# Problem Set 6

Problems 1-5 correspond to "A simple linear classifier"

## Problem 1

3/3 points (graded)

A linear classifier on $\mathbb{R}^2$ is specified by $w = (-1, 3)$ and $b = -6$.

a) At what point does the decision boundary intersect the $x_1$-axis? (Just give the $x_1$-intercept, a real number.)

-6 ✔

$-6$

b) At what point does the decision boundary intersect the $x_2$-axis? (Just give the $x_2$-intercept.)

2 ✔

$2$

c) What label, $1$ or $-1$, is assigned to the point $(1, 1)$?

-1 ✔

$-1$

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## Problem 2

1/1 point (graded)

A particular line in $\mathbb{R}^2$ passes through the points $(0,1)$ and $(2,0)$ and is specified by equation $w \cdot x + b = 0$, where $b = -2$ and $w \in \mathbb{R}^2$. What is $w$?

- $w = (0,1)$
- $w = (0,2)$
- **$w = (1,2)$**
- $w = (2,-1)$

✔

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## Problem 3

1/1 point (graded)

The Perceptron algorithm makes an update whenever it encounters a data point $(x,y)$ that is "misclassified" by the current $w, b$. What does this mean, precisely? Choose the best option from this list.

- $y\,(w \cdot x + b) = 0$
- $y\,(w \cdot x + b) < 0$
- **$y\,(w \cdot x + b) \leq 0$**
- $y\,(w \cdot x + b) > 0$

✔

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## Problem 4

1/1 point (graded)

A particular data set of $n$ points is randomly permuted and then the Perceptron algorithm is run on it, repeatedly cycling through the points until convergence. It converges after $k$ updates. Which of the following must be true? Select all that apply.

- [ ] $n \geq k$
- [x] If this process were repeated with a different random permutation, then the algorithm would again converge.
- [ ] If this process were repeated with a different random permutation, then the algorithm would again make $k$ updates before convergence.
- [x] The data is linearly separable.

✔

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## Problem 5

1/1 point (graded)

The Perceptron algorithm is run on a data set, and converges after performing $p+q$ updates. Of these updates, $p$ are on data points whose label is $-1$ and $q$ are on data points whose label is $+1$. What is the final value of parameter $b$?

- (•) $q-p$
- ( ) $p+q$
- ( ) $p-q$
- ( ) $q$

✔

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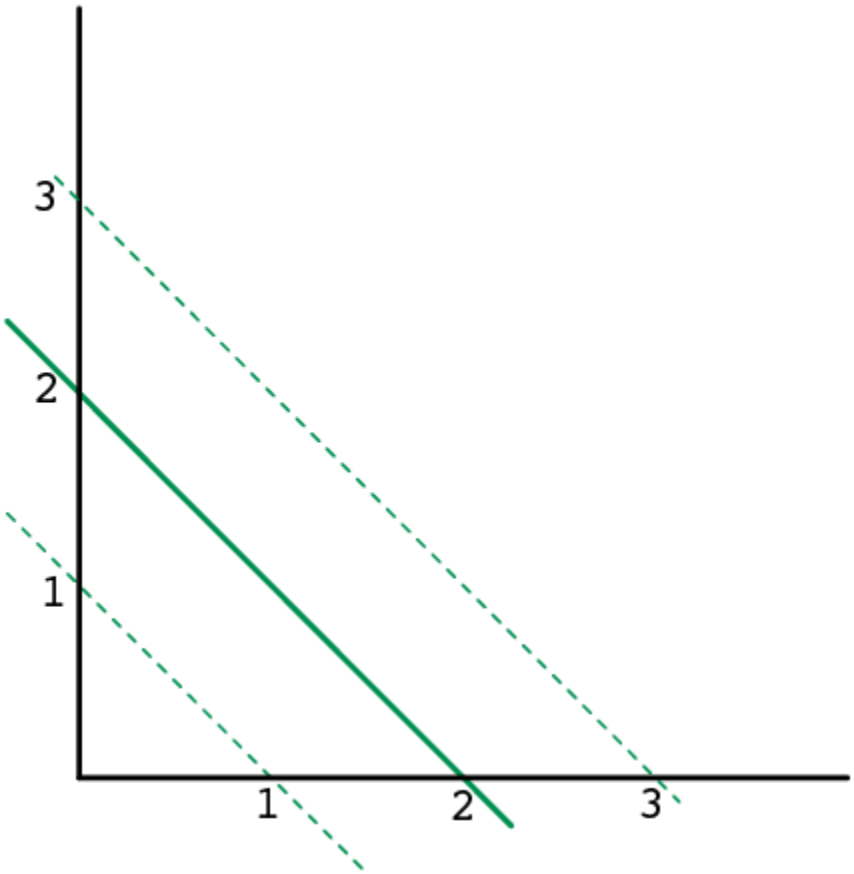
Problems 6-8 correspond to "Support vector machines I"

