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Variance

Video

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- Hello, and welcome back.

So, we have talked about the expectation

of a random variable

and what's the average of a random variable,

and now we want to talk about what's the difference

in a random variable, call it variance.

So, we're talking in general about distribution properties

and recall that these are deterministic

0:49 / 0:00 1.0x

7.6_Variance

POLL

Which of the following is greater (≥) for a random variable X?

RESULTS

E[X^2]	**45%**
Depends on X	**38%**
E[X]^2	**17%**

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Results gathered from 211 respondents.

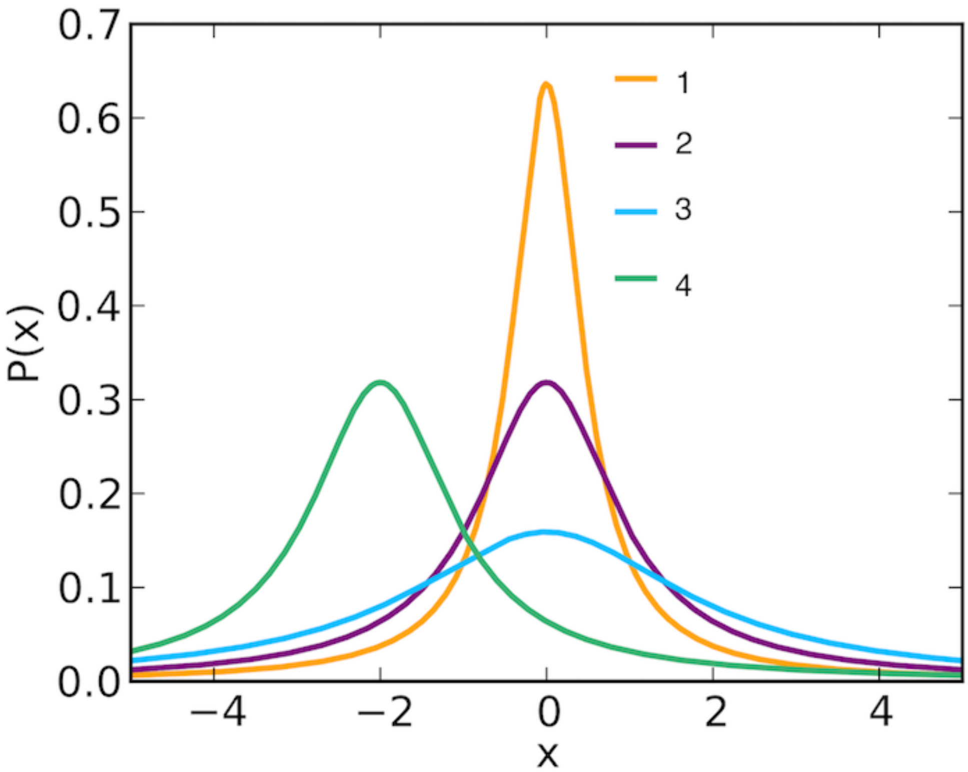
FEEDBACK

E[X^2] will be greater. Since V(X)=E[X^2]-E[X]^2, and V(X) is always non-negative.

1

0 points possible (ungraded)

Given 4 probability density functions, which one shows the greatest variance?



- 1
- 2
- 3 ✔
- 4

Answer
Correct: Video: Variance

Explanation
Variance measures how far a set of (random) numbers are spread out from their average value. 3 is the brodest one.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
A random variable X is distributed over $\{-1, 0, 1\}$ according to the p.m.f. $P(X = x) = \frac{|x|+1}{5}$.

Find its expectation $E(X)$

0 ✔ **Answer:** 0

0

Explanation

The pmf is symmetric around 0, hence the mean is 0.

and variance $V(X)$

0.32 ✖ **Answer:** 4/5

0.32

Explanation
By definition, Var$(X) = \frac{2}{5} \times (-1-0)^2 + \frac{1}{5} \times (0-0)^2 + \frac{2}{5} \times (1-0)^2 = \frac{2}{5} + 0 + \frac{2}{5} = \frac{4}{5}$
Or, Var$(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 4/5 - 0 = 4/5$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

4.0/4.0 points (graded)
Let random variable X be distributed according to the p.m.f

x	1	2	3
$P(x)$	0.3	0.5	0.2

- If $Y = 2^X$, what are

$E[Y]$

4.2 ✔ **Answer:** 4.2

4.2

Explanation
$E(Y) = E(2^X) = 2 \times 0.3 + 4 \times 0.5 + 8 \times 0.2 = 4.2$

$\text{Var}(Y)$

4.36 ✔ **Answer:** 4.36

4.36

Explanation
For any random variable Z, $V(Z) = E(Z^2) - E(Z)^2$. Here $E(Y^2) = E(2^{2X}) = 4 \times 0.3 + 16 \times 0.5 + 64 \times 0.2 = 22$
Thus $V(Y) = 22 - 4.2^2 = 4.36$

- If $Z = aX + b$ has $E[Z] = 0$ and $\text{Var}(Z) = 1$, what are:

$|a|$

1.4282856857 ✔ **Answer:** 1.42857

1.4282856857

$|b|$

2.71374280284628299 ✔ **Answer:** 2.714285

2.71374280284628299

Explanation
First, $E(X) = 0.3 \times 1 + 0.5 \times 2 + 0.2 \times 3 = 1.9 E(X^2) = 0.3 \times 1 + 0.5 \times 4 + 0.2 \times 9 = 4.1$and thus
$\mathrm{Var}(X) = E(X^2) - E(X)^2 = 4.1 - 1.9^2 = 0.49$
Now, by linearity of expectation, $0 = E(Z) = aE(X) + b = 1.9 \cdot a + b$Further, we know
$1 = \mathrm{Var}(Z) = \mathrm{Var}(aX + b) = a^2 \cdot \mathrm{Var}(X) = a^2 \cdot 0.49$Solving these two equations gives $|a| = 1.42857$, $|b| = 2.71485$.

