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(1) Use Taylor's series to find the expansion of $\log_e x$ in Powers of $(x-1)$ Hence find The Value of $\log 1.1$

$a = 1$

=> Let

$f(x) = \log_e x$

$f(x) = f(1)$

$= \log 1$

$= 0$

$f'(x) = \frac{1}{x} \Rightarrow f'(1) = \frac{1}{(1)^1} = 1$

$f''(x) = \frac{-1}{x^2} \Rightarrow f''(1) = \frac{-1}{(1)^2} = -1$

$f'''(x) = \frac{2}{x^3} \Rightarrow f'''(1) = \frac{2}{(1)^3} = 2$

$f^{iv}(x) = \frac{-6}{x^4} \Rightarrow f^{iv}(1) = \frac{-6}{(1)^4} = -6$

We know that the Taylor's series is

$f(x) = f(a) + (x-1) + f'(a) + \frac{(x-1)^2}{2!} + f''(a) + \frac{(x-1)^3}{3!}$

$f''' + \frac{(x-1)^4}{4!} + f^{iv}(x)$

$f(x) = 0 + (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4}$

Put $x = 1.1$

$$f(1.1) = (1.1-1) - \frac{(1.1-1)^2}{2} + \frac{(1.1-1)^3}{3} - \frac{(1.1-1)^4}{4}$$

$$= 0.1 - 0.005 + 0.0003 - 0.0002$$

$$f(1.1) = 0.09532$$

② Expand $\log \tan(\frac{\pi}{4}+x)$ in Powers of $x$ by using the taylor's Series.

$\Rightarrow$ We know that

$$f(a+h) = f(a) + h \cdot f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \dots \quad (1)$$

Let $a = \frac{\pi}{4}$ and $h = x$

$$\therefore f(a+x) = \log \tan\left(\frac{\pi}{4}+x\right)$$

$$f'(x) = \frac{1}{\tan x} \cdot \sec^2 x = \frac{\sec^2 x}{\tan x} \Rightarrow f'\left(\frac{\pi}{4}\right) = f'(a)$$

$$= \frac{\sec^2 \pi/4}{\tan \pi/4}$$

$$= \frac{(\sqrt{2})^2}{1}$$

$$= 2$$

Now

$$f'(x) = \frac{\sec^2 x}{\tan x}$$

$$= \frac{\tan^2 x + 1}{\tan x}$$

$$= \frac{\tan^2 x}{\tan x} + \frac{1}{\tan x}$$

$f'(x) = \tan x + \cot x$

$f''(x) = \sec^2 x - \operatorname{cosec}^2 x$

$f''(\frac{\pi}{4}) = \sec^2(\pi/4) - \operatorname{cosec}^2 \frac{\pi}{4}$

$= (\sqrt{2})^2 - (\sqrt{2})^2$

$= 2 - 2$

$= \underline{\underline{0}}$

$f'''(x) = (2\sec x)(\sec x \cdot \tan x) - (2\operatorname{cosec} x)$

$(-\operatorname{cosec} x \cdot \cot x)$

$= 2\sec^2 x \tan x + 2\operatorname{cosec}^2 x \cdot \cot x$

$\Rightarrow f'''(a) = f'''(\frac{\pi}{4})$

$= 2\sec^2 \frac{\pi}{4} \cdot \tan \frac{\pi}{4} + 2\operatorname{cosec}^2 \frac{\pi}{4} \cdot \cot \frac{\pi}{4}$

$= 2(\sqrt{2})^2(1) + 2(\sqrt{2})^2 \cdot (1)$

$= (2 \times 2) + (2 \times 2)$

$= \underline{\underline{8}}$

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$* \log \tan\left(\frac{\pi}{4}+x\right) = f(a) + x \cdot f'(a) + \frac{x^2}{2!} f''(a) + \frac{x^3}{3!} f'''(a)$

$= f\left(\frac{\pi}{4}\right) + x f'\left(\frac{\pi}{4}\right) + \frac{x^2}{2} f''\left(\frac{\pi}{4}\right) + \frac{x^3}{3!} f'''\left(\frac{\pi}{4}\right) + \dots$

