

Course > Week 9... > Proble... > Proble...

# **Problem Set 9**

Problems 1-6 correspond to "Linear Projections"

### Problem 1

1/1 point (graded)

In  $\mathbb{R}^2$  , what is the unit vector corresponding to the  $x_1$ -direction?











Submit

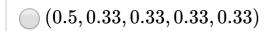
## Problem 2

1/1 point (graded)

What is the unit vector in the same direction as (3,2,2,2,2)?

(1, 0.67, 0.67, 0.67, 0.67)

(0.6,	0.4	0.4	0.4	0.4
(0.0,	0.4,	0.4,	0.4,	U.4





Submit

### Problem 3

1/1 point (graded)

What is the projection of the vector (3,5,-9) onto the direction (0.6,-0.8,0)?

-2.2

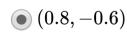
Submit

-2.2

# Problem 4

1/1 point (graded)

What is the (unit) direction along which the projection of  $\left(4,-3\right)$  is largest?



$$\bigcirc (-0.6, -0.8)$$

$$(-0.8, 0.6)$$

~

Submit

### Problem 5

1/1 point (graded)

What is the (unit) direction along which the projection of (4,-3) is smallest?

- $\bigcirc \ (0.8,-0.6)$
- (-0.6, -0.8)
- (-0.8, 0.6)
- (0.8, 0.6)



Submit

# Problem 6

1/1 point (graded)

The projection of vector x onto direction u is exactly zero. Which of the following statements is necessarily true? Select all that apply.

- ightharpoonup u is orthogonal to x.
- $\begin{tabular}{l} \hline u \end{tabular}$  is in the opposite direction to x.
- ightharpoonup u is at right angles to x.
- It is not possible to have a projection of zero.



Submit

Problems 7-8 correspond to "Principal component analysis I: one-dimensional projection"

#### Problem 7

4/4 points (graded)

A three-dimensional data set has covariance matrix

$$\Sigma = \left(egin{array}{ccc} 4 & 2 & -3 \ 2 & 9 & 0 \ -3 & 0 & 9 \end{array}
ight).$$

a) What is the variance of the data in the  $x_1$ -direction?



b) What is the correlation between  $x_1$  and  $x_3$ ?



c) What is the variance in the direction (0,-1,0)?



d) What is the variance in the direction of (1,1,0)?



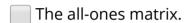
Submit

### Problem 8

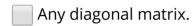
1/1 point (graded)

Which of the following covariance matrices has the property that the variance is the same in any direction? Select all that apply.











Submit

Problems 9-11 correspond to "Principal component analysis II: the top k directions"

# Problem 9

8/8 points (graded)

Let  $u_1,u_2\in\mathbb{R}^d$  be two vectors with  $\|u_1\|=\|u_2\|=1$  and  $u_1\cdot u_2=0$ . Define U to be the matrix whose columns are  $u_1$  and  $u_2$ .

What are the dimensions of the following matrices?

a)  ${\it U}$ 

# of Rows =

d



# of Columns =	Prob
2	<b>~</b>
2	
b) $U^T$	
# of Rows =	
2	<b>~</b>
2	
# of Columns	
d	<b>~</b>
c) $UU^T$	
# of Rows =	
d	<b>~</b>
# of Columns =	
d	~
d) $u_1u_1^T$	
# of Rows =	
d	<b>~</b>

# of Columns =

d

**~** 

Submit

#### Problem 10

1/1 point (graded)

Continuing from the previous problem, let  $u_1,u_2\in\mathbb{R}^d$  be two vectors with  $\|u_1\|=\|u_2\|=1$  and  $u_1\cdot u_2=0$ , and define U to be the matrix whose columns are  $u_1$  and  $u_2$ .

Which of the following linear transformations sends points  $x \in \mathbb{R}^d$  to their (two-dimensional) projections onto directions  $u_1$  and  $u_2$ ? Select all that apply.

$$lacksquare x \mapsto (u_1 \cdot x, u_2 \cdot x)$$

$$lacksquare x\mapsto (u_1\cdot x)\,u_1+(u_2\cdot x)\,u_2$$

$$lacksquare x \mapsto U^T x$$

$$lacksquare x\mapsto UU^Tx$$



Submit

### Problem 11

2/2 points (graded)

For a particular four-dimensional data set, the top two eigenvectors of the covariance matrix are:

$$rac{1}{2}inom{1}{1}{1}{1}{1}, rac{1}{2}inom{-1}{1-1}{1}.$$

a) What is the PCA projection of point (2,4,2,6) into two dimensions? Write it in the form (a,b).





