First Semester 2023-24
Data Structures and Algorithms Design (Merged-SEZG519/SSZG519)

Exercises (Analyzing Algorithms)

1. Use Master's theorem to solve the following division functions.
   a. $T(n) = 3T(n/2) + n^2$

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = 3, b = 2, g(n) = n^2 \Rightarrow$ so $k = 2, p = 0$

      $a < b^k$ and $p >= 0 \Rightarrow O(n^k \log^p n) = O(n^2)$

   b. $T(n) = 4T(n/2) + n$^2

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = 4, b = 2, g(n) = n^2 \Rightarrow$ so $k = 2, p = 0$

      $a = b^k$ and $p > -1 \Rightarrow O(n^k \log^{p+1} n) = O(n^2 \log n)$

   c. $T(n) = 16T(n/4) + n$

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = 16, b = 4, g(n) = n \Rightarrow$ so $k = 1, p = 0$

      $a > b^k \Rightarrow O(n^{\log_b a}) = O(n^{\log_4 16}) = O(n^2)$

   d. $T(n) = 2T(n/2) + n\log n$

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = 2, b = 2, g(n) = n\log n \Rightarrow$ so $k = 1, p = 1$

      $a = b^k$ and $p > -1 \Rightarrow O(n^k \log^{p+1} n) = O(n\log^2 n)$

   e. $T(n) = 2T(n/4) + n^{0.51}$

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = 2, b = 4, g(n) = n^{0.51} \Rightarrow$ so $k = 0.51, p = 0$

      $a < b^k$ and $p >= 0 \Rightarrow O(n^k \log^p n) = O(n^{0.51})$

   f. $T(n) = \sqrt{2}T(n/2) + \log n$

      $T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

      $a = \sqrt{2}, b = 2, g(n) = \log n \Rightarrow$ so $k = 0, p = 1$

      $a > b^k$ and $p >= 0 \Rightarrow O(n^{\log_b a}) = O(n^{\log_2 \sqrt{2}}) = O(n^{0.5})$

g. $T(n) = 6T(n/3) + n^2 \log n$

$T(n) = aT(n/b) + g(n)$ where $g(n) = n^k \log^p n$

$a = 6, b = 3, g(n) = n^2 \log n \Rightarrow$ so $k = 2, p = 1$

$a < b^k$ and $p >= 0 \Rightarrow O(n^k \log^p n) = O(n^2 \log n)$

2. Use Master's theorem to solve the following decreasing functions.

a. $T(n) = 0.5T(n-1) + n$

$T(n) = aT(n-b)+g(n)$, where $g(n) = n^k$
$a = 0.5, b = 1, g(n) = n \Rightarrow$ so $k = 1$

$a < 1, O(n^k) = O(n)$

b. $T(n) = 2/3T(n-1) + n^2$

$T(n) = aT(n-b)+g(n)$, where $g(n) = n^k$
$a = 2/3, b = 1, g(n) = n^2 \Rightarrow$ so $k = 2$

$a < 1, O(n^k) = O(n^2)$

c. $T(n) = T(n-1) + n^2$

$T(n) = aT(n-b)+g(n)$, where $g(n) = n^k$
$a = 2/3, b = 1, g(n) = n^2 \Rightarrow$ so $k = 2$

$a = 1, O(n^{k+1}) = O(n^3)$

d. $T(n) = 2T(n-1) + n^2$

$T(n) = aT(n-b)+g(n)$, where $g(n) = n^k$
$a = 2, b = 1, g(n) = n^2 \Rightarrow$ so $k = 2$

$a > 1, O(n^k a^{n/b}) = O(n^2 2^n)$

e. $T(n) = 3T(n-2) + n$

$T(n) = aT(n-b)+g(n)$, where $g(n) = n^k$
$a = 3, b = 2, g(n) = n \Rightarrow$ so $k = 1$

$a > 1, O(n^k a^{n/b}) = O(n\ 3^{n/2})$