Combinations

Video

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- Hello again, everyone.

Last time we talked about permutations,

and in this lecture we'll discuss combinations.

So what are they?

So first we're going to look at subsets of a set.

And so a subset of size k is called a k-subset.

And we'll denote all subsets of size k

16:39 / 0:00 1.0x

4.3_Combinations

POLL

Which of the following is larger for k≤n?

RESULTS

- [x] **The number of k-permutations of an n-set** **84%**
- [ ] **The number of k-subsets of an n-set** **16%**

Submit

Results gathered from 355 respondents.

FEEDBACK

The number of k-permutations is larger.

In selecting subsets, the order doesn't matter, hence the number of k-subsets is the number of k-permutations divided by k!

1

0 points possible (ungraded)

In how many ways can a basketball coach select 5 starting players form a team of 15?

- [x] $\frac{15!}{5!10!}$ ✔
- [ ] $\frac{15!}{10!}$
- [ ] $\frac{15!}{5!}$
- [ ] None of the above

Explanation

It can be deducted from partial permutation, but the order does not matter. It is $\binom{15}{5} = \frac{15^{\underline{5}}}{5!} = \frac{15!}{5!10!}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

- In how many ways can you select a group of 2 people out of 5?

- [x] 10 ✔
- [ ] 25
- [ ] 125
- [ ] None of the above

Explantion

$binom52 = 10.$

- In how many ways can you select a group of 3 people out of 5?

- [x] 10 ✔
- [ ] 25
- [ ] 125
- [ ] None of the above

Explantion

$binom53 = 10.$

- In how many ways can you divide 5 people into two groups, where the first group has 2 people and the second has 3?

- [x] 10 ✔
- [ ] 25
- [ ] 125

None of the above

Explantion

After we determine the group of 2, the group of 3 is determined as well, hence the answer is $binom52 = 10$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Ten points are placed on a plane, with no three on the same line. Find the number of:

- lines connecting two of the points,

45 **Answer:** 45

45

Explanation

Choosing any 2 points out of the 10 points can make a line: $\binom{10}{2}$

- these lines that do not pass through two specific points (say A or B),

20 **Answer:** 28

20

Explanation

Choosing any 2 points out of the remaining 8 points (except A, B): $\binom{8}{2}$

- triangles formed by three of the points,

10 **Answer:** 120

10

Explanation

As no three on the same line, choosing any 3 points out of the 10 points make a triangle: $\binom{10}{3}$

- these triangles that contain a given point (say point A),

9

✖ **Answer:** 36

9

Explanation

With point A fixed, choosing any 2 points out of the remaining 9 points make a triangle: $\binom{9}{2}$

- these triangles contain the side AB.

66

✖ **Answer:** 8

66

Explanation

With point A and B fixed, choosing any 1 point out of the remaining 8 points make a triangle: $\binom{8}{1}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

The set $\{1, 2, 3\}$ contains 6 nonempty intervals: $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, and $\{1, 2, 3\}$.

How many nonempty intervals does $\{1, 2, \ldots, 10\}$contain?

2037

✖ **Answer:** 55

2037

Explanation

$\{1, 2, \ldots, n\}$contains $\binom{n}{1}$ singleton intervals and $\binom{n}{2}$ intervals of 2 or more elements. Hence the total number of intervals is $\binom{n}{2} + \binom{n}{1}$. By Pascal's identity $\binom{n}{2} + \binom{n}{1} = \binom{n+1}{2}$. This can also be seen by considering the $n+1$ midpoints $\{0.5, 1.5, \ldots n + 0.5\}$. Any pair of these points defines an interval in $\{1, 2, \cdots n\}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

A rectangle in an $m \times n$ chessboard is a cartesian product $S \times T$, where S and T are nonempty intervals in $\{1, \ldots, m\}$and $\{1, 2, \ldots, n\}$respectively. How many rectangles does the 3×6 chessboard have?

05 ✖ **Answer:** 126

05

Explanation

Repeating the same analysis as the above question, but for two different intervals, we have $\binom{4}{2} \cdot \binom{7}{2} = 126.$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6

4.0/8.0 points (graded)

A standard 52-card deck consists of 4 suits and 13 ranks. Find the number of 5-card hands where:

- any hand is allowed (namely the number of different hands),

2598960 ✔ **Answer:** 2598960

2598960

Explanation

This is simply $\binom{52}{5}$.

- all five cards are of same suit,

5148 ✔ **Answer:** 5148

5148

Explanation

There are 4 suits in total and 13 cards in each suit, hence $4 \cdot \binom{13}{5}$ hands.

- all four suits are present,

14077700 ✘ **Answer:** 685464

14077700

Explanation

One of the 4 suits will appear twice, hence $4 \cdot \binom{13}{2} \cdot 13^3$ hands.

- all cards are of distinct ranks.

154440 ✘ **Answer:** 1317888

154440

Explanation

First pick 5 out of 13 ranks, then choose their suits. Therefore there are $\binom{13}{5} \cdot 4^5$ hands.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

7

2.0/2.0 points (graded)

A company employs 4 men and 3 women. How many teams of three employees have at most one woman?