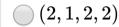


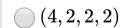


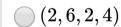


b) What is the reconstruction, from this projection, to a point in the original four-dimensional space? Write it in the form (a,b,c,d)

loom (2, 5, 2, 5)









Submit

Problems 12-14 correspond to "Linear algebra V: eigenvalues and eigenvectors"

## Problem 12

0/2 points (graded)

Consider the 2 imes 2 matrix  $M = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$  .

a) One of its eigenvectors is $\frac{1}{\sqrt{2}}\begin{pmatrix} 1\\ 1 \end{pmatrix}$	. )	. What is the corresponding eigenvalue?
$\sqrt{2} \setminus 1$	. /	1 0 0



b) Its other eigenvector is  $\frac{1}{\sqrt{2}} \binom{1}{-1}$  . What is the corresponding eigenvalue?



Submit

### Problem 13

6/6 points (graded)

A  $2 \times 2$  matrix M has eigenvalues 10 and 5.

a) What are the eigenvalues of 2M (that is, each entry of M is multiplied by 2)?

Larger eigenvalue =



Smaller eigenvalue =

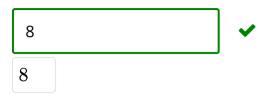


b) What are the eigenvalues of M+3I, where I is the 2 imes 2 identity matrix?

Larger eigenvalue =

13

Smaller eigenvalue =



c) What are the eigenvalues of  $M^2=MM$ ?

Larger eigenvalue =

100

Smaller eigenvalue =



Submit

# Problem 14

5/7 points (graded)

A certain three-dimensional data set has covariance matrix

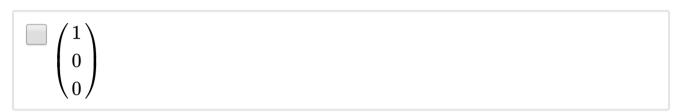
$$\begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

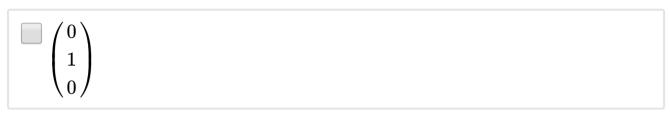
•

a) Consider the direction  $u=\left(1,1,1\right)/\sqrt{3}$ . What is variance of the projection of the data onto direction u?



b) Which of the following are eigenvectors of the covariance matrix? Select all that apply.







$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1\\1\\0 \end{pmatrix}$$

$$egin{array}{c} egin{array}{c} egin{array}{c} 0 \ 1 \ 1 \end{array} \end{pmatrix}$$

$$\begin{array}{c} \checkmark \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \end{array}$$

c) Find the eigenvalues of the covariance matrix. List them in decreasing order.

Problems 15-17 correspond to "Linear algebra VI: spectral decomposition"

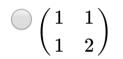
# Problem 15

1/1 point (graded)

M is a 2 imes 2 real-valued symmetric matrix with eigenvalues  $\lambda_1 = 6, \lambda_2 = 1$  and corresponding eigenvectors

$$u_1=rac{1}{\sqrt{5}}inom{2}{1}, \ \ u_2=rac{1}{\sqrt{5}}inom{-1}{2}.$$

What is M?



$$\bigcirc \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$$



Submit

## Problem 16

1/1 point (graded)

For a certain data set in d-dimensional space, the covariance matrix has the following interesting property: there are k positive eigenvalues and the rest are zero (where k < d). What can we conclude from this? Select all that apply.

lacksquare Each of the data points has at most $k$ nonzero coordinates.					
$ lap{igg }$ Each data point can be expressed as a linear combination of the top $k$ eigenvectors.					
It is possible to discard $d-k$ of the coordinates without losing any of the variance in the data.					
Submit					
Problem 17					
1/1 point (graded) A data set in $\mathbb{R}^d$ has a covariance matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$ . Under which of the following conditions is PCA most likely to be effective as a form of dimensionality reduction? Select all that apply.					
When the $\lambda_i$ are approximately equal.					
$lacksquare$ When most of the $\lambda_i$ are close to zero.					
When most of the $\lambda_i$ are close to 1.					
$lacksquare$ When the sequence $\lambda_1,\lambda_2,\ldots$ is rapidly decreasing.					
When the sequence $\lambda_1, \lambda_2, \ldots$ is rapidly decreasing.					

© All Rights Reserved