Expectation

Video

Start of transcript. Skip to the end.

26:46 / 0:00 1.0x

- Hello and welcome back.
In the last lecture we talked
about the cumulative distribution function
and now we would like to move on
and calculate expectations.
This picture and images that we'll get to
later on are taken from the Daily Mirror.
So, first of all let's see,
when we have random variables,
let's see what matters, what we care about.

7.3_Expectation

POLL

The expectation of a random variable X must be a number X can take.

RESULTS

Not true	**80%**
True	**20%**

Submit

Results gathered from 236 respondents.

FEEDBACK

The expectation of a die roll is 3.5.

1

0 points possible (ungraded)

Which 2 of the following are true about the expectation of a random variable?

- [x] Not random ✔

- [ ] Random value
- [x] Property of the distribution ✔
- [ ] Independent of the distribution

✔

Answer
Correct:
Video: Expectation
Video: Expectation
Video: Expectation
Video: Expectation

Explanation
An expectation of a distribution is a constant, which can be deducted by the distribution.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2

2.0/2.0 points (graded)
A quiz-show contestant is presented with two questions, question 1 and question 2, and she can choose which question to answer first. If her initial answer is incorrect, she is not allowed to answer the other question. If the rewards for correctly answering question 1 and 2 are $200 and $100 respectively, and the contestant is 60% and 80% certain of answering question 1 and 2, which question should she answer first as to maximize the expected reward?

Question 2 ✔ **Answer:** Question 2

Explanation
The expected reward if Question 1 is answered first is given by
$300 \times 0.6 \times 0.8 + 200 \times 0.6 \times 0.2 + 0 = 168$
and if Question 2 is chosen to be answered first,
$300 \times 0.8 \times 0.6 + 100 \times 0.8 \times 0.4 + 0 = 176$
Thus she should choose to answer Question 2 first.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

3

0 points possible (ungraded)
If we draw cards from a 52-deck with replacement 100 times, how many times can we expect to draw a black king?

- (•) 3.846 ✔
- ( ) 1.923
- ( ) 0.038
- ( ) 7.692

Answer
Correct: Video: Expectation

Explanation

Create 100 random variables $X_1, X_2, \cdots, X_{100}$, each of which is a binary number, with 1 denotes we get a black king and 0 otherwise. It is easy to show that $E[X_i] = \frac{2}{52}$.
The times we expect to draw a black king can be calculated using
$E[X_1 + X_2 + \cdots + X_{100}] = E[X_1] + E[X_2] + \cdots + E[X_{100}] = \frac{200}{52} = 3.846$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

4

2.0/2.0 points (graded)
Each time you play a die rolling game you must pay $1. If you roll an even number, you win $2. If you roll an odd number, you lose additional $1. What is the expected value of your winnings?

- $-\$0.50$ ✔
- $+\$0.50$
- $+\$0.00$
- $+\$1.00$
- $-\$1.00$

Answer
Correct: Video: Expectation

Explanation
Since each time you need to pay $1 for the game, the quesion is equivalent to "If you roll an even number, you win $1. If you roll an odd number, you lose $2."
With $P(\text{even}) = P(\text{odd}) = \frac{1}{2}$, the expetation is $1 \times \frac{1}{2} + (-2) \times \frac{1}{2} = -0.5$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
Choose a random subset of $\{2^1, 2^2, \cdots, 2^{10}\}$ by selecting each of the 10 elements independently with probability $1/2$. Find the expected value of the smallest element in the subset (e.g. the subset can be $\{2^1, 2^3, 2^4, 2^7\}$. The smallest element is 2^1).

0.1 ✖ **Answer:** 10

0.1

Explanation
An element 2^j, $(j \in \{1, \cdots, 10\})$ is the smallest if and only if all elements less than it have not been chosen and j is chosen. The probability of this happening is $1/2^j$. Therefore the expectation is $\sum_{j=1}^{10} 1/2^j \cdot 2^j = 10$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

An edX assignment has 50 multiple-choice questions, each with four choices of which one is correct. A student gets 3 points for solving a question correctly, and loses a point for an incorrect answer. What is the expected score of a student who answers all questions uniformly at random?

