EDA Homework part 1

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1.) Show that $m(a+bX) = a + b \times m(X)$

$$= \frac{1}{N} \sum_{i=1}^{N} (a + bx_i)$$

$$= \frac{1}{N} \left( \sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i \right)$$

$$= \frac{1}{N} \left( Na + b \sum_{i=1}^{N} x_i \right)$$

$$= a + b \cdot \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$m(a+bX) = a + b \cdot m(X)$$

2.) Show that $cov(X, a+bY) = b \times cov(X,Y)$

$$cov(x, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))((a+by_i) - m(a+bY))$$

$$= \frac{1}{N} \sum (x_i - m(X))(a + by_i - (a + bm(Y)))$$

$$= \frac{1}{N} \sum (x_i - m(X)) b(y_i - m(Y))$$

$$= \therefore cov(X, a+bY) = b\, cov(X,Y)$$

3. Show that $cov(a+bX, a+bX) = b^2 cov(X,X)$
& in particular that $cov(X,X) = s^2$

$$= \frac{1}{N} \sum_{i=1}^{N} ((a+bx_i) - m(a+bX))^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (b(x_i - m(x)))^2$$

$$= b^2 \cdot \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2$$

$$= b^2 cov(X,X)$$

$$Cov(X,X) = \frac{1}{N} \sum (x_i - m(X))^2 = S^2$$

4.) $2+5 \times X$ or arcsinn$(X)$

If $g$ is strictly $>$

$$\text{Median}(g(X)) = g(\text{Median}(X))$$

Strictly increasing func. Keep the order of the data the same so the median stays the same

$$M = x + g(m) = g(x) + P(g(m) \geq g(x)) = P(x \leq m) = .5$$
$$X = m + g(x) = g(m) + P(g(x) = \geq g(m)) = P(\leq m) = .5$$

* If $g$ is only non decreasing, then median of $X$ might be any value in a median interval $[m-, m+]$

Quantile:

yes it applis to any quantile level $p \in (0,1)$

strictly $>$: $Q_p(g(X)) = g(Q_p(X))$

IQR:

$$IQR(g(X)) = g(Q_{0.75}(X)) - g(Q_{.25}(X))$$

IQR is: $Q_{.75} - Q_{.25}$

Range:

$$\text{Range}(g(X)) = g(\max X) - g(\min X)$$

5.)

5.)

Not always true.

If $g(x) = a + bx$ then linearity holds because it's under affine transforms

$$m(g(x)) = \frac{1}{2}(0^2 + 2^2) = 2$$

BUT

$$g(m(X)) = 1^2 = 1$$

$$\therefore m(g(x)) \neq g(m(x))$$