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Answers are displayed within the problem

4

5.0/5.0 points (graded)
Consider two games. One with a guaranteed payout $P_1 = 90$, and the other whose payout P_2 is equally likely to be 80 or 120. Find:

- $E(P_1)$

90 ✔ **Answer:** 90

90

Explanation
The distribution of P_1 is $P(P_1 = 90) = 1$. Hence, $E(P_1) = 1 \times 90 = 90$

- $E(P_2)$

100 ✔ **Answer:** 100

100

Explanation
The distribution of P_2 is $P(P_2 = 80) = P(P_2 = 120) = \frac{1}{2}$. Hence, $E(P_2) = \frac{1}{2} \times 80 + \frac{1}{2} \times 120 = 100$

- $\mathrm{Var}(P_1)$

0 ✔ **Answer:** 0

0

Explanation
By definition, $\mathrm{Var}(P_1) = 1 \times (90 - 90)^2 = 0$

- $\mathrm{Var}(P_2)$

400 ✔ **Answer:** 400

400

Explanation
By definition, $\mathrm{Var}(P_2) = \frac{1}{2} \times (80 - 100)^2 + \frac{1}{2} \times (120 - 100)^2 = 400$

- Which of games 1 and 2 maximizes the `risk-adjusted reward' $E(P_i) - \sqrt{\mathrm{Var}(P_i)}$?

- [x] 1 ✔
- [ ] 2

Explanation
By definition, $E(P_1) - \sqrt{\mathrm{Var}(P_1)} = 90$, $E(P_2) - \sqrt{\mathrm{Var}(P_2)} = 80$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

5

0.0/2.0 points (graded)
Which of the following are always true for random variables X, Y and real numbers a, b?

- [x] The variance of X is always non-negative. ✔
- [x] The standard deviation of X is always non-negative. ✔
- [x] If $V(X) = V(Y)$, then $V(X+a) = V(Y+b)$. ✔
- [ ] If $V(aX) = V(bX)$ for $a \neq 0$ and $b \neq 0$, then $a = b$.
- [ ] If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $X = Y$.
- [ ] If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $E[X^2] = E[Y^2]$. ✔

✖

Explanation
- True.
- True. Standard deviation is defined by $\sqrt{V(X)}$, which is also non-negative.
- True. Adding a constant a to random varianle X will not affect its varaince.
$V(X+a) = E((X+a-E(X+a))^2) = E((X+a-E(X)-a)^2) = E((X-E(X))^2) = V(X)$
- False. When $V(X) = 0$, this does not hold.
- False. Consider two random variables X, Y with pmf, $P(X=x) = \begin{cases} \frac{1}{2}, x=-1, \\ \frac{1}{2}, x=1 \end{cases}$ and $P(Y=y) = \begin{cases} \frac{1}{8}, y=-2 \\ \frac{3}{4}, y=0 \\ \frac{1}{8}, y=2 \end{cases}$. Now
$E(X) = E(Y) = 0, V(X) = V(Y) = 1$ However, $X \neq Y$.
- True. As $E(X^2) = V(X) + E^2[X]$, if $E(X) = E(Y)$ and $V(X) = V(Y)$, then $E(X^2) = E(Y^2)$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)
We say X_A is an indicator variable for event A: $X_A = 1 \text{ if A occurs, } X_A = 0 \text{ if A does not occur.}$

If $P(A) = 0.35$, what is:

- $E(X_A)$?

12

✖ **Answer:** 0.35

12

Explanation
The distribution of X_A is $P(X_A = 1) = P(A) = 0.35, P(X_A = 0) = 1 - P(A) = 0.65$The epectation is $E(X_A) = 0.35 \times 1 + 0.65 \times 0 = 0.35$

- $\mathrm{Var}(X_A)$?

✖ **Answer:** 0.2275

Explanation
The variance of X_A is $\mathrm{Var}(X_A) = 0.35 \times (1 - 0.35)^2 + 0.65 \times (0 - 0.35)^2 = 0.2275$

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Answers are displayed within the problem

7

0 points possible (ungraded)
Let X denote the number when rolling a fair six-sided die, then what is:

- $\mathrm{Var}(X)$?

2.92

✔ **Answer:** 35/12

2.92

Explanation
The expectation of X is $E(X) = 3.5$. The variance of X is
$\mathrm{Var}(X) = \frac{1}{6} \times (1 - 3.5)^2 + \frac{1}{6} \times (2 - 3.5)^2 + \frac{1}{6} \times (3 - 3.5)^2 + \frac{1}{6} \times (4 - 3.5)^2 + \frac{1}{6} \times (5 - 3.5)^2 + \frac{1}{6} \times (6 - 3.5)^2 = \frac{35}{12}$

- σ_X?

1.71

✔ **Answer:** 1.7078

1.71

Explanation
The standard deviation of X is $\sigma_X = \sqrt{\mathrm{Var}(X)} = 1.7078$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

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