Stars and Bars
Video

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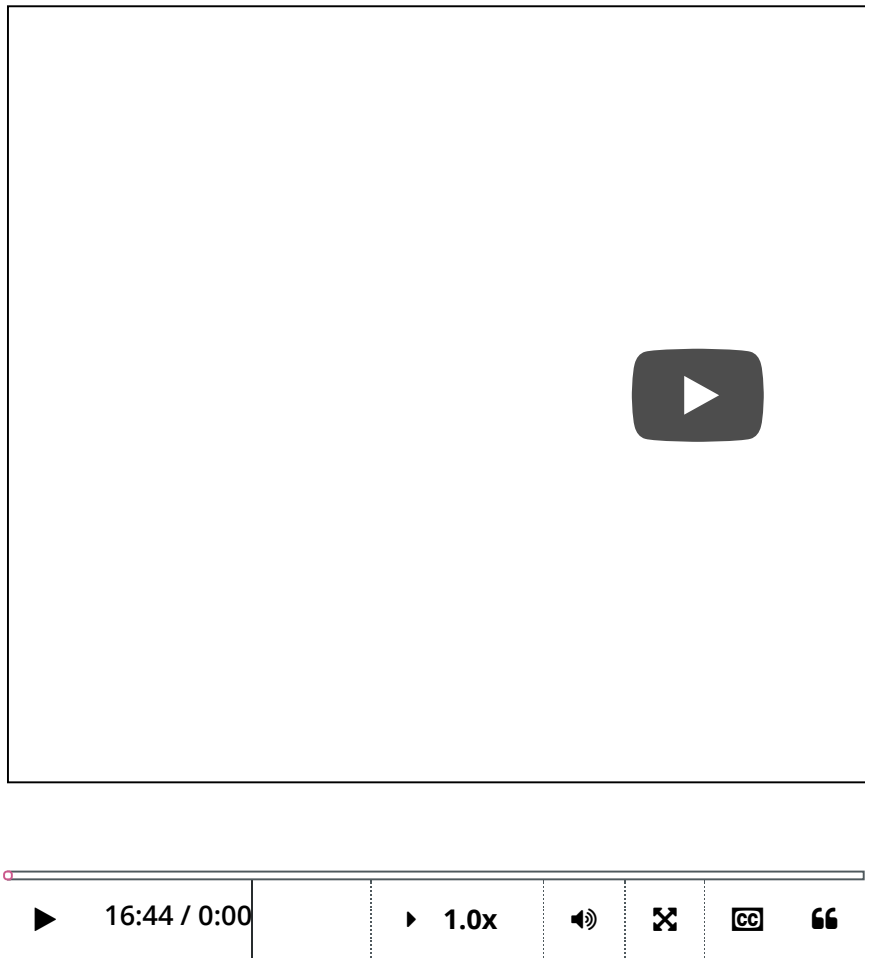

- Welcome back.

So far, we've been talking about

different applications of combinatorics.

We've talked about permutations, about combinations,

the binomial coefficient,

and counting different things using them.

And today, we're going to

4.8_Stars_and_Bars

POLL

In how many different ways can you write 11 as a sum of 3 **positive** integers if order matters?

- **28**
- **36**
- **45**
- **None of the above**

Submit

1

0 points possible (ungraded)

If $a+b+c+d=10$ how many ordered integer solutions (a,b,c,d) are there, when all elements are

- non-negative,

5 ✖ **Answer:** 286

5

Explanation

$\binom{13}{3}$, follows from the lecture.

- positive?

✖ **Answer:** 84

Explanation

$\binom{9}{3}$, follows from the lecture.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2

2/3 points (graded)

In how many ways can we place 10 idential red balls and 10 identical blue balls into 4 distinct urns if:

- there are no constraints,

81796 ✔ **Answer:** 81796

81796

Explanation
$\binom{13}{3} \cdot \binom{13}{3}$, by combining stars and bars for both balls evaluated separately.

- the first urn has at least 1 red ball and at least 2 blue balls,

✖ **Answer:** 36300

Explanation
First place 1 red ball and 2 blue balls in the first urn, and then repeat the above part with 9 red balls and 8 blue balls, resulting in $\binom{12}{3} \cdot \binom{11}{3}$.

- each urn has at least 1 ball? Hint: use complement and inclusion exclusion?

65094 ✔ **Answer:** 65094

65094

Explanation
There are $\binom{12}{2}^2$ ways to place the balls so that urn 1 is empty, $\binom{11}{1}^2$ ways so that urns 1 and 2 are empty and $\binom{10}{0}^2 = 1$ so that urns 1 2 and 3 are empty. By inclusion exclusion, there are $\binom{4}{1}\binom{12}{2}^2 - \binom{4}{2}\binom{11}{1}^2 + \binom{4}{3}\binom{10}{0}^2$ placements where at least one urn is empty. And by the complement rule, the answer is
$\binom{13}{3}^2 - \binom{4}{1}\binom{12}{2}^2 + \binom{4}{2}\binom{11}{1}^2 - \binom{4}{3}\binom{10}{0}^2 = 65,094$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

2.0/4.0 points (graded)

How many 6-digit sequences are:

- strictly ascending, as 024579 or 135789, but not 011234,

210 ✔ **Answer:** 210

210

Explanation

Every six-digit strictly asending sequence corresponts to 6 distinct digits. There are $\binom{10}{6} = 210$ ways to choose them.

- ascending (not necessarily strictly), as 023689, 033588, or 222222.

504 ✖ **Answer:** 5005

504

Explanation

Every six-digit (not necessarily striclty) assending sequence corresponds to a collection of 6 digits, possibly with repetition. Let x_i denote the number of times digit i is included in the number. Using stars and bars, the number of ways of assigning $x_0 + x_1 + \cdots + x_9 = 6$is $\binom{6+10-1}{6} = 5005.$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

How many terms are there in the expansion of $(x + y + z)^{10}$?

16 ✖ **Answer:** 66

16

Explanation

Any coefficient corresponds to a term of the form $x^{a_1} y^{a_2} z^{a_3}$ with $a_1 + a_2 + a_3 = 10$ such that $a_i \geq 0$. The number of possible solutions to this problem is given by $\binom{12}{2} = 66$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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