edX

Bayes' Rule

Video

For example,

D1 is the outcome for the first die,

and we let S be the sum of D1 and D2,

of the two faces that we observe,

so we can see that the probability, for example,

that the sum is five, given that the first die is two,

that's the same as the probability

that the second die was three,

'cause then the sum will be five.

Probability of the second die is three is 1/6.

This one is easy.

But conversely, calculating the probability

that the first die turned up two, given the sum is five,

is a little more difficult,

so this is the calculation we want to do,

and we're going to use Bayes' rule for that.

2:01 / 0:00 1.0x

6.5_Bayes_Rule

POLL

Monty Hall Problem:

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car and behind the others are goats. You pick a door, say door 1. The host knows what is behind each door. He opens another door, say door 3, which has a goat. He then says to you, "Do you want to change your selection to door 2?" Is it to your advantage to switch your choice?

RESULTS

It is better to switch to door 2.	**62%**
There is no difference.	**30%**
It is better to keep my choice of door 1.	**8%**

Submit

Results gathered from 231 respondents.

FEEDBACK

It is better to switch.

See the explanation to the Monte Hall problem here.

1

0 points possible (ungraded)

A rare disease occurs randomly in one out of 10,000 people, and a test for the disease is accurate 99% of the time, both for those who have and don't have the disease. You take the test and the result is postive. The chances you actually have the disease are approximately:

- 10%
- 1% ✔
- 0.1%
- 0.01% ✖

Explanation

Let H and $D)$ be the events that you Have and Don't have the disease, respectively, and let S be the event that the result is poSitive.

By the streamlined version of Bayes' Rule, $P(H|S) = \frac{P(H,S)}{P(S)} = \frac{P(H,S)}{P(H,S)+P(D,S)}$.

Now, $P(H,S) = P(H) \cdot P(S|H) = 0.0001 \cdot 0.99 \approx 0.0001$ and

$P(D,S) = P(D) \cdot P(S|D) = 0.9999 \cdot 0.01 \approx 0.01$

Hence $P(H|S) = \frac{0.0001}{0.0001+0.01} \approx 0.01$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

A car manufacturer has three factories producing 21%, 35%, and 44% of its cars, respectively. Of these cars, 7%, 6%, and 2%, respectively, are defective. A car is chosen at random from the manufacturer's supply.

- What is the probability that the car is defective?

2 ✖ **Answer:** 0.0445

2

Explanation

Let F_1, F_2, F_3 be the events that the care is made by the first, second, and third factory, respectively, and let D be the event that the car is defective. By the law of total probability,

$P(D) = P(F_1) \cdot P(D|F_1) + P(F_2) \cdot P(D|F_2) + P(F_3) \cdot P(D|F_3) = 0.21 \cdot 0.07 + 0.35 \cdot 0.06 + 0.44 \cdot 0.02 = 0.0445$

.

- Given that the car is defective, what is the probability that was produced by the first factory?

✖ **Answer:** 0.3303

Explanation

By Bayes' Rule and using $P(D)$ from above, $P(F_1|D) = \frac{P(F_1)\cdot P(D|F_1)}{P(D)} = \frac{0.21\cdot 0.07}{0.0445} = 0.3303$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

2.0/2.0 points (graded)

A college graduate is applying for a job and has 3 interviews. She passes the first, second, and third interviews with probabilities 0.9, 0.8, and 0.7, respectively. If she fails any interview, she cannot proceed with subsequent interview(s) and will not get the job. If she didn't get the job, what is the probability that she failed the second interview?

0.3629032258 ✔ **Answer:** 45/124

0.3629032258

Explanation

Let F, S, and T denote the events that the applicant passed the first, second, and third interviews, respectively. The probability that she failed the second interview given that she didn't get the job is $P(\overline{S}|\overline{FST}) = P(F\overline{S}|\overline{FST}) = \frac{P(F\overline{S}\wedge\overline{FST})}{P(\overline{FST})} = \frac{P(F\overline{S})}{P(\overline{FST})} = \frac{0.9\cdot 0.2}{1-0.9\cdot 0.8\cdot 0.7}$ where the first equality follows as the applicant fails the second interview iff she passes the first interview and fails the second.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

An ectopic pregnancy is twice as likely to develop when a pregnant woman is a smoker than when she is a nonsmoker. If 32% of women of childbearing age are smokers, what fraction of women having ectopic pregnancies are smokers?

52 ✖ **Answer:** 16/33

52

Explanation

Let S and E denote the events that a pregnant woman is a smoker, and has an ectopic pregnancy, respectively. We are told that $P(S) = .32$ and $P(E|\overline{S}) = .5 \cdot P(E|S)$. By Bayes' Rule,
$P(S|E) = \frac{P(E|S)\cdot P(S)}{P(E|S)\cdot P(S)+P(E|\overline{S})\cdot P(\overline{S})} = \frac{P(E|S)\cdot P(S)}{P(E|S)\cdot P(S)+.5\cdot P(E|S)\cdot P(\overline{S})} = \frac{P(S)}{P(S)+.5\cdot P(\overline{S})} = \frac{.32}{.32+.5\cdot(1-.32)} = \frac{16}{33}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

3.0/3.0 points (graded)

Each of Alice, Bob, and Chuck shoots at a target once, and hits it independently with probabilities 1/6, 1/4, and 1/3, respectively. If only one shot hit the target, what is the probability that Alice's shot hit the target?

