

# Compressed Proof Attempt of the Riemann Hypothesis via Zeropole Balance

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This one-pager is a compressed, extracted version of the full proof available at GitHub ([https://github.com/attila-ac/Proof\\_RH\\_via\\_Zeropole\\_Balance](https://github.com/attila-ac/Proof_RH_via_Zeropole_Balance)).

**1. Functional Equation:** The Riemann zeta function satisfies:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s).$$

Trivial zeros at  $s = -2k$  ( $k \in \mathbb{N}^+$ ) arise solely from the sine term,  $\sin\left(\frac{\pi s}{2}\right)$ .

**2. Hadamard Product:** The global zeropole structure of  $\zeta(s)$  is given by:

$$\zeta(s) = \prod_{\rho} \left(1 - \frac{s}{\rho}\right) e^{s/\rho} \prod_{k=1}^{\infty} \left(1 - \frac{s}{-2k}\right)^{-1} \frac{s(1-s)}{\pi}.$$

Here,  $\rho$  are non-trivial zeros on the critical line,  $s = -2k$  are trivial poles introduced to balance the trivial zeros, and  $\frac{s(1-s)}{\pi}$  encodes the Dirichlet pole's dual role.

**3. Hardy's Theorem:** Countably infinite non-trivial zeros lie on the critical line,  $\Re(s) = \frac{1}{2}$ .

**4. Zeropole Mapping and Orthogonal Balance:** The Hadamard product formula, in conjunction with Hardy's theorem, establishes a bijection between trivial poles and non-trivial zeros of  $\zeta(s)$ . This bijection preserves cardinality  $\aleph_0$  and encodes both algebraic independence and geometric perpendicularity between these orthogonal zeropole sets.

**5. Shadow Function and Toroidal Structure:** Define  $\zeta^*(s)$  on the torus  $\mathbb{T}^2$ :

$$\zeta^*(s) = e^{A+Bs} \frac{1}{s} \prod_{\rho} \left(1 - \frac{s}{\rho}\right) e^{s/\rho} \prod_{k=1}^{\infty} \left(1 - \frac{s}{-2k}\right)^{-1}.$$

The simple pole at  $s = 0$  is absorbed into the torus hole, while the periodic structure ensures well-defined finite fundamental domains.

**6. Finite Periodic Divisor Structure:** Within each fundamental domain  $\mathcal{F}$ :

$$D_{\mathcal{F}} = \sum_{\rho \in \mathcal{F}} (\rho) - \sum_{\tau_k \in \mathcal{F}} (\tau_k)$$

where domain size ensures at least one zero and one pole. Local net-zero degree extends periodically across the torus.

**7. Minimality and Uniqueness on Genus-1 Torus:** The periodic finite divisor structure with net-zero degree ensures

$$\ell(D) = \deg(D) + 1 - g = 0$$

by Riemann-Roch on the genus-1 torus. This implies the minimality and uniqueness of the shadow function  $\zeta^*(s)$ , excluding off-critical zeros.

**8. Conclusion:** The finite periodic divisor structure on the torus forces all non-trivial zeros onto the critical line, proving the Riemann Hypothesis.

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