Comparing Object Correlation Metrics for Effective Space Traffic Management

Marvin Peña Theodore Y. Faust Adam Q. Jaffe Julie Y. Zhang

Institute of Pure and Applied Mathematics, University of California, Los Angeles

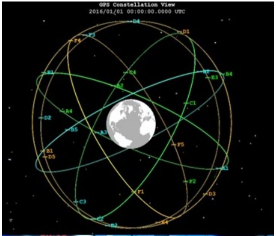
Sponsored by The Aerospace Corporation

Industry Mentors: Dr. Daniel P. Lubey, Dr. Andrew J. Binder, Mr. James E. Gidney

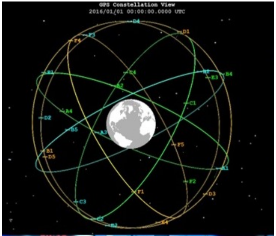
Academic Mentor: Minh Pham

August 23, 2018

The Aerospace Corporation



The Aerospace Corporation



- Established in 1960 as a California nonprofit corporation

The Aerospace Corporation

- Established in 1960 as a California nonprofit corporation
- Operates a federally funded research and development center

The Aerospace Corporation

- Established in 1960 as a California nonprofit corporation
- Operates a federally funded research and development center
- Performs objective technical analyses and assessments

The Aerospace Corporation

- Established in 1960 as a California nonprofit corporation
- Operates a federally funded research and development center
- Performs objective technical analyses and assessments
- Specialized laboratories test, analyze, and troubleshoot virtually every aspect of rocket and satellite systems

Motivation

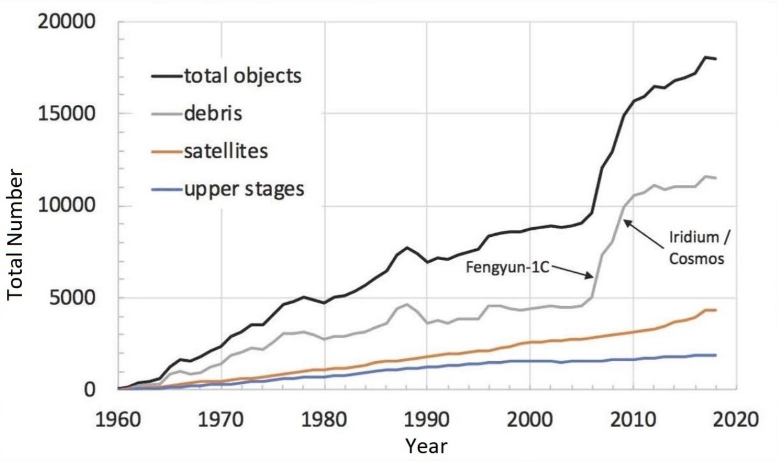


Figure 1. Catalogued Objects in Space Surveillance Network

Motivation

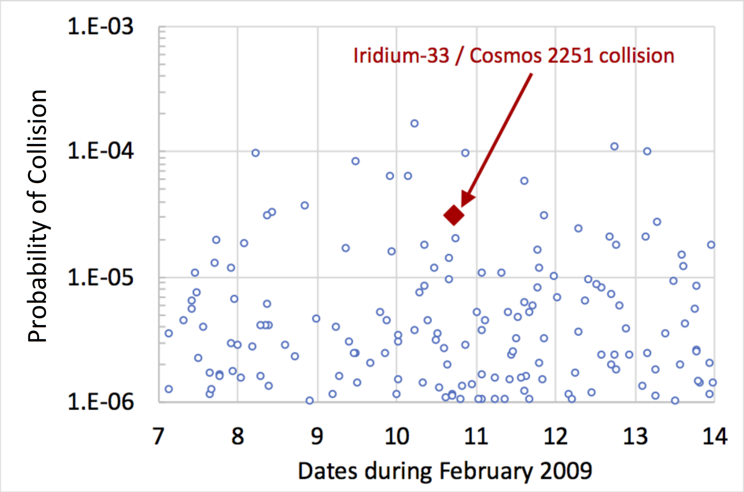


Figure 2. Collision Probabilities

Non-Cooperative Space Traffic Management

- Estimate state of objects in orbit without a communication channel

Non-Cooperative Space Traffic Management

- Estimate state of objects in orbit without a communication channel
- Space-based and ground-based sensors contain measurement error

Non-Cooperative Space Traffic Management

- Estimate state of objects in orbit without a communication channel
- Space-based and ground-based sensors contain measurement error
- Consider distribution of possible states

Non-Cooperative Space Traffic Management

- Estimate state of objects in orbit without a communication channel
- Space-based and ground-based sensors contain measurement error
- Consider distribution of possible states
- Optimally assign measurements to objects

Non-Cooperative Space Traffic Management

- Estimate state of objects in orbit without a communication channel
- Space-based and ground-based sensors contain measurement error
- Consider distribution of possible states
- Optimally assign measurements to objects
- Comprehensive comparison of likelihood-of-coincidence metrics

Overview

Simulation Framework

`aeropackage`

True State Propagation

- To propagate the true satellite states (positions and velocities) over time, we numerically solve the following ODE (where $\mathbf{r}$ is the position of the satellite):

True State Propagation

- To propagate the true satellite states (positions and velocities) over time, we numerically solve the following ODE (where $\mathbf{r}$ is the position of the satellite):

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} + \mathbf{f}_{\text{perturb}} \tag{1}$$

True State Propagation

- To propagate the true satellite states (positions and velocities) over time, we numerically solve the following ODE (where $\mathbf{r}$ is the position of the satellite):

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} + \mathbf{f}_{\text{perturb}} \tag{1}$$

- $\mathbf{f}_{\text{perturb}}$ consists of various perturbations that deviate the path of the satellite from that given in two-body gravity.

