

Assignment : 2

Course Code : DAA

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Q:1 Master Method

$$(a) \quad T(n) = 4T(n/3) + n$$

$$a=4, \quad b=3 \quad f(n)=n$$

$$n^{\log_b a} = n^{\log_3 4}$$

$$f(n) = n^{\log_b a}$$

$$T(n) = \Theta(n^{\log_b a})$$

Recursion Tree:

$$T(n) = 4T(n/3) + n$$

$$T(n) = 4\left(4T\left(\frac{n}{9}\right) + \frac{n}{3}\right) + n$$

$$= 16T\left(\frac{n}{27}\right) + 4\left(\frac{n}{3}\right) + n$$

$$= 64T\left(\frac{n}{81}\right) + 16\left(\frac{n}{9}\right) + \left(\frac{4}{3}\right)n$$

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Generalize:

$$T(n) = 4^k T\left(\frac{n}{3^k}\right) + n \sum_{i=0}^{k-1} \left(\frac{4^i}{3^i}\right)$$

Recursion stops at $\frac{n}{3^k} = 1$

when $k = \log_3 n$

$$\sum_{i=0}^{k-1} \left(\frac{4}{3}\right)^i = \Theta\left(n^{\log_3 4}\right)$$

$$T(n) = \Theta\left(n^{1.2619}\right)$$

b) $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + cn$

$$T(n) = \left[T\left(\frac{n}{9}\right) + T\left(\frac{2n}{9}\right) + c\left(\frac{n}{3}\right)\right] +$$

$$\left[T\left(\frac{2n}{9}\right) + T\left(\frac{4n}{9}\right) + c\left(\frac{2n}{3}\right)\right] +$$

$$cn$$

$$= T(n/27) + T(2n/27) + T(2n/27) + T(4n/27) + T(2n/9) + T(4n/9) + cn(1 + 1/3 + 2/3)$$

$$cn(1 + \frac{1}{3} + \frac{2}{3} + \frac{1}{9} + \frac{2}{9} + \dots)$$

Recursion stops at $\frac{n}{3^k} = 1$

$$k = \log_3 n$$

- $O(n)$ at each level.
- $T(n) = \Theta(n \log n)$

For Master Method:

$$a_1 = 1, a_2 = 1$$

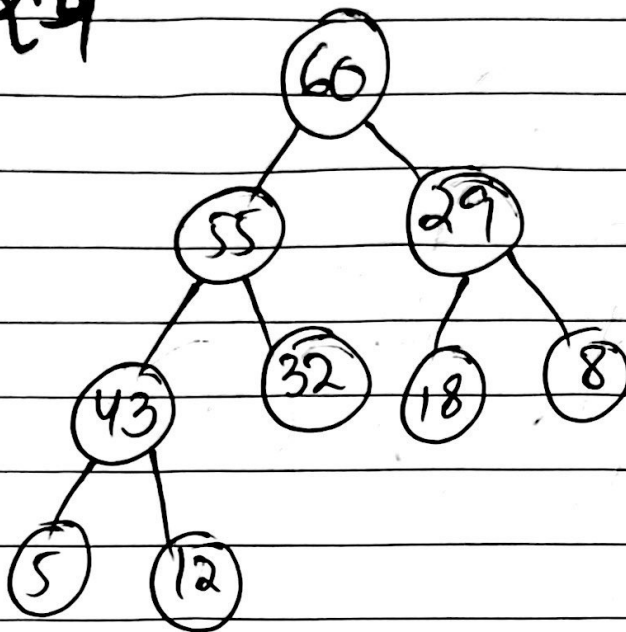
$$b_1 = 3, b_2 = 3$$

$$f(n) = O(n)$$

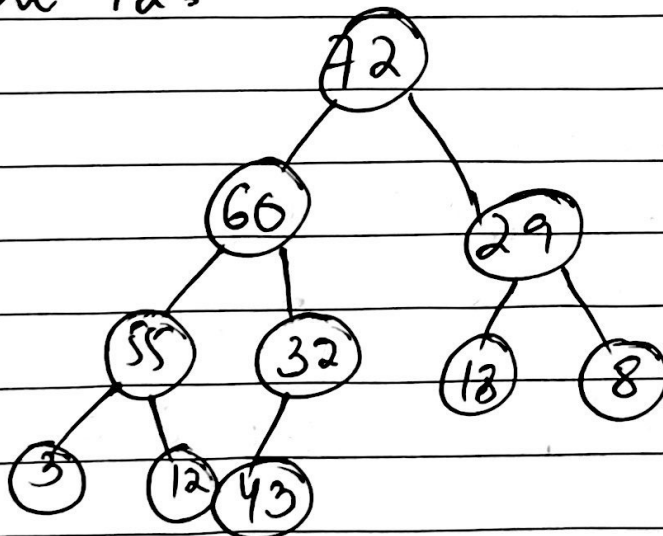
$$n^{\log_b a} = n^{\log_3 1}$$

• 2 recursive with diff sizes. So, Master Method not apply.

