Course › Week 2... › Proble... › Proble...

Problem Set 2

Problems 1-2 correspond to "The generative approach to classification"

Problem 1

1/1 point (graded)

Which of the following accurately describes the generative approach to classification, in the case where there are just two labels?

○ Fit a model to the boundary between the two classes.

● Fit a probability distribution to each class separately.

✓

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Problem 2

1/1 point (graded)

In a generative model with \(k\) classes, the class probabilities are \(\pi_1, \ldots, \pi_k\) (summing to 1) and the individual class distributions are \(P_1(x), \ldots, P_k(x)\). In order to classify a new point \(x\), we should pick the label \(j\) that maximizes which of the following quantities?

○ \(\pi_j\)

○ \(P_j(x)\)

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○ \(\pi_j + P_j(x)\)

● \(\pi_j P_j(x)\)

✓

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Problems 3-8 correspond to "Probability review I: probability spaces, events, conditioning"

Problem 3

3/3 points (graded)

What is the \({\bf size}\) of the \({\bf sample \space space}\) in each of the following experiments?

a) A fair coin is tossed.

2 ✓

\(\)

Answer

Correct: The possible outcomes are 0 and 1.

b) A fair die is rolled.

6 ✓

\(\)

Answer

Correct: The possible outcomes are 1,2,3,4,5,6.

c) A fair coin is tossed ten times in a row.

1024 ✓

\(\)

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Correct:
For each of the coins, there are two possible outcomes. For all ten coins together, there are \(2 \times 2 \times \cdots \times 2 = 1024\) outcomes.

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Problem 5

3/3 points (graded)
Two fair dice are rolled. What is the probability that:

a) Their sum is 10, given that the first roll is a 6?

0.166666 ✓

\(\)

Answer
Correct:
If the first roll is a 6, the second needs to be a 4, which happens with probability 1/6.

b) Their sum is 10, given that the first roll is an even number?

0.11 ✓

\(\)

Answer
Correct:
The probability that the sum is 10 *given that* the first roll is even is, by the basic conditioning formula, equal to Pr(sum is 10 AND first roll is even) divided by Pr(first roll is even). Let's compute these two separately. Pr(sum is 10 AND first roll is even) correspond to just two possible outcomes, (4,6) and (6,4); the probability that one of these occurs is 2/36 = 1/18. Meanwhile, Pr(first roll is even) is 1/2. Now divide.

c) They have the same value?

0.1666666 ✓

\(\)

Answer
Correct:
Whatever the first roll is, the probability that the second roll is exactly that number is

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Problem 6

0/1 point (graded)

A certain genetic disease occurs in 5% of men but just 1% of women. Let's say there are an equal number of men and women in the world. A person is picked at random and found to possess the disease. What is the probability, given this information, that the person is male?

0.02 ✗

\(\)

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Problem 7

2 points possible (graded)

The TryMe smartphone company has three factories making its phones. They are all fairly unreliable: 10% of the phones from factory 1 are defective, 20% of the phones from factory 2 are defective, and 24% of the phones from factory 3 are defective. The factories do not produce the same numbers of phones: factory 1 produces \(1/2\) of TryMe's phones, while factories 2 and 3 each produce \(1/4\).

a) What is the probability that a TryMe phone chosen at random is defective?

\(\)

b) Given that a TryMe phone is defective, what is the probability that it came from factory 1?

\(\)

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Problem 8

0/1 point (graded)

Here are some statistics collected by a doctor about patients who walk into her office.

\(\bullet\) 25% of the patients have the flu.

\(\bullet\) Among patients with the flu, 75% have a fever.

\(\bullet\) Among patients who don't have the flu, 50% have a fever.

A new person walks into the doctor's office and turns out to have a fever. What is the probability that he has the flu?

0.25 ✖

\(\)

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Problems 9-12 correspond to "Generative modeling in one dimension"

Problem 9

2/2 points (graded)

Suppose we have one-dimensional data points lying in \(X = [-1,1]\), that have associated labels in \(Y = \{1,2,3\}\). The individual classes have weights

\[\pi_1 = \frac{1}{3}, \ \ \pi_2 = \frac{1}{6}, \ \ \pi_3 = \frac{1}{2}\]

and densities \(P_1, P_2, P_3\) as shown below. (For instance, \(P_1\) is the density of the points whose label is \(1\); in particular, this means that \(P_1\) integrates to \(1\).)

