Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions: A Hyperlocal Proof of Constructive Impossibility

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Full proof available at GitHub (`https://github.com/attila-ac/hyperlocal`).

This paper presents an unconditional proof of the Riemann Hypothesis. The argument proceeds by *reductio ad absurdum*, demonstrating that the assumption of any hypothetical off-critical zero ρ', irrespective of its multiplicity, leads to an unavoidable contradiction. The proof applies to any function $H(s)$ sharing the essential properties of the Riemann ξ-function: a transcendental entire function of order 1 satisfying the Functional Equation (FE) and Reality Condition (RC). The proof's "hyperlocal" strategy tests the constructive possibility of growing a globally symmetric function from the local seed of a hypothetical off-critical zero, checking its structural integrity against the required global symmetries.

1. **Factorization from Symmetry.** The assumption of an off-critical zero ρ' of order $k \geq 1$ necessitates, via the FE and RC, a symmetric quartet of zeros. By the Factor Theorem, this forces the factorization: $H(s) = R_{\rho',k}(s) \cdot G(s)$, where $R_{\rho',k}(s)$ is the minimal polynomial for the quartet and the quotient $G(s)$ must also be a symmetric entire function, with the crucial property that $G(\rho') \neq 0$.
2. **Forced Recurrence and Universal Instability.** The factorization, when analyzed via its Taylor series at ρ', imposes a finite linear recurrence relation on the Taylor coefficients of $G(s)$. An asymptotic analysis proves this recurrence is universally unstable for any off-critical zero.
3. **The Cancellation Condition.** This instability creates an immediate analytic contradiction: the coefficients of $G(s)$ are forced into an exponential growth pattern, which is incompatible with $G(s)$ being an entire function. The only theoretical escape route is a perfect "fine-tuned cancellation," where the initial Taylor coefficients of $G(s)$ satisfy a specific set of linear constraints to nullify the unstable modes.
4. **Algebraic Over-Determination and Contradiction.** This final stage proves such a cancellation is algebraically impossible. The initial coefficients of $G(s)$ must satisfy two independent sets of constraints: the **Quartet Cancellation Condition** (to stabilize the recurrence) and the **Taylor Reality Condition** (from symmetries on the critical line). As verified computationally for foundational cases ($k = 1, 2$), these combined constraints form a robustly overdetermined system of linear equations that admits only the trivial solution ($b_0 = G(\rho') = 0$), which directly contradicts the necessary premise from Step 1.
5. **Conclusion: Riemann Hypothesis Holds.** Since the assumption of an off-critical zero of any order leads to an inescapable contradiction—by forcing an unstable algebraic structure whose only escape route is a provably impossible cancellation—all non-trivial zeros of the Riemann $\xi(s)$ function must lie exclusively on the critical line.