True State Propagation

- To propagate the true satellite states (positions and velocities) over time, we numerically solve the following ODE (where $\mathbf{r}$ is the position of the satellite):

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} + \mathbf{f}_{\text{perturb}} \tag{1}$$

- $\mathbf{f}_{\text{perturb}}$ consists of various perturbations that deviate the path of the satellite from that given in two-body gravity.
- Without the $\mathbf{f}_{\text{perturb}}$ term, models two-body gravity:
  1. Perfectly spherical earth with symmetric mass distribution

True State Propagation

- To propagate the true satellite states (positions and velocities) over time, we numerically solve the following ODE (where $\mathbf{r}$ is the position of the satellite):

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} + \mathbf{f}_{\text{perturb}} \tag{1}$$

- $\mathbf{f}_{\text{perturb}}$ consists of various perturbations that deviate the path of the satellite from that given in two-body gravity.
- Without the $\mathbf{f}_{\text{perturb}}$ term, models two-body gravity:
  1. Perfectly spherical earth with symmetric mass distribution
  2. Only force acting on satellite is Earth's gravity

True State Propagation

- In particular,

$$\mathbf{f}_{\text{perturb}} = \mathbf{f}_{\text{obl}} + \mathbf{f}_{\text{drag}} + \mathbf{f}_{\text{SRP}} + \mathbf{f}_{\text{3B}}$$

  where

  - $\mathbf{f}_{\text{obl}}$ is the force due to the oblateness of the Earth

True State Propagation

- In particular,

$$\mathbf{f}_{\text{perturb}} = \mathbf{f}_{\text{obl}} + \mathbf{f}_{\text{drag}} + \mathbf{f}_{\text{SRP}} + \mathbf{f}_{\text{3B}}$$

  where

  - $\mathbf{f}_{\text{obl}}$ is the force due to the oblateness of the Earth
  - $\mathbf{f}_{\text{drag}}$ is the force due to atmospheric drag

True State Propagation

- In particular,

$$\mathbf{f}_{\text{perturb}} = \mathbf{f}_{\text{obl}} + \mathbf{f}_{\text{drag}} + \mathbf{f}_{\text{SRP}} + \mathbf{f}_{3\text{B}}$$

  where

  - $\mathbf{f}_{\text{obl}}$ is the force due to the oblateness of the Earth
  - $\mathbf{f}_{\text{drag}}$ is the force due to atmospheric drag
  - $\mathbf{f}_{\text{SRP}}$ is the force due to solar radiation pressure.

True State Propagation

- In particular,

$$\mathbf{f}_{\mathsf{perturb}} = \mathbf{f}_{\mathsf{obl}} + \mathbf{f}_{\mathsf{drag}} + \mathbf{f}_{\mathsf{SRP}} + \mathbf{f}_{\mathsf{3B}}$$

  where

  - $\mathbf{f}_{\mathsf{obl}}$ is the force due to the oblateness of the Earth
  - $\mathbf{f}_{\mathsf{drag}}$ is the force due to atmospheric drag
  - $\mathbf{f}_{\mathsf{SRP}}$ is the force due to solar radiation pressure.
  - $\mathbf{f}_{\mathsf{3B}}$ is the force due to third-body gravity

Summary of Perturbations

- Earth Oblateness: Due to its rotation, the Earth has greater mass near the equator than at the poles.

Summary of Perturbations

- Earth Oblateness: Due to its rotation, the Earth has greater mass near the equator than at the poles.
- Atmospheric Drag: The force due to the interaction of atmospheric particles with the satellite.

Summary of Perturbations

- Earth Oblateness: Due to its rotation, the Earth has greater mass near the equator than at the poles.
- Atmospheric Drag: The force due to the interaction of atmospheric particles with the satellite.
- Solar radiation pressure (SRP): force caused by the impact, reflection, absorption, and re-emission of photons.

Summary of Perturbations

- Earth Oblateness: Due to its rotation, the Earth has greater mass near the equator than at the poles.
- Atmospheric Drag: The force due to the interaction of atmospheric particles with the satellite.
- Solar radiation pressure (SRP): force caused by the impact, reflection, absorption, and re-emission of photons.
- Third-body gravity: The impact on the motion of a satellite by the gravity of other bodies such as the Sun, Moon, or planets.

Sensors

- Two main types of sensors:

Sensors

- Two main types of sensors:
  - Ground-based: Fixed to specific position (latitude and longitude) on surface of earth

Sensors

- Two main types of sensors:
  - Ground-based: Fixed to specific position (latitude and longitude) on surface of earth
  - Space-based: Essentially satellites with sensing capability; movement modeled by the ODE (1)

Sensors

- Two main types of sensors:
  - Ground-based: Fixed to specific position (latitude and longitude) on surface of earth
  - Space-based: Essentially satellites with sensing capability; movement modeled by the ODE (1)

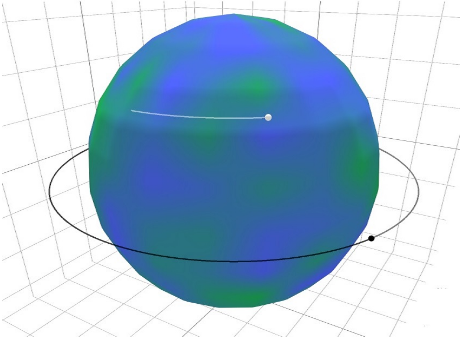


Figure 3. Tracks of Sensor Movement

Sensors

- Additionally, sensors can take different types of measurements of an object:

Sensors

- Additionally, sensors can take different types of measurements of an object:
  - Range: The Euclidean distance from the sensor to the satellite

Sensors

- Additionally, sensors can take different types of measurements of an object:
  - Range: The Euclidean distance from the sensor to the satellite
  - Angle: Represented in azimuth and elevation (also called altitude), the angle (as related to the tangent plane to the Earth at the sensor location) of the satellite.

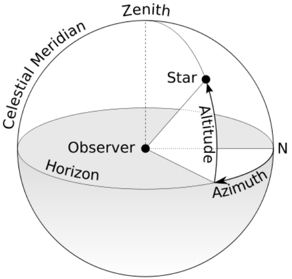


Figure 4. Azimuth/Elevation Diagram

Propagating Uncertainty

- Due to the error in measurements, we cannot determine the exact state (position and velocity) of a satellite. Only know the distribution of the state of a satellite

Propagating Uncertainty

- Due to the error in measurements, we cannot determine the exact state (position and velocity) of a satellite. Only know the distribution of the state of a satellite
- We assume that this state distribution is a multivariate Gaussian: $(\mu^{(t_0)}, \Sigma^{(t_0)})$ at time t_0.

