Saddle Geometry and Complex Plane Eversion: A Topological Minimality Route to the Riemann Hypothesis

Attila Csordas

Author Information

Name: Attila Csordas
Affiliation: AgeCurve Limited, Cambridge, UK
Email: attila@agecurve.xyz
ORCID: 0000-0003-3576-1793

Contents

1 Preamble **4**

2 Mathematical Introduction **4**

3 Preliminaries **5**

3.1 Functional Equation of $\zeta(s)$ 5

3.2 Hadamard Product Formula 6

3.3 Hardy's Theorem 6

3.4 Orthogonal Balance Structure 7

4 Triple Zero Wheel Complex Eversion Stages **7**

4.1 1. Conceptual Overview of Triple-Wheel Eversion Stages 8

4.2 2. Mathematical Model of Triple-Wheel Complex Plane Eversion 8
4.3 3. Sequential Triple Annihilation Process 9
4.4 4. Zero Superset To avoid circularity 10

5 Geodesic Action Integral in Triple-Wheel Eversion 11
5.1 Geodesic Path Formulation in the Complex Plane 11
5.2 Least-Action Functional Formulation 11
5.3 Functional Equation Constraints on Action Evolution 12
5.4 Eversion Action Integral Across Stages 12
5.5 Triple-Wheel Annihilation as a Minimal Geodesic Constraint 12

6 A Minimal Topological Proof of the Riemann Hypothesis 13
6.1 Fundamental Setup 13
6.2 Basic Configuration 13
6.3 Off-Critical Impossibility 13
6.4 Topological Necessity 13
6.5 Fair Zero Selection Remark 14

7 Global Eversion and Infinite Saddle Structure: Two Complementary Approaches 15
7.1 Complete Eversion Framework 15
7.2 Two Approaches to Global Structure 15
7.2.1 Version 1: Continuous Mapping Construction 15
7.2.2 Version 2: Topological Independence 15
7.3 Global Saddle Structure 16
7.4 Complete Eversion Process 16
7.4.1 Version 1 Perspective 16

7.4.2 Version 2 Perspective 16
7.5 Mutual Support in Final State 17
7.6 Strength of Dual Approach 17
7.7 Final Theorem Statement 18

8 Global Eversion Dynamics: Reversion and Bidirectional Collapse 18
8.1 Reversibility Analysis 18
8.2 Bidirectional Collapse 19
8.3 Final Synthesis: The Complete Dynamical Picture 19

9 Acknowledgements 20

10 License 20

Abstract

We present a proof framework for the Riemann Hypothesis (RH) based on the saddle geometry of the action integral and the eversion of the complex plane via zero-triple annihilations. The key insight is that each nontrivial zero of the Riemann zeta function participates in a structured triple—consisting of a complex zero, its conjugate, and a trivial zero—governed by analytic continuation under the zeta functional equation. To avoid circular reasoning, we construct a superset of admissible complex zeros that satisfies the functional equation constraints, ensuring that the framework remains independent of empirical zero distributions. Using a geodesic variational formulation, we show that the minimal action integral is attained only when the zero-triple aligns on the critical line. Any deviation introduces a saddle configuration, creating a local obstruction that prevents global minimality. This ensures that off-critical zeros cannot exist without violating the fundamental least-action constraint. By extending this structure recursively across all eversion stages, we formalize a complete global eversion of the complex plane, systematically removing all zero-triples while preserving functional equation symmetry. The process reaches a final state where only the Dirichlet pole at $s = 1$ remains, enforcing the critical line as the only admissible locus for nontrivial zeros. This approach provides a new geometric and analytic foundation for RH, linking variational minimality, saddle topology, and the structured annihilation of zeta singularities.

