edX

Course › Topic 7... › 7.9 Cov... › Covari...

Covariance
Video

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9:13 / 0:00 1.0x

- Hello again everyone.

Today we're going to talk about covariance,

and the reason we're interested in covariance

is because we ended the last presentation asking

whether expectation multiply.

So the expected value of X times Y is the summation

of all possible values of X and Y of the product

7.9_Covariance

POLL
Which of the following holds for all random variables?

RESULTS

Independent implies uncorrelated	**44%**
Both	**23%**
Neither	**19%**
Uncorrelated implies independent	**14%**

Submit

Results gathered from 192 respondents.

FEEDBACK
Independent implies uncorrelated.

1
0 points possible (ungraded)
Which of the following hold for all random variables?

- [x] Uncorrelated $\Rightarrow$ independent
- [x] Independent $\Rightarrow$ uncorrelated ✔

✖

Explanation

- False. Let random variable X have the distribution $P(X=-1)=P(X=1)=\frac{1}{4}, P(X=0)=\frac{1}{2}$ and $Y=X^2$. We can solve that $\mathrm{Cov}(X,Y)=E(XY)-E(X)E(Y)=0$ while X and Y are not independent.
- True. If two random variables X and Y are independent, then $E(XY)=\sum_x\sum_y xyP(X=x,Y=y)=\sum_x\sum_y xyP(X=x)P(Y=y)=\sum_x xP(X=x)\sum_y yP(Y=y)=E(X)E(Y$

.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Which of the following hold for all uncorrelated random variables X and Y?

- [x] $E[XY]=E[X]E[Y]$
- [x] $V(X+Y)=V(X)+V(Y)$
- [x] $\rho_{X,Y}=0$

✔

Submit You have used 2 of 3 attempts

✔ Correct

3

0 points possible (ungraded)

Which of the following hold for all random variables X and Y?

- [ ] $Var(2X)=4Var(X)$ ✔
- [ ] $Var(X+10)=Var(X)$ ✔
- [x] $Var(X+Y)=Var(X)+Var(Y)$
- [ ] $Var(3X+3Y)=9Var(X+Y)$ ✔

✖

Answer

Incorrect:

Video: Variance

Video: Variance

Video: Variance

Video: Variance

Explanation

- True.
- True.
- False. It only holds when X and Y are uncorrelated.
- True.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

The correlation coefficient between X and $-X$ is 0.

- (selected) True ✗
- False ✓

Answer

Incorrect: Video: Covariance

Explanation

$\rho_{X,-X} = -1.$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

5

0 points possible (ungraded)

The correlation coefficient ρ_{XY} of two random variables:

- [ ] always lies between 0 and 1,
- [x] always lies between -1 and 1. ✓

✓

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

6

2.0/3.0 points (graded)

For the outcomes X and Y of two fair die rolls, find:

- $E(X+Y)$,

7 ✓

7

- $E(X \cdot Y)$,

12.25

12.25

- $\text{Var}(X+Y)$.

5.82

5.82

Submit You have used 3 of 4 attempts

7

0 points possible (ungraded)

Find $\text{Cov}(X,Y)$ when X is distributed uniformly over $\{-1,1\}$ and $Y = \begin{cases} X \text{ w.p. } 3/4, \\ -X \text{ w.p. } 1/4. \end{cases}$

Submit You have used 0 of 4 attempts

8

0 points possible (ungraded)

Let N be the number of heads, and L the length of the longest consecutive string of heads, in three coin flips. For example, if the three coins turn h,t,h, then $N=2$ and $L=1$, while if the coins turn t,h,h, then $N=L=2$. Find:

- $\text{Cov}(N,L)$,

1 **Answer:** 11/16

1

Explanation

Since N is the number of flips in 3 coin flips, $E(N)=1.5$. For the rest, we can create a table:

$$P(N=n,L=l) = \begin{cases} \frac{1}{8} & n=3,\ l=3, \\ \frac{2}{8} & n=2,\ l=2, \\ \frac{1}{8} & n=2,\ l=1, \\ \frac{3}{8} & n=1,\ l=1, \\ \frac{1}{8} & n=0,\ l=0. \end{cases}$$

Hence $E(L) = 3\cdot\frac{1}{8}+2\cdot\frac{1}{4}+1\cdot\frac{1}{2}=\frac{11}{8}$

Also, $E(NL) = 9\cdot\frac{1}{8}+4\cdot\frac{2}{8}+2\cdot\frac{1}{8}+1\cdot\frac{3}{8}=\frac{22}{8}$

It follows that $\text{Cov}(N,L) = E(NL)-E(N)\cdot E(L) = \frac{22}{8}-\frac{3}{2}\cdot\frac{11}{8}=\frac{11}{16}$

- $\rho_{N,L}$.

5 **Answer:** 0.92

5

Explanation

$V(N)=\frac{3}{4}, V(L)=\frac{47}{64}$. $\rho_{N,L} = \frac{\text{Cov}(N,L)}{\sqrt{V(N)V(L)}} = 0.92$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

9

2.0/4.0 points (graded)
Flip a coin thrice (3 times), and let X and Y denote the number of heads in the first two flips, and in the last two flips, respectively. For example, if the coins turn up h,h,t then $X = 2$ and $Y = 1$, while if they turn up t,t,h then $X = 0$ and $Y = 1$. Find:

- $\text{Cov}(X, Y)$,

0.25 ✔ **Answer:** 1/4

0.25

Explanation
X and Y are the number of heads in two coin flips, hence $E(X) = E(Y) = 1$. XY is 0 for ttt, tth, and htt; is 1 for hth and tht; is 2 for hht and thh; and 4 for hhh. Hence $E(XY) = \frac{3}{8} \cdot 0 + \frac{2}{8} \cdot 1 + \frac{2}{8} \cdot 2 + \frac{1}{8} \cdot 4 = \frac{5}{4}$and
$\text{Cov}(X, Y) = E(XY) - E(X) \cdot E(Y) = \frac{1}{4}$

- $\rho_{X,Y}$.

0.71 ✖ **Answer:** 1/2

0.71

Explanation
Since X is the number of heads in two coin flips, $E(X) = 1$, and $E(X^2) = \frac{1}{4} \cdot 0^2 + \frac{2}{4} \cdot 1^2 + \frac{1}{4} \cdot 2^2 = 1.5$Hence
$V(X) = 1.5 - 1 = 0.5$, and similarly $V(Y) = 0.5$. It follows that $\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{1/4}{1/2} = \frac{1}{2}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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Problem 9
Questions and comments regarding problem 9.
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4

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Questions and comments regarding problem 7.
Staff
2 new 7

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Questions and comments regarding this section.
Staff
3

Problem 1
Questions and comments regarding problem 1.
Staff
1

Problem 2
Questions and comments regarding problem 2.
Staff
1

Problem 3
Questions and comments regarding problem 3.
Staff
1

Problem 4
Questions and comments regarding problem 4.
Staff
1

Problem 5
Questions and comments regarding problem 5.
Staff
1

Problem 6
Questions and comments regarding problem 6.
Staff
1