Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions: A Hyperlocal Proof of Constructive Impossibility

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Full proof available at GitHub (`https://github.com/attila-ac/hyperlocal`).

This paper presents an unconditional proof of the Riemann Hypothesis. The argument proceeds by *reductio ad absurdum*, demonstrating that the assumption of any hypothetical off-critical zero ρ' of any order for a function $H(s)$ in the class of the Riemann ξ-function—transcendental, entire, of order 1, and satisfying the Functional Equation (FE) and Reality Condition (RC)—leads to an unavoidable algebraic contradiction. The proof's strategy is a hyperlocal analysis of the consequences of such a zero.

1. **Factorization from Symmetry.** The assumption of an off-critical zero ρ' of order $k \geq 1$ necessitates, via the Functional Equation (FE) and Reality Condition (RC), the existence of a symmetric quartet of zeros. By the Factor Theorem, this forces the factorization:
$$H(s) = R_{\rho',k}(s) \cdot G(s)$$
where $R(s)$ is the minimal polynomial for the quartet and the quotient $G(s)$ must also be a function in the same class, with the crucial property that $G(\rho') \neq 0$.
2. **The Unstable Recurrence.** The factorization, when analyzed via its Taylor series at ρ', imposes a finite linear recurrence relation on the coefficients of $G(s)$. A direct asymptotic analysis of this recurrence's characteristic polynomial proves that it is **unstable** for any off-critical zero. This instability is the foundational engine of the proof.
3. **The Necessary Cancellation and Final Contradiction.** The instability implies that for $G(s)$ to be entire, its initial Taylor coefficients must satisfy a precise "Cancellation Condition" to prevent exponential growth. The final step of the proof is to show this condition is impossible to satisfy.
   - For the system to be consistent, the Cancellation Condition must be satisfiable at all four points of the symmetric quartet.
   - This requirement, combined with additional independent constraints derived from the full symmetries of $G(s)$, generates an **overdetermined system of linear equations** on the initial Taylor coefficients of $G(s)$.
   - As verified by the computational proof in Appendix D, this system has full rank and admits only the **trivial solution** (e.g., $b_0 = 0$).
   - The conclusion $b_0 = G(\rho') = 0$ directly contradicts the necessary premise that $G(\rho') \neq 0$.
4. **Conclusion: Riemann Hypothesis Holds.** Since the assumption of an off-critical zero leads to an inescapable algebraic contradiction, all non-trivial zeros of the Riemann $\xi(s)$ function must lie exclusively on the critical line.