- **Nothing, Sam is correct.** **7%**
- **Made a multiplication error.** **3%**

Submit

Results gathered from 381 respondents.

FEEDBACK
Sam subtracted the sequence QQ from the total twice.

The problems in this section are either variations or combinations of techniques we have learned.

1

3.0/3.0 points (graded)
An n-variable Boolean function maps $\{0,1\}^n$ to $\{0,1\}$. How many 4-variable Boolean functions are there?

- 16
- 256
- $65,536$ ✔

Explanation
From the video, there are 2^{2^n} Boolean functions of n-variables, and $2^{2^4} = 2^{16} = 65,436$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

6.0/6.0 points (graded)
A word that reads the same from left to right and right to left is a *palindrome*. For example, "I", "noon" and "racecar" are palindromes. In this question, we consider *palindromic integers* such as 101 and 66. Note that the first digit of an integer **cannot** be 0.

How many positive 5-digit integer palindromes

- are there?

900 ✔ **Answer:** 900

900

Explanation

Since the number is palindromic, it is determined by its first three digits. The first digit must be 1,2,...,9, while the second and third are unconstrained. Hence the total number is $9 \cdot 10 \cdot 10 = 900$

- are even, for example 29192?

400

✔ **Answer:** 400

400

Explanation
The first digit can be only 2,4,6,8, hence $4 \cdot 10 \cdot 10 = 400$

- contain 7 or 8, for example 27172 or 38783?

452

✔ **Answer:** 452

452

Explanation
$7 \cdot 8 \cdot 8 = 448$palindromic integers do not contain 7 or 8. By the complement rule, 900-448=452 contain 7 or 8.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)
How many 5-digit ternary strings are there without 4 consecutive 0s, 1s or 2s?

For example, 01210 and 11211 are counted, but 20000, 11112, and 22222 are excluded.

4

✘ **Answer:** 228

4

Explanation
First count the number of excluded sequences.
A 5-digit string contains 4 consecutive 0's if it starts with 0000, or ends with 0000, or both.
The number of 5-digit ternary strings that start with 0000 is 3, the number that end with 0000 is 3, and the number that both starts and ends with 0000 is 1 (the sequence 00000). By inclusion exclusion, there are 3+3-1=5 sequences containing 0000.
Similarly there are 5 sequencess with 1111 and 5 sequences with 2222.
These sequences are disjoint, hence by the sum rule, the total number of forbidden sequences is 5+5+5=15.
The total number of 5-digit ternary strings is $3^5 = 243$.
By the subtraction rule, the number of allowed sequences is $243 - 15 = 228$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

A password consists of four or five characters, each an upper-case letter, a lower-case letter, or a digit. How many passwords are there if each of the three character types must appear at least once?

100 ✖ **Answer:** 443888640

100

Explanation

For each of length 4 and 5, we use inclusion-exclusion to count the number of password that do not contain at least one of the three character types, then use the complement rule.

Let U, L, and D denote the sets of passwords that do not contain upper-case letters, lower-case letters, and digits, respectively, and let n be the password length. Then $|U| = |L| = 36^n$, and $|D| = 52^n$. Furthermore, $|L \cap U| = 10^n$, while $|L \cap D| = |U \cap D| = 26^n$, and $U \cap L \cap D = \emptyset$. Finally, the number of length-n character sequences is 62^n.

$|U \cup L \cup D|$ is the number of sequences that have at least one character type missed.

By the 3-set inclusion-exclusion principle,

$|U \cup L \cup D| = |U| + |L| + |D| - |L \cap U| - |L \cap D| - |U \cap D| + |U \cap L \cap D|$ Hence, there are $(62^4 - 36^4 - 36^4 - 52^4 + 26^4 + 26^4 + 10^4) + (62^5 - 36^5 - 36^5 - 52^5 + 26^5 + 26^5 + 10^5) = 443,888,640$ valid passwords.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

In the US, telephone numbers are 7-digit long. While real phone numbers have some restrictions, for example, they cannot start with 0 or 1, here we assume that all 7-digit sequences are possible, even 0000000.

