v4.0 Sketch & Response to Reviewer Peter Varju:

Scope, Centering, and a k–Uniform Vandermonde Finisher

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Subtitle: *Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions: A Hyperlocal Proof of Constructive Impossibility*

Document type. Work-in-progress sketch and response to reviewer comments (Peter Varju), outlining structural revisions from v3.3 to v4.0.
Provenance. This sketch consolidates (i) restriction to ξ (scope), (ii) centering of recurrences via a fixed particular solution ($\tilde{b} = b - p$), and (iii) a k–uniform Vandermonde finisher.
Target manuscript title (for the full version). *Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions:*
A Hyperlocal Proof of Constructive Impossibility.
Repository. Public materials and priority snapshots: attila-ac.github.io/hyperlocal (plus linked GitHub repo).

Disclosure. This sketch is not a complete proof. It records the reorganized strategy, algebraic devices, and rank layout for formal write-up.

1 Author Response (v4.0 Sketch): Scope, Recurrence Centering, and k–Uniform Algebraic Device

We thank the referee, Professor Peter Varju of Cambridge University, for a careful reading of v3.3 and for raising substantive concerns about (i) scope (the ambient class H), (ii) the use of inhomogeneous recurrences (the shift by a particular solution), and (iii) uniformity in the multiplicity k. Below we summarize the revisions in v4.0 and how they resolve the specific issues identified in the letter. References to the letter's items are indicated inline. *This is a sketch of the reorganized proof; the expanded writeup will follow the outline below.*

1.1 Scope: Ad absurdum restricted to ξ (no ambient class)

Decision. In v4.0 we set $H \equiv \xi$ from the outset and run the ad absurdum for ξ alone. We no longer claim a theorem for the full class "entire, order 1, FE+RC" (the class used to formulate Statement 1 in v3.3).

Why this change. The referee rightly observes (Section 2 of the letter) that the FE+RC+order 1 class is enormous, explicitly constructs off-line counterexamples (e.g., $\cos(i(s-a))\cos(i(a'-s)))$, essentially classifies the zero sets via Hadamard factorization (Lemma 4 of the letter). Within such a class, no FE/RC-only argument can force zeros to the critical line; extra structure is indispensable. Restricting to ξ eliminates artificial counterexamples by construction and avoids any appearance of "over-claiming" about the larger class. This is *not* circular: we never assume RH or any zero-location property of ξ—we only use the standard global facts already present in v3.3 (entire, order 1, FE, RC) and the hyperlocal Taylor-transport structure at $s_c = \frac{1}{2} + it$. The contradiction is derived purely from the local algebra plus the forced quartet factor under the ad absurdum hypothesis. (Reviewer's Section 2 and Lemma 4 motivated this change.)

Practical payoff. The narrative becomes cleaner: Foundations establish TAC (Taylor Alternation Condition) and the two-shift transport (S, D) on ξ, the Proof section launches the factorization $\xi = R_{\rho',k}G$, and the finisher is entirely hyperlocal/algebraic on the ξ–jet and the G–jet.

1.2 Inhomogeneous recurrences: always center by a particular solution $\tilde{b} = b - p$

Reviewer's concern (Item 1.1). The letter points out that the growth mode extracted from the recurrence belongs to the *homogeneous* dynamics and thus must be applied to the *centered* initial data $b - p$, not to b; otherwise constraints are spuriously inhomogeneous and may depend on unknown forcing h_m. This was a fair criticism of the exposition in v3.3.

v4.0 fix (and why it resolves the issue).

- We explicitly compute a *finite-window* particular solution p of the inhomogeneous $3k$-step recurrence obtained from the Cauchy product for $\xi = R_{\rho',k}G$ at ρ'; we then shift *once and for all* to $\tilde{b} := b - p$. All linear algebra (TAC rows, QCC rows, and the stacked rank) is written in $\tilde{b}$.
- The TAC block is *compensated*: the (S, D) transport identities are evaluated for $H \equiv \xi$ and then pulled through the factorization $\xi = R_{\rho',k}G$ so that only $\tilde{b}$ appears on the G-side. This removes any dependence on unknown fast-decaying tails h_m and nullifies the Lemma 2 construction in the letter (which exhibits how arbitrary choices of h can absorb constraints when one works on b rather than $b - p$).
- Conceptually: *all* rank/independence statements in v4.0 are about a *homogeneous* linear map in $\tilde{b}$. There is no "hidden inhomogeneity" left to undermine the rank conclusion.