## Problem 6

1/1 point (graded)

The figure below shows a two-dimensional linear separator $w \cdot x + b = 0$, along with the parallel lines $w \cdot x + b = -1$ and $w \cdot x + b = 1$.




What is the margin of this classifier?

- (•) A number between 0.5 and 1.
- ( ) 1.
- ( ) A number between 1 and 2.
- ( ) 2.

A number greater than 2.

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## Problem 7

4/5 points (graded)

A support vector machine classifier is learned for a data set in $\mathbb{R}^2$. It is given by $w = (3, 4)$ and $b = -12$.

a) What is the $x_1$-intercept of the decision boundary?

4

$4$

b) What is the $x_2$-intercept of the decision boundary?

3

$3$

c) What is the margin of this classifier?

0.2

$0.2$

d) It turns out that the data set has two distinct support vectors of the form $(1, ?)$. What are they?

(give the missing $x_2$ coordinates for the support vectors with the smaller $x_2$ value first)

2

2

1 ✖

1

Submit

## Problem 8

4/4 points (graded)

Consider the following small data set in $\mathbb{R}^2$:

Points $(1,2)$, $(2,1)$, $(2,3)$, $(3,2)$ have label $-1$.

Points $(4,5)$, $(5,4)$, $(5,6)$, $(6,5)$ have label $+1$.

Now, suppose (hard margin) SVM is run on this data.

a) What is the $x_1$-intercept of the decision boundary?

7 ✔

7

b) What is the $x_2$-intercept of the decision boundary?

7 ✔

7

c) What is $w$?

- $w = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$w = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$w = \begin{pmatrix} 2 \\ 1/2 \end{pmatrix}$

$w = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ ✔

d) What is $b$?

-3.5 ✔

$-3.5$

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Problems 9-12 correspond to "Support vector machines II"

## Problem 9

4/4 points (graded)

Here is the optimization problem for the soft-margin SVM.

$$\min_{w\in\mathbb{R}^d, b\in\mathbb{R}, \xi\in\mathbb{R}^n} \quad \|w\|^2 + C\sum_{i=1}^{n}\xi_i$$
$$\text{s.t.: } y^{(i)}(w\cdot x^{(i)} + b) \geq 1 - \xi_i \quad \text{for all } i = 1, 2, \ldots, n$$
$$\xi \geq 0$$

a) How many slack variables are there? The answer should be a function of $n$ and/or $d$.

n ✔

b) What setting of $C$ will recapture the hard-margin SVM?

- Very small $C$
- Very large $C$
- There is no value of $C$ that will do this

✔

**Answer**
Correct: Larger $C$ imposes a heavier penalty on slack.

c) As $C$ is increased, what happens to the margin of the linear classifier that is returned?

- The margin gets larger.
- The margin gets smaller.
- The margin is unchanged.

- The way in which the margin changes is unpredictable.

✔

**Answer**
Correct:
As $C$ grows, the optimization problem places more emphasis on classifying the training data correctly and less on having a big margin.

d) Suppose we have a data set that is linearly separable and we use it to train both a hard-margin SVM $(w_H, b_H)$ and a soft-margin SVM $(w_S, b_S)$. Which of the following statements is true? Select all that necessarily apply.

- [ ] Both linear classifiers have zero training error.
- [x] $\|w_H\| \geq \|w_S\|$
- [ ] $\|w_H\| \leq \|w_S\|$
- [x] The margin achieved by $(w_H, b_H)$ is at most the margin achieved by $(w_S, b_S)$.