$= 0 + x(2) + \frac{x^2}{2}(0) + \frac{x^3}{6}(8) + \dots$

$= 2x + \frac{4}{3}x^3 + \dots$

(3) Using Taylor's Series find $\sqrt{36.12}$

GTU Dec-14

Solu$^n$

$\Rightarrow$ Let $f(a+h) = \sqrt{a+h} = \sqrt{36+0.12}$

$a = 36,\ h = 0.12$

Let $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(a) = f'(36) = \frac{1}{2\sqrt{36}} = \frac{1}{12}$

$f''(x) = \frac{-1}{4} x^{-3/2} \Rightarrow f''(a) = f''(36) = \frac{-1}{4}[36]^{-3/2}$

$= \frac{-1}{4}[6^2]^{-\frac{3}{2}}$

$= \frac{-1}{4} \times \frac{1}{6^3}$

$= \frac{-1}{4} \times \frac{1}{216}$

$= -\frac{1}{864}$

$f'''(x) = \frac{3}{8} x^{-\frac{5}{2}}$

$\Rightarrow f'''(a) = f'''(36)$

$= \frac{3}{8} [36]^{-\frac{5}{2}}$

$= \frac{3}{8} [6^2]^{-\frac{5}{2}}$

$= \frac{3}{8} \times \frac{1}{6^5} = \frac{3}{62208}$ and so on

Putting these values in the Taylor's series

$f(a+h) = f(a) + f'(a) \cdot h + f''(a) \cdot \frac{h^2}{2!} + f'''(a) \cdot \frac{h^3}{3!} + \ldots$

$= 6 + \frac{1}{12}(0.12) - \frac{1}{864} \frac{(0.12)^2}{2} + \frac{3}{62208} \frac{(0.12)^3}{6} \ldots$

$= 6 + 0.01 - 0.0000083 + 0.000000$

$= 6.0100083$ (Approximate)

(4) Expand the function $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in Power of $(x-3)$

Soln:-

$$f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$$
$$f(3) = (3)^4 - 11(3)^3 + 43(3)^2 - 60(3) + 14$$
$$= 81 - 297 + 387 - 180 + 14$$
$$f(3) = 5$$

$f(a) = x - 3$
$x = 3$
$f(a) = 3$

~~f(3) =~~

$$f'(a) = 4x^3 - 33x^2 + 86x - 60$$
$$f'(3) = 4(3)^3 - 33(3)^2 + 86(3) - 60$$
$$= 96 - 297 + 258 - 60$$
$$f'(3) = 9$$

$$f''(x) = 12x^2 - 66x + 86$$
$$f''(3) = 12(3)^2 - 66(3) + 86$$
$$= 108 - 198 + 86$$
$$f''(3) = -4$$

$$f'''(x) = 24x - 66$$
$$f'''(3) = 24(3) - 66$$
$$= 72 - 66$$
$$= 6$$

$f^{iv}(x) = 24$

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$$f'(a+h) = f(a) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3)$$

$$+ \frac{(x-3)^3}{3!}f'''(3) + \frac{(x-3)^4}{4!}f^{iv}(3)$$

$$= 5 + 9(x-3) - 2(x-3)^2 + (x-3)^3 + (x-3)^4$$

(5) Find the Taylor's series expansion of $f(x) = x^3 - 2x + 4$ at $a = 2$

GTU JUN-2010

Solu$^n$

=>

Since $f(x) = x^3 - 2x + 4 \Rightarrow f(a) = f(2)$

$$= 2^3 - 2(2) + 4$$

$$= 8$$

$f'(x) = 3x^2 - 2 \Rightarrow f'(a) = f'(2) = 3(2)^2 - 2$

$$= 10$$

$f''(x) = 6x \quad \Rightarrow f''(a) = f''(2) = 6 \times 2 = 12$

$f'''(x) = 6 \quad \Rightarrow f'''(a) = f'''(2) = 6$

$f^{iv}(x) = 0$

All other derivatives gives the value zero.

we know that Taylor series expansion is

$$f(x) = f(a) + (x-a)\,f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

$$= f(2) + (x-2)\,f'(2) + \frac{(x-2)^2}{2!} f''(2) + \frac{(x-2)^3}{3!} f'''(2) + 0$$

$$= 8 + (x-2)\,10 + \frac{(x-2)^2}{2}(12) + \frac{(x-2)^3}{6}(6) + 0$$

$$= 8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$$

(6) Find the Maclaurin Series Expansion of $f(x) = \frac{1}{\sqrt{4-x}}$

Dec-11

Soln:

