

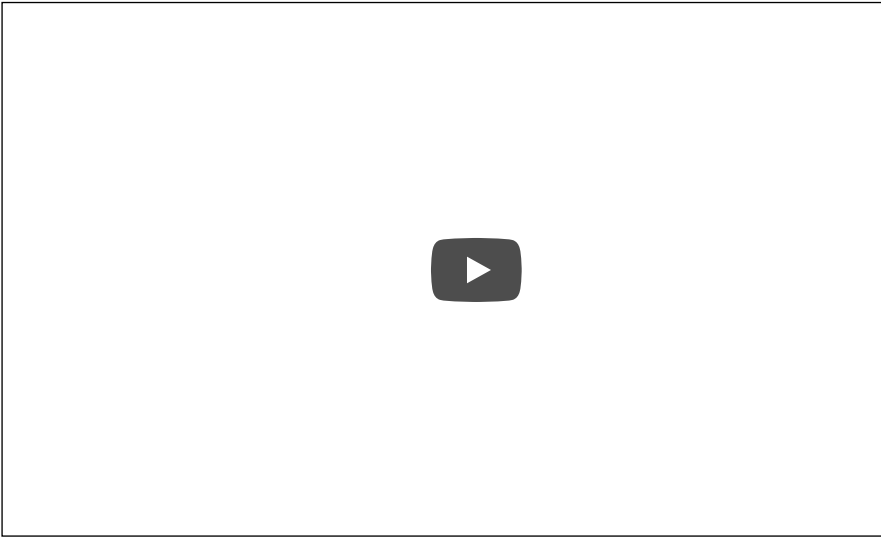


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Functions of Random Variables

Video

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- Hello and welcome back.
In the last lecture, we talked about Random Variables,
and now we would like to start modifying them.
We'll talk about Functions of Random Variables, okay.
So just like we noticed,
observed for discrete random variables,
it's very useful to talk about
Functions of Random Variables,

9.2 Functions of Random Variables

POLL
Let X be a continuous random variable. What type of function g will make the random variable $g(X)$ discrete?

RESULTS

- | | |
|---|-----|
| <input type="radio"/> step | 75% |
| <input type="radio"/> linear | 13% |
| <input type="radio"/> increasing | 8% |
| <input checked="" type="radio"/> decreasing | 4% |

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Results gathered from 147 respondents.

FEEDBACK
A step function will make $g(X)$ discrete, as it will take only the y -values that correspond to the steps.

1

3.0/3.0 points (graded)
Let (X, Y) be distributed over $[0, 1] \times [0, 1]$ according to $f(x, y) = 6xy^2$. Find $P(XY^3 \leq 1/2)$.

0.75

✔ Answer: 0.75

0.75

ExplanationLet $Z = XY^3$.For any $z \in (0, 1)$, $Z = XY^3 \leq z$ iff $Y \leq \min\{(z/X)^{1/3}, 1\}$. Therefore

$$P(Z \leq z) = P(XY^3 \leq z) = \int_0^z \int_0^1 f(x, y) dy dx + \int_z^1 \int_0^{(z/x)^{1/3}} f(x, y) dy dx = \int_0^z \int_0^1 6xy^2 dy dx + \int_z^1 \int_0^{(z/x)^{1/3}} 6xy^2 dy dx = z^2$$

Plugging in $z = 1/2$ gives the answer.

Submit

You have used 3 of 4 attempts

Answers are displayed within the problem

2

3.0/4.0 points (graded)

A random variable X follows the distribution

$$f_X(x) = \begin{cases} Cx^2 & -1 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

and $Y = X^2$. Calculate• C

0.3333333333

✔ Answer: 1/3

0.3333333333

ExplanationSince $1 = \int_{-1}^2 f_X(x) dx = \int_{-1}^2 Cx^2 dx = 3C$ we must have $C = 1/3$.• $P(X \geq 0)$

0.888888888888889

✔ Answer: 8/9

0.888888888888889

Explanation

$$P(X \geq 0) = \int_0^2 f_X(x) dx = \int_0^2 \frac{1}{3} x^2 dx = \frac{1}{9} \cdot x^3 \Big|_0^2 = \frac{8}{9}$$

• $E[Y]$

2.2

✔ Answer: 11/5

2.2

Explanation

$$E(Y) = E(X^2) = \int_{-1}^2 x^2 f_X(x) dx = \int_{-1}^2 \frac{1}{3} \cdot x^4 dx = 33/15 = 11/5$$

• $V(Y)$

3.94

✘ Answer: 228/175

3.94

Explanation

First, $E(Y^2) = E(X^4) = \int_{-1}^2 x^4 f_X(x) dx = \int_{-1}^2 \frac{1}{3} \cdot x^6 dx = \frac{129}{21}$
Hence $V(Y) = E(Y^2) - E(Y)^2 = \frac{129}{21} - \left(\frac{11}{5}\right)^2 = \frac{228}{175}$

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You have used 4 of 4 attempts

i Answers are displayed within the problem

3

0 points possible (ungraded)

Let X be distributed according to $f(x) = ce^{-2x}$ over $x > 0$. Find $P(X > 2)$.

11

✖ Answer: 0.0183

11

Explanation

Since $\int_0^\infty f(x) dx = 1$, we have $c = 2$.

$P(X > 2) = \int_2^\infty f(x) dx = \int_2^\infty 2e^{-2x} dx = e^{-4}$.

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You have used 4 of 4 attempts









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