AHMEDABAD INSTITUTE OF TECHNOLOGY

BCA Semester – I
MATHEMATICS – 1
Unit – 3 Assignment
Nrusinh Patel (NP)

Q 1. Find the order (dimensions) of the following matrices:

1. $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 4 \end{bmatrix}$
2. $B = \begin{bmatrix} -2 & 0 & 7 & 1 \\ 3 & 1 & 0 & 5 \end{bmatrix}$
3. $D = \begin{bmatrix} 6 \end{bmatrix}$
4. $E = \begin{bmatrix} 9 & 2 & -1 \\ 2 & 5 & 0 \end{bmatrix}$
5. $C = \begin{bmatrix} 4 & -2 \\ 1 & 0 \\ 5 & 3 \end{bmatrix}$

Q 2. Consider the following matrices: $X = \begin{bmatrix} 2a-3b & a+2c \\ 3d-b & 4c \end{bmatrix}$ and $Y = \begin{bmatrix} 5 & 2 \\ 1 & 8 \end{bmatrix}$
If $X = Y$, find the values of a, b, c, and d.

Q 3. Consider the following matrices: $A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & -2 \end{bmatrix}$
Find the following expressions involving matrices A and B:

1. $4A + 2B$
2. $3A - B$

Q 4. Consider the following matrices: $A = \begin{bmatrix} 4 & 2 \\ -1 & 6 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 7 \\ 2 & -5 \end{bmatrix}$ $C = \begin{bmatrix} 9 & -1 \\ 0 & 3 \end{bmatrix}$
Find the matrix X that satisfies the equation: $2(X + A) - 2(B - 3C) = C$

Q 5. Find AB and BA, whichever is possible.

1. $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$
2. $A = \begin{bmatrix} -2 & 1 & 5 \\ 3 & 2 & 2 \\ 2 & 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 & 8 \\ 1 & -3 & 7 \end{bmatrix}$
3. $A = \begin{bmatrix} -1 & 2 & 8 \\ 0 & 3 & 5 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 8 \\ -2 & 0 & 0 \\ -5 & -7 & 3 \end{bmatrix}$
4. $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$
5. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \\ 1 & 0 \end{bmatrix}$
6. $A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

Q 6. Find the transpose of the following matrices:

1. $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ 2. $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \end{bmatrix}$ 3. $C = \begin{bmatrix} 4 & -1 & 2 \\ 2 & 3 & 0 \\ 1 & 2 & -2 \end{bmatrix}$ 4. $D = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Q 7. For each of the following square matrices, determine whether it is a symmetric matrix, a skew symmetric matrix, or neither:

1. $A = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$ 3. $C = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 0 & 5 \\ 2 & 5 & 3 \end{bmatrix}$ 5. $E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & 4 \end{bmatrix}$

2. $B = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 4 & 3 \\ -2 & 3 & 2 \end{bmatrix}$ 4. $D = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 3 \\ 0 & 3 & -4 \end{bmatrix}$

Q 8. Represent each of the following matrices as a sum of a symmetric matrix and a skew symmetric matrix:

1. $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ 3. $C = \begin{bmatrix} 4 & -1 \\ -1 & 0 \end{bmatrix}$

2. $B = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 4 & 3 \\ -2 & 3 & 2 \end{bmatrix}$ 4. $D = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 3 \\ 0 & 3 & -4 \end{bmatrix}$

Q 9. Prove the following property for matrices A and B: $(AB)^T = B^T A^T$
Where: $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 0 & 2 \end{bmatrix}$

Q 10. Let $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 4 & 1 \end{bmatrix}$. Calculate the products AA^T and $A^T A$.

Q 11. Find A^2, A^3 and A^4 if $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

Q 12. Find B^2 and B^3 if $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 2 & 0 & 1 \end{bmatrix}$.

Q 13. Given a matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. Verify the equation:

$$A^2 - 4A + I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where I is the 2×2 identity matrix.

Q 14. Given a matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix}$. Verify the equation:

$$A^3 - 7A^2 + 23A - 30I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Where I is the 3×3 identity matrix.

