Functions of Random Variables

Video

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- Hello and welcome back.

In the last lecture, we talked about Random Variables,

and now we would like to start modifying them.

We'll talk about Functions of Random Variables, okay.

So just like we noticed,

observed for discrete random variables,

it's very useful to talk about

Functions of Random Variables,

3:47 / 0:00 1.0x

9.2_Functions_of_Random_Variables

POLL

Let X be a continuous random variable. What type of function g will make the random variable g(X) discrete?

RESULTS

step	**75%**
linear	**13%**
increasing	**8%**
decreasing	**4%**

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Results gathered from 147 respondents.

FEEDBACK

A step function will make g(X) discrete, as it will take only the y-values that correspond to the steps.

1

3.0/3.0 points (graded)

Let (X, Y) be distributed over $[0,1] \times [0,1]$ according to $f(x,y) = 6xy^2$. Find $P(XY^3 \le 1/2)$.

0.75

✔ **Answer:** 0.75

0.75

Explanation

Let $Z = XY^3$.

For any $z \in (0,1)$, $Z = XY^3 \leq z$ iff $Y \leq \min\{(z/X)^{1/3}, 1\}$. Therefore

$P(Z \leq z) = P(XY^3 \leq z) = \int_0^z \int_0^1 f(x,y)\,dydx + \int_z^1 \int_0^{(z/x)^{1/3}} f(x,y)\,dydx = \int_0^z \int_0^1 6xy^2dydx + \int_z^1 \int_0^{(z/x)^{1/3}} 6xy^2dydx = z^2$

Plugging in $z = 1/2$ gives the answer.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

2

3.0/4.0 points (graded)

A random variable X follows the distribution

$$f_X(x) = \begin{cases} Cx^2 & -1 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

and $Y = X^2$. Calculate

- C

0.33333333333

✔ **Answer:** 1/3

0.33333333333

Explanation

Since $1 = \int_{-1}^2 f_X(x)\,dx = \int_{-1}^2 Cx^2dx = 3C$ we must have $C = 1/3$.

- $P(X \geq 0)$

0.8888888888888889

✔ **Answer:** 8/9

0.8888888888888889

Explanation

$P(X \geq 0) = \int_0^2 f_X(x)\,dx = \int_0^2 \frac{1}{3}x^2dx = \frac{1}{9} \cdot x^3\big|_0^2 = \frac{8}{9}$

- $E[Y]$

2.2

✔ **Answer:** 11/5

2.2

Explanation

$E(Y) = E(X^2) = \int_{-1}^2 x^2 f_X(x)\,dx = \int_{-1}^2 \frac{1}{3} \cdot x^4dx = 33/15 = 11/5$

- $V(Y)$

3.94

✖ **Answer:** 228/175

3.94

Explanation

First, $E(Y^2) = E(X^4) = \int_{-1}^{2} x^4 f_X(x)\,dx = \int_{-1}^{2} \frac{1}{3} \cdot x^6 dx = \frac{129}{21}$
Hence $V(Y) = E(Y^2) - E(Y)^2 = \frac{129}{21} - \left(\frac{11}{5}\right)^2 = \frac{228}{175}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)
Let X be distributed according to $f(x) = ce^{-2x}$ over $x > 0$. Find $P(X > 2)$.

11 ✖ **Answer:** 0.0183

11

Explanation
Since $\int_0^\infty f(x)\,dx = 1$, we have $c = 2$.
$P(X > 2) = \int_2^\infty f(x)\,dx = \int_2^\infty 2e^{-2x} dx = e^{-4}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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