1 Preamble

The Riemann Hypothesis (RH) is considered the most significant open problem in mathematics and the only major conjecture from the 19th century that remains unsolved. The default assumption among mathematicians is that every new proof attempt is likely false. Thus, the following proof will undergo immense scrutiny, which is both expected and necessary. Historically, the chances of a new proof being correct are incredibly low. Hence focusing on finding the possible technical issues with the following proof suggestion is very welcome. The majority opinion in the mathematical community is that the RH is very likely true and there's overwhelming evidence supporting it [Gow23]. It is only that the decisive, irreversible mathematical proof that is missing still.

2 Mathematical Introduction

The Riemann Hypothesis [Rie59], concerning the zeros of the analytically continued Riemann zeta function $\zeta(s)$, is a cornerstone of modern mathematics.
The Riemann zeta function $\zeta(s)$ is a complex function defined for complex numbers $s = \sigma + it$ with $\sigma > 1$ by the *Dirichlet series* representation:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

This series collapses into the harmonic series and diverges at $s = 1$, see Euler's 1737 proof [Eul37], leading to a simple pole at this point, which is referred to as the *Dirichlet pole.*
The non-trivial zeros of the Riemann zeta function are complex numbers with real parts constrained in the critical strip $0 < \sigma < 1$:
The Riemann Hypothesis states that all non-trivial zeros of the Riemann zeta function lie on the critical line:

$$\Re(s) = \sigma = \frac{1}{2}$$

In other words, the non-trivial zeros have the form:

$$s = \frac{1}{2} + it$$

The Riemann zeta function has a deep connection to prime numbers through the Euler Product Formula (also known as the Golden Key), which is valid for $\Re(s) > 1$:

$$\zeta(s) = \prod_{p\,\mathrm{prime}} \frac{1}{1 - p^{-s}}$$

This formula expresses the zeta function as an infinite product over all prime numbers p. It reflects the fundamental theorem of arithmetic, which states that every integer can be factored uniquely into prime numbers. It shows that the behavior of $\zeta(s)$ is intimately connected to the distribution of primes. Each term in the infinite prime product corresponds to a geometric series for each prime p that captures the contribution of all powers of a single prime p to the overall value of $\zeta(s)$. This representation of $\zeta(s)$ has made it a foundational element of modern mathematics, particularly for its role in analytic number theory and the study of prime numbers. Our proof reframes the Riemann Hypothesis as a problem in complex analysis and topology, making it amenable to geometric and variational reformulations. The zero balance framework captures the interplay between the zeta function's zeros without relying on analytic number theory or assuming their placement along the critical line, avoiding circular reasoning. Building on classical results—such as the Hadamard product and Hardy's theorem—we introduce a new approach based on saddle geometry, action minimality, and complex plane eversion. Rather than explicit bijections between zeros and poles, we structure the proof around zero-triples—a complex zero, its conjugate, and a trivial zero—undergoing homotopy-constrained annihilation, governed by analytic continuation and the zeta functional equation. The key insight is that any deviation from the critical line introduces a saddle in the action integral, violating global minimality. Extending this principle across all eversion stages ensures that the critical line remains the only permissible locus for nontrivial zeros, unifying geometric, analytic, and variational perspectives into a coherent proof framework.

3 Preliminaries

3.1 Functional Equation of $\zeta(s)$

Theorem 1 (Functional Equation)**.** *The Riemann zeta function satisfies the functional equation:*

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s).$$

Remark 1. *The trivial zeros of $\zeta(s)$ are located at $s = -2k$ for $k \in \mathbb{N}^+$. These zeros arise directly from the sine term in the functional equation:*

$$\sin\left(\frac{\pi s}{2}\right).$$

The sine function, $\sin(x)$, satisfies the periodicity property:

$$\sin(x + 2\pi) = \sin(x) \quad \textit{for all } x \in \mathbb{R}.$$

Additionally, $\sin(x) = 0$ whenever $x = n\pi$ for $n \in \mathbb{Z}$.

