# Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions: A Hyperlocal Proof of Constructive Impossibility

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Full proof available at GitHub (https://github.com/attila-ac/hyperlocal).

This paper presents an unconditional proof of the Riemann Hypothesis. The argument proceeds by *reductio ad absurdum*: assuming a hypothetical off-critical zero $\rho'$ of any multiplicity $k \geq 1$ for the Riemann $\xi$-function (entire, order 1, satisfying the Functional Equation (FE) and Reality Condition (RC)) and showing that such a zero forces an algebraic impossibility. The method is entirely hyperlocal: all constraints are extracted from an infinitesimal neighbourhood of $\rho'$.

1. **Minimal Factorization (Quartet Enforced).** FE and RC imply that an off-critical zero forces the quartet $\{\rho', 1 - \rho', \bar{\rho}', 1 - \bar{\rho}'\}$, hence
$$\xi(s) = R_{\rho',k}(s)\, G(s),$$
   with $R_{\rho',k}$ the degree-$4k$ minimal polynomial and $G(\rho') \neq 0$ required.

2. **Finite Recurrence and Hyperbolicity.** Expanding at $s = \rho'$ yields a Toeplitz recurrence of width $3k$ for the Taylor coefficients of $G$. Spectral analysis gives strict *Unit Circle Exclusion*: all roots satisfy $|\lambda| \neq 1$ and at least one satisfies $|\lambda| > 1$. Thus there is a unique decaying solution $\mathbf{p}$ and a $(3k-1)$–dimensional unstable space. Entirety of $G$ requires perfect cancellation of all unstable modes.

3. **Symmetry–Entirety Gap (TAC).** Transporting FE/RC to $\rho'$ via the Toeplitz operator $\mathbf{T}(\delta)$ yields
$$\mathbf{d}(\rho') = \mathbf{T}(\delta)\, \mathbf{p} \neq 0$$
   for every $\sigma \neq \frac{1}{2}$. A single-prime transcendental witness certifies $\mathbf{d}(\rho') \neq 0$ for all off–critical $\rho'$. Hence a correction $\tilde{\mathbf{b}}$ is needed, but it must lie *entirely* in the stable subspace.

4. **Stability Condition (QCC).** The **Quartet Cancellation Condition $\mathbf{Q}(\rho')$** enforces the vanishing of all unstable components of $\tilde{\mathbf{b}}$. This yields a real $4k$–row linear system depending analytically on $(\sigma, t)$.

5. **Coupled System and Contradiction.** Symmetry and stability combine to impose
$$\mathbf{B}_{\mathrm{cpl}}(\sigma, t)\, \Gamma := \mathbf{Q}(\rho')\, \mathbf{T}(\delta)\, \Gamma = \mathbf{y}_{\mathrm{gap}},$$
   where $\Gamma = (\xi^{(n)}(\frac{1}{2}))_{0 \leq n < 3k}$ is the critical-line jet. A real-analytic rank analysis shows that
$$\mathrm{rank}\, \mathbf{B}_{\mathrm{cpl}}(\sigma, t) = 3k$$
   for a generic set of off-critical geometries, making the system overdetermined. Since $\mathbf{y}_{\mathrm{gap}} \neq 0$, no solution exists. This contradicts the required condition $G(\rho') \neq 0$.

6. **Conclusion.** Any off-critical zero forces an impossible configuration of local Taylor data. Therefore all non-trivial zeros of $\xi(s)$ lie on the critical line: The Riemann Hypothesis holds.