FEEDBACK
Only the geometric distribution.

Several of the following questions ask about the number of experiments performed till a certain outcome is observed. Unless otherwise stated, include the final experiment (where the outcome is observed) in the count. For example, the number of coin tosses till observing a heads in the sequence t, t, h, is 3.

1

0 points possible (ungraded)
A die is rolled until the number 1 turns up. The expected number of rolls is

- ○ 2,
- ○ 4,
- ◉ 6, ✔
- ○ 8.

Explanation
$E(X) = \frac{1}{p} = 6.$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
A pair of dice are repeatedly rolled till the two sum to ≥ 10. For example (6,3), (2,4), (5,5), stopping after three pair rolls. The expected number of times the pair is rolled is:

- ○ 2,
- ○ 4,
- ◉ 6, ✔
- ○ 8.

Explanation
There are 6 outcomes where a pair of dice sums to at least 10: (4,6), (5,5), (5,6), (6,4), (6,5), (6,6).
Hence the probability of this large sum is 6/36=1/6.
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The number of times we rol the pair till we observe ≥ 10, is distributed $G_{1/6}$.

The expected number is 6.

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3

3.0/3.0 points (graded)

A G_p random variable is odd with probability

- $\frac{1-p}{2-p}$,
- $\frac{p}{2-p}$,
- $\frac{1}{2-p}$, ✔
- $p + (1-p)^2 \cdot p$

Explanation

There are two natural ways to find the probability that $X \sim G_p$ is odd.

The first is "brute force".

Recall that $1 + q + q^2 + \ldots = \frac{1}{1-q}$

Hence,

$P(X \text{ is odd}) = P(X = 1) + P(X = 3) + \ldots = p + \bar{p}^2 \cdot p + \bar{p}^4 \cdot p + \ldots = \frac{p}{1-\bar{p}^2} = \frac{p}{1-(1-p)^2} = \frac{p}{2p-p^2} = \frac{1}{2-p}$

.

The second method is by relating $(P(X \text{ is even})$ to $P(X \text{ is odd})$.

$P(X \text{ is even}) = P(X \text{ is even} \cap X > 1) = P(X > 1) \cdot P(X \text{ is even} | X > 1) = P(X > 1) \cdot P(X \text{ is odd})$

X is even or odd, hence $1 = P(X \text{ is odd}) + (1-p) \cdot P(X \text{ is odd}) = (2-p) \cdot P(X \text{ is odd})$

Hence $P(X \text{ is odd}) = \frac{1}{2-p}$.

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Answers are displayed within the problem

4

0 points possible (ungraded)

Find the expected number of coin tosses till the third heads appears, (e.g., for h, t, h, t, h, five coins were tossed).

10 ✖ **Answer:** 6

10

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Explanation

For $1 \le i \le 3$, let X_i be the number of tosses between the $i-1$th and ith heads.
For example, for t,h,t,t,h,h, then $X_1 = 2$, $X_2 = 3$, and $X_3 = 1$.
Each X_i is a distributed $G_{1/2}$, hence has expectation 2.
The number of coin tosses till the third head appears is $X_1 + X_2 + X_3$, and by the linearity of expectations,
$E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3) = 6$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
X is the random number of times a coin with heads probability $1/4$ is tossed till the first heads appears, find:

- $E(X)$,

✖ **Answer:** 4

Explanation

$E(X) = \frac{1}{p} = 4$

- $E(X^2)$,

66 ✖ **Answer:** 28

66

Explanation

$E(X^2) = V(X) + E^2(X) = 28$

- $V(X)$,

✖ **Answer:** 12

Explanation

$V(X) = \frac{1-p}{p^2} = 12$

- σ_X,

✖ **Answer:** 3.4614

Explanation

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- $P(X \le 10)$,

✖ **Answer:** 0.9437

Explanation
$P(X \le 10) = \sum_{i=0}^{9} pq^i = 0.9437$

- $P(X > 5)$.

✖ **Answer:** 0.2373

Explanation
$P(X > 5) = \sum_{i=6}^{\infty} pq^i = 1 - \sum_{i=0}^{4} qp^i = 0.2373$

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Answers are displayed within the problem

6

6/9 points (graded)
Two coins with heads probabilities $1/3$ and $1/4$ are alternately tossed, starting with the $1/3$ coin, until one of them turns up heads. Let X denote the total number of tosses, including the last. Find:

- $P(X = 5)$,

0.08333 ✔ **Answer:** 1/12

0.08333

Explanation
$P(X = 5) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{12}$

- $P(X \text{ odd})$,

0.666666 ✔ **Answer:** 2/3

0.666666

Explanation
Similar to Problem 3, this can be done in two ways. Brute force or relating two probabilities.
For the brute force, $P(X \text{ is odd}) = P(X = 1) + P(X = 3) + \ldots = \frac{1}{3} + \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{3} + \ldots$
$= \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{3} + \ldots = \frac{1}{3} \cdot \left(1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\right) = \frac{1}{3} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{2}{3}$
Alternatively,
$P(X \text{ is odd}) = P(X = 1) + P(X \text{ is odd} \cap X \ge 3) = P(X = 1) + P(X \ge 3) \cdot P(X \text{ is odd} | X \ge 3)$
$= P(X = 1) + P(X \ge 3) \cdot P(X \text{ is odd}) = \frac{1}{3} + \frac{2}{3} \cdot \frac{3}{4} \cdot P(X \text{ is odd}) = \frac{1}{3} + \frac{1}{2} \cdot P(X \text{ is odd})$.
Generating Speech Output ld$) = \frac{1}{3}$, or $P(X \text{ is odd}) = \frac{2}{3}$,

- $E(X)$.

12 ✖ **Answer:** 10/3

12

Explanation

$E(X) = \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=3}^{\infty} i \cdot P(X = i)$.

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=1}^{\infty} (i+2) \cdot P(X = i+2, X > 2)$

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=1}^{\infty} (i+2) \cdot P(X = i+2 | X > 2) \cdot P(X > 2)$

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \frac{2}{3} \cdot \frac{3}{4} \cdot \sum_{i=1}^{\infty} (i+2) \cdot P(X = i)$

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \frac{2}{3} \cdot \frac{3}{4} \cdot (E(X) + 2)$

Hence $E(X) \cdot (1 - \frac{1}{2}) = \frac{1}{3} + \frac{1}{3} + 1 = \frac{5}{3}$
And therefore $E(X) = \frac{10}{3}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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Problem 4