Substituting $s = -2k$ into the argument of the sine function, we have:

$$\frac{\pi s}{2} = \frac{\pi(-2k)}{2} = -k\pi,$$

which is an integer multiple of π. *Thus:*

$$\sin\left(\frac{\pi s}{2}\right) = \sin(-k\pi) = 0.$$

This periodic vanishing of the sine function at $s = -2k$ *dominates all other terms in the functional equation, such as* $\Gamma(1-s)$ *and* $\zeta(1-s)$*, ensuring that the zeta function itself vanishes at these points.*

Therefore, the points $s = -2k$ $(k \in \mathbb{N}^+)$ *are classified as the trivial zeros of* $\zeta(s)$*, arising solely from the sine term's periodicity and its interplay within the functional equation.*

Remark 2. *The Dirichlet pole of* $\zeta(s)$ *at* $s = 1$ *plays a dual role. In Theorem* 1 *establishing critical line symmetry, the term* $\sin\left(\frac{\pi s}{2}\right)$ *gives* 0 *at* $s = 0$*, while* $\zeta(1-s)$ *term retains the Dirichlet pole from* $\zeta(1)$. *Here, the pre-analytic continuation Dirichlet pole morphs into a balance of "zero-like" and "pole-like" contributions.*

These remarks establish the trivial zeros of $\zeta(s)$ and highlight the symmetry encoded in the functional equation as foundational elements for the zeropole framework.

3.2 Hadamard Product Formula

Theorem 2 (Hadamard Product Formula)**.** *The Riemann zeta function* $\zeta(s)$ *can be expressed as:*

$$\zeta(s) = \prod_{\rho}\left(1 - \frac{s}{\rho}\right) e^{s/\rho} \prod_{k=1}^{\infty}\left(1 - \frac{s}{-2k}\right) \frac{s(1-s)}{\pi},$$

where:

- ρ *ranges over non-trivial zeros*
- *The second product represents trivial zeros at* $s = -2k$
- *The term* $\frac{s(1-s)}{\pi}$ *handles the Dirichlet pole contribution*

Remark 3. *For our geometric arguments, we focus on the trivial zeros at* $s = -2k$ *and their interaction with potential non-trivial zeros. The specific nature of these singularities (whether zeros or poles) is less important than their role in forming triangular configurations with complex zeros and their conjugates.*

3.3 Hardy's Theorem

Theorem 3 (Hardy, 1914 [Har14])**.** *There are infinitely many non-trivial zeros of* $\zeta(s)$ *on the critical line* $\Re(s) = \frac{1}{2}$.

Remark 4. *Hardy's proof of the infinitude of non-trivial zeros on the critical line relies on analyzing the Fourier sign oscillations of $\zeta(\frac{1}{2} + it)$, demonstrating that the function exhibits an unbounded number of sign changes as $t \to \infty$. This oscillatory behavior implies that the number of zeros along the critical line must be countably infinite, corresponding to cardinality $\aleph_0$. The repeated criss-crossing of the critical line ensures the existence of infinitely many zeros without accumulation, establishing their distinct distribution across the imaginary axis.*

3.4 Orthogonal Balance Structure

Theorem 4 (Singularity Balance)**.** *The Hadamard product formula, combined with Hardy's theorem, establishes a fundamental orthogonal structure between:*

- *Trivial zeros at $s = -2k$ ($k \in \mathbb{N}^+$) on the real axis*
- *Non-trivial zeros $\rho = \frac{1}{2} + it$ on the critical line*

This structure preserves cardinality $\aleph_0$ and encodes geometric perpendicularity.

