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Axioms

Axioms

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Hello and welcome back.

So we have talked about different properties of probability

and now we would like to start in some sense afresh

by defining a set of axioms

and showing that we can prove many of the results,

that we have observed before.

So we'll have only three

5.6_Probability_Axioms

POLL

Does P(A)=0 imply that A is the empty set?

RESULTS

- Not necessarily 82%
- Yes 18%

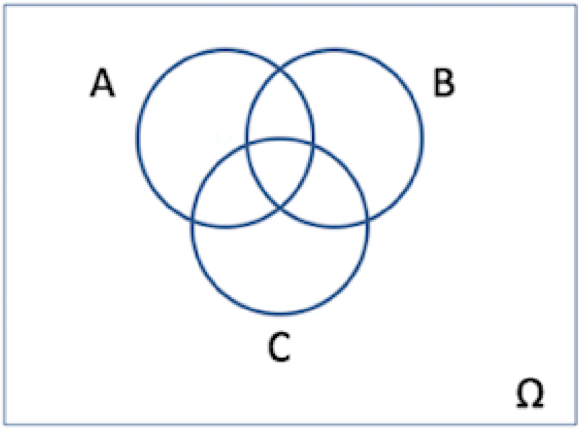
Submit

Results gathered from 292 respondents.

FEEDBACK
It is possible that P(A)=0 for a non-empty set A.

1

0 points possible (ungraded)
For any three events A, B, and C, we have $P(B) =$



- $P(A \cap B) + P(B \cap C) + P(B \cap A^c \cap C^c)$
- $P(A \cap B) + P(B \cap C) - P(A \cap B \cap C) + P(B \cap A^c \cap C^c)$ ✔
- $P(A^c \cap C^c) + P(A \cap B) + P(B \cap C)$
- $P(\Omega) - P(A) - P(C) + P(A \cap B \cap C)$

Answer
Correct: Video: Total Probability

Explanation

- False. It is $P(B) + P(A \cap B \cap C)$

- True.

- False. This includes the events outside of the three circles.

- False. Same as above.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

1/1 point (graded)

Under which of the following probability assignments does $S = \{a_1, a_2, a_3\}$ become a probability space?

- [ ] $P(a_1) = 0.2,\ P(a_2) = 0.3,\ P(a_3) = 0.4$
- [x] $P(a_1) = 0.2,\ P(a_2) = 0.3,\ P(a_3) = 0.5$ ✔
- [ ] $P(a_1) = 0.3,\ P(a_2) = -0.2,\ P(a_3) = 0.9$
- [x] $P(a_1) = 0.2,\ P(a_2) = 0,\ P(a_3) = 0.8$ ✔

✔

Explanation

Two necessary conditions:

1. The probability P of the events satisfies $0 \leq P \leq 1$.
2. All Ps sum up to 1.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Which of the following **always** holds?

- [ ] $A \subseteq B \quad \Rightarrow \quad P(A) < P(B)$
- [ ] $A \subseteq B \quad \Rightarrow \quad P(A) \leq P(B)$ ✔
- [x] $A \subset B \quad \Rightarrow \quad P(A) < P(B)$
- [ ] $A \subset B \quad \Rightarrow \quad P(A) \leq P(B)$ ✔

✖

Explanation

The only tricky part may be the third. Note that because elements may have 0 probabilities, non-empty events may also have zero probability. Hence A may be a strict subset of B and yet have the same probability. For example, if the sample space is $\{a, b\}$ and $P(a) = 1$ while $P(b) = 0$, then $P(\{a\}) = P(\{a, b\})$.

Submit

You have used 3 of 3 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Which of the following statements are true?

- [ ] If $P(E) = 0$ for event E, then $E = \emptyset$.
- [ ] If $E = \emptyset$, then $P(E) = 0$. ✔
- [ ] If $E_1 \cup E_2 = \Omega$, then $P(E_1) + P(E_2) = 1$.
- [x] If $P(E_1) + P(E_2) = 1$, then $E_1 \cup E_2 = \Omega$.
- [x] If $E_1 \uplus E_2 = \Omega$, then $P(E_1) + P(E_2) = 1$. ✔
- [ ] If $P(E_1) + P(E_2) = 1$, then $E_1 \uplus E_2 = \Omega$.