Propagating Uncertainty

- Due to the error in measurements, we cannot determine the exact state (position and velocity) of a satellite. Only know the distribution of the state of a satellite
- We assume that this state distribution is a multivariate Gaussian: $(\mu^{(t_0)}, \Sigma^{(t_0)})$ at time t_0.
- Need to use the dynamics of the system (1) to determine what the distribution will be at a future time $t_1 > t_0$

Propagating Uncertainty

- Due to the error in measurements, we cannot determine the exact state (position and velocity) of a satellite. Only know the distribution of the state of a satellite
- We assume that this state distribution is a multivariate Gaussian: $(\mu^{(t_0)}, \Sigma^{(t_0)})$ at time t_0.
- Need to use the dynamics of the system (1) to determine what the distribution will be at a future time $t_1 > t_0$
- Use the ODE solver to propagate $\mu^{(t_0)}$ to time t

Propagating Uncertainty

- Due to the error in measurements, we cannot determine the exact state (position and velocity) of a satellite. Only know the distribution of the state of a satellite
- We assume that this state distribution is a multivariate Gaussian: $(\mu^{(t_0)}, \Sigma^{(t_0)})$ at time t_0.
- Need to use the dynamics of the system (1) to determine what the distribution will be at a future time $t_1 > t_0$
- Use the ODE solver to propagate $\mu^{(t_0)}$ to time t
- Use the State Transition Matrix Φ to propagate $\Sigma^{(t_0)}$ as follows:

$$\Sigma^{(t_1)} = \Phi(t_1, t_0)\Sigma^{(t_0)}\Phi(t_1, t_0)^T.$$

Example of Uncertainty Propagation

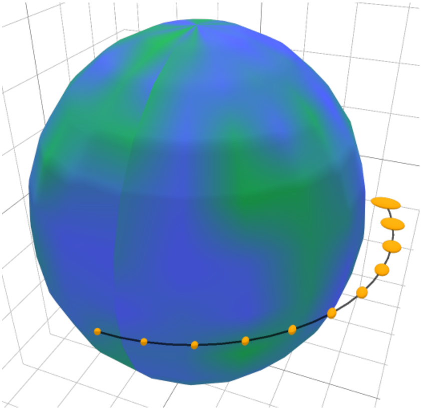


Figure 5. Propagation of Mean and Covariance

Object Correlation

Measurement Space Object Correlation

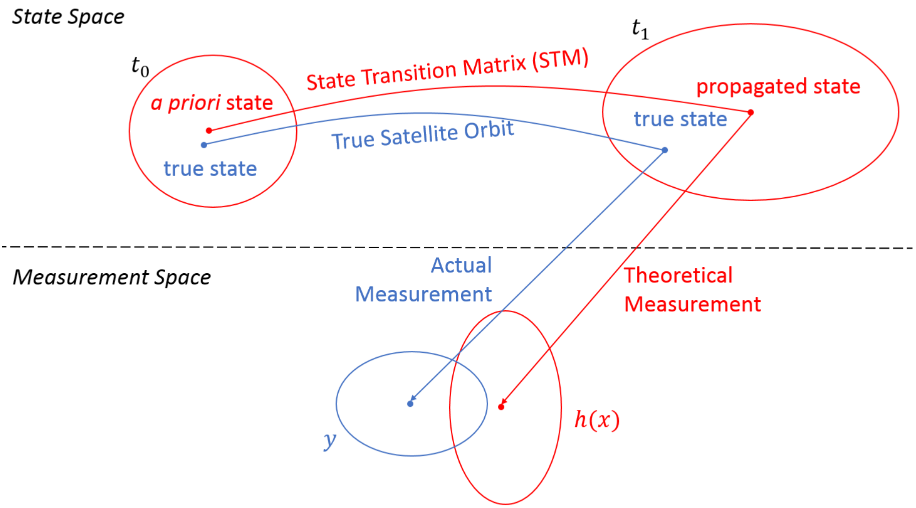


Figure 6. Object Correlation in Measurement Space

Distance Metrics in Measurement Space

Let $D_1 \overset{d}{=} N(\mu_1, \Sigma_1)$ and $D_2 \overset{d}{=} N(\mu_2, \Sigma_2)$, with dimension k.

Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)$$

Distance Metrics in Measurement Space

Let $D_1 \overset{d}{=} N(\mu_1, \Sigma_1)$ and $D_2 \overset{d}{=} N(\mu_2, \Sigma_2)$, with dimension k.

Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)$$

Bhattacharyya:

$$\begin{aligned} d_B(D_1, D_2) = &\frac{1}{4}(\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1) \\ &+ \frac{1}{2} \log \left(\frac{\det(\Sigma_1 + \Sigma_2)}{\sqrt{\det \Sigma_1 \det \Sigma_2}} \right) - \frac{k}{2} \log 2 \end{aligned}$$

Distance Metrics in Measurement Space

Kullback-Liebler divergence is a nonsymmetric function, so we define two versions of it.

KL1:

$$
\begin{aligned}
d_{KL}(D_1, D_2) = {} & \frac{1}{2}(\mu_2 - \mu_1)^T \Sigma_2^{-1} (\mu_2 - \mu_1) \\
& + \frac{1}{2} \log\left(\frac{\det \Sigma_2}{\det \Sigma_1}\right) + \frac{1}{2} \mathsf{Tr}(\Sigma_2^{-1}\Sigma_1) - \frac{k}{2}
\end{aligned}
$$

