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Mean and Variance Video

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UCSDSE212017-V018700

0:00 / 0:00 1.0x

- Hello and welcome back.

Now that we know what we're going to do,

we're going to start by looking

at estimating the first property,

and so then talk about parameter estimation.

We'll also have a mean example.

First, if we want to talk about

POLL

A distribution has mean 5 and variance 10. If we collect a sample by making 20 independent observations, what is the variance of the sample mean?

○ **2**

○ **1/2**

○ **1/4**

○ **1/40**

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11.2_Parameter_Estimation

1

0 points possible (ungraded)

If an estimator is unbiased, then

☐ its value is always the value of the parameter,

☐ its expected value is always the value of the parameter,

☐ it variance is the same as the variance of the parameter.

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2

1/1 point (graded)

If $\{X_1, \ldots, X_n\}$ are the observed values of n sample items, which of the following are unbiased estimators for distribution mean?

☑ X_1 ✓

☑ $\frac{1}{n}\sum_{i=1}^{n} X_i$ ✓

☐ $\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2}$

✓

Explanation

Denote the distribution mean as μ
- True. $E\left(X_1\right) = \mu$.
- True. $E\frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{n}E\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n}n\mu = \mu$
- False. $E\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2}\right) \neq \mu$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

As the sample size n grows, the sample mean estimates the distribution mean better. Because

☐ its bias decreases,

☑ its variance decreases, ✓

☐ none of the above.

✓

Explanation

$V\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{V(X)}{n}$. The variance of the sample mean decreases as n grows.

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Answers are displayed within the problem

4

1.0/1.0 point (graded)

A sample of size n has sample mean 20.20. After adding a new observed value 21, the sample mean increases to 20.25. What is n?

15 ✓

15

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5

0 points possible (ungraded)

To estimate the average alcohol consumption of UCSD students, we take three random samples of 40, 45 and 50 students respectively, and their sample means turn out to be 3.15, 3.20 and 2.76 pints per week respectively. What is the sample mean of the collection of all three samples?

0.5 ✗ **Answer:** 3.022222222

0.5

Explanation

Let the total sum of samples be S. Clearly, $S = 40 \cdot 3.15 + 45 \cdot 3.20 + 50 \cdot 2.76 = 408$

The sample mean is thus $\overline{X} = \frac{S}{n} = 408/135 = 3.022222222$

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Answers are displayed within the problem

6

3.0/3.0 points (graded)

Let $X_1, X_2, \ldots, X_n$ be independent samples from a distribution with pdf $f_X(x) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}} \, (X \geq 0)$. Which of the following is an unbiased estimator for θ?

○ $\bar{X}$

● $\frac{\bar{X}}{2}$ ✓

○ $\frac{\bar{X}}{3}$

○ $\frac{\bar{X}}{6}$

Explanation

By linearity of expectation, $E\left(\bar{X}\right) = E\left(X_1\right) = \int_0^\infty x \frac{1}{\theta^2} x e^{-\frac{x}{\theta}} dx = 2\theta$ Thus $E\left(\frac{\bar{X}}{2}\right) = \theta$ and is therefore an unbiased estimator for θ.

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Answers are displayed within the problem

7

0 points possible (ungraded)

For $i \in \{1, \cdots, n\}$ let $X_i \sim \mathrm{U}\left(0, W\right)$ independently of each other, and let $M_n = \max_{i \in \{1, \cdots, n\}} X_i$. For what value of c is $c \cdot M_n$ an unbiased mean estimator?

● $\frac{n+1}{2n}$ ✓

○ $\frac{n}{2(n-1)}$

○ $\frac{2n+1}{4n}$

○ $\frac{2n}{4n-1}$

Explanation

Consider M_n. For some $m \in (0, W)$, $M_n \le m$ if and only if $X_i \le m \, \forall i$. This gives $P\left(M_n \le m\right) = P\left(X_1 \le m \cap \cdots \cap X_n \le m\right) = \prod_{i=1}^n P\left(X_i \le m\right) = \left(\frac{m}{W}\right)^n$. Differentiating the previous expression w.r.t. m, the density of M_n is given by $f_{M_n}\left(m\right) = \frac{nm^{n-1}}{W^n}$. Thus $E\left(M_n\right) = \int_0^W z \cdot f_{M_n}\left(z\right) dz = \int_0^W z \cdot \frac{nz^{n-1}}{W^n} dz = \frac{n}{n+1} \cdot W$ Therefore $\frac{n+1}{2n} \cdot M_n$ is an unbiased estimator for $W/2$.

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Answers are displayed within the problem

8

0 points possible (ungraded)

Let X be distributed $\text{Poisson}\,(\lambda)$. Which of the following is an unbiased estimator for λ^2.

○ X^2

● $X^2 - X$ ✓

○ $2X^2 - X$

○ $3X^2 - 2X$

Explanation

For X that is distributed $\text{Poisson}\,(\lambda)$, we know that $E\,(X) = Var\,(X) = \lambda$ Thus $E\,(X^2) = Var\,(X) + E(X)^2 = \lambda + \lambda^2$and therefore $E\,(X^2 - X) = \lambda^2$.

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Answers are displayed within the problem

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