



Linear Algebra

Introduction to Vectors

Linear Algebra

- Linear algebra is the study of lines and planes, vector spaces, and mappings, all of which are necessary for linear transformations.
- It is a branch of mathematics, but the truth of it is that linear algebra is the mathematics of data. Matrices and vectors are the language of data.

Linear Algebra For Machine Learning

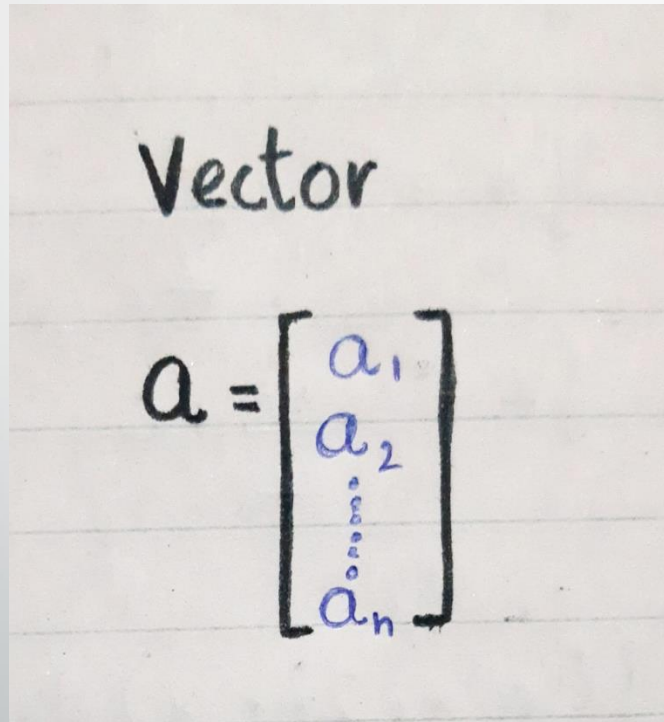
- It is a key foundation to the field of machine learning, from notations used to describe the operation of algorithms to the implementation of algorithms in code.

Applications of Linear Algebra

- As linear algebra is the mathematics of data, the tools of linear algebra are used in many domains.
- Matrices in Engineering, such as a line of springs.
- Graphs and Networks, such as analyzing networks.
- Markov Matrices, Population, and Economics, such as population growth.
- Linear Programming, the simplex optimization method.
- Fourier Series: Linear Algebra for Functions, used widely in signal processing.
- Linear Algebra for Statistics and Probability, such as least squares for regression.
- Computer Graphics, such as the various translation, scaling and rotation of images.

Defining a Vector

- Vectors are an array of numbers in which the order of the numbers is important.
- A lowercase bold letter, such as **a**, is commonly used to indicate Vectors.



A photograph of a piece of lined paper with handwritten text. The word "Vector" is written in black ink at the top. Below it, the equation $a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ is written in blue ink. The vector a is represented by a lowercase letter, and the components a_1, a_2, \dots, a_n are listed vertically within square brackets.

Vector Addition and Multiplication

Vector Addition

$$a = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

$$a + b = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 + 1 \\ 4 + 3 \\ 6 + 5 \\ 8 + 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 11 \\ 15 \end{bmatrix}$$

Vector Multiplication:

$$2a = 2 \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$$

$$2a = \begin{bmatrix} 2 \times 2 \\ 2 \times 4 \\ 2 \times 6 \\ 2 \times 8 \end{bmatrix}, 2a = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix}$$

Norm of a Vector

- The magnitude $||a||$ of a vector is measured by its norm.

Norm of a Vector:

$$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$||a|| = \sqrt{(2)^2 + (4)^2} = \sqrt{4 + 16}$$

$$= \sqrt{20} \approx 4.472$$

Dot product of two vectors

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots a_n b_n.$$

$$a = \begin{bmatrix} -2 \\ 4 \\ 8 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

$$a \cdot b = (-2 \times 1) + (4 \times 2) + (8 \times 3)$$

$$a \cdot b = (-2 + 8 + 24).$$

$$a \cdot b = 30$$