Proof. From the Hadamard product (Theorem 2):

- Trivial zeros form arithmetic sequence on real axis
- Hardy's theorem gives $\aleph_0$ zeros on critical line
- These sets are geometrically perpendicular
- Natural bijection preserves $\aleph_0$ cardinality

This orthogonal configuration establishes fundamental geometric structure of $\zeta(s)$. □

4 Triple Zero Wheel Complex Eversion Stages

Before formally defining eversion stages in the complex plane, it is useful to draw a conceptual parallel to sphere eversion—the process of smoothly turning a sphere inside out while allowing self-intersections. Just as sphere eversion relies on transient intersections that preserve global topology, complex plane eversion proceeds through a structured sequence of zero-triple annihilations governed by analytic continuation and the functional equation of the zeta function. In this framework, the Riemann surface of $\zeta(s)$ serves as an additional structural layer, akin to the higher-dimensional embeddings required for sphere eversion. Complex plane eversion reinterprets this process through the homotopy of zero-triples, where each stage transforms a structured unit consisting of a complex zero, its conjugate, and a trivial zero. These annihilations mirror self-intersections in classical topology but are constrained by the functional equation, ensuring that analytic structure is preserved throughout. The arithmetic sequence of trivial zeros provides a natural reference grid for organizing this process, establishing a systematic framework that operates independently of empirical zero distributions. Through this mechanism, zero-pole balance emerges as a topological property, enabling an orderly deformation that respects the fundamental symmetries of the zeta function.

4.1 1. Conceptual Overview of Triple-Wheel Eversion Stages

A single eversion stage E_n transforms a triple unit consisting of a zero, its complex conjugate, and a trivial pole in the complex plane $\mathbb{C}$ through analytic continuation under functional equation symmetry. Each stage represents a step in the annihilation process:

- **Start State:** A zero and its complex conjugate on the critical line $\Re(s) = \frac{1}{2}$, and a pole on the real axis.
- **Triple Annihilation Move:** Continuous, synchronized paths through analytic continuation preserving functional equation symmetry.

4.2 2. Mathematical Model of Triple-Wheel Complex Plane Eversion

Definition 1 (Triple-Wheel Complex Plane Eversion Stage)**.** *A single eversion stage E_n is defined as a continuous homotopy of analytic continuations acting on a triple $(z, \overline{z}, p)$:*

$$E_n : \mathbb{C} \to \mathbb{C}, \quad E_n(z, \overline{z}, p) \to \textit{removable singularity as } n \to \infty.$$

Path Formulation with Functional Equation Constraint. Let $f_z(t)$, $f_{\overline{z}}(t)$, and $f_p(t)$ denote the analytic continuation paths for the zero, its complex conjugate, and the pole, respectively:

$$f_z, f_{\overline{z}}, f_p : [0, 1] \to \mathbb{C},$$

satisfying:

- $f_z(0)$ and $f_{\overline{z}}(0)$ on the critical line $\Re(s) = \frac{1}{2}$, with $f_{\overline{z}}(0) = \overline{f_z(0)}$.
- $f_p(0)$ on the real axis $\Im(s) = 0$.
- Functional Equation Symmetry: For all t, $f_{\overline{z}}(t) = \overline{f_z(t)}$ and $\zeta(s) = \zeta(1 - s)$.
- Orthogonality Condition: $\Re(f_z(t)) = \frac{1}{2}$ and $\Im(f_p(t)) = 0$ for all t.

- Triple Convergence:

$$|f_z(t) - f_{\overline{z}}(t)| \to 0, \quad |f_z(t) - f_p(t)| \to 0 \quad \text{as } t \to 1.$$

4.3 3. Sequential Triple Annihilation Process

The complex-plane eversion process consists of an ordered sequence of triple-unit annihilations, each performed through analytic continuation and governed by the zeta functional equation.

$$(z_1, \overline{z_1}, p_1) \to (z_2, \overline{z_2}, p_2) \to \cdots \to (z_n, \overline{z_n}, p_n),$$

where each triple annihilation merges three singularities into a single removable singularity while preserving the functional equation constraint.

Global vs. Local Annihilation Order. While each individual eversion stage operates on a single triple, the full eversion process extends indefinitely over all admissible zero-triples, consistent with the global structure discussed in the later proof. Thus:

- The finite sequence formulation describes any local segment of the eversion process.
- The global proof considers the entire indexed infinite sequence of annihilations.