- 21
- 22 ✔
- 23
- 24

Explanation

There are $\binom{4}{3} = 4$ teams with 0 women and $\binom{3}{1} \times \binom{4}{2} = 3 \times 6 = 18$teams with 1 woman, for a total of 22.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

8

5.0/5.0 points (graded)

A (tiny) library has 5 history texts, 3 sociology texts, 6 anthropology texts and 4 psychology texts. Find the number of ways a student can choose:

- one of the texts,

18 ✔ **Answer:** 18

18

Explanation

- two of the texts,

153 ✔ **Answer:** 153

153

Explanation

- one history text and one other type of text,

65

✔ **Answer:** 65

65

Explanation

The student can choose 5 different history texts, and $3+6+4=13$other texts, by the product rule there are $5 \cdot 13 = 65$ways of doing that.

- one of each type of text,

360

✔ **Answer:** 360

360

Explanation

The student selects one text of each type, by the product rule this can be done in $5 \cdot 3 \cdot 6 \cdot 4 = 360$ways.

- two different types of text.

119

✔ **Answer:** 119

119

Explanation

There are $5 \cdot 3 = 15$ ways to choose one history and one sociology text, $5 \cdot 6 = 30$ ways to choose one history and one anthropology text, etc. In total there are $5 \cdot 3 + 5 \cdot 6 + 5 \cdot 4 + 3 \cdot 6 + 3 \cdot 4 + 6 \cdot 4 = 119$ways.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

In how many ways can 7 distinct red balls and 5 distinct blue balls be placed in a row such that

- all red balls are adjacent,

0

❌ **Answer:** 3628800

0

Explanation

There are 6 ways to place 7 red balls adjacent. Hence the number of ways is
$6 \times 7! \times 5! = 3628800$

- all blue balls are adjacent,

❌ **Answer:** 4838400

Explanation

There are 8 ways to place 5 red balls adjacent. Hence the number of ways is
$8 \times 7! \times 5! = 4838400$

- no two blue balls are adjacent.

❌ **Answer:** 33868800

Explanation

First, decide on the locations of the red and blue balls. Arrange all 7 red balls in a line, we can then choose 5 out of the 8 gaps (including those at the beginning and end) to place the blue balls. Since the balls are distinct we can permute the blue balls, and the red balls, for a total of $\binom{8}{5}7!5!$ arrangements.

Submit

You have used 4 of 4 attempts

ℹ Answers are displayed within the problem

10

0 points possible (ungraded)

For the set $\{1, 2, 3, 4, 5, 6, 7\}$find the number of:

- subsets,

✖ **Answer:** 2^7

Explanation

There are 7 elements in the set. The number of subsets is 2^7.

- 3-subsets,

10

✖ **Answer:** 35

10

Explanation

Choose 3 elements out of 7. The number of ways is $\binom{7}{3} = 35$.

- 3-subsets containing the number 1,

✖ **Answer:** 15

Explanation

1 is fixed.

Choose 2 elements out of 6. The number of ways is $\binom{6}{2} = 15$.

- 3-subsets not containing the number 1.

✖ **Answer:** 20

Explanation

Choose 3 elements out of 6 (excluding 1). The number of ways is $\binom{6}{3} = 20$.

Submit

You have used 4 of 4 attempts

Answers are displayed within the problem

11 Functions.

0 points possible (ungraded)

A function $f : X \to Y$ is *injective* or *one-to-one* if different elements in X map to different elements in Y, namely,

$$\forall x \neq x' \in X, \quad f(x) \neq f(x').$$

A function $f : X \to Y$ is *surjective* or *onto* if all elements in Y are images of at least one element of X, namely,

$$\forall y \in Y \quad \exists x \in X, \quad f(x) = y.$$

For sets $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$, find the number of

- functions from A to B,

0

✖ **Answer:** 4^3

0

Explanation

As we saw in the lecture, there are $|B|^{|A|} = 4^3 = 64$ functions from A to B.

- functions from B to A,

✖ **Answer:** 3^4

Explanation

$|A|^{|B|} = 3^4 = 81.$

- one-to-one functions from A to B,

✖ **Answer:** 24

Explanation

There are 4 possible values for $f(1)$. Once $f(1)$ is determined, 3 options for $f(2)$ will keep f one-to-one. And after $f(1)$ and $f(2)$ are determined, two options for $f(3)$ will keep f one-to-one. Hence the total number of one-to-one functions is $4^{\underline{3}} = 4 \cdot 3 \cdot 2 = 24$

- onto functions from B to A.

✖ **Answer:** 36

Explanation
If a mapping from B to A is onto, then two elements of B map to a single element in A, while the other two elements of B map to the remaining 2 elements of A. There are $\binom{4}{2} = 6$ ways to choose the two elements with the same image, and then $3! = 6$ ways to to associate the pair and two other elements of B with the three elements of A. The total number of onto functions from B to A is therefore $6 \cdot 6 = 36$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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