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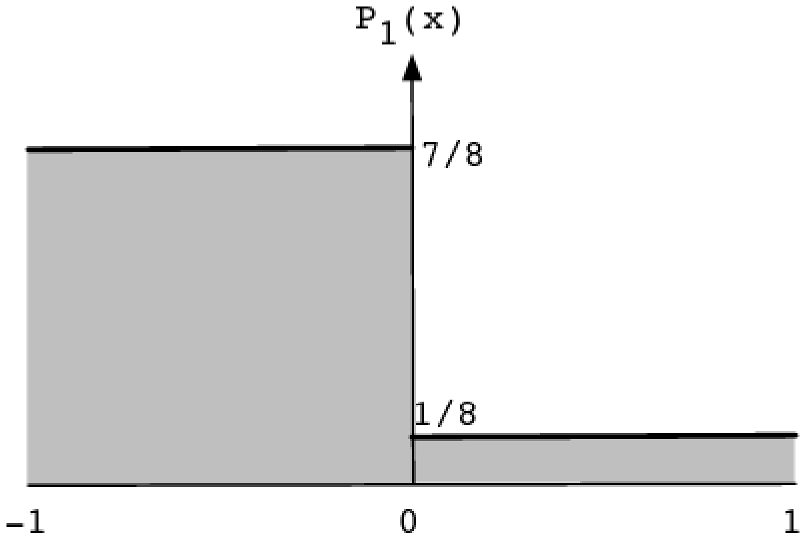

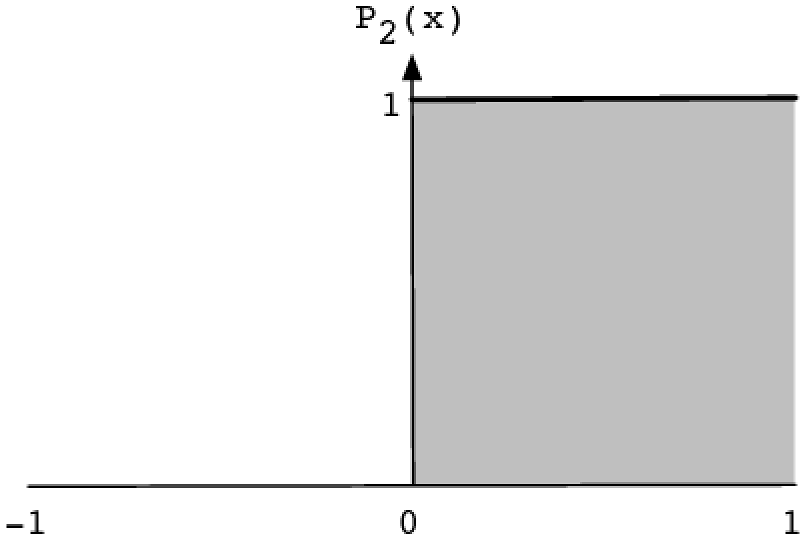

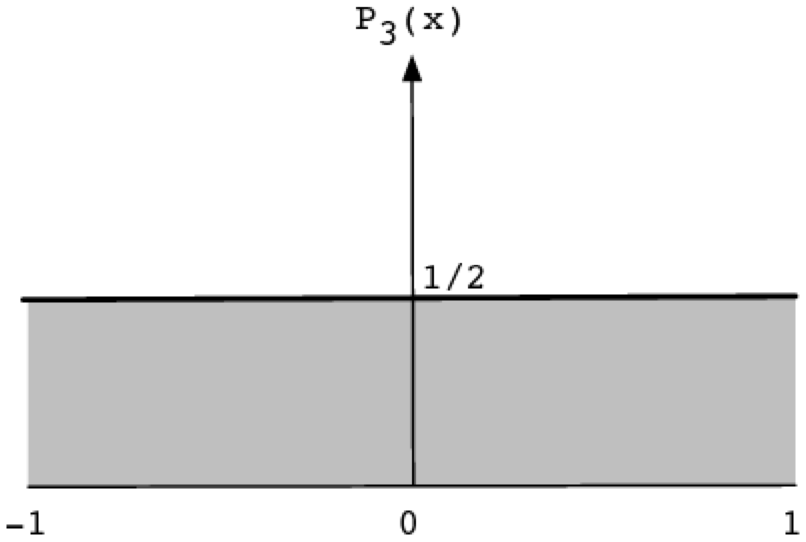

Based on this information, what labels should be assigned to the following points?

a) \(-1/2\)

1 ✓

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b) \(1/2\)

3 ✓

\(\)

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Problem 10

2/2 points (graded)

A set of 100 data points in \(\mathbb R\) have mean of \(20\) and standard deviation of \(10\). We want to fit a Gaussian \(N(\mu, \sigma^2)\) to this data. What \(\mu\) and \(\sigma^2\) should we pick?

a) \(\mu\) =

20 ✓

\(\)

b) \(\sigma^2\) =

100 ✓

\(\)

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Problem 11

1/1 point (graded)

A generative approach is used for a binary classification problem and it turns out that the resulting classifier predicts \(+\) at \({\bf all}\) points \(x\) in the input space. What can we conclude for sure? Check all that apply.