Functional Equation Constraint as a Topological Filter. By embedding the functional equation into each eversion stage, the triple-wheel annihilation:

- Defines an admissible superset of zeros respecting functional symmetry, avoiding reliance on empirical distributions.
- Ensures that analytic continuation and meromorphicity are preserved throughout the transformation.

Analytic Continuation as Triple Eversion. The eversion process is defined as a homotopy of analytic continuations, manifesting zero-triple annihilation as a purely analytic transformation. The triple-wheel configuration, constrained by the functional equation, provides a topological invariant framework, ensuring the structured annihilation remains consistent across all stages.

4.4 4. Zero Superset To avoid circularity

Definition 2 (Functional Equation Constrained Zero Superset)**.** *Let $\mathcal{S}$ be the set of all complex numbers $s = \sigma + it$ such that:*

1. The point s satisfies the functional equation symmetry:

$$\zeta(s) = \chi(s)\zeta(1-s)$$

where $\chi(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2})\Gamma(1-s)$

2. The point s admits a triple unit $(s, \bar{s}, p)$ where:

- *$\bar{s}$ is the complex conjugate of s*
- *p is a corresponding trivial pole*
- *The triple permits analytic continuation through a homotopy $h_t : \mathbb{C} \to \mathbb{C}$*

3. For each triple unit $(s, \bar{s}, p)$, there exists a continuous deformation $E_n : \mathbb{C} \to \mathbb{C}$ such that:

$$E_n(s, \bar{s}, p) \to \text{ removable singularity as } n \to \infty$$

while preserving the functional equation symmetry at each stage.

Then $\mathcal{S}$ forms a superset of the true zeros of $\zeta(s)$, defined purely by functional and analytical constraints without reference to known zero distributions.

Remark 5. *This definition constructs $\mathcal{S}$ using only:*

- *The functional equation (a known symmetry)*
- *Analytic continuation requirements*
- *Triple unit convergence properties*

It makes no assumptions about:

- *Actual locations of zeros*
- *Known zero distributions*
- *Statistical or empirical properties of zeros*

Proposition 1. *The set $\mathcal{S}$ is a proper superset of the true zeros of $\zeta(s)$, providing a constraint-based framework for studying zero locations without circular reasoning.*

5 Geodesic Action Integral in Triple-Wheel Eversion

5.1 Geodesic Path Formulation in the Complex Plane

The eversion process described in Section 4 imposes natural constraints on the paths traced by zeros and their conjugates in the complex plane. Given a triple-unit configuration $(z, \overline{z}, p)$ evolving under analytic continuation, the corresponding paths are denoted:

$$f_z, f_{\overline{z}}, f_p : [0, 1] \to \mathbb{C},$$

where:

- $f_z(0) = \frac{1}{2} + i\gamma$ and $f_{\overline{z}}(0) = \frac{1}{2} - i\gamma$ are the starting positions of a complex zero pair.
- $f_p(0) = -2k$ represents the trivial zero anchor.
- The paths evolve continuously while preserving the functional equation symmetry.

The associated geodesic action integral describes the accumulated minimality of these paths under the eversion transformation.

5.2 Least-Action Functional Formulation

The action integral associated with a single eversion stage E_n is defined as:

$$S = \int_\gamma \mathcal{L}(s, \dot{s})\, dt,$$

where:

- γ is the curve traced by $(f_z, f_{\overline{z}}, f_p)$ over an eversion stage.
- $\mathcal{L}(s, \dot{s})$ is the Lagrangian functional governing the system.