✖

Explanation
- False. E is not necessary to be $\emptyset$.
- True.
- False. Let $\Omega = \{1,2,3\}, E_1 = \{1,2\}, E_2 = \{2,3\}$ $E_1 \cup E_2 = \Omega$, but $P(E_1) + P(E_2) = \frac{4}{3}$.
- False. Let $\Omega = \{1,2,3\}, E_1 = \{1,2\}, E_2 = \{1\}$ $P(E_1) + P(E_2) = 1$, but $E_1 \cup E_2 \neq \Omega$.
- True.
- False. Same as option 4.

Submit

You have used 3 of 3 attempts

Answers are displayed within the problem

5

7/7 points (graded)

Suppose A, B are events that $P(A) = 0.65$, $P(B) = 0.5$ and $P(A \cap B) = 0.25$. What are the following probabilities?

- $P(A^c)$

0.35

✔ **Answer:** 0.35

0.35

Explanation
$P(A^c) = 1 - P(A) = 0.35$

- $P(B^c)$

0.5

✔ **Answer:** 0.5

0.5

Explanation
$P(B^c) = 1 - P(B) = 0.5$

- $P(A \cup B)$

0.9 ✔ **Answer:** 0.9

0.9

Explanation
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.9$

- $P(A - B)$

0.4 ✔ **Answer:** 0.4

0.4

Explanation
$P(A - B) = P(A \cup B) - P(B) = 0.4$

- $P(B - A)$

0.25 ✔ **Answer:** 0.25

0.25

Explanation
$P(B - A) = P(A \cup B) - P(A) = 0.25$

- $P(A \Delta B)$

0.65 ✔ **Answer:** 0.65

0.65

Explanation
$P(A \Delta B) = P(A \cup B) - P(A \cap B) = 0.65$

- $P((A \cup B)^c)$

0.1 ✔ **Answer:** 0.1

0.1

Explanation
$P((A \cup B)^c) = 1 - P(A \cup B) = 0.1$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Let P be a probability function on $S = \{a_1, a_2, a_3\}$. Find $P(a_1)$ if:

- $P(\{a_2, a_3\}) = 3P(a_1)$

30

✖ **Answer:** 0.25

30

Explanation

We have $P(\{a_2, a_3\}) = 3P(a_1)$ and $P(a_1) + P(\{a_2, a_3\}) = 1$
Solving the equations we have $P(a_1) = 0.25$.

- $P(a_1) = 2P(a_2) = 3P(a_3)$

✖ **Answer:** 0.5454

Explanation

We have $P(a_1) = 2P(a_2) = 3P(a_3)$ and $P(a_1) + P(a_2) + P(a_3) = 1$
Solving the equations we have $P(a_1) = 6/11$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

Let X be distributed over $\{1, 2, \ldots, 100\}$ with $P(X = i) = \frac{i}{k}$ for some integer k. Find:

- k

15

✖ **Answer:** 5050

15

Explanation

$K = 1 + 2 + \ldots + 100 = \frac{(1+100)\cdot 100}{2} = 5050$

- $|E|$ where $E = \{\text{integer multiples of } 3\}$,

✖ **Answer:** 33

Explanation

$E = \{3, 6, 9, \ldots, 96, 99\}$ hence $|E| = 33$.

- $P(E)$.

✖ **Answer:** 0.333

Explanation

$\frac{3+6+\ldots+96+99}{5050} = \frac{102\cdot 33}{2\cdot 5050} = \frac{1683}{5050} = 0.333267$.

Note that, as could be expected, this probability is very close to $1/3$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)

Consider a die where the probability of rolling $1, 2, 3, 4, 5$ and 6 are in the ratio $1 : 2 : 3 : 4 : 5 : 6$ What is the probability that when this die is rolled twice, the sum is 7?

15

✖ **Answer:** 0.12698

15

Explanation

Let p be the probability of rolling a 1, then for $i = 1, 2, 3, \ldots, 6$ the probability of rolling i is $i \cdot p$.

These probabilities sum to $21p$, which must be 1, hence $p = 1/21$.

Hence The probability that the sum is 7 is $\frac{2 \cdot (1 \cdot 6 + 2 \cdot 5 + 3 \cdot 4)}{21 \cdot 21} = \frac{56}{21^2} = \frac{8}{63}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

Jack solves a Math problem with probability 0.4, and Rose solves it with probability 0.5. What is probability that at least one of them can solve the problem?

- 0.7
- 0.9
- 0.6
- Not enough information ✔

Explanation

Let A be the event that Jack solves the problem, B be the event that Rose solves the problem. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ but $P(A \cap B)$ is missed here.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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