Since $f(x) = \frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}}$

$f(0) = (4-0)^{-\frac{1}{2}}$

$= (2^2)^{-\frac{1}{2}}$

$a = 0$

$= \frac{1}{2}$

$$f'(x) = \frac{-1}{2}(4-x)^{-\frac{1}{2}-1}(-1)$$

$$= \frac{1}{2}(4-x)^{-\frac{3}{2}}$$

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$f'(0) = \frac{1}{2}[4-0]^{-\frac{3}{2}}$

$= \frac{1}{2}[2^2]^{-\frac{3}{2}}$

$= \frac{1}{2} \times \frac{1}{2^3} = \frac{1}{2^4} = \frac{1}{16}$

$f''(x) = \frac{1}{2} \times \left(-\frac{3}{2}\right) \frac{[4-x]^{-\frac{3}{2}-1}(-1)}{(-1)}$

$= -\frac{3}{4} \times (-1)\ (4-x)^{-\frac{5}{2}}$

$= \frac{3}{4}(4-x)^{-\frac{5}{2}}$

$f''(0) = \frac{3}{4}[4-0]^{-\frac{5}{2}}$

$= \frac{3}{4}[2^2]^{-\frac{5}{2}}$

$= \frac{3}{2^2} \times \frac{1}{2^5} = \frac{3}{2^7} = \frac{3}{128}$

and so on

we know that the maclarin's series is

$f(x) = \left(f(0) + x \cdot f'(0) + \frac{x^2}{2!} f''(0)\right) + \frac{x^3}{3!} f'''(0) + \ldots$

$= \frac{1}{2} + x\left(\frac{1}{16}\right) + \frac{3}{2!(128)} x^2 + \ldots]$

$= \frac{1}{2}\left[1 + \frac{1}{8}x + \frac{3}{2!\,64} x^2 + \ldots\right)$

(7) Expand $5+4(x-1)^2-3(x-1)^3+(x-1)^4$ in ascending Powers of $x$

Dec-13 GTV ⇒ Solu^n

since $a=0$

$f(x)=5+4(x-1)^2-3(x-1)^3+(x-1)^4$

$f(0)=5+4(0-1)^2-3(0-1)^3+(0-1)^4$

$=5+4-3(-1)+1$

$=5+4+3+1$

$=13$

$f'(x)=0+8(x-1)-9(x-1)^2+4(x-1)^3$

$f'(0)=8(0-1)-9(0-1)^2+4(0-1)^3$

$=8(-1)-9(1)+4(-1)$

$=-8-9-4$

$=-21$

$f''(x)=8-18(x-1)+12(x-1)^2$

$f''(0)=8-18(0-1)+12(0-1)^2$

$=8+18+12$

$=38$

$f'''(x)=-18+24(x-1)$

$f'''(0)=-18+24(0-1)$

$=-18-24$

$=-42$

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$f^{iv}(x) = 24$
$f^{iv}(0) = 24$

we know that the maclaurin's series is

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0)$$

$$= 13 + x(-21) + \frac{x^2}{2}(38) + \frac{x^3}{6}(-42) + \frac{x^4}{24}(24)$$

$$= 13 - 21x + 19x^2 - 7x^3 + x^4$$

(8) Express $(x-1)^4 + 2(x-1)^3 + 5(x-1)' + 2$ in ascending Power of $x$

GTU JUN-16

$a = 0$

Solu$^n$!

Since

$$f(x) = (x-1)^4 + 2(x-1)^3 + 5(x-1) + 2$$

$$f(a) \Rightarrow f(0) = (0-1)^4 + 2(0-1)^3 + 5(0-1) + 2$$

$$= 1 + 2(-1) + 5(-1) + 2$$

$$= 1 - 2 - 5 + 2$$

$$= 3 - 7$$

$$= -4$$

$$f'(x) = 4(x-1)^3(1) + 6(x-1)^2(1-0) + 5(1) + 0$$

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$$f'(0) = 4(0-1)^3 + 6(0-1)^2 + 5$$
$$= 4(-1) + 6(1) + 5$$
$$= -4 + 6 + 5$$
$$= 11 - 4$$
$$= 7$$

