edX

Total Probability
Video

Start of transcript. Skip to the end.

UCSDSE212017-V014300

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Hello, and welcome back.

In the last lecture, we talked about sequential probability,

and we would like to talk about total probability.

So sometimes it's easier to split an event

into different parts

and calculate the probability of each part of the event

and then add them up.

6.4_Total_Probability

POLL

60% of our students are American (born), and 40% are foreign (born). 20% of the Americans and 40% of the foreigners speak two languages. What is the probability that a random student speaks two languages?

RESULTS

⚪ **0.28**		**90%**
⚪ **0.18**		**4%**
⚪ **0.34**		**4%**
⚫ **0.45**		**2%**

Submit

Results gathered from 230 respondents.

FEEDBACK

The probability is 0.6 * 0.2 + 0.4 * 0.4 = 0.28.

1

0 points possible (ungraded)

Three 100-marble bags are placed on a table. One bag has 60 red and 40 blue marbles, one as 75 red and 25 blue marbles, and one has 45 red and 55 blue marbles.

You select one bag at random and then choose a marble at random. What is the probability that the marble is red?

- ⚪ 0.2025
- ⚪ 0.33
- ⚪ 0.50
- ⚫ 0.60 ✔

Answer
Correct: Video: Total Probability

Explanation
The probability is $\frac{1}{3}(0.6 + 0.75 + 0.45) = 0.60$

Submit   You have used 1 of 2 attempts

ℹ   Answers are displayed within the problem

2

2.0/2.0 points (graded)

Each of Alice and Bob has an identical bag containing 6 balls numbered 1, 2, 3, 4, 5, and 6. Alice randomly selects one ball from her bag and places it in Bob's bag, then Bob randomly select one ball from his bag and places it in Alice's bag. What is the probability that after this process the content in two bags remain unchanged?

| 0.28571428571 | ✔ **Answer:** 2/7

0.28571428571

Explanation

The two bags will remain unchanged if the ball Bob picks has the same number as the one Alice placed there. Once Alice puts a ball numbered n in Bob's bag, the probability the Bob picks a ball numbered n is $\frac{2}{7}$. The total probability is $\sum_{n=1}^{6} \frac{1}{6} \cdot \frac{2}{7} = \frac{2}{7}$.

| Submit | You have used 2 of 4 attempts

ℹ   Answers are displayed within the problem

3. Two subsets

0 points possible (ungraded)

Let A and B be two random subsets of $\{1, 2, 3, 4\}$. What is the probability that $A \subseteq B$?

| 5 | ✘ **Answer:** 0.3164

5

Explanation

This can be calculated via total probability, conditioning on the possible sizes of A from 0 to 4, namely

$$\left(\binom{4}{0} 2^0 + \binom{4}{1} 2^1 + \cdots + \binom{4}{4} 2^4 \right) / (2^4)^2 = 0.3164$$

A cleaner derivation is to observe that $A \subseteq B$ iff for every element x, $x \in A \to x \in B$, namely $x \notin A$ and $x \notin B$, or $x \notin A$ and $x \in B$, or $x \in A$ and $x \in B$. (Just $x \in A$ and $x \notin B$ is excluded.) Since any given x is in A and B independently at random, the

probability that this holds for x is $3/4$, and the probability that it holds for all four elements is $(3/4)^4 = 0.3164$.

Submit   You have used 4 of 4 attempts

ⓘ   Answers are displayed within the problem

4

2.0/2.0 points (graded)

Eight equal-strength players, including Alice and Bob, are randomly split into 4 pairs, and each pair plays a game, resulting in four winners. What is the probability that exactly one of Alice and Bob will be among the four winners?

| 0.57142857142 | ✔ **Answer:** 4/7 |

0.57142857142

Explanation

Here are two ways of solving the problem. One using total probability, the other by symmetry.

Total Probability.

Let E be the desired event that exactly one of Alice or Bob is a winner. We divide the sample space into two disjoint events, E_1, E_2. E_1 is the event that Alice and Bob play against each other and E_2 is the complimentary event that Alice and Bob play against other players. Since Alice is equally likely to play with any of the seven other players, $P(E_1) = 1/7$, hence $P(E_2) = 6/7$. Now $P(E|E_1) = 1$, while $P(E|E_2) = 1/2$ since Alice and Bob each play an independent game where the probability of winning is $1/2$. Therefore

$$P(E) = P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2) = 1/7 \cdot 1 + 6/7 \cdot 1/2 = 4/.7$$

Symmetry.

In the end, 4 of the 8 players will be declared winners. There are $\binom{8}{4}$ such 4-winner "quartets", all equally likely.

The number of "quartets" that contain exactly one of Alice and Bob is $\binom{2}{1} \cdot \binom{6}{3}$.

Hence the probability that this occurs is $\dfrac{\binom{2}{1} \cdot \binom{6}{3}}{\binom{8}{4}} = \dfrac{4}{7}$.

Submit   You have used 2 of 4 attempts

ⓘ   Answers are displayed within the problem

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The following code doesn't run: common_pow = num_powers.sort_values(ascending=False)[0:5].inde…   **2**

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Questions and comments regarding problem 2.   **3**
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