KL2:

$$d_{KL}(D_2, D_1)$$

Measurement Space Object Correlation

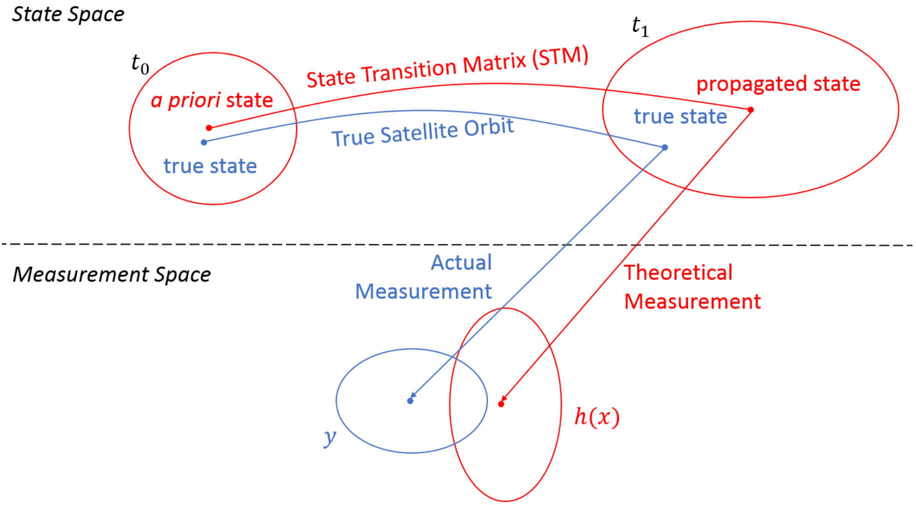


Figure 7. Object Correlation in Measurement Space

State Space Object Correlation

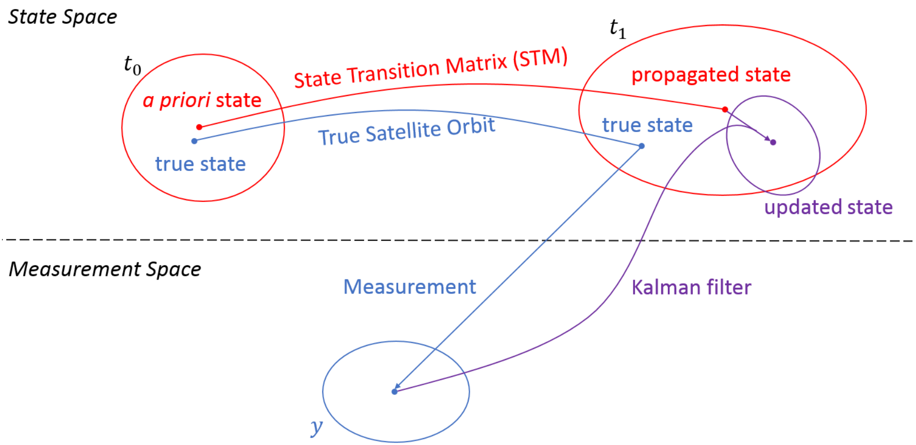


Figure 8. Object Correlation in State Space

Distance Metrics in State Space

- Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2 - 2\mathsf{Cov}(D_1, D_2))^{-1} (\mu_2 - \mu_1)$$

Essentially the same as measurement space, but includes a term to account for the correlation between the two states

Distance Metrics in State Space

- Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2 - 2\text{Cov}(D_1, D_2))^{-1} (\mu_2 - \mu_1)$$

  Essentially the same as measurement space, but includes a term to account for the correlation between the two states
- Optimal control distance (OCD):
  - The effect of a control function u on the system dynamics is

$$\ddot{r} = f(t, r(t)) + u(t)$$

Distance Metrics in State Space

- Mahalanobis:

$$d_M(D_1, D_2) = (\mu_2 - \mu_1)^T(\Sigma_1 + \Sigma_2 - 2\mathsf{Cov}(D_1, D_2))^{-1}(\mu_2 - \mu_1)$$

  Essentially the same as measurement space, but includes a term to account for the correlation between the two states
- Optimal control distance (OCD):
  - The effect of a control function u on the system dynamics is

$$\ddot{r} = f(t, r(t)) + u(t)$$

  - The metric is

$$d_{OCD}((t_0, s_0), (t_1, s_1)) = \inf_{u \in U} \int_{t_0}^{t_1} \frac{1}{2}\|u(t)\|^2 dt$$

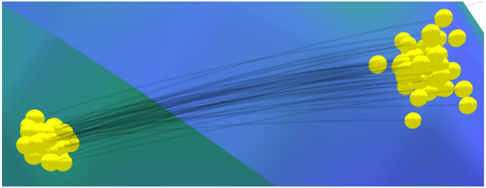
    where U is the space of all control functions that take state s_0 at time t_0 to state s_1 at time t_1

Experiment Design

Experiment Design

How do the metrics perform on clusters of satellites?

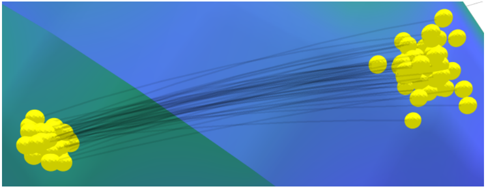
- `CLUSTER-OUT`: Satellites begin in a cluster at time t_0 and disperse as they are propagated to a later time t_1



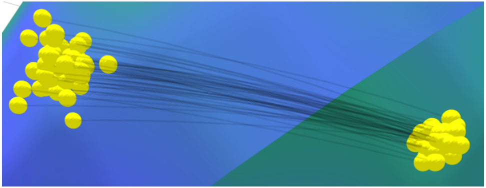
Experiment Design

How do the metrics perform on clusters of satellites?

- CLUSTER-OUT: Satellites begin in a cluster at time t_0 and disperse as they are propagated to a later time t_1



- CLUSTER-IN: Satellites begin dispersed at time t_0 and become clustered as they are propagated to a later time t_1



Specifications

- 10 space-based sensors at equal longitudinal intervals above the equator in geosynchronous orbit

Specifications

- 10 space-based sensors at equal longitudinal intervals above the equator in geosynchronous orbit
- 20 ground-based sensors at major cities chosen to have a relatively even distribution of latitudes and longitudes

Specifications

- 10 space-based sensors at equal longitudinal intervals above the equator in geosynchronous orbit
- 20 ground-based sensors at major cities chosen to have a relatively even distribution of latitudes and longitudes
- Generate cluster of satellites at time t_0 by the following process:
  - Randomly select cluster mean state μ_C at a given altitude alt_C
  - Set the state covariance of the cluster to be

$$\Sigma_C = \begin{pmatrix} \sigma_r^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_r^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_r^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_v^2 \end{pmatrix}$$

  - For each i, sample a state deviation $\xi_i \sim N(\vec{0}, \Sigma_C)$ and set the state of satellite i to be $\vec{x}_i = \mu_C + \xi_i$.