$$f''(x) = 12(x-1)^2 + 12(x-1)(1-0) + 0$$
$$f''(0) = 12(0-1)^2 + 12(0-1)$$
$$= 12(1) + 12(-1)$$
$$= 12 - 12$$
$$= 0$$

$$f'''(x) = 24(x-1)(1-0) + 12(1-0)$$
$$f'''(0) = 24(0-1) + 12$$
$$= -24 + 12$$
$$= -12$$

$$f^{IV}(x) = 24(1-0) + 0$$
$$= 24$$
$$f^{IV}(0) = 24$$

Putting these values in the maclaurin's series

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{IV}(0) + \cdots$$

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$$= -4 + 7x + 0 - 12\frac{x^3}{6} + 24\frac{x^4}{24}$$

$$f(x) = -4 + 7x - 2x^3 + x^4$$

(9) Find the Maclaurin's series of $e^x$

Solu$^n$

$f(x) = e^x$

$f(0) = e^0$

$f(0) = 1$

$f(x) = e^x$

$f'(0) = e^0$

$f'(0) = 1$

$f'' = e^x$

$f''(0) = e^0$

$f''(0) = 1$

$f''' = e^x$

$f'''(0) = e^0$

$f'''(0) = 1$

$$f(x) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x)$$

$$= 1 + (0)(1) + \frac{(0)^2}{2}(1) + \frac{(0)^3}{6}(1)$$

$f(x) = 1$

(10) Find the Taylor Polynomial of order 3 for about $x = 4$

GTU Jun-13

$f(x) = \sqrt{x}$

Solu$^n$

Let $f(x) = \sqrt{x} \Rightarrow f(4) = \sqrt{4} = 2$

$= (x)^{1/2}$

$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \times 2} = \frac{1}{4}$

$f''(x) = \frac{-1}{4 \cdot x^{3/2}} \Rightarrow f''(4) = \frac{-1}{4[4]^{3/2}} = \frac{-1}{4[2^2]^{3/2}}$

$= \frac{-1}{4 \times 8} = \frac{-1}{32}$

$f'''(x) = \frac{3}{8} \cdot \frac{1}{x^{5/2}} \Rightarrow f'''(4) = \frac{3}{8} \frac{1}{[4]^{5/2}}$

$= \frac{3}{8}\left[\frac{1}{(2^2)^{5/2}}\right]$

$= \frac{3}{8} \times \frac{1}{2^5} = \frac{3}{256}$

We know that Taylor Expansion is

$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \ldots$

Taylor Polynomical of order 3. is

$P_3(x) = f(4) + (x-4) f'(4) + \frac{(x-4)^2}{2} f''(4) + \frac{(x-4)^3}{6} f'''(4)$

$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

(11) Expand log $\sec x$ in maclaurin's serie

⇒ Solu^n

since $f(x) = \log(\sec x)$

$f(a) = f(0)$

$= \log \sec 0$

$= \log 1$

$= 0$

$f'(x) = \frac{1}{\sec x}(\sec x \cdot \tan x)$

$f'(x) = \tan x \Rightarrow f'(a) = f'(0) = \tan 0 = 0$

$f''(x) = \sec^2 x \Rightarrow f''(0) = \sec^2 0 = 1$

$f'''(x) = (2 \sec x \ (\sec x \cdot \tan x)$

$f'''(x) = 2 \cdot \sec^2 x \cdot \tan x$

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$f'''(a) = 2\sec^2(0) \cdot \tan 0$
$= 2(1)(0)$
$= 0$

$f''''(x) = 2\sec^2 x(\sec^2 x) + 2\tan x(2\sec x \cdot \sec x \tan x)$

$= 2\sec^4 x + 4\tan^2 x \cdot \sec^2 x$

$f^{IV}(a) = f^{IV}(0)$
$= 2\sec^4 0 + 4\tan^2 0 \sec^2 0$
$= 2(1) + 4(0)(1)$
$= 2 + 0$
$= 2$

We know that the maclaurin's series is

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{IV}(0) + \dots$$

$$\log\sec(x) = 0 + x(0) + \frac{x^2}{2}(1) + \frac{x^3}{6}(0) + \frac{x^4}{24}(2) + \dots$$

$$= \frac{x^2}{2} + \frac{x^4}{12} + \dots$$

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(12) Prove that $\lim_{x \to 0} \log_x \tan x = 1$