Q 15. Calculate the determinants of the following matrices:

1. $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$
2. $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$
3. $C = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$
4. $D = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}$

Q 16. Calculate the determinants of the following matrices:

1. $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$
2. $B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$
3. $C = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 0 & 5 \\ 2 & 5 & 3 \end{bmatrix}$
4. $D = \begin{bmatrix} 0 & 1 & -3 \\ 1 & -2 & 4 \\ -3 & 4 & 0 \end{bmatrix}$

Q 17. Calculate the adjoints of the following matrices:

1. $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$
2. $B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$
3. $C = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 0 & 5 \\ 2 & 5 & 3 \end{bmatrix}$

Q 18. Calculate the inverse matrices for the following matrices:

1. $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$
2. $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$
3. $C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 4 \\ -1 & 0 & -1 \end{bmatrix}$
4. $D = \begin{bmatrix} 5 & -2 & 3 \\ -1 & 4 & 1 \\ 2 & 3 & 6 \end{bmatrix}$
5. $E = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & 3 & -2 \end{bmatrix}$

Q 19. Use Cramer's Rule and Matrix Inversion Method both to find the solutions to the following systems of linear equations:

1. $\begin{aligned} 2x + 3y &= 11 \\ -4x + y &= 2 \end{aligned}$
2. $\begin{aligned} 3x - 2y &= 7 \\ 5x + 2y &= 12 \end{aligned}$
3. $\begin{aligned} x + 2y &= 4 \\ -3x - y &= -9 \end{aligned}$

Q 20. Use Cramer's Rule and Matrix Inversion Method both to find the solutions to the following systems of linear equations:

1. $\begin{aligned} 2x - y + 3z &= 7 \\ x + 2y + z &= 4 \\ 3x + y - 2z &= 5 \end{aligned}$
2. $\begin{aligned} 4x + 3y - z &= 5 \\ -2x + 2y + 3z &= 1 \\ 2x + y - 2z &= 6 \end{aligned}$
3. $\begin{aligned} x - 2y + z &= 2 \\ 2x + y - 3z &= 3 \\ x - 2y + 2z &= 4 \end{aligned}$

Q-25 → Find the rank of the matrix using elementary row operations.

(1) $A = \begin{bmatrix} 2 & 3 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 4 & 1 & 1 & -2 \\ 1 & 5 & 3 & -1 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -1 & 2 & 12 \\ 2 & 2 & -1 & 9 \\ 2 & 1 & -1 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \\ 2 & 4 & 3 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}$

Q-26 Find the rank of matrix by converting it into normal form.

1) $\begin{bmatrix} 2 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 4 & 2 & 4 & 3 \\ 4 & 6 & 7 & 3 \end{bmatrix}$

3) $\begin{bmatrix} 4 & 6 & 3 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ (4) $\begin{bmatrix} 3 & 4 & 3 \\ 4 & 2 & 4 \\ 2 & 1 & 3 \end{bmatrix}$

Q-23 Find the rank of the matrix

$$A = \begin{bmatrix} -1 & 1 & 1 & -2 \\ 1 & 0 & 1 & 2 \\ 1 & 5 & 3 & -1 \\ 2 & 3 & 0 & 1 \end{bmatrix}$$

Q-24 Find the rank of the matrix using determinant method.

1) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

4) $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ (5) $\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 3 \\ 3 & -3 & 9 \end{bmatrix}$

6) $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -2 & 2 & 0 \end{bmatrix}$

Q-21 Convert the following matrix into row echlon form using elementary row operations

1) $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

(2) $\begin{bmatrix} 3 & 2 & 7 & 4 \\ -1 & 3 & 2 & 2 \\ 2 & 5 & 3 & 6 \\ 5 & 7 & 4 & 10 \end{bmatrix}$

3) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

(4) $\begin{bmatrix} 3 & 3 & -3 & 3 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

5) $\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$

Q-22 Convert the following matrix into reduce row echlon form using elementary row operation.

1) $\begin{bmatrix} -2 & -4 & -2 & -10 & 0 \\ 3 & 6 & 1 & 13 & -4 \\ 2 & 4 & 1 & 9 & -2 \end{bmatrix}$

(2) $\begin{bmatrix} 2 & 1 & -1 & 4 \\ 1 & -1 & 2 & 12 \\ 2 & 2 & -1 & 9 \end{bmatrix}$