Coarse Parameter Search

- Need to determine most impactful parameters

Coarse Parameter Search

- Need to determine most impactful parameters
- Set to these values when varying:
  - Observation gap $t_1 - t_0$ (seconds): $300, 1800, 3600, 7200$
  - Altitude alt$_C$ (km): $350, 762.5, 1175, 1587.5, 2000$
  - Dispersal σ_r^2 (m^2): $10^6, 10^8, 10^{10}$
  - Satellite *a priori* uncertainty σ_{ap}^2 (m^2): $10^0, 10^2, 10^4$
  - Sensor measurement error σ_{meas}^2 (m^2): $10^0, 10^2, 10^4$

Coarse Parameter Search

- Need to determine most impactful parameters
- Set to these values when varying:
    - Observation gap $t_1 - t_0$ (seconds): $300, 1800, 3600, 7200$
    - Altitude alt$_C$ (km): $350, 762.5, 1175, 1587.5, 2000$
    - Dispersal σ_r^2 (m^2): $10^6, 10^8, 10^{10}$
    - Satellite *a priori* uncertainty σ_{ap}^2 (m^2): $10^0, 10^2, 10^4$
    - Sensor measurement error σ_{meas}^2 (m^2): $10^0, 10^2, 10^4$
- Set to nominal values when not varying:
    - Observation Gap: 3600 s
    - Altitude: 750 km
    - Dispersal: 10^8 m^2
    - Satellite *a priori* uncertainty: 10^2 m^2
    - Sensor measurement error: 10^2 m^2

Fine Parameter Analysis

- If we pick 2 parameters to vary, we can vary them in higher resolution

Fine Parameter Analysis

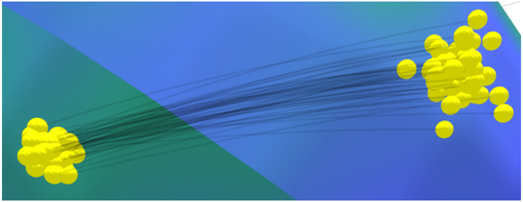
- If we pick 2 parameters to vary, we can vary them in higher resolution
- Use same nominal values when fixed

Fine Parameter Analysis

- If we pick 2 parameters to vary, we can vary them in higher resolution
- Use same nominal values when fixed
- Vary across these ranges in 20 equal increments:
  - Observation gap: $[300, 7200]$ s
  - Altitude: $[350, 2000]$ km
  - Dispersal: $[10^4, 10^{10}]$ m^2 (log scale)
  - Satellite *a priori* uncertainty: $[10^0, 10^4]$ m^2 (log scale)
  - Sensor measurement error: $[10^0, 10^4]$ m^2 (log scale)

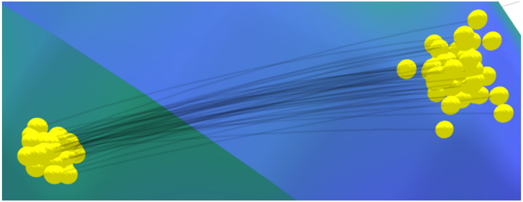
Results

CLUSTER-OUT: Determining Impactful Variables



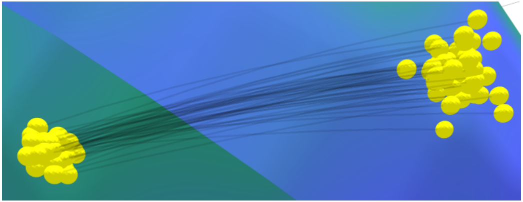
- Run 30 trials, with 50 satellites leaving a cluster

CLUSTER-OUT: Determining Impactful Variables



- Run 30 trials, with 50 satellites leaving a cluster
- For each metric and modality, use Least Absolute Shrinkage and Selection Operator (LASSO) to determine 3 most impactful parameters

CLUSTER–OUT: Determining Impactful Variables



- Run 30 trials, with 50 satellites leaving a cluster
- For each metric and modality, use Least Absolute Shrinkage and Selection Operator (LASSO) to determine 3 most impactful parameters
- Will run finer pairwise investigation on observation gap, dispersal, and sensor variance

CLUSTER-OUT: Grid Plots

- Run 30 trials of each test

CLUSTER-OUT: Grid Plots

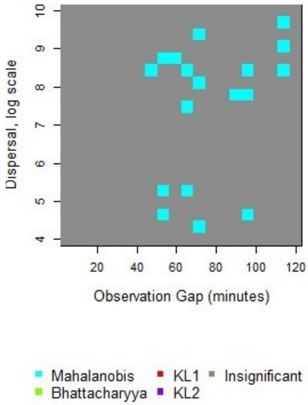
- Run 30 trials of each test
- For each parameter pair, find the metric with the highest average success rate

CLUSTER-OUT: Grid Plots

- Run 30 trials of each test
- For each parameter pair, find the metric with the highest average success rate
- Conduct a one-sided paired t-test with $\alpha = 0.05$ for determining whether the winning metric has a significantly higher mean than the losing metrics

CLUSTER-OUT: Grid Plots

- Run 30 trials of each test
- For each parameter pair, find the metric with the highest average success rate
- Conduct a one-sided paired t-test with $\alpha = 0.05$ for determining whether the winning metric has a significantly higher mean than the losing metrics
- If the null hypothesis is rejected for all losing metrics, color the cell according to the metric
- Else, color the cell gray since the win is insignificant