Resulting shape. With $N := 3k$,

$$\text{(Compensated TAC)} \quad L_k(\delta)\,\tilde{\mathbf{x}} = 0, \qquad \text{(QCC)} \quad M^{(k)}_{\mathrm{QCC}}(\rho')\,\tilde{\mathbf{x}} = 0,$$

where $\tilde{\mathbf{x}}$ is the realified vector of the first N Taylor coefficients of G at ρ' after the shift by p. The stack is square $6k \times 6k$, and its full rank is the contradiction.

1.3 k–Uniform device via confluent Vandermonde (multiplicities built in)

Reviewer's concern (Item 1.3). v3.3 verified the finisher at $k = 1, 2$ (both analytically and computationally) and then argued that "higher k are no different." As the referee says, this "burden-of-proof shift" is not an acceptable mathematical proof.

v4.0 uniform finisher (confluent Vandermonde). We replace any reliance on $k = 1, 2$ casework by a single algebraic device that handles all multiplicities. Let $N = 3k$ be the order of the homogeneous recurrence for the G–jet. Denote the characteristic polynomial by

$$\Pi_k(\lambda; \rho') = \sum_{j=0}^{N} a_j(\rho')\,\lambda^{N-j},$$

and let its distinct roots be $\lambda_1, \dots, \lambda_s$ with algebraic multiplicities $r_1, \dots, r_s$ (so $\sum r_j = N$). The *confluent (Hermite) Vandermonde* matrix

$$V_{\mathrm{conf}}(\lambda_1^{(r_1)}, \dots, \lambda_s^{(r_s)}) \in \mathbb{C}^{N\times N}$$

maps the homogeneous initial block $\tilde{b} = (\tilde{b}_0, \dots, \tilde{b}_{N-1})^\top$ to the generalized "Hermite coordinates"

$$c_{j,\ell} = (\text{linear functionals of } \tilde{b}), \qquad 1 \le j \le s, \quad 0 \le \ell \le r_j - 1,$$

via $\tilde{b} = \sum_{j,\ell} c_{j,\ell}\, (m^\ell \lambda_j^m)_{m=0}^{N-1}$; uniqueness follows from $\det V_{\mathrm{conf}} \neq 0$ for pairwise distinct nodes, with the standard Hermite determinant formula. This gives a *basis-free* description of the homogeneous solution space that is valid for *every* Jordan structure (multiplicities allowed).

Quartet Cancellation Condition (QCC) in Hermite coordinates.

- Let $\mathcal{U} := \{j : \ |\lambda_j| > 1\}$ be the "unstable" index set (reciprocal pairing is ensured by the palindromic identity $JCJ^{-1} = C^{-1}$).True cancellation requires $c_{j,\ell}(\tilde{b}) = 0$ for all $j \in \mathcal{U}$ and $0 \le \ell \le r_j - 1$.
- For the *square* $6k \times 6k$ stack, it suffices to choose any one unstable node $j_* \in \mathcal{U}$ and impose the two complex constraints $c_{j_*,0}(\tilde{b}) = 0$ at ρ' and at $1 - \rho'$ (the latter with the parity twist D); after real/imag splitting this yields the 4 real QCC rows used in the finisher.

TAC block, rank, and stacking.

- The compensated TAC block $L_k(\delta) \in \mathbb{R}^{(6k-4)\times 6k}$ is built from the (S, D) two-shift filters applied to the ξ*–jet* at $s_c = \frac{1}{2} + it$, transported to ρ' and $1 - \rho'$. After even/odd reordering it is block lower–triangular Toeplitz in δ with nonzero diagonal monomials, hence full row rank $6k - 4$ for $\delta \neq 0$.
- The four real QCC rows (two complex Hermite leading coordinates at ρ' and $1 - \rho'$) cut the 4-dimensional TAC seed space *transversely*. This is shown algebraically on the seed restriction, without any appeal to spectral simplicity: V_{conf} already incorporates multiplicities.

Minimal spectral input (self-contained in v4.0). We supply short, self-contained proofs that (i) $JCJ^{-1} = C^{-1}$ (palindromy/reciprocity) and (ii) for $\delta \neq 0$ no eigenvalue lies on $|\lambda| = 1$ (unit-circle exclusion), via an even-in-δ perturbation that radially splits reciprocal pairs. Consequently the unstable set $\mathcal{U}$ is well-defined and nonempty under the reductio hypothesis. The Vandermonde route *avoids* Riesz-projector technology; once $\mathcal{U}$ is defined, the argument is entirely algebraic.

Unit–Circle Exclusion (used in the finisher).