☐ There are no \(-\) points in the training set.

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☐ The \(+\) points are spread out over the space, while the \(-\) points are concentrated in a small region.

☑ There are fewer \(-\) points than \(+\) points in the training set.

☐ The density of \(+\) points is greater than the density of \(-\) points everywhere in the space.

✓

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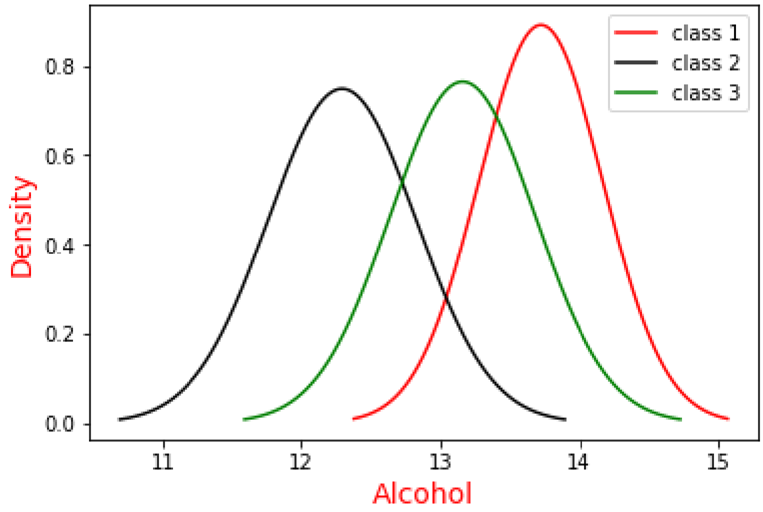
Problem 12

5/5 points (graded)

For the winery example from lecture, the densities obtained are reproduced here:



The class probabilities are \(\pi_1 = 0.33, \pi_2 = 0.39, \pi_3 = 0.28\). What labels would be assigned to the following points?

a) 12.0

2 ✓

\(\)

b) 12.5

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2 ✓

\(\)

c) 13.0

3 ✓

\(\)

d) 13.5

1 ✓

\(\)

e) 14.0

1 ✓

\(\)

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Problems 13-15 correspond to "Probability review II: random variables, expected value, and variance"

Problem 13

4 points possible (graded)

A fair die is rolled twice. Let \(X_1\) and \(X_2\) denote the outcomes, and define random variable \(X\) to be the minimum of \(X_1\) and \(X_2\).

a) How many possible values are there for \(X\)?

\(\)

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b) What is the probability that \(X = 1\)?

\(\)

c) What is \(E(X)\)?

\(\)

d) What is \(\mbox{var}(X)\)?

\(\)

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Problem 14

1/2 points (graded)

In a series of ten independent experiments, a random variable \(X\) takes on values

\[1, 1, 2, 5, 0, 1, 2, 2, 1, 1 .\]

a) Give an estimate of \(E(X)\).

1.6 ✓

\(\)

b) Give an estimate of \(\mbox{var}(X)\).

✗

\(\)

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Problem 15

1/1 point (graded)

Which of the following random variables has \({\bf zero \space variance}\)? Check all that apply.

☐ \(X\) takes on values \(-1\) and \(1\) with equal probability.

☑ \(X\) always takes on value 1.

☐ \(X\) is always equal to \(X^2\).

☑ \(X\) is always zero.

✓

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Problems 16-18 correspond to "Probability review III: modeling dependence"

Problem 16

4/4 points (graded)

In each of the following cases, say whether \(X\) and \(Y\) are dependent or independent.

a) Randomly pick a card from a pack of 52 cards. Define \(X\) to be 1 if the card is a Jack, and 0 otherwise. Define \(Y\) to be 1 if the card is a spade, and 0 otherwise.

○ dependent

● independent

✓

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b) Randomly pick two cards from a pack of 52 cards. \(X\) is 1 if the first card is a spade, and 0 otherwise. \(Y\) is 1 if the second card is a spade, and 0 otherwise.