CLUSTER−OUT: Results

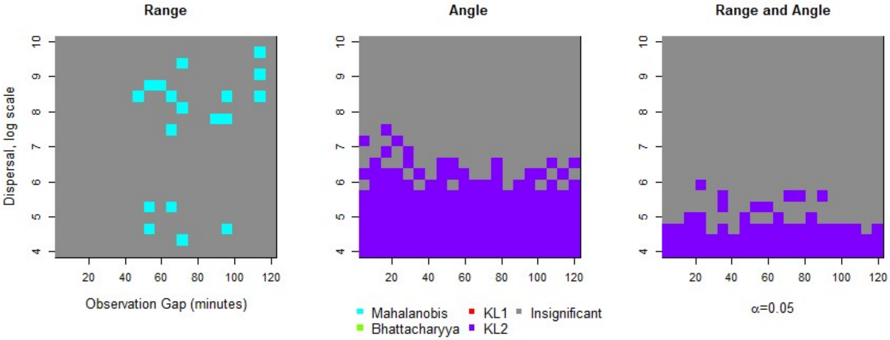


Figure 9. CLUSTER−OUT Test: Observation Gap vs Cluster Dispersal

CLUSTER-OUT: Results

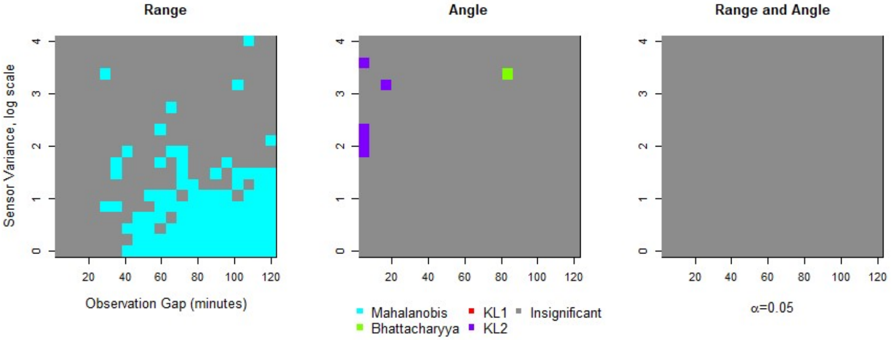


Figure 10. CLUSTER-OUT: Observation Gap vs Sensor Variance

CLUSTER−OUT: Results

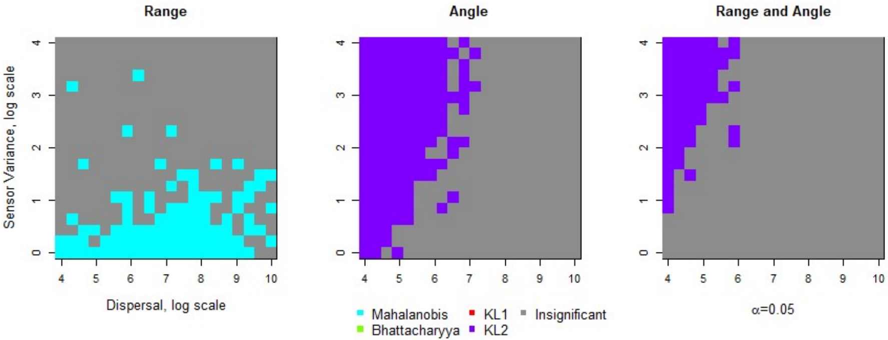


Figure 11. CLUSTER−OUT: Cluster Dispersal vs Sensor Variance

CLUSTER-OUT: Results

- Range measurements: Mahalanobis wins for sensor variance $\log(\sigma^2_{meas}) < 1.5$, and in other regions hard to describe

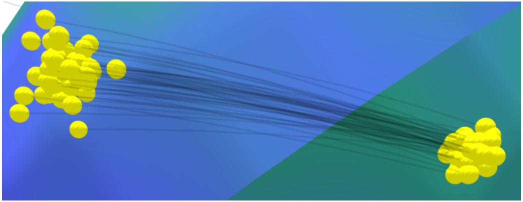
CLUSTER-OUT: Results

- Range measurements: Mahalanobis wins for sensor variance $\log(\sigma_{meas}^2) < 1.5$, and in other regions hard to describe
- Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6.5$

CLUSTER–OUT: Results

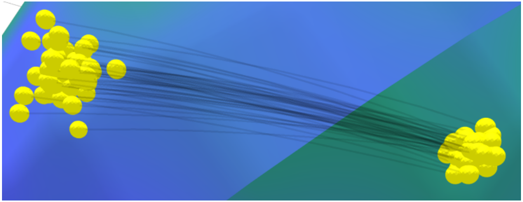
- Range measurements: Mahalanobis wins for sensor variance $\log(\sigma^2_{meas}) < 1.5$, and in other regions hard to describe
- Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6.5$
- Range and angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 5.5$

CLUSTER-IN: Determining Impactful Variables



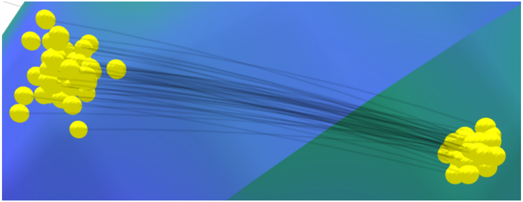
- Run 30 trials, with 50 satellites going towards a cluster

CLUSTER-IN: Determining Impactful Variables



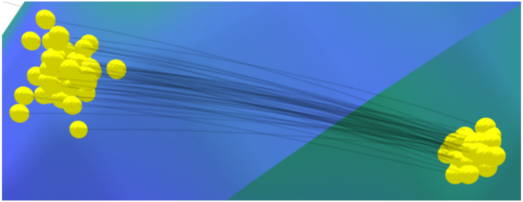
- Run 30 trials, with 50 satellites going towards a cluster
- For each metric and modality, use LASSO to determine 3 most impactful parameters

CLUSTER-IN: Determining Impactful Variables



- Run 30 trials, with 50 satellites going towards a cluster
- For each metric and modality, use LASSO to determine 3 most impactful parameters
- Dispersal and sensor variance appear in all models
- Altitude appears 7 out of the 12 times, with observation gap appearing 5 times

CLUSTER-IN: Determining Impactful Variables



- Run 30 trials, with 50 satellites going towards a cluster
- For each metric and modality, use LASSO to determine 3 most impactful parameters
- Dispersal and sensor variance appear in all models
- Altitude appears 7 out of the 12 times, with observation gap appearing 5 times
- Will run finer pairwise investigation on altitude, dispersal, and sensor variance