Soln

Let $l = \lim_{x \to 0} \log_x = \tan x$

$$= \lim_{x \to 0} \frac{\log \tan x}{\log x} \quad \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{\tan x} \cdot \sec^2 x}{1/x}$$

$$= \lim_{x \to 0} \frac{x}{\tan x} \cdot \lim_{x \to 0} \sec^2 x$$

$$= 1 \times \sec^2 0$$

$$= 1 \times 1$$

$$\boxed{l = 1}$$

(13) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$

$\Rightarrow$ Soln $\log l = \lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$

$$= \lim_{x \to \frac{\pi}{2}} \cot x \log \sec x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \sec x}{1/\cot x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \sec x}{\tan x} \quad \left(\frac{\infty}{\infty} \text{ Form}\right)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\sec x} \cdot \sec x \tan x}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan x}{\sec^2 x} \quad \left(\frac{\infty}{\infty} \text{ Form}\right)$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec^2 x}{2 \sec x \cdot \tan x \cdot \sec x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{2 \tan x}$$

$$= \frac{1}{2 \tan \frac{\pi}{2}}$$

$$= \frac{1}{2(\infty)}$$

$$\log l = \frac{1}{\infty}$$

$$l = e^0$$
$$l = 1$$

(14) find $\lim\limits_{x \to 0} \left[\frac{1}{\sin x} - \frac{1}{x}\right]$

$$l = \lim_{x \to 0} \left[\frac{1}{\sin x} - \frac{1}{x}\right].$$

$$= \lim_{x \to 0} \left[\frac{x - \sin x}{x \cdot \sin x}\right] \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \left[\frac{x - \sin x}{x \cdot x} \cdot \frac{1 \times x}{\sin x}\right]$$

$$= \lim_{x \to 0} \left(\frac{x - \sin x}{x^2}\right) \lim_{x \to 0} \frac{x}{\sin x}$$

$$= \lim_{x \to 0} \frac{x - \sin x}{x^2}$$

$$= \lim_{x \to 0} = \frac{1 - \cos x}{2x}$$

$$= \lim_{x \to 0} \frac{0 - (\sin x)}{2} \quad \left(\frac{0}{0} \text{ from}\right)$$

$$= \lim_{x \to 0} \frac{\sin x}{2} \quad \left(\frac{0}{0} \text{ from}\right)$$

$= \lim_{x \to 0} \frac{\sin 0}{2}$

$= 0/2$

$\underline{l = 0}$

(15) Prove that $\lim_{x \to \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}} = 1$

<u>Solⁿ</u>

$\Rightarrow$ Let $l = \lim_{x \to \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$ ($\infty^0$ form)

$\log l = \lim_{x \to \infty} \frac{1}{x} \log\left(\frac{1}{x}\right)$

$= \lim_{x \to \infty} \frac{-\log x}{x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= -\lim_{x \to \infty} \frac{x}{1}$

$= 0$

$l = e^0 = 1$

(16) Evaluate $\lim_{x \to 0} \left[\frac{1}{x^2} - \cot^2 x\right]$

Soluⁿ

$$l = \lim_{x \to 0} \left[\frac{1}{x^2} - \cot^2 x\right]$$

$$= \lim_{x \to 0} \left(\frac{1 - \cot^2 x}{x^2}\right) \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x \to 0} \frac{\tan^2 x}{x^2}$$

$$= \lim_{x \to 0} 1$$

$$l = 1$$

(17) Evaluate: $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$

Soluⁿ

$$\Rightarrow \log l = \lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \cos x \cdot \log \cos x$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\log \cos x}{\frac{1}{\cos x}}$$