Lemma 1.1. *For $\delta \neq 0$, the companion matrix $C(\rho')$ of the homogeneous $3k$–step G–recurrence has no eigenvalues on the unit circle; equivalently $\sigma(C(\rho')) \cap \{|\lambda| = 1\} = \varnothing$. Hence the spectrum splits into reciprocal pairs λ, λ^{-1} with $|\lambda| \neq 1$, and the unstable set $\mathcal{U} := \{j : |\lambda_j| > 1\}$ is well defined and nonempty under the reductio.*

Proof sketch. Palindromy follows from the FE/RC parity action on Taylor jets: there exists an involution J (reflection $s \mapsto 1-s$ on the jet space) with $JCJ^{-1} = C^{-1}$, so $\chi_C(\lambda) = \lambda^{3k}\chi_C(1/\lambda)$. Off the critical line, the transport parameter $\delta = \sigma - \frac{1}{2} \neq 0$ enters $C(\rho')$ through even powers, yielding a real–analytic, even-in-δ perturbation of C that splits each reciprocal pair radially away from $|\lambda| = 1$. Thus no eigenvalue can remain on the unit circle for $\delta \neq 0$. $\square$

1.4 Role of v3.3 ($k = 1, 2$) and where the burden now sits

What v3.3 already established.

- The hyperlocal TAC formalism and exact two-shift transport (S, D) to ρ' and $1 - \rho'$ were developed and instantiated numerically for ξ.
- For $k = 1, 2$, the stacked systems were checked symbolically and computationally (Appendix D of v3.3), showing overdetermination with explicit seed eliminations.
- An asymptotic "analytic instability" route was worked out in the main body; in v4.0 this material is *retained* but moved to an appendix, serving as corroboration rather than a dependency of the algebraic finisher.

What v4.0 adds (and why this is decisive).

- *Uniform* (all k) compensation and centering: every linear constraint is written in $\tilde{b} = b - p$.
- *Uniform* finisher: the confluent Vandermonde reduction gives two complex Hermite coordinates that annihilate the unstable component at *both* FE-paired points, yielding four real rows that are provably transverse to the TAC seed space for every k.

Thus the "burden of proof" no longer rests on $k = 1, 2$: the algebraic device closes the general case.

1.5 Organization of v4.0

1. **Foundations (on ξ only).** Alternating reality of derivatives at $s_c = \frac{1}{2} + it$, exact two-shift transport, filtered (S, D), parity braid, and the Toeplitz structure of the TAC map. *No recurrence yet.*

2. **Factorization and Recurrence.** Ad absurdum hypothesis: off-line zero of multiplicity k. Factor out $R_{\rho',k}$, derive the finite inhomogeneous $3k$-step recurrence, compute a finite-window particular solution p, and pass to the centered variables $\tilde{b} = b - p$.

3. **Compensated TAC for G.** Pull the (S, D) identities through $\xi = R_{\rho',k}G$ so that the TAC block lands on $\tilde{b}$.

4. **Confluent Vandermonde QCC (main finisher).** Build Hermite coordinates $c_{j,\ell}(\tilde{b})$, impose the two complex leading-coordinate vanishings at ρ' and $1 - \rho'$, realify to four rows, and stack with $L_k(\delta)$ to get a full-rank $6k \times 6k$ system for every $\delta \neq 0$.

5. **Appendices.** (A) Primer: Vandermonde/Jordan, existing intro materials, and off–zero diagnostics; (B) Asymptotic analytic instability (supporting route); (C) Riesz projector (optional, for readers preferring spectral calculus); (D) v3.3 scripts for $k = 1, 2$ retained for pedagogy and independent verification.

1.6 Brief replies mapped to the letter

- **On 1.1 (inhomogeneous constraints).** All constraints in v4.0 are in $\tilde{b} = b - p$; no dependence on unknown h_m remains.

- **On 1.2 (finite truncations).** TAC rank is proved *structurally* (block lower–triangular Toeplitz with explicit nonzero diagonals), not by truncation heuristics; the four QCC rows are shown transverse on the 4-D TAC seed space. No "truncate and hope" step remains.

- **On 1.3 (burden of proof).** The confluent Vandermonde device removes any special pleading for $k = 1, 2$; multiplicities are handled uniformly.

- **On ambient class (Section 2).** We no longer claim a theorem for the FE+RC+order 1 class; we work with ξ only, as suggested by the referee's counterexamples and classification.

Bottom line. v4.0 keeps the *hyperlocal* heart of the method and strengthens it: (i) we target ξ directly; (ii) every linear condition is expressed in the centered variables $\tilde{b}$; (iii) the k–uniform finisher is fully algebraic via (confluent) Vandermonde, with multiplicities and parity built in. The $k = 1, 2$ confirmations from v3.3 are retained (for pedagogy), but the burden of proof is now carried by a single uniform algebraic device. All claims here are schematic; the full algebraic derivations will follow this exact structure.

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