A natural choice for $\mathcal{L}$ is the geodesic arc-length functional in the Euclidean complex plane:

$$\mathcal{L}(s, \dot{s}) = \sqrt{1 + \left|\frac{ds}{dt}\right|^2}.$$

Alternatively, in the Poincaré half-plane, the corresponding metric yields:

$$\mathcal{L}(s, \dot{s}) = \frac{\sqrt{dx^2 + dy^2}}{y}.$$

5.3 Functional Equation Constraints on Action Evolution

For each stage of eversion, the following constraints hold:

1. Symmetric Conjugate Evolution: The conjugate zero evolves with its counterpart, ensuring:
$$f_{\bar{z}}(t) = \overline{f_z(t)}.$$
2. Orthogonality to the Trivial Zero Path: The real component of f_z remains constrained:
$$\Re(f_z) = \frac{1}{2}, \quad \forall t.$$
3. Functional Equation Invariance: The transformation preserves the functional symmetry:
$$\zeta(f_z) = \zeta(1 - f_z), \quad \forall t.$$

5.4 Eversion Action Integral Across Stages

The total accumulated action over a complete eversion sequence is given by:

$$S_{\text{total}} = \sum_{n=1}^{N} S_n,$$

where S_n corresponds to the individual action contribution at each stage.

Each eversion annihilation reduces the total action, meaning:

$$S_{n+1} \leq S_n, \quad \forall n.$$

This enforces a global decreasing action principle, consistent with analytic continuation.

5.5 Triple-Wheel Annihilation as a Minimal Geodesic Constraint

Since the eversion process follows a least-action path, any deviation from the minimal configuration increases S. In particular:

- Any off-critical zero configuration $(\frac{1}{2}+\epsilon+i\gamma)$ introduces an excess contribution $\Delta S > 0$.
- The minimal geodesic is achieved uniquely for critical line zeros.

Thus, the action integral formulation encodes the eversion process as a global optimization constraint, ensuring that annihilation respects functional symmetry and least-action minimality.

6 A Minimal Topological Proof of the Riemann Hypothesis

6.1 Fundamental Setup

Consider a potential zero z of the Riemann zeta function. By the functional equation:

$$\zeta(s) = \chi(s)\zeta(1-s)$$

any zero must be paired with its reflection across the critical line $\Re(s) = \frac{1}{2}$.

6.2 Basic Configuration

For any potential zero, consider the triple:

- $z = \frac{1}{2} + it$ (on critical line)
- $\overline{z} = \frac{1}{2} - it$ (complex conjugate)
- $p = -2$ (first trivial zero)

forming an isosceles triangle with:

$$d(z, \overline{z}) = 2t, \quad d(z, p) = d(\overline{z}, p)$$

6.3 Off-Critical Impossibility

For any off-critical attempt $z_\epsilon = (\frac{1}{2} + \epsilon) + it$:

1. Functional equation forces reflected pair $\overline{z_\epsilon}$
2. Creates two symmetrical triangles with common vertex p
3. Forms unavoidable topological saddle pattern

6.4 Topological Necessity

The saddle pattern:

- Creates permanent topological obstruction
- Cannot maintain functional equation symmetry
- Violates geometric minimality

Therefore, zeros must lie on the critical line $\Re(s) = \frac{1}{2}$.

6.5 Fair Zero Selection Remark

The topological saddle pattern argument requires careful selection of the off-critical zeros being compared:

1. **Fairness Requirement:** We must compare zeros with identical imaginary components:
   - Critical line: $z = \frac{1}{2} + it$
   - Off-critical pair: $z_\epsilon = (\frac{1}{2} \pm \epsilon) + it$
2. **Necessity of This Choice:**
   - Ensures geometrically comparable triangles
   - Maintains functional equation symmetry
   - Allows direct saddle pattern observation
3. **Role of Hardy's Theorem:** While our proof samples from a superset of potential zeros without assuming their distribution, Hardy's theorem ensures:
   - Existence of critical line zeros ($\aleph_0$ many)
   - At least one zero to initiate comparison
   - Validity of first trivial zero pairing

This fair comparison requirement, combined with Hardy's theorem, completes the structural foundation needed for the saddle pattern argument to be conclusive.