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$$\lim_{x \to \frac{\pi}{2}} \frac{\log \cos x}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\tan x}{\sec^2 x}$$

$$= \frac{-0}{1}$$

$\log e = 0$

$e = e^0$

$\underline{e = 1}$

(18) Evaluate $\lim_{x \to 1} (1-x) \tan \frac{\pi}{2} x$

<u>Soln</u>

$$l = \lim_{x \to 1} (1-x) \tan \frac{\pi}{2} x$$

$$= \lim_{x \to 1} \frac{(1-x)}{\left(\frac{1}{\tan \frac{\pi}{2} x}\right)}$$

$$= \lim_{x \to \pi} \lim_{x \to 1} \frac{(1-x)}{\cot \frac{\pi}{2} x}$$

$$\lim_{x \to 1} \frac{(-1)}{-\operatorname{cosec}^2(\frac{\pi}{2}x)(\frac{\pi}{2})}$$

$$= \frac{2}{\pi} \lim_{x \to 1} \frac{1}{\operatorname{cosec}^2(\frac{\pi}{2}x)}$$

$$= \frac{2}{\pi} \lim_{x \to 1} \sin^2(\frac{\pi}{2}x)$$

$$= \frac{2}{\pi} (\sin^2(\frac{\pi}{2}))$$

$$= \frac{2}{\pi}(1)$$

$$= \frac{2}{\pi}$$

(19) Evaluate $\lim_{x \to 0} \frac{1}{x}(1 - x \cot x)$

Sol$^n$

$\Rightarrow$ we have the Limit is an indeterminate form $\infty - \infty$ thus by L' Hospitals rule

$$l = \lim_{x \to 0} \frac{1}{x} [1 - x \cot x]$$

$$= \lim_{x \to 0} \left[ \frac{1}{x} - \frac{x}{x} \cot x \right]$$

(2)

$$= \lim_{x \to 0} \left[ \frac{1}{x} - \cot x \right] \quad (\infty - \infty \text{ form})$$

$$= \lim_{x \to 0} \left[ \frac{1}{x} - \frac{1}{\tan x} \right]$$

$$= \lim_{x \to 0} \left[ \frac{\tan x - x}{x \cdot \tan x} \right]$$

$$= \lim_{x \to 0} \left( \frac{\tan x - x}{x} \right) \cdot \frac{1}{\tan x}$$

$$= \lim_{x \to 0} \left( \frac{\tan x - x}{x \times x} \right) \cdot \lim_{x \to 0} \frac{x}{\tan x}$$

$$= \lim_{x \to 0} \left( \frac{\tan x - x}{x^2} \right) \cdot 1 \quad \left( \frac{0}{0} \text{ form} \right)$$

Diff $= \lim_{x \to 0} \frac{\sec^2 x - 1}{2x} \quad \left( \frac{0}{0} \text{ form} \right)$

$$\lim_{x \to 0} \frac{2 \sec x \cdot (\sec x \cdot \tan x) - 0}{2}$$

$$= \frac{2 \sec^2 0 \cdot \tan 0}{2}$$

$$= 2(1)^2 (0)$$

$$l = 0$$

(20) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos x}$

Soln

=) we have the limit is an indeterminate form $\left(\frac{0}{0}\right)$

thus by L'Hospital rule

$$\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2}{-\sin x} = \frac{-2}{\phantom{x}}$$

(21) Find $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$

Soln

=) Let $l = \lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$ ($\infty - \infty$ Form)

$$= \lim_{x \to 0} \left(\frac{\sin^2 x - x^2}{x^2 \sin^2 x}\right)$$

$$= \lim_{x \to 0} \left(\frac{\sin^2 x - x^2}{x^4}\right)$$

$$= \lim_{x \to 0} \left(\frac{2 \sin x \cos x - 2x}{4x^3}\right)$$

$$= \lim_{x \to 0} \left( \frac{\sin 2x - 2x}{4x^3} \right)$$

$$= \lim_{x \to 0} \left( \frac{2\cos 2x - 2}{12x^2} \right)$$

$$= \lim_{x \to 0} \left( \frac{-4\sin 2x}{24x} \right)$$

$$= \lim_{x \to 0} \left( \frac{-8\cos 2x}{24} \right)$$

$$= \frac{-8}{24}$$

$$= -\frac{1}{3}$$

(22) Evaluate $\lim_{x \to 0} (\cos x)^{\cot x}$

Solu$^n$

$\Rightarrow$ Let $\quad l = \lim_{x \to 0} (\cos x)^{\cot x} \quad (1^\infty \text{ form})$

$$\log l = \lim_{x \to 0} \cot x \cdot \log(\cos x)$$

$$= \lim_{x \to 0} \frac{\log(\cos x)}{\tan x} \quad \left( \frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \to 0} \frac{\frac{1}{\cos x}(-\sin x)}{\sec^2 x}$$

$$= 0$$

$$l = e^0 = 1$$