42 ✖ **Answer:** 0

42

Explanation

Since the probability of solving a question correctly here is $1/4$, the expected score is $50 \cdot (3 \cdot 1/4 - 1 \cdot 3/4) = 0$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

Which of the following statements are true for a random variable X?

- [x] $E(X)$ must be in the range $(0, 1)$
- [ ] $E(X)$ can take a value that X does not take ✔
- [x] $P(X \leq E(X)) = 1/2$
- [ ] $E(X) = \frac{1}{2}(x_{\max} + x_{\min})$

✖

Explanation

- False.
- True. For random variable X uniformly distributed over $\{-1, 1\}$, the expection is $E(X) = 0$, which cannot be taken by X.
- False. For random variable X uniformly distributed over $\{-1, 0, 1\}$, the expection is $E(X) = 0$. Then $P(X \leq E(X)) = P(X \leq 0) = \frac{2}{3}$
- False. For random variable X uniformly distributed over $\{-2, 0, 1\}$, the expection is $E(X) = -\frac{1}{3} \neq \frac{1}{2}(x_{\max} + x_{\min}) = -0.5$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)

A bag contains five balls numbered 1 to 5. Randomly draw two balls from the bag and let X denote the sum of the numbers.

- What is $P(X \leq 5)$?

12 ✖ **Answer:** 0.4

12

Explanation

The total number of ways to draw balls is $\binom{5}{2} = 10$.

There are 4 ways to draw 2 balls with sum smaller or equal to 5 (i.e. $(1, 2), (1, 3), (1, 4), (2, 3)$).

Thus $P(X \leq 5) = \frac{4}{10} = 0.4$

- What is $E(X)$?

✖ **Answer:** 6

Explanation
Find out the distribution of X, which is
$P(X=3)=0.1, P(X=4)=0.1, P(X=5)=0.2, P(X=6)=0.2, P(X=7)=0.2, P(X=8)=0.1, P(X=9)=0.1$
.
The expectation is $E(X) = 0.1 \times 3 + 0.1 \times 4 + 0.2 \times 5 + 0.2 \times 6 + 0.2 \times 7 + 0.1 \times 8 + 0.1 \times 9 = 6$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)
A player flips two fair coins. The player wins $\$3$ if 2 heads occur and $\$1$ if 1 head occurs. How much money (in $\$$) should the player lose when no heads occur for the game to be fair (expected gain is 0)?

0

✖ **Answer:** 5

0

Explanation
The prability distribution is $P(2 \text{ heads}) = P(\text{no heads}) = \frac{1}{4}, P(1 \text{ head}) = \frac{1}{2}$
Suppose the the player loses $\$x$ when no heads occur. To make the game fair, $E(X) = \frac{1}{4} \times 3 + \frac{1}{2} \times 1 + \frac{1}{4} \times (-x) = 0$Hence we have $x = 5$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

10

0 points possible (ungraded)
There are 3 classes with 20, 22 and 25 students in each class for a total of 67 students. Choose one out of the 67 students is uniformly at random, and let X denote the number of students in his or her class. What is $E(X)$?

0.2

✖ **Answer:** 22.5224

0.2

Explanation
The probability distrbution is,
$P(\text{from the class with 20 students}) = \frac{20}{67}, P(\text{from the class with 22 students}) = \frac{22}{67}, P(\text{from the class with 25 students}) =$
.
Hence $E(X) = 20 \cdot \frac{20}{67} + 22 \cdot \frac{22}{67} + 25 \cdot \frac{25}{67} = 22.5224$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

Discussion

Topic: Topic 7 / Expectation

Hide Discussion

Add a Post

Show all posts ▼ by recent activity ▼

Problem 2
Questions and comments regarding problem 2.
Staff
3

General Comments
Questions and comments regarding this section.
Staff
3

Problem 4
Questions and comments regarding problem 4.
Staff
7

Problem 5
Questions and comments regarding problem 5.
Staff
3

Problem 9
Questions and comments regarding problem 9.
Staff
3

Problem 1
Questions and comments regarding problem 1.
Staff
1

Problem 3
Questions and comments regarding problem 3.
Staff
1

Problem 6
Questions and comments regarding problem 6.
Staff
1

Problem 7
Questions and comments regarding problem 7.
Staff
1