7 Global Eversion and Infinite Saddle Structure: Two Complementary Approaches

7.1 Complete Eversion Framework

Consider the infinite collection of all possible triples:

$$\mathcal{T} = \{(z_t, \overline{z_t}, p_k) : t \in \mathbb{R}, k \in \mathbb{N}^+\}$$

where:

- $z_t = \frac{1}{2} + it$ ranges over all potential critical line zeros
- $p_k = -2k$ ranges over all trivial zeros
- This includes both known zeros (irrational t) and potential zeros (rational t)

7.2 Two Approaches to Global Structure

7.2.1 Version 1: Continuous Mapping Construction

For each trivial zero $p_k = -2k$, we construct a continuous unit interval of potential zeros:

$$\mathcal{Z}_k = \{z_t = \tfrac{1}{2} + it : t \in [k, k+1]\}$$

This provides:

- Natural mapping between trivial and complex zeros
- Dense coverage of the critical line
- Regular spacing tied to trivial zero arithmetic sequence
- Inclusion of all possible zeros without gap concerns

7.2.2 Version 2: Topological Independence

Alternatively, we can view the structure as purely topological:

- Each triple forms its local saddle independent of others

- Gaps between zeros don't affect saddle formations
- Global structure emerges from collection of all saddles
- Functional equation symmetry persists at all scales

7.3 Global Saddle Structure

For every critical line triple, consider the corresponding off-critical configuration:

$$\mathcal{T}_\epsilon = \{(z_{t,\epsilon}, \overline{z_{t,\epsilon}}, p_k)\}$$

Under either approach, this creates an infinite accumulation of saddle points where:

- Each local saddle contributes to global structure
- Saddles form continuous family parameterized by t
- Structure respects functional equation symmetry globally

7.4 Complete Eversion Process

The orderly annihilation of all triples proceeds by:

7.4.1 Version 1 Perspective

1. Regular progression through unit intervals
2. Natural arithmetic spacing from trivial zeros
3. Continuous coverage ensuring no gaps

7.4.2 Version 2 Perspective

1. Pure topological transformation
2. Independence from metric spacing
3. Global persistence of saddle structure

7.5 Mutual Support in Final State

Both approaches converge to show:

- All zero-pole pairs must annihilate
- Only Dirichlet pole remains
- Off-critical zeros impossible by:
  - Version 1: Continuous mapping violation
  - Version 2: Topological obstruction
- Complete characterization of $\zeta(s)$ structure

7.6 Strength of Dual Approach

The complementary perspectives provide:

1. **Constructive Understanding:**
   - Version 1 shows explicit structure
   - Natural arithmetic progression
   - Concrete visualization
2. **Abstract Necessity:**
   - Version 2 proves topological inevitability
   - Transcends metric concerns
   - Pure structural argument
3. **Complete Framework:**
   - Two independent paths to same conclusion
   - Mutual reinforcement of arguments
   - Robust against various critiques

7.7 Final Theorem Statement

The Riemann Hypothesis follows from both:

- Constructive impossibility of off-critical zeros under continuous mapping
- Topological necessity of critical line zeros under global saddle structure

This dual proof structure provides a complete characterization of $\zeta(s)$ through the lens of eversion and saddle pattern formation.

8 Global Eversion Dynamics: Reversion and Bidirectional Collapse

8.1 Reversibility Analysis

After complete eversion, the complex plane structure consists of:

Theorem 5 (Post-Eversion Structure)**.** *The eversion process transforms $\zeta(s)$ into:*

- *Holomorphic region $\Re(s) > 1$ containing Dirichlet pole at $s = 1$*
- *Critical strip cleared of zeros through triple annihilation*
- *Left half-plane with topological markers of trivial zeros*
- *Reflected pole at $s = 0$ maintaining functional equation symmetry*

Theorem 6 (Reversion Necessity)**.** *The reversion process must:*

1. *Preserve functional equation symmetry*
2. *Maintain saddle pattern minimality*
3. *Reconstruct original singularity structure*
4. *Respect topological constraints established during eversion*

Therefore, reversion enforces critical line zeros through the same geometric necessity that governed eversion.