● dependent

○ independent

✓

c) Toss a coin ten times. \(X\) is the number of heads and \(Y\) is the number of tails.

● dependent

○ independent

✓

d) Roll a fair die. \(X\) is 1 if the outcome is even, and 0 otherwise. \(Y\) is 1 if the outcome is \(\geq 3\), and zero otherwise.

○ dependent

● independent

✓

Submit

Problem 17

2 points possible (graded)

Random variables \(X,Y\) take on values in the range \(\{-1,0,1\}\) and have the following joint distribution.

\(\begin{array}{cc|ccc} & & & Y & \\ & & -1 & 0 & 1 \\ \hline & -1 & 0 & 0 & 1/3 \\ X & 0 & 0 & 1/3 & 0 \\ & 1 & 1/3 & 0 & 0 \end{array}\)

a) What is the covariance between \(X\) and \(Y\)?

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\(\)

b) What is the correlation between \(X\) and \(Y\)?

\(\)

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Problem 18

2/2 points (graded)

Random variables \(X,Y\) take on values in the range \(\{-1,0,1\}\) and have the following joint distribution.

\(\begin{array}{cc|ccc} & & & Y & \\ & & -1 & 0 & 1 \\ \hline & -1 & 1/6 & 0 & 1/6 \\ X & 0 & 0 & 1/3 & 0 \\ & 1 & 1/6 & 0 & 1/6 \end{array}\)

a) Are \(X\) and \(Y\) independent?

● dependent

○ independent

✓

b) Are \(X\) and \(Y\) uncorrelated?

○ correlated

● uncorrelated

✓

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Problems 19-20 correspond to "Two-dimensional generative modeling with the bivariate Gaussian"

Problem 19

2/2 points (graded)

Each of the following scenarios describes a joint distribution \((x,y)\). In each case, give the parameters of the (unique) bivariate Gaussian that satisfies these properties.

a) \(x\) has mean 2 and standard deviation 1, \(y\) has mean 2 and standard deviation 0.5, and the correlation between \(x\) and \(y\) is \(-0.5\).

○ \(\mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\)

○ \(\mu = \begin{pmatrix} 2 \\ -1 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & -1 \\ -1 & \frac{1}{2} \end{pmatrix}\)

● \(\mu = \begin{pmatrix} 2 \\ 2 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}\)

✓

b) \(x\) has mean 1 and standard deviation \(1\), and \(y\) is equal to \(x\).

○ \(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\)

● \(\mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\)

○ \(\mu = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\)

○ \(\mu = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\) , \(\Sigma = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\)

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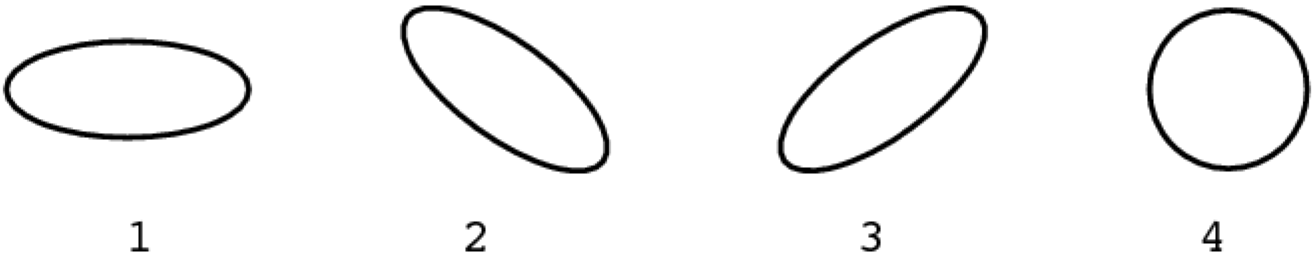
Problem 20

3/3 points (graded)

Here are four possible shapes of Gaussian distributions:



For each of the following Gaussians \(N(\mu,\Sigma)\), indicate which of these shapes (1,2,3,4) is the best approximation.

a) \(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\) and \(\Sigma = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}\)

1 ✓

\(\)

b) \(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\) and \(\Sigma = \begin{pmatrix} 9 & 2 \\ 2 & 1 \end{pmatrix}\)

3 ✓

\(\)

c) \(\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\) and \(\Sigma = \begin{pmatrix} 1 & -0.75 \\ -0.75 & 1 \end{pmatrix}\)

2 ✓

\(\)

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