Compressed Proof Attempt of the Riemann Hypothesis via Zeropole Balance

Attila Csordas, AgeCurve, Cambridge, UK, 26/12/2024 - 05/02/2025
This one-pager is a compressed, extracted version of the full proof available at GitHub (`https://github.com/attila-ac/Proof_RH_via_Zeropole_Balance`).

1. Functional Equation: The Riemann zeta function satisfies:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s).$$

Trivial zeros at $s = -2k$ ($k \in \mathbb{N}^+$) arise solely from the sine term, $\sin\left(\frac{\pi s}{2}\right)$.
2. Hadamard Product: The global zeropole structure of $\zeta(s)$ is given by:

$$\zeta(s) = \prod_\rho \left(1 - \frac{s}{\rho}\right) e^{s/\rho} \prod_{k=1}^{\infty} \left(1 - \frac{s}{-2k}\right)^{-1} \frac{s(1-s)}{\pi}.$$

Here, ρ are non-trivial zeros on the critical line, $s = -2k$ are trivial poles introduced to balance the trivial zeros, and $\frac{s(1-s)}{\pi}$ encodes the Dirichlet pole's dual role.
3. Hardy's Theorem: Countably infinite non-trivial zeros lie on the critical line, $\Re(s) = \frac{1}{2}$.
4. Zeropole Mapping and Orthogonal Balance: The Hadamard product formula, in conjunction with Hardy's theorem, establishes a bijection between trivial poles and non-trivial zeros of $\zeta(s)$. This bijection preserves cardinality $\aleph_0$ and encodes both algebraic independence and geometric perpendicularity between these orthogonal zeropole sets.
5. Shadow Function and Toroidal Structure: Define $\zeta^*(s)$ on the torus $\mathbb{T}^2$:

$$\zeta^*(s) = e^{A+Bs} \frac{1}{s} \prod_\rho \left(1 - \frac{s}{\rho}\right) e^{s/\rho} \prod_{k=1}^{\infty} \left(1 - \frac{s}{-2k}\right)^{-1}.$$

The simple pole at $s = 0$ is absorbed into the torus hole, while the periodic structure ensures well-defined finite fundamental domains.
6. Finite Periodic Divisor Structure: Within each fundamental domain $\mathcal{F}$:

$$D_{\mathcal{F}} = \sum_{\rho \in \mathcal{F}} (\rho) - \sum_{\tau_k \in \mathcal{F}} (\tau_k)$$

where domain size ensures at least one zero and one pole. Local net-zero degree extends periodically across the torus.
7. Minimality on Genus-1 Torus: By Riemann-Roch on the genus-1 torus:

$$\ell(D) = \deg(D) + 1 - g = 0$$

The periodic finite divisor structure with net-zero degree ensures minimality, excluding off-critical zeros. The shadow function $\zeta^*(s)$ is unique in this toroidal framework.
8. Conclusion: The finite periodic divisor structure on the torus forces all non-trivial zeros onto the critical line, proving the Riemann Hypothesis.