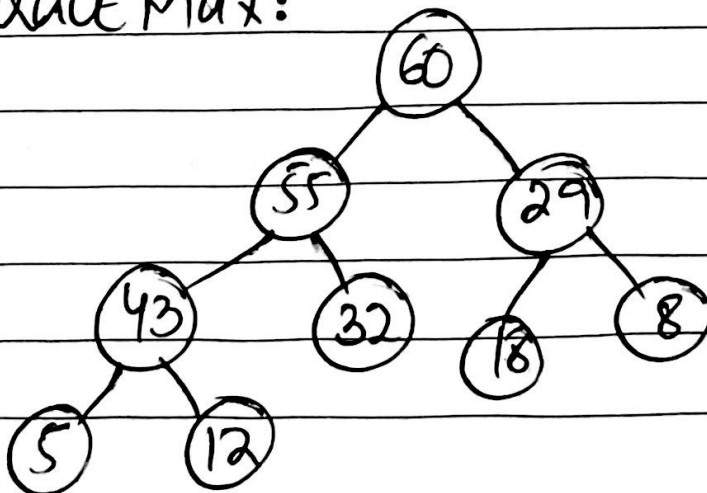
Q:3



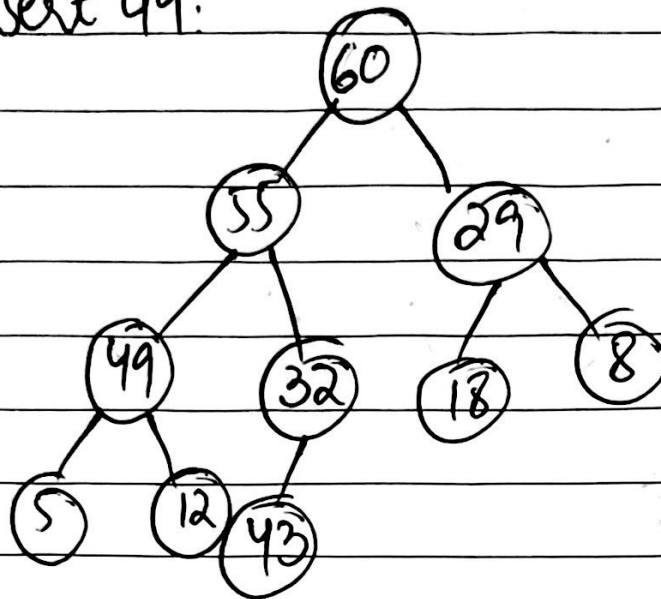
Insert 72:



Extract max:



Insert 49:



```

void minHeapify(int arr[], int n, int i)
{
    int small = i;
    int left = 2 * i + 1;
    int right = 2 * i + 2;
  
```

```

    if (left < n && arr[left] < arr[small])
    {
        small = left;
  
```

```

    if (right < n && arr[right] < arr[small])
    {
        small = right;
  
```

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if (small != i) {
    swap (arr[i], arr[small]);
    min Heapify (arr, n, small);
}
void build MinHeap (int arr[], int n)
for (int i = n/2 - 1; i >= 0; i--) {
    min Heapify (arr, n);
}

```

•) Heapify takes $O(\log n)$ per node.

Total nodes to Heapify = $\frac{n}{2}$

$T(n) = O(n)$

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Q:3

Node* insert (Node* root, int Key) {

if (root == NULL)
return new Node(Key);

if (Key < root->key) {

root->left = insert(root->left, Key);

} else {

root->right = insert(root->right, Key);

} return root;

} void InOrder (Node* root) {

if (root) {

InOrder (root->left);

cout << root->key;

InOrder (root->right);

}

}. Lucrum }

Insertion takes:

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$$T(n) = T\left(\frac{n}{2}\right) + O(1)$$

Inorder takes:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(1)$$

Master Method.

$$a = 2, \quad b = 2, \quad f(n) = O(1)$$

$$n \log_2 2 = n$$

$$\begin{aligned} T(n) &= \Theta(n^{\log_b a}) \\ &= \Theta(n) \end{aligned}$$

$$\text{Best case} = O(\log n)$$

$$\text{Worst case} = O(n)$$

$$\text{InOrder} = O(n)$$

$$O(n \log n) + O(n) = O(n \log n)$$

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Question: 2

height = 2

$$2^3 - 1 = 7$$

10 nodes. So height 2 not possible.

height = 3

$$2^4 - 1 = 15$$

15 nodes. So height 3 is possible.

height = 4, 5, 6, 7 \Rightarrow possible