- 31/72
- 6/31 ✔
- 10/31
- 15/31

Explanation

Let A, B, and C, be the events that Alice, Bob, and Chuck hit the target, respectively, and let $E = A\overline{BC} \cup \overline{A}B\overline{C} \cup \overline{AB}C$ be the event that only one shot hit the target.

Then $P(E) = \frac{1}{6} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{31}{72}$

By Bayes' Rule, $P(A|E) = \frac{P(AE)}{P(E)} = \frac{P(A\overline{BC})}{P(E)} = \frac{6/72}{31/72} = \frac{6}{31}$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Jack has two coins in his pocket, one fair, and one "rigged" with heads on both sides. Jack randomly picks one of the two coins, flips it, and observes heads. What is the probability that he picked the fair coin?

- 3/4 ✖
- 2/3
- 1/3 ✔
- 1/4

Explanation

Let F and R be the events that Jack picked the fair and rigged coin, respectively, and let H be the event that he obvserved heads.

By the "streamlined" Bayes' rule, $P(F|H) = \frac{P(F,H)}{P(H)} = \frac{P(F,H)}{P(F,H)+P(R,H)}$.

Now, $P(F,H) = P(F) \cdot P(H|F) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$while $P(R,H) = P(R) \cdot P(H|R) = \frac{1}{2} \cdot 1 = \frac{1}{2}$

Hence $P(F|H) = \frac{1/4}{1/4+1/2} = \frac{1}{3}$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

It rains in Seattle one out of three days, and the weather forecast is correct two thirds of the time (for both sunny and rainy days). You take an umbrella if and only if rain is forecasted.

- What is the probability that you are caught in the rain without an umbrella?

11

✖ **Answer:** 1/9

11

Explanation

Let R be the event that it rains, and C the event that the forecast is correct. We are told that $P(R) = \frac{1}{3}$ and $P(C|R) = P(C|\overline{R}) = \frac{2}{3}$. The probability you are caught in the rain without an umbrella is $P(R \wedge \overline{C}) = P(R) \cdot P(\overline{C}|R) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

- What is the probability that you carry an umbrella and it does not rain?

✖ **Answer:** 2/9

Explanation

The probability you are carry and umbrella and it does not rain is $P(\overline{R} \wedge \overline{C}) = P(\overline{R}) \cdot P(\overline{C}|\overline{R}) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)

On any night, there is a 92% chance that an burglary attempt will trigger the alarm, and a 1% chance of a false alarm, namely that the alarm will go off when there is no burglary. The chance that a house will be burglarized on a given night is $1/1000$. What is the chance of a burglary attempt if you wake up at night to the sound of your alarm?

0

✖ **Answer:** 0.084

0

Explanation

Let A be the event of triggering alarm, B be the event of an burglary attempt, NB be the event of no attempts. As $P(A|B) = 0.92, P(A|NB) = 0.01, P(B) = 0.001$ Following Bayes rule $P(B|A) = 0.084$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

An urn labeled "heads" has 5 white and 7 black balls, and an urn labeled "tails" has 3 white and 12 black balls. Flip a fair coin, and randomly select on ball from the "heads" or "tails" urn according to the coin outcome. Suppose a white ball is selected, what is the probability that the coin landed tails?

0 ✖ **Answer:** 12/37

0

Explanation

Let H and T be the events that the coin turnd up heads and tails, and let W be the event of selecting a white ball. $P(H) = P(T) = 0.5, P(W|H) = \frac{5}{12}, P(W|T) = \frac{3}{15}$ Following Bayesian rule $P(T|W) = \frac{12}{37}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

10

0 points possible (ungraded)

A car manufacturer receives its air conditioning units from 3 suppliers. 20% of the unitws come from supplier A, 30% from supplier B, and 50% from supplier C. 10% of the units from supplier A are defective, 8% of units from supplier B are defective, and 5% of units from supplier C are defective. If a unit is selected at random and is found to be defective.

What is the probability that a unit came from supplier A if it is:

defective,

✖ **Answer:** 0.289

non-defective,

0 ✖ **Answer:** 0.193

0

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

11

0 points possible (ungraded)

Suppose that 15% of the population have cancer, 50% of the population smokes, and 75% of those with cancer smoke. What fraction of smokers have cancer?

- 0.05625
- 0.225 ✔
- 0.25
- 0.75

Explanation

Let S and C be the events that a person smokes, and has cancer, respectively.
Then $P(C|S) = \frac{P(S|C)P(C))}{P(S)} = 0.225$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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