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Bernoulli
Video

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- Hello and welcome back.

So in this lecture, we're going to start talking

about the first of the families

of distribution that we're going to present,

and we'll talk about the Bernoulli Distributions.

And what we're going to show is we're going

to mention that they are the simplest

▶ 16:57 / 0:00 ▸ 1.0x

8.1_Bernoulli

POLL

Every random variable distributed over {0, 1} is Bernoulli.

RESULTS

●	**Yes**	**56%**
○	**Not necessarily**	**44%**

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Results gathered from 211 respondents.

FEEDBACK

Every random variable over {0,1} attains the value 1 with some probability (p) and 0 with the remaining probability (1-p), hence is (B_p). So the answer is Yes.

1

1.0/1.0 point (graded)

$X \sim B_p$ with $p > 0.5$ and $V(X) = 0.24$ Find

- p,

0.6 ✓ **Answer:** 0.6

0.6

Explanation

For a Bernoulli distribution, $E(X^2) = E(X) = p$ Thus $V(X) = E(X^2) - E(X)^2 = p - p^2 = p(1-p)$ Since $0.24 = V(X) = p(1-p)$ and $p \geq 0.5$, we must have $p = 0.6$.

- $E[X]$.

0.6 ✓ **Answer:** 0.6

0.6

Explanation

$E(X) = p = 0.6$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Which of the following hold for two Bernoulli variables?

☑ Independent implies uncorrelated, ✓

☐ Uncorrelated implies independent. ✓

✗

Explanation

- True. It is trivial.
- True. Let $X \sim B_{p_x}, Y \sim B_{p_y}$.

If X and Y are uncorrelated, $\mathrm{Cov}\left(X,Y\right)=E\left(XY\right)-E\left(X\right)E\left(Y\right)$.

$$
\begin{aligned}
&=\sum_{x=0}^{1}\sum_{y=0}^{1}xyP\left(X=x,Y=y\right)-p_xp_y\\
&=P\left(X=1,Y=1\right)-p_xp_y\\
&=P\left(X=1|Y=1\right)P\left(Y=1\right)-p_xp_y\\
&=\left(P\left(X=1|Y=1\right)-p_x\right)p_y\\
&=0
\end{aligned}
$$

Hence, $P\left(X=1|Y=1\right)=p_x=P\left(X=1\right)$and similarly $P\left(Y=1|X=1\right)=p_y=P\left(Y=1\right)$ From that, we have

$P\left(X=0|Y=1\right)=\frac{P(Y=1|X=0)P(X=0)}{P(Y=1)}=1-p_x=P\left(X=0\right)\Rightarrow P\left(Y=1|X=0\right)=p_y=P\left(Y=1\right)$, and similarly $P\left(X=1|Y=0\right)=p_x=P\left(X=1\right)$ Thus, X and Y are independent.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

3

1/1 point (graded)

Consider ten independent $B_{0.3}$ trials. Which of the following is the most probable?

- ● 0000000000 ✓
- ○ 1111111111
- ○ 1110000000
- ○ 0001111111

Explanation

Under $B_{0.3}$, the probability of sequence with w ones and $n-w$ zeros is $0.3^w\cdot0.7^{(n-w)}=0.7^n\cdot(3/7)^w$, which decreases with w.

Hence 0000000000 is the most likely sequence with probability 0.7^{10}, while 1111111111 is least likely with probability 0.3^{10}. This is also logical as under $B_{0.3}$, every bit is more likely to be a 0 than a 1.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Consider ten independent $B_{0.3}$ trials. Which of the following is the most probable?

Try to reconcile with the previous question.

○ 10 zeros

○ 10 ones

● 3 ones and 7 zeros ✓

○ 3 zeros and 7 ones.

Explanation
First, inutitively, for $B_{0.3}$ we expect to see roughly 30% 1's.
Slightly more rigorously, while individually, a sequence with 10 zeros is the most likely among all sequences, there is only one such sequence. When you balance the probability of each sequence with the number of such sequence, you see that observing a sequence with 3 ones and 7 zeros is most likely.
We will do this calculation formally when we study binomial distributions in the next section.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

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5 Bernoulli modifications

0 points possible (ungraded)
Let $X \sim B_{0.2}$. Find the Bernoulli parameter for the following random variables. Write -1 if they are not Bernoulli.

- X^2,

-1 ✖ **Answer:** 0.2

-1

Explanation
Since $X \in \{0,1\}$, we have $X^2 = X$.

