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Partial Permutations

Video

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- Welcome again everyone.

Last lecture, we talked about permutations

and now we want to talk about partial permutations.

These are permutations where you don't want to

arrange all the objects that you have,

but just some subset.

So we know that the number of ways

to arrange n objects is n factorial

6:58 / 0:00 1.0x

4.2_Partial_Permutations

POLL

How many 2-permutations do we have for set {1,2,3,4}?

RESULTS

12	**84%**
16	**12%**
8	**4%**

Submit

Results gathered from 370 respondents.

FEEDBACK
The answer is 12.

1

0 points possible (ungraded)
In how many ways can 5 cars - a BMW, a Chevy, a Fiat, a Honda, and a Kia - park in 8 parking spots?

6720 ✔ **Answer:** 6720

6720

Explanation
There are 8 locations for the BMW, the 7 for the Chevy, etc, so the total number of ways is
$8^{\underline{5}} = 6720.$

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

2

0 points possible (ungraded)
In how many ways can 5 people sit in 8 numbered chairs?

6720 ✔ **Answer:** 6720

6720

Explanation
The first person can sit in any of the 8 chairs, the second in one of the remaining 7, etc. Hence
$8^{\underline{5}} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

3

6.0/6.0 points (graded)
Find the number of 7-character (capital letter or digit) license plates possible if no character can repeat and:

- there are no further restrictions,

42072307200 ✔ **Answer:** 42072307200

42072307200

Explanation
$36^7 = 42,072,307,200$

- the first 3 characters are letters and the last 4 are numbers,

78624000 ✔ **Answer:** 78624000

78624000

Explanation
Choose 3 from capital letters, and 4 from digits, where the order matters. The result is $26^3 \cdot 10^4 = 78,624,000$

- letters and numbers alternate, for example A3B9D7Q or 0Z3Q4A9.

336960000 ✔ **Answer:** 336960000

336960000

Explanation
Such plates contain either four letters and three digits, or the other way. The two sets are disjoint. Hence
$26^3 \cdot 10^4 + 26^4 \cdot 10^3 = 336,960,000$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

4

2.0/2.0 points (graded)
A derangement is a permutation of the elements such that none appear in its original position. For example, the only derangements of $\{1, 2, 3\}$ are $\{2, 3, 1\}$ and $\{3, 1, 2\}$. How many derangements does $\{1, 2, 3, 4\}$ have?

9 ✔ **Answer:** 9

9

Explanation
Let F_1 be the set of permutations of $\{1, 2, 3, 4\}$, where 1 is in location 1, for example 1324. Similarly let F_2 be the set of permutations where 2 is in location 2, for example 3214, etc.
Then $F_1 \cup F_2 \cup F_3 \cup F_4$ is the set of all 4-permutations where at least one element remains in its initial location. The set of permutations where no elements appears in its initial location is the complement of this set.
Note that $|F_i|| = 3!$, for distinct i, j $|F_i \cap F_j| = 2!$, for distinct i, j, k $|F_i \cap F_j \cap F_k| = 1!$ and $|F_1 \cap F_2 \cap F_3 \cap F_4| = 0!$
Hence by inclusion exclusion,
$|F_1 \cup F_2 \cup F_3 \cup F_4| = \binom{4}{1} \cdot 3! - \binom{4}{2} \cdot 2! + \binom{4}{3} \cdot 1! - \binom{4}{4} \cdot 0! = 4^3 - 4^2 + 4^1 - 4^0 = 24 - 12 + 4 - 1 = .15$
It follows that the number of derangements is $4! - 15 = 9$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

Eight books are placed on a shelf. Three of them form a 3-volume series, two form a 2-volume series, and 3 stand on their own. In how many ways can the eight books be arranged so that the books in the 3-volume series are placed together according to their correct order, and so are the books in the 2-volume series? Noted that there is only one correct order for each series.

120

✔ **Answer:** 120

120

Explanation

Since the 3-volume books must be placed in a unique order, we can view them as a just one "super book", similarly for the 2-volume books. We therefore have a total of 5 books that we can arrange freely, and we can do so in $5! = 120$ ways.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

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