✔

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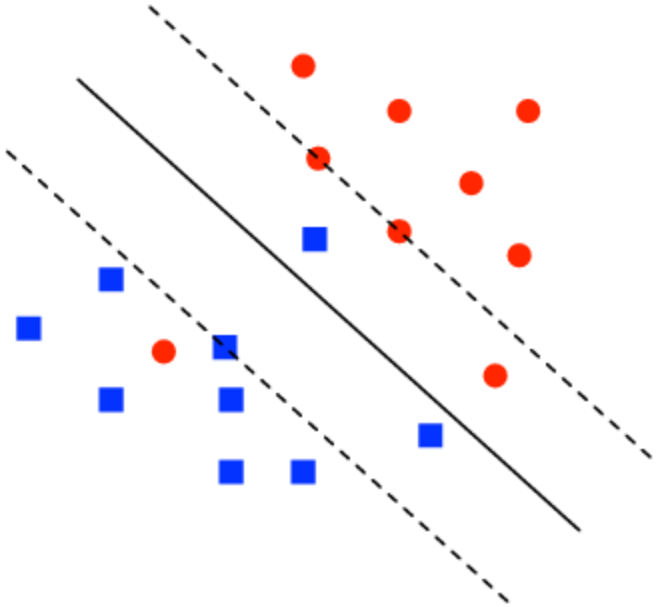
## Problem 10

4/4 points (graded)
The picture below shows the decision boundary obtained upon running soft-margin SVM on a small data set of blue squares and red circles.



a) How many support vectors are there?

7 ✔

7

b) What is the largest slack value on a red (circle-shaped) point, roughly?

2.5 ✔

2.5

c) What is the largest slack value on a blue (square-shaped) point, roughly?

1.5 ✔

1.5

d) Suppose the factor $C$ in the soft-margin SVM optimization problem were increased. Would you expect the margin to **`increase`** or **`decrease`**?

decrease ✔

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## Problem 11

1/1 point (graded)

Would it ever make sense to use the soft-margin SVM on a linearly separable data set? Select all that apply.

- [ ] No, unless you are unsure whether the data is linearly separable.
- [x] Yes, because it may lead to a larger margin and better generalization.
- [ ] No, because it might fail to perfectly separate the training set.

✔

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## Problem 12

1/1 point (graded)

The soft-margin SVM involves a constant $C$ that needs to be set. Which of the following is an appropriate way of setting it? Select all that apply.

- [ ] The output of the SVM is not very sensitive to the choice of $C$, so it doesn't really matter how $C$ is set.
- [ ] Try various settings, and pick the one that yields the smallest training error.
- [x] Try various settings, and pick the one that yields the smallest cross-validation error.
- [ ] Try various settings, and pick the one that yields the largest margin.

✔

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Problems 13-16 correspond to "Duality"

## Problem 13

1/1 point (graded)

The dual form of the Perceptron algorithm is run on a data set of four points $x \in \mathbb{R}^2$ with labels $y \in \{-1, 1\}$. The very first update takes place on the first data point, $x^{(1)} = (3, 2)$, which has label $-1$. What are the values of $\alpha$ and $b$ right after this first update?

- $\alpha = (-1, 0, 0, 0)\,,\ b = 1$
- $\alpha = (1, 0, 0, 0)\,,\ b = 1$
- **$\alpha = (1, 0, 0, 0)\,,\ b = -1$** ✔
- $\alpha = (0, 0, 0, 0)\,,\ b = -1$

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## Problem 14

4/4 points (graded)

The dual form of the Perceptron algorithm is used to learn a binary classifier, based on $n$ training points. It converges after $k$ updates, and returns a vector $\alpha$. For each of the following statements, indicate whether it is necessarily true or possibly false.

a) Each $\alpha_i$ is either $0$ or $1$.

possibly false ✔

b) $\sum_i \alpha_i = k$.

necessarily true ✔

c) $\alpha$ has at most $k$ nonzero coordinates.

necessarily true ✔

d) The training data must be linearly separable.

necessarily true ▾ ✔

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## Problem 15

1/1 point (graded)

The dual form of the hard-margin SVM returns a vector $\alpha$. Which data points $x^{(i)}$ are the support vectors in this solution?

- Those with $\alpha_i = 0$
- Those with $\alpha_i > 0$
- Those with $\alpha_i \geq 0$
- The support vectors cannot be determined simply by looking at $\alpha$

✔

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## Problem 16

1/2 points (graded)

Consider the primal and dual forms of the soft-margin SVM for binary classification. Suppose they are used on a training set of $n$ points, where each point is $d$-dimensional.

a) How many real-valued variables are there in the primal optimization problem? (Don't use spaces in your expression.)

n+1 ✖

b) How many real-valued variables are there in the dual optimization problem?

n ✔

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