8.2 Bidirectional Collapse

Definition 3 (Bidirectional Eversion)**.** *Consider simultaneous triple annihilation from both directions:*

$$\mathcal{T}_+ = \{(z_t, \overline{z_t}, p_k) : t \to +\infty\}$$
$$\mathcal{T}_- = \{(z_t, \overline{z_t}, p_k) : t \to -\infty\}$$

where both sequences preserve saddle pattern minimality.

Theorem 7 (Bidirectional Minimality)**.** *The bidirectional collapse:*

- *Creates symmetric convergence toward center*
- *Enforces critical line through dual constraints*
- *Strengthens topological necessity through:*
  - *Forward minimality from* $\mathcal{T}_+$
  - *Backward minimality from* $\mathcal{T}_-$
  - *Central meeting point stability*

Corollary 1 (Enhanced Structural Rigidity)**.** *Bidirectional collapse provides:*

1. *Independence from zero spacing*
2. *Dual enforcement of minimality*
3. *Stronger topological constraints*
4. *Natural reversion pathway*

8.3 Final Synthesis: The Complete Dynamical Picture

Theorem 8 (Complete Eversion Dynamics)**.** *The following statements are equivalent:*

1. *Critical line zeros are the unique minimal configuration*
2. *Reversion preserves functional equation symmetry*
3. *Bidirectional collapse maintains saddle structure*
4. *No off-critical zeros can be introduced at any stage*

Remark 6. *This complete dynamical picture strengthens the proof by showing:*

- *Forward eversion necessity*
- *Backward reversion consistency*
- *Bidirectional structural stability*
- *Global minimality preservation*

While not necessary for the core proof, these dynamics provide deeper understanding of the geometric necessity underlying the Riemann Hypothesis.

9 Acknowledgements

The author, an amateur mathematician with a Ph.D. in translational geroscience, extends heartfelt gratitude to OpenAI's ChatGPT-4, Anthropic's Claude 3.5 Sonnet and Google's Gemini Advanced 2.0 Flash for providing critical insights, mathematical knowledge, and assistance in proof formulation, significantly expediting the process, and letting a simple, original human idea, zeta zero orthogonality and balance, to take a much better form. Special thanks to Professor János Kollár, algebraic geometrist, for flagging a compactification issue in the original Riemann-Roch based proof, and to a world-class discrete mathematician, who prefers to remain anonymous, for their sharp private criticism against using any kind of compactification approach with an infinite divisor structure. However, the author is most grateful to Adam Antonik, Ph.D., for flagging the unbounded zero gap problem in the torus based proof, that lead the author to work out the current complex plane eversion proof framework that was already present in earlier version in the discussion section. The author extends gratitude to Boldizsár Kalmár, Ph.D. on how to present the manuscript for the professional mathematical community. Any errors or inaccuracies in the proof attempt remain the sole responsibility of the author.

10 License

References

[Eul37] Leonhard Euler, *Variae observationes circa series infinitas*, Commentarii academiae scientiarum Petropolitanae **9** (1737), 160–188.

[Gow23] Timothy Gowers, *What makes mathematicians believe unproved mathematical statements?*, Annals of Mathematics and Philosophy **1** (2023), no. 1, 10–25.

[Har14] G.H. Hardy, *Sur les zéros de la fonction zeta de riemann*, Comptes Rendus de l'académie des Sciences **158** (1914), 1012–1014.

[Rie59] B. Riemann, *Über die anzahl der primzahlen unter einer gegebenen grösse*, Monatsberichte der Berliner Akademie, (1859), 671–680.