- $+\sqrt{X}$,

✖ **Answer:** 0.2

Explanation
Since $X \in \{0,1\}$, we have $+\sqrt{X} = X$.

- $1 - X$,

✖ **Answer:** 0.8

Explanation
$1 - X$ takes values in $\{0, 1\}$, hence is Bernoulli, and $1 - X = 1$ iff $X = 0$, which happens with probability 0.8.

- $-X$.

✗ **Answer:** -1

Explanation
$-X$ takes values in $\{0, -1\}$, hence is not Bernoulli.

Submit You have used 4 of 4 attempts

ℹ Answers are displayed within the problem

6 Bernoulli pairs

0 points possible (ungraded)
Let $X \sim B_{0.4}$, $Y \sim B_{0.2}$, and they are independent. Find the Bernoulli parameter for the following random variables. Write -1 if they are not Bernoulli.

- $X \cdot Y$,

0.08 ✓ **Answer:** 0.08

0.08

Explanation
$X \cdot Y$ takes values in $\{0, 1\}$, hence is Bernoulli. It is 1 iff $X = Y = 1$ which happens with probability $0.4 \cdot 0.2 = 0.08$.

- X^Y, recall that $0^0 = 1$,

✗ **Answer:** 0.88

Explanation
X^Y takes values in $\{0, 1\}$, hence is Bernoulli. It is 0 iff $X = 0$ and $Y = 1$, which happens with probability $0.6 \cdot 0.2 = 0.12$, hence it is 1 with probability 0.88.

- $|X - Y|$,

✗ **Answer:** 0.44

Explanation

$|X-Y|$ takes values in $\{0,1\}$, hence is Bernoulli. It is 1 iff $X \neq Y$, which happens with probability $0.6 \cdot 0.2 + 0.4 \cdot 0.8 = 0.44$

- $X + Y$.

✖ **Answer:** -1

Explanation

$X + Y$ takes values in $\{0,1,2\}$, hence is not Bernoulli.

Submit You have used 4 of 4 attempts

ℹ Answers are displayed within the problem

7 Bernoulli sum

0 points possible (ungraded)

$X = U + V$, where U and V are independent Bernoulli variables with different expectations but the same variance 0.21. Find:

- $E(X)$,

✖ **Answer:** 1

Explanation

Let $U \sim B_p$ and $V \sim B_q$. Since U and V have the same variance, $p \cdot (1-p) = q \cdot (1-q)$ and since $p \neq q$, we must have $q = 1 - p$. Hence $E(X) = E(U+V) = E(U) + E(V) = p + q = p + (1-p) = .1$

- $V(X)$,

0.1

✖ **Answer:** 0.42

0.1

Explanation

$V(X) = V(U) + V(V) = 0.42$

- σ_X.

✖ **Answer:** 0.6481

Explanation

$\sigma_X = \sqrt{V(X)} = 0.6481.$

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Answers are displayed within the problem

8

2/3 points (graded)

Let X be the number of heads when flipping four coins with heads probabilities 0.3, 0.4, 0.7, and 0.8. Find:

- $P\left(X=1\right)$,

0.1872 ✓ **Answer:** 0.1872

0.1872

Explanation

$P\left(X=1\right)=0.3\cdot0.6\cdot0.3\cdot0.2+0.7\cdot0.4\cdot0.3\cdot0.2+0.7\cdot0.6\cdot0.7\cdot0.2+0.7\cdot0.6\cdot0.3\cdot0.8=0.1872$

- $E\left(X\right)$,

2.1992 ✓ **Answer:** 2.2

2.1992

Explanation

$E\left(X\right)=0.3+0.4+0.7+0.8=2.2$

- $V\left(X\right)$.

1.89359145 ✗ **Answer:** 0.82

1.89359145

Explanation

$V\left(X\right)=0.21+0.24+0.21+0.16=0.82$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

9 Light bulbs

0 points possible (ungraded)

Every light bulb is defective with 2% probability. What is the probabity that a package of 8 bulbs will not suffice for a project requiring 7?

1 ✗ **Answer:** 0.0103

1

Explanation

Let $X \sim B(0.02, 8)$ be the number of defective bulbs in a package.

The box will not suffice if there are 2 or more defective bulbs, which happens with probability.

$$P(X < 7) = 1 - P(X = 8) - P(X = 7) = 1 - (1 - 0.02)^8 - 8 \cdot (1 - 0.02)^7 \cdot 0.02 = 0.0103.$$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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