Moment Generating Video

Start of transcript. Skip to the end.

▶ 14:12 / 0:00 1.0x

- Hello and welcome back.

We have so far talked about Markov's

and Chebyshev's inequality and we want to move

to more sophisticated and stronger inequalities.

But before we do that I want to spend this lecture

preparing the background for the stronger inequalities.

So we're going to talk

10.4a_Moment_Generating_Functions

10.4b_Moment_Generating_Functions_Examples

POLL

If M(t) is a moment generating function, then what is M(0)?

RESULTS

1	**63%**
depends on the distribution	**27%**
0	**6%**
infinity	**3%**

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Results gathered from 126 respondents.

FEEDBACK

M(0)=E[e^0]=1

1

0 points possible (ungraded)

If X has moment generating function $M_X(t) = (1-3t)^{-1}$, what is $V(X)$?

- 6
- 9 ✔
- 12 ✖

Explanation

$E(X) = \frac{\partial M_X(t)}{\partial t}|_{t=0} = \frac{3}{(1-3t)^2}|_{t=0} = 3.$

$E(X^2) = \frac{\partial^2 M_X(t)}{\partial t^2}|_{t=0} = \frac{18}{(1-3t)^3}|_{t=0} = 18.$

$V(X) = E(X^2) - E^2(X) = 9$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2

0 points possible (ungraded)

Let $M_X(t)$ be the MGF of X. Which of the following hold for all X and Y?

- [x] $M_X(0) = 1$ ✔
- [x] $M_X(t) \geq 0$ for all t ✔
- [x] $M_{3X+2}(t) = e^{2t} \cdot M_X(3t)$ ✔
- [ ] $M_{X+Y}(t) = M_X(t) M_Y(t)$

✔

Explanation

- True. $M_X(0) = E(e^{0X}) = E(1) = 1$
- True. As $e^{tx} \geq 0$ for all t, $M_X(t) = E(e^{tX}) \geq 0$
- True. $M_{3X+2}(t) = E(e^{t(3X+2)}) = e^{2t} E(e^{3tX}) = e^{2t} \cdot M_X(3t)$
- False. It only holds when X and Y are independent.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

If X is a non-negative continuous random variable with moment generating function

$$M_X(t) = \frac{1}{(1-2t)^2}, \quad t < \frac{1}{2}$$

Calculate

- $E[X]$

4

✔ Answer: 4

4

Explanation

Recall that $E(X) = M'(0)$.

$M'(t) = (-2) \cdot (1-2t)^{-3} \cdot (-2) = 4 \cdot (1-2t)^{-3}$.

Hence $E(X) = 4$.

- $V(X)$

8

✔ Answer: 8

8

Explanation

Similar to the first part $E(X^2) = M''(0)$.

$M''(t) = (-12) \cdot (1-2t)^{-4} \cdot (-2) = 24 \cdot (1-2t)^{-4}$.

Hence $E(X^2) = 24$, and $V(X) = E(X^2) - E^2(X) = 24 - 16 = 8$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Let $X_1, X_2, \ldots$ be independent $B_{1/2}$ random variables, and let $M \sim P_4$, namely Poisson with mean 4. Which of the following is the MGF of $X_1 + X_2 + \ldots + X_M$?

- $e^{2(1+e^t)}e^{-4}$ ✔
- $e^{1+e^t}e^{-2}$
- $\frac{1+e^t}{2}$
- $\frac{1+e^{2t}}{2}$ ✖

Explanation

Let $Y = X_1 + X_2 + \ldots + X_M$, then $P(Y=k|M=m) = \binom{m}{k}\frac{1}{2}^m$ and using the product rule we get

$P(Y=k, M=m) = \binom{m}{k}\frac{1}{2}^m e^{-4}\frac{4^m}{m!} = \binom{m}{k}2^m\frac{e^{-4}}{m!}$, and $P(Y=k) = \sum_{m=k}^{\infty} P(Y=k, M=m)$

Thus the moment generating funtion of Y is given by

$M_Y(t) = E_Y[e^{kt}] = \sum_{k=0}^{\infty} e^{kt} P(Y=k) = \sum_{k=0}^{\infty} e^{kt} \sum_{m=k}^{\infty} P(Y=k, M=m) = \sum_{m=0}^{\infty}\sum_{k=0}^{m} e^{kt} P(Y=k, M=m) = \sum$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

5

3.0/3.0 points (graded)

Let X be a random variable with MGF $M_X(t) = \frac{1}{3}e^{-t} + \frac{1}{6} + \frac{1}{2}e^{2t}$. What is $P(X \le 1)$?

0.5 ✔ **Answer:** 0.5

0.5

Explanation

The pmf of X is $P(X=x)=\begin{cases}\frac{1}{2}, x=2\\ \frac{1}{6}, x=0\\ \frac{1}{3}, x=-1\end{cases}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Let $M_X(t)$ be an MGF, which of the following are valid MGF's?

- [x] $M_X(2t)\,M_X(7t)$ ✔
- [x] $e^{-5t}M_X(t)$ ✔
- [ ] $3M_X(t)$

✔

Explanation

- True.
- True. $e^{-5t}M(t)=E\left(e^{t(X-5)}\right)$.
- False. $3M(0)=3\neq 1$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

If $M_X(t)=e^{-5(1-e^t)}$, find $V(X)$.

5 ✔ **Answer:** 5

5

Explanation

$E(X)=\frac{\partial M_X(t)}{\partial t}|_{t=0}=5.$

$E(X^2)=\frac{\partial^2 M_X(t)}{\partial t^2}|_{t=0}=30.$

$V(X)=E(X^2)-E^2(X)=5$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

3.0/3.0 points (graded)

Find the MGF of $(X_1 + X_2 + X_3 + X_4)/3$where each X_i is an independent $B_{1/2}$ random variable?

- [x] $\left((1+e^{t/3})/2\right)^4$ ✔
- [ ] $\left((1+e^{t})/2\right)^4$
- [ ] $\left((2/3+e^{t}/3)\right)^4$
- [ ] $\left((2/3+e^{t/3}/3)\right)^4$

Explanation

$E\left(e^{\frac{tX_1}{3}}\right) = \frac{\left(1+e^{\frac{t}{3}}\right)}{2}$.

$M_X(t) = E\left(e^{\frac{tX_1}{3}}\right)E\left(e^{\frac{tX_2}{3}}\right)E\left(e^{\frac{tX_3}{3}}\right)E\left(e^{\frac{tX_4}{3}}\right) = \left(\frac{1+e^{\frac{t}{3}}}{2}\right)^4$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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1