Off-Critical Riemann Zeta Zeros Cannot Seed Symmetric Entire Functions: A Hyperlocal Proof of Constructive Impossibility

Attila Csordas, AgeCurve Limited, Cambridge, UK, 26/06/2025 - 04/07/2025
Full proof available at GitHub (`https://github.com/attila-ac/hyperlocal`).

This paper presents an unconditional proof of the Riemann Hypothesis. The argument proceeds by *reductio ad absurdum*, showing that the assumption of any off-critical zero ρ' for an entire function $H(s)$ sharing the properties of the Riemann ξ-function—its symmetries (FE & RC) and its full class of growth constraints (finite order, vertical decay, and polynomial horizontal growth)—leads to a terminal contradiction for a zero of any order $k \geq 1$.

The core of the proof is an analytical engine that translates the function's global properties into a fatal local constraint. We first prove that for any such function, its derivative $H'(s)$ must be purely imaginary on the critical line (the Imaginary Derivative Condition, IDC). We then prove a powerful Affine Constraint Proposition: an entire function satisfying the IDC on an *offset* vertical line and possessing the required growth properties is rigorously forced to be an affine polynomial.

The main argument, **"Clash of Natures"** uses this engine to create a "pincer movement" that establishes two mutually exclusive properties for the derivative $H'(s)$ of a hypothetical *transcendental* function with an off-critical zero:

- **Prong 1 (from Structure): It Cannot Be Affine.** The necessary factorization of the function around the mandated zero quartet, $H(s) = R_{\rho',k}(s)G(s)$, imposes a structure on the derivative $H'(s)$. We prove that if $H(s)$ is transcendental, its derivative *cannot* be an affine polynomial without violating the analytic properties of the quotient function $G(s)$.
- **Prong 2 (from Symmetries): It Must Be Affine.** The analytical engine, when applied at the off-critical zero, rigorously forces this same derivative $H'(s)$ to be an affine polynomial.

A function cannot simultaneously be and not be an affine polynomial. This direct contradiction refutes the existence of off-critical zeros in any such transcendental function. The appendix provides a second, independent proof track that is purely algebraic. It uses the same analytical engine but shows that the affine structure is also directly incompatible with the Taylor series expansion of the derivative at an off-critical zero. This provides a parallel validation of the result. Since the assumption of an off-critical zero leads to a definitive contradiction via at least two independent lines of reasoning, no such zeros can exist for any function in our defined class. As the Riemann $\xi(s)$ function is a member of this class, all of its non-trivial zeros must lie exclusively on the critical line. The Riemann Hypothesis therefore holds unconditionally.