Conditional Probability

Video

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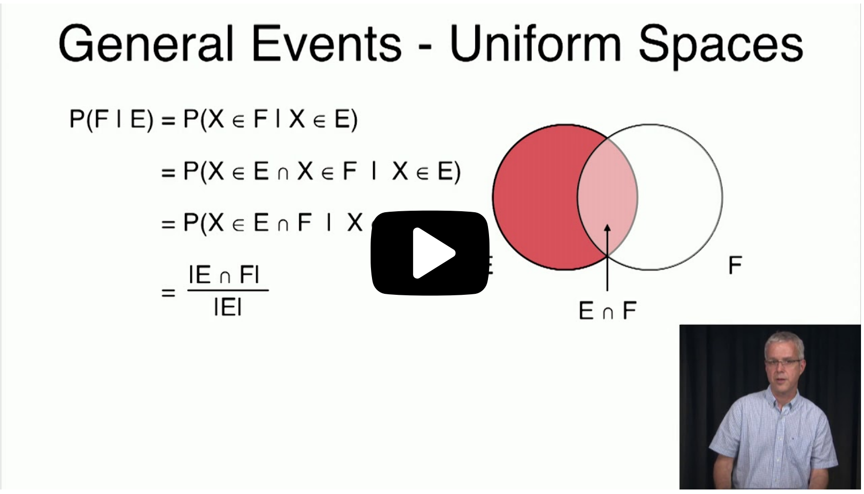

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- Hello again, everyone.

In the last few videos we talked about probability

and today we want to look at the nuance version

called conditional probability.

So, first let's motivate it.

So, often we have partial information about the world

6.1_Conditional_Probability

POLL

Let A and B be two positive-probability events. Does P(A|B)>P(A) imply P(B|A)>P(B)?

RESULTS

Not necessarily	**69%**
Yes	**31%**

Submit

Results gathered from 264 respondents.

FEEDBACK

Yes.

P(A|B)=P(A,B) / P(B) and P(B|A)=P(A,B) / P(A).

Hence, P(A|B)>P(A) iff P(A,B)>P(A) * P(B) iff P(B|A)>P(B).

1

0 points possible (ungraded)

Suppose $P(A) > 0$. Find $P(B|A)$ when:

- $B = A$,

10 ✖ **Answer:** 1

10

Explanation

Given that A happens, B must happens. Hence $P(B|A) = 1$.

- $B \supseteq A$,

✖ **Answer:** 1

Explanation

Same as above.

- $B = \Omega$,

✖ **Answer:** 1

Explanation

Same as above.

- $B = A^c$,

✖ **Answer:** 0

Explanation
Given that A happens, B can never happens. Hence $P(B|A) = 0$.

- $A \cap B = \emptyset$,

✖ **Answer:** 0

Explanation
Same as above.

- $B = \emptyset$.

✖ **Answer:** 0

Explanation
Same as above.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
If A and B are disjoint positive-probablity events, then $P(A|B)$=

- [ ] $P(A)$,
- [x] $P(B|A)$, ✔
- [ ] $P(A \cup B)$,
- [ ] $P(A \cap B)$. ✔

✖

Explanation

Since A and B are disjoint, $P(A|B) = 0$.
$P(A \cap B) = P(B|A) = 0$ while $P(A)$ and $P(A \cup B)$ are positive as A and B are positive-probablity events.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

3

3/4 points (graded)

Given events A, B with $P(A) = 0.5$, $P(B) = 0.7$, and $P(A \cap B) = 0.3$ find:

- $P(A|B)$,

0.42857142857 ✔ **Answer:** 3/7

0.42857142857

Explanation

$P(A|B) = P(A \cap B)/P(B) = 0.3/0.7 = 3/7$

- $P(B|A)$,

0.6 ✔ **Answer:** 3/5

0.6

Explanation

$P(B|A) = P(B \cap A)/P(A) = 0.3/0.5 = 3/5$

- $P(A^c|B^c)$,

0.3 ✖ **Answer:** 1/3

0.3

Explanation

$P(A^c|B^c) = P(A^c \cap B^c)/P(B^c) = 0.1/0.3 = 1/3$

- $P(B^c|A^c)$.

0.2 ✔ **Answer:** 1/5

0.2

Explanation

$P(B^c|A^c) = P(B^c \cap A^c)/P(A^c) = 0.1/0.5 = 1/5$

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Answers are displayed within the problem

4

0 points possible (ungraded)

Find the probability that the outcome of a fair-die roll is at least 5, given that it is at least 4.

