quad \int \sin(u)\ du$ $\qquad \because \int \sin(x) = -\cos(x)$

$\Rightarrow \quad -\cos(u) + C.$

$\Rightarrow \quad -\cos(x^2) + C$

why +C ?

- It is the "Constant of Integration". It is there because of **all the functions whose derivative is 2x**:
- The derivative of $\mathbf{x^2}$ is **2x**,
- The derivative of $\mathbf{x^2+4}$ is also **2x**,
- The derivative of $\mathbf{x^2+99}$ is also **2x**,
- Because the derivative of a constant is zero.
- So when we **reverse** the operation (to find the integral) we only know **2x**, but there could have been a **constant of any value**.