How many phone numbers:

- start or end with two identical digits, for example 0012345, 1234511, or 2222222,

100 ✖ **Answer:** 1900000

100

Explanation

$10 \cdot 10^5 = 10^6$ pohne numbers start with two identical digits, and the same number end with two identical digits, furthermore $10^2 \cdot 10^3 = 10^5$ numbers start and end with two identical digits. By inclusion-exclusion, the answer is $2 \cdot 10^6 - 10^5 = 1,900,000$

- contain a substring of 5 consecutive digits. For example 0034567, 2567892, or 0123456.

155 ✖ **Answer:** 1700

155

Explanation
Let L, M, and R, be the sets of phone numbers whose 5 left, middle, and right digits, respectively, are consecutive. The set of phone numbers with five consecutive digits is $L \cup M \cup R$, and we'll determine its size. $|L| = |M| = |R| = 6 \cdot 10^2 = 600$ while $|L \cap M| = |M \cap R| = 5 \cdot 10 = 50$ and $|L \cap R| = |L \cap M \cap R| = 4$ By inclusion-exclusion, $|L \cup M \cup R| = 3 \cdot 600 - 2 \cdot 50 - 4 + 4 = 1700$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6

6/6 points (graded)
How many ordered pairs (A, B), where A, B are subsets of $\{1, 2, 3, 4, 5\}$ have:

- $A \cap B = \emptyset$

243 ✔ **Answer:** 243

243

Explanation
Represent each pair (A, B) of disjoint subssets of $\{1, 2, 3, 4, 5\}$ as a ternary sequence of length 5, where 1 in location i indicaes that $i \in A$, 2 in location i indicates that $i \in B$, and 0 in location i indicates that i is in neither A nor B. For example, 10201 corresponds to $A = \{1, 5\}$ and $B = \{3\}$. The number of disjoint subset pairs is therefore the same as the number of ternary sequences of length 5, namely 3^5.

- $A \cap B = \{1\}$

81 ✔ **Answer:** 81

81

Explanation
Include 1 in both A, B and then repeat above part with the set $\{2, 3, 4, 5\}$.

- $|A \cap B| = 1$

405 ✔ **Answer:** 405

405

Explanation
Same as the above part, except the element in common can be any from $\{1, 2, \ldots, 5\}$ and therefore $5 \cdot 3^4 = 405$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

How many ordered pairs (A, B), where A, B are subsets of $\{1, 2, 3, 4, 5\}$, are there if:

- $A \cup B = \{1, 2, 3, 4, 5\}$

✖ **Answer:** 243

Explanation

By De Morgan's Law, $A \cup B = \{1, 2, 3, 4, 5\}$iff $A^C \cap B^c = \emptyset$. By part (a) there are $3^5 = 243$ ways to select the sets A^C and B^C, hence there are also 243 ways to select A and B.
Or we can represent each pair (A, B) of subssets of $\{1, 2, 3, 4, 5\}$whose union is the whole set as a ternary sequence of length 5, where 1 in location i indicaes that $i \in A$, 2 in location i indicates that $i \in B$, and 0 in location i indicates that i is in both A nor B. For example, 10201 corresponds to $A = \{1, 2, 4, 5\}$and $B = \{2, 3, 4\}$. The number of pairs whose union is $\{1, 2, 3, 4, 5\}$is therefore the same as the number of ternary sequences of length 5, namely 3^5.

- $|A \cup B| = 4$

4

✖ **Answer:** 405

4

Explanation

Choose one of the 5 elements to be in neither in A nor in B. Say this element is 1. Then, we need to choose subsets A, B of $\{2, 3, 4, 5\}$whose union is $\{2, 3, 4, 5\}$. Repeating the argument in Q6, we see that there are 3^4 ways of doing that, hence the total number is $5 \cdot 3^4 = 405$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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