M T W T F S S

70142254.
Attiya Sajid
Assignment: 1

Q:1(a) Total Execution Cost:

```
void print Pattern (int n).
 for (int i=0 ; i<n ; i++)
  for (int j=i ; j<n ; j++)
   cout <<"      ";

for (int k=1; k<i+1; k++)
  cout << " @ " ;
  cout << endl;
```

Now; $= \left(\frac{n^2-n}{2}\right) + \left(\frac{n^2+3n}{2}\right) + 3$

$= n^2 + 2$ total cost

$O(n^2)$ time complexity.

(b)

void matrix Multiplication(int A[] N[],
int B[N] int([][N])   1 times

for int $i=0$; $i<n$; $i++$ n times
for int $j=0$; $j<n$; $j++$ $n \times n$ times
[i][j] = 0 $n \times n$ times
for(int $K=0$; $K<N$; $K++$) $n \times n \times n$ times
[i][j] += A[i][K] * B[K][j] $n \times n \times n$ times

Total Cost :

$$n^3 + 2n + 1$$

Q:2 $2^{n+1} = O(2^n)$

$f(n) \leq c \cdot g(n)$

$2^{n+1} \leq c \cdot 2^n$

$\frac{2^{n+1}}{2^n} \leq c$

$2 \leq c$

Now;

$f(n) \leq c \cdot g(n)$

M T W T F S S

$2^{n+1} \leq 2 \cdot 2^n$

$\therefore n = 1$

$2^{1+1} \leq 2 \cdot 2^{(2)}$

$8 \leq 8$

So; $O(2^n)$ is valid for 2^{n+1}

is $2^n = O(2^n)$.

$f(n) \leq c \cdot g(n)$

$2^{2n} \leq c \cdot 2^n$

$2^n \leq c$

$\therefore$ $n = 1$ $\qquad$ $n = 2$

$c = 2$ $\qquad$ $c = 4$

now for $n = 1$

$2^2 \leq 2 \cdot 2^1$

$4 \leq 4$

now for $n = 2$

$2^4 \leq 2 \cdot 2^2$

$16 \leq 8$.

$O(2^n)$ is not valid for 2^{2n}.

Q:3 Recursive Relation

(a) $T_{(n)} = 2T(n/2 + 17) + n$

$T(n/2 + 17) = 2T(n/4 + 34) + \frac{n}{2} + 17$

Substituting backwards:

$T(n) = 2\left[2T\left(\frac{n}{4}\right) + 34 + \frac{n}{2} + 17\right] + n$

$= 4T\left[\left(\frac{n}{4}\right) + 34\right] + n + 34 + n$

$= 4T\left(\frac{n}{4} + 34\right) + 2n + 34.$

Value of $T\left(\frac{n}{4} + 34\right)$:

$T\left(\frac{n}{4} + 34\right) = 2T\left(\frac{n}{8} + 51\right) + \left(\frac{n}{4} + 34\right)$

Again Substitute backwards.

$T(n) = 4\left[2T\left(\frac{n}{8} + 51\right) + \left(\frac{n}{4} + 34\right)\right] + 2n + 34$

$= 8T\left(\frac{n}{8} + 51\right) + n + 2n + 170$

$= 8T\left(\frac{n}{8} + 51\right) + 3n + 170$

M T W T F S S

Expected Pattern:

$T(n) = 2^k T\left(\frac{n}{2}k + 17k\right) + kn + 17(2^k - 1)$

for Base case 'k'

$\frac{n}{2}k + 17k = 1$

$2^n = \frac{n}{1-17k}$

log property.

$k = \log_2\left(\frac{n}{1-17k}\right)$

$k \approx \log_2 n$

Using $2^k = n$ and $k = \log_2 n$

$T(n) = nT(1) + n\log_2(n) + 17n - 17$

M T W T F S S

$T(n) = O(n \log_{n} n)$.

(b) $T(n) = 4T\left(\frac{n}{2} + O(n)\right)$

$T\left(\frac{n}{2}\right) = 4T\left(\frac{n}{4}\right) + O\left(\frac{n}{2}\right)$

Substitute

$T(n) = 4\left(4T\left(\frac{n}{4}\right) + O\left(\frac{n}{2}\right)\right) + O(n)$.

$= 16T\left(\frac{n}{4}\right) + 2\,O(n) + O(n)$

$= 16T\left(\frac{n}{4}\right) + 3\,O(n)$

Now

$T\left(\frac{n}{4}\right)$:

$T\left(\frac{n}{4}\right) = 4T\left(\frac{n}{8}\right) + O\left(\frac{n}{4}\right)$

Substitute:

$T(n) = 16\left(4T\left(\frac{n}{8}\right) + O\left(\frac{n}{4}\right)\right) + 3\,O(n)$

$= 64T\left(\frac{n}{8}\right) + 4\,O(n) + 3\,O(n)$

$= 64T\left(\frac{n}{8}\right) + 7O(n)$

M T W T F S S

Expected Pattern:

$$T(n) = 4^K T\left(\frac{n}{2}K\right) + (4^K - 1)O(n)$$

Base (K)

$$\frac{n}{2}K = 1 \Rightarrow 2^K = n$$

$$K = \log_2 n.$$

2^K and K

$$= 4^{\log_2 n} T(1) + (4^{\log_2 n} - 1)\, O(n).$$

$$= n^2 T(1) + (n^2 - 1)\, O(n)$$

$$T(n) = O(n^2)$$