CLUSTER-IN: Results

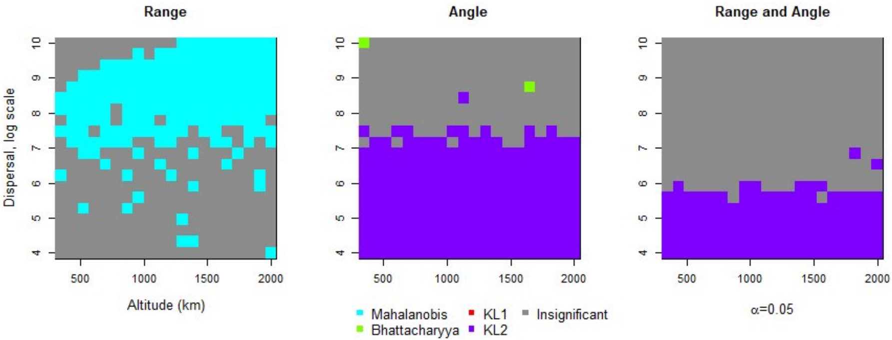


Figure 12. CLUSTER-IN: Altitude vs Cluster Dispersal

CLUSTER-IN: Results

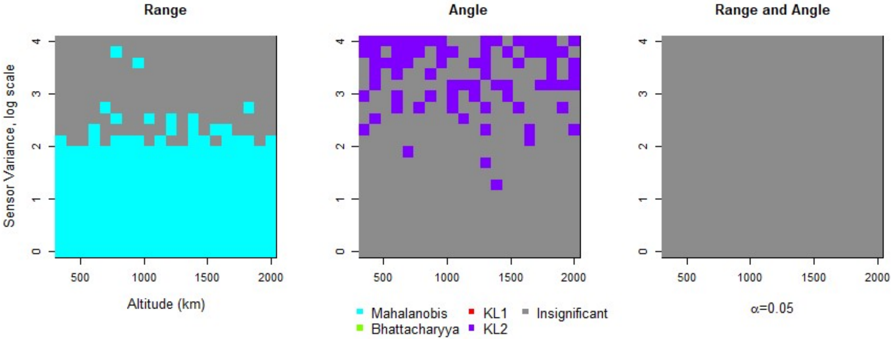


Figure 13. CLUSTER-IN: Altitude vs Sensor Variance

CLUSTER−IN: Results

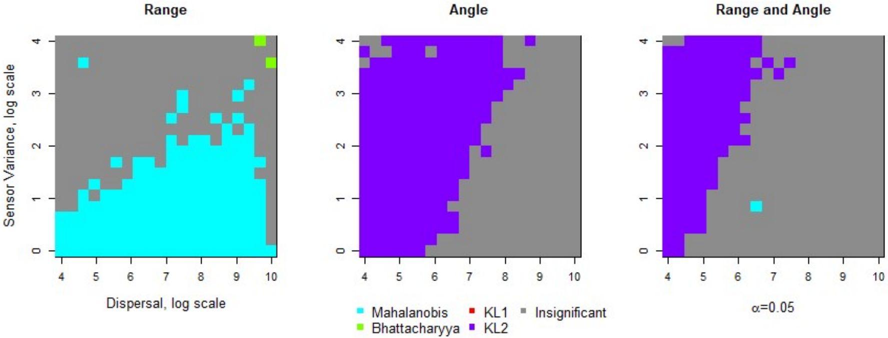


Figure 14. CLUSTER−IN: Cluster Dispersal vs Sensor Variance

CLUSTER-IN: Results

- Range measurements: Mahalanobis wins for dispersal $\log(\sigma_r^2) > 7$, all altitudes, and sensor variance $\log(\sigma_{meas}^2) < 2$

CLUSTER-IN: Results

- Range measurements: Mahalanobis wins for dispersal $\log(\sigma_r^2) > 7$, all altitudes, and sensor variance $\log(\sigma_{meas}^2) < 2$
- Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 7$, any altitude, and sensor variance $\log(\sigma_{meas}^2) > 2$

CLUSTER-IN: Results

- Range measurements: Mahalanobis wins for dispersal $\log(\sigma_r^2) > 7$, all altitudes, and sensor variance $\log(\sigma_{meas}^2) < 2$
- Angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 7$, any altitude, and sensor variance $\log(\sigma_{meas}^2) > 2$
- Range and angle measurements: KL2 wins for dispersal $\log(\sigma_r^2) < 6$ and any altitude

OPTIMAL-CONTROL-DISTANCE

- Same experiment setup as `CLUSTER-IN`, but only with < 5 satellites
- Perform object correlation in state space
- Consider only Mahalanobis and OCD

- Same experiment setup as `CLUSTER-IN`, but only with < 5 satellites
- Perform object correlation in state space
- Consider only Mahalanobis and OCD
- OCD always achieves 100% accuracy, while Mahalanobis achieves varied results across trials
- Due to time constraints, it is difficult to examine the behavior of OCD for larger clusters

SIMULATING-LEO: Experiment Specifications

- A more realistic space environment: 1000 satellites
- Generate 30 clusters of 25 satellites
  - 15 clusters as in CLUSTER-OUT
  - 15 clusters as in CLUSTER-IN
- Generate 250 single satellites with randomly selected elliptical orbits
- Run 30 trials of this test, averaging the success rates across trials for each metric and modality