- $\frac{2}{3}$ ✔
- $\frac{2}{4}$ ✖
- $\frac{1}{3}$
- $\frac{1}{2}$

Explanation

$P(\text{at least } 5|\text{at least } 4) = \frac{P(\text{at least } 5 \cap \text{at least } 4)}{P(\text{at least } 4)} = \frac{P(\text{at least } 5)}{P(\text{at least } 4)} = \frac{2}{3}.$

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Answers are displayed within the problem

5

0 points possible (ungraded)

Two balls are painted red or blue uniformly and independently. Find the probability that both balls are red if:

- at least one is red,

12 ✖ **Answer:** 1/3

12

Explanation

$P(2R|\exists R) = \frac{P(2R \wedge \exists R)}{P(\exists R)} = \frac{P(2R)}{P(\exists R)} = \frac{1/4}{3/4} = \frac{1}{3}.$

- a ball is picked at random and it is pained red.

✖ **Answer:** 1/2

Explanation

$P(2R|\text{random ball is R}) = \frac{P(2R \wedge \text{random ball is R})}{P(\text{random ball is R})} = \frac{P(2R)}{P(\text{random ball is R})} = \frac{1/4}{1/2} = \frac{1}{2}.$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6

2.0/3.0 points (graded)

Three fair coins are sequentialy tossed. Find the probability that all are heads if:

- the first is tails,

0.25 ✖ **Answer:** 0

0.25

Explanation

If the fisrt coin is tails, it's impossible for all coins to be heads, hence the probability is 0.

More formally, $P(X_1 \wedge X_2 \wedge X_3|\overline{X_3}) = \frac{P(X_1 \wedge X_2 \wedge X_3 \wedge \overline{X_3})}{P(\overline{X_3})} = \frac{P(\emptyset)}{P(\overline{X_3})} = \frac{0}{1/2} = 0$

- the first is heads,

0.25 ✔ **Answer:** 1/4

0.25

Explanation

First intuitively, if the first coin is heads, then all are heads iff the second and third coins are heads, which by independence of coin flips happens with probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

A bit more formally, let X_1, X_2, X_3 be the events that the first, second, and third coin is heads. Then $P\left(X_1 \wedge X_2 \wedge X_3 | X_1\right) = \frac{P(X_1 \wedge X_2 \wedge X_3 \wedge X_1)}{P(X_1)} = \frac{P(X_1 \wedge X_2 \wedge X_3)}{P(X_1)} = \frac{1/8}{1/2} = \frac{1}{4}$

- at least one is heads.

0.1428571429 ✔ **Answer:** 1/7

0.1428571429

Explanation

First intuitively, there are seven possible outcome triples where at least one of the coins is heads, and only one of them has all heads. Hence the probability of all heads given that one is heads is $1/7$.

More formally,

$$P\left(X_1 \wedge X_2 \wedge X_3 | X_1 \vee X_2 \vee X_3\right) = \frac{P((X_1 \wedge X_2 \wedge X_3) \wedge (X_1 \vee X_2 \vee X_3))}{P(X_1 \vee X_2 \vee X_3)} = \frac{P(X_1 \wedge X_2 \wedge X_3)}{P(X_1 \vee X_2 \vee X_3)} = \frac{1/8}{7/8} = \frac{1}{7}$$

.

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Answers are displayed within the problem

7

0 points possible (ungraded)

A 5-card poker hand is drwan randomly from a standard 52-card deck. Find the probability that:

- all cards in the hand are ≥ 7 (7, 8,..., K, Ace), given that the hand contains at least one face card (J, Q, or K),

✘ **Answer:** 0.0957

Explanation

There are where $4 \cdot (13-3) = 40$non-face cards, hence $\binom{40}{5}$ hands without face cards. Therefore, of the $\binom{52}{5}$ hands, (\binom{52}{5}-\binom{40}{5}\) hands contain a face card.

Similarly, there are $\binom{32}{5}$ hands consisting of cards ≥ 7, of which $\binom{20}{5}$ contain no face cards, and $\binom{32}{5} - \binom{20}{5}$ hands contain a face card.

Hence, the requested probability is $\frac{\binom{32}{5}-\binom{20}{5}}{\binom{52}{5}-\binom{40}{5}} = 0.0957$

- there are exactly two suits given that the hand contains exactly one queen.

10 ✖ **Answer:** 0.156

10

Explanation

There are $4 \cdot \binom{48}{4}$ hands with exactly one queen.

To count the number of hands with exactly one queen and two suites, observe that there are 4 ways to choose the queen, then 3 ways to select the other suit, and from the $26 - 2 = 24$ non-queens of these two suits, $\binom{24}{4}$ ways to select the remaining 4 cards, but of those, $\binom{12}{4}$ hands will have all cards of the same suit as the queen. Hence there are $4 \cdot 3 \cdot \left(\binom{24}{4} - \binom{12}{4}\right)$ ways to select cards with exactly one queen and two suits.

The desired probability is therefore, $\frac{4 \cdot 3 \cdot \left(\binom{24}{4}-\binom{12}{4}\right)}{4 \cdot \binom{48}{4}} = 0.156.$

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Answers are displayed within the problem

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3

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Questions and comments regarding problem 3.
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? With P(A|B), both A and B must be subsets of the same Omega set, correct?
In other words, if we say P(A) is the probability of a roll of 4 from a single die roll, then Omega is {1,2,3,4,5,...
2

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