SIMULATING-LEO: Results

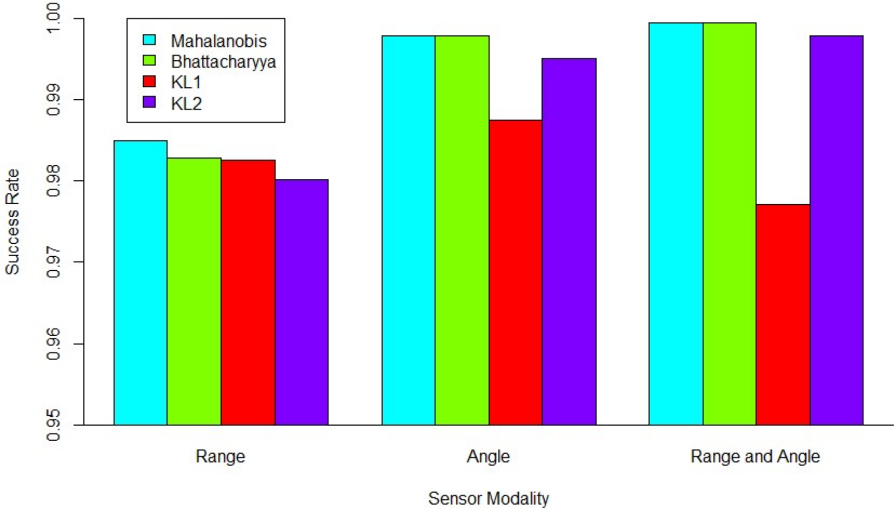


Figure 15. SIMULATING-LEO

SIMULATING-LEO: Results

- Success rates for range are lower than success rates for angle and range-and-angle by $\approx 1\%$

SIMULATING-LEO: Results

- Success rates for range are lower than success rates for angle and range-and-angle by $\approx 1\%$
- Mahalanobis wins for all modalities, but ties with Bhattacharyya for angle

SIMULATING-LEO: Results

- Success rates for range are lower than success rates for angle and range-and-angle by $\approx 1\%$
- Mahalanobis wins for all modalities, but ties with Bhattacharyya for angle
- Bhattacharyya performs surprisingly well, given its absence in previous figures

SIMULATING-LEO: Results

- Success rates for range are lower than success rates for angle and range-and-angle by $\approx 1\%$
- Mahalanobis wins for all modalities, but ties with Bhattacharyya for angle
- Bhattacharyya performs surprisingly well, given its absence in previous figures
- For range KL1 beats KL2 slightly, while KL2 beats KL1 by a wider margin for angle and range-and-angle

Conclusion

General Results

- `CLUSTER-IN` and `CLUSTER-OUT`:
  - Mahalanobis is the best for range measurements
  - KL2 is the best for angle and range-and-angle measurements

General Results

- `CLUSTER-IN` and `CLUSTER-OUT`:
  - Mahalanobis is the best for range measurements
  - KL2 is the best for angle and range-and-angle measurements
- `OPTIMAL-CONTROL-DISTANCE`: OCD performs very well, but is extremely slow computationally

General Results

- `CLUSTER-IN` and `CLUSTER-OUT`:
  - Mahalanobis is the best for range measurements
  - KL2 is the best for angle and range-and-angle measurements
- `OPTIMAL-CONTROL-DISTANCE`: OCD performs very well, but is extremely slow computationally
- `SIMULATING-LEO`: Mahalanobis is consistently the best metric

Deliverables

- Simulation framework
  - Dynamical model
  - Satellite and sensor classes
  - Distance metrics and object correlation

Deliverables

- Simulation framework
  - Dynamical model
  - Satellite and sensor classes
  - Distance metrics and object correlation
- Experiment scripts and data processing scripts

Deliverables

- Simulation framework
  - Dynamical model
  - Satellite and sensor classes
  - Distance metrics and object correlation
- Experiment scripts and data processing scripts
- Raw and processed data

Deliverables

- Simulation framework
  - Dynamical model
  - Satellite and sensor classes
  - Distance metrics and object correlation
- Experiment scripts and data processing scripts
- Raw and processed data
- Final results

Future Work

- Efficiently implementing optimal control distance
- More realistic dynamical system
- Range and angle rate sensors

Acknowledgements

Questions?

References

[1] A. M. Hawkins, *Fundamental Orbital Mechanics for RIPS*. Corporate Presentation, July 2017.

[2] K. Hill, K. Alfriend, and C. Sabol, *Covariance-Based Uncorrelated Track Association*, in AIAA/AAS Astrodynamics Specialist Conference and Exhibit, AIAA Paper 2008-7211, 2008.

[3] M. Holzinger, D. Scheeres, and K. Alfriend, *Object Correlation, Maneuver Detection, and Characterization Using Control Distance Metrics*, AIAA Journal of Guidance, Control, and Dynamics, 35 (July - August 2012), pp. 1312–1325.

[4] G. James, D. Witten, T. Hastie, and R. Tibshirani, *An Introduction to Statistical Learning with Applications in R*, Springer, New York, 2013.

[5] T. Kailath, *The Divergence and Bhattacharyya Distance Measures in Signal Selection*, IEEE Transactions on Communication Technology, 15 (1967), pp. 52–60.

[6] S. Kullback, *Information Theory and Statistics*, Wiley, New York, 1959.

[7] P. Mahalanobis, *On the Generalized Distance in Statistics*, in Proceedings of the National Institute of Sciences of India, vol. 2, 1932, pp. 49–55.

[8] G. Peterson, M. Sorge, and W. Ailor, *Space Traffic Management in the Age of New Space*, tech. rep., The Aerospace Corporation, April 2018.

[9] B. Tapley, B. Schutz, and G. Born, *Statistical Orbit Determination*, Elsevier, Amsterdam, 2004.

[10] D. Vallado, *Fundamentals of Astrodynamics and Applications*, Microcosm Press joint with Springer, California and New York, third ed., 2007.

[11] P. Zarchan and H. Musoff, *Fundamentals of Kalman Filtering: A Practical Approach*, American Institute of Aeronautics and Astronautics, Inc., United States of America, fourth ed., 2013.

Figures

Figure 1. Catalogued Objects in Space Surveillance Network. G. Peterson, M. Sorge, and W. Ailor, *Space Traffic Management in the Age of New Space*, tech. rep., The Aerospace Corporation, April 2018.

Figure 2. Collision Probabilities. G. Peterson, M. Sorge, and W. Ailor, *Space Traffic Management in the Age of New Space*, tech. rep., The Aerospace Corporation, April 2018.

Figure 5 Joshua Cesa (https://commons.wikimedia.org/wiki/File:Azimut_altitude.svg), "Azimut altitude", Text, https://creativecommons.org/licenses/by/3.0/legalcode

Figures 3, 4, and 6-15 were generated by our own simulation framework, using Plot.ly and R.