edX

Course › Topic 1... › 10.3 La... › Weak L...

Weak Law of Large Numbers Video

Start of transcript. Skip to the end.

9:10 / 0:00 1.0x

- Hello and welcome back.

In the previous lectures,

we talked about Markov and Chebyshev's inequality

and now we would like to apply them

to get the Weak Law of Large Numbers, okay.

So first, a little bit of motivation,

so probability theory is based on the fact that

10.3_Law_of_Large_Numbers

POLL

You have two fair coins, and you toss the pair 10,000 times (so you get 10,000 outcome pairs). Roughly how many pairs will not show any tails?

- 0
- 1250
- 2500
- 5000

Submit

1

1/1 point (graded)

In plain terms, the Weak Law of Large Numbers states that as the number of experiments approaches infinity, the difference between the sample mean and the distribution mean can be as small as possible.

- True ✔
- False

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2

0 points possible (ungraded)

Given n iid random varibles $X_1, X_2, \cdots, X_n$ with mean μ, standard deviation $\sigma < \infty$, and the sample mean $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, is it true that $\lim_{n\to\infty} E\left((S_n - \mu)^2\right) = 0$?

- True ✔
- False

Explanation

$\lim_{n\to\infty} E\left((S_n - \mu)^2\right) = \lim_{n\to\infty} V\left(S_n\right) = \lim_{n\to\infty} \frac{\sigma^2}{} = 0$ This means S_n converges to the true mean μ in mean square sense.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

3

3.0/3.0 points (graded)

The height of a person is a random variable with variance ≤ 5 inches^2. According to Mr. Chebyshev, how many people do we need to sample to ensure that the sample mean is at most 1 inch away from the distribution mean with probability $\geq 95\%$?

100 ✔ **Answer:** 100

100

Explanation

Recall from the proof of the weak law of large numbers that if $X_1, \ldots X_n$ are iid samples each with variance σ^2, then the variance of the sample mean $\overline{X^n}$ is σ^2/n. Therefore, if we sample n people, the sample mean of their heights will have a variance $\leq 5/n$ inches ^{2}.

By Chebyshev's Inequality, the probability that the sample mean will be at least 1 inch away from the mean is at most $\frac{5/n}{1^2} = \frac{5}{n}$, hence the probability that the sample mean will be at most 1 inch away is at least $1 - \frac{5}{n}$. We would like to have $1 - \frac{5}{n} \geq 0.95$, hence $\frac{5}{n} \leq 0.05$, or $n \geq 100$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

For $i = 1, 2, \cdots, n$ let $X_i \sim \mathcal{U}(0, 4)$, $Y_i \sim \mathcal{N}(2, 4)$, and they are independent. Calculate,

$E\left(X_i\right)$

✖ **Answer:** 2

Explanation

For $X_i \sim \mathcal{U}(a,b)$, $E(X_i) = \frac{a+b}{2} = 2$

$V(X_i)$

0

✖ **Answer:** 4/3

0

Explanation

For $X_i \sim \mathcal{U}(a,b)$, $V(X_i) = \frac{(b-a)^2}{12} = \frac{4}{3}$.

$E(Y_i)$

✖ **Answer:** 2

Explanation

For $Y_i \sim \mathcal{N}(\mu, \sigma^2)$, $E(Y_i) = \mu$.

$V(Y_i)$

✖ **Answer:** 4

Explanation

For $Y_i \sim \mathcal{N}(\mu, \sigma^2)$, $E(Y_i) = \sigma^2$.

Find the limit in probability of when $n \to \infty$

$\frac{1}{n}\sum_{i=1}^{n}(X_i + Y_i)$

✖ **Answer:** 4

Explanation

According to WLLN, when $n \to \infty$,
$\frac{1}{n} \sum_{i=0}^{n} (X_i + Y_i) \to E(X_i + Y_i) = E(X_i) + E(Y_i) = 4$

$\frac{1}{n} \sum_{i=1}^{n} (X_i Y_i)$

✖ **Answer:** 4

Explanation

According to WLLN, when $n \to \infty$, $\frac{1}{n} \sum_{i=0}^{n} (X_i Y_i) \to E(X_i Y_i) = E(X_i) E(Y_i) = 4$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

Flip a fair coin n times and let X_n be the number of heads. Is it true that $P\left(|X_n - \frac{n}{2}| > 1000\right) < 0.99$?

- True
- False ✔

Explanation

Very roughly, $|X_n - \frac{n}{2}| \approx \frac{\sqrt{n}}{2}$, which grows as n grows. We cannont guarantee $P\left(|X_n - \frac{n}{2}| > 1000\right) < 0.99$

Does the result above contradict with the WLLW?

- Yes
- No ✔

Explanation

The WLLN shows that $\lim_{n\to\infty} P(|\frac{X_n}{n} - \frac{1}{2}| > \epsilon) = 0 \quad \Rightarrow P(|X_n - \frac{n}{2}| > \epsilon n) = 0$
ϵn increases when n increases, but 1000 is not changed.

Submit

You have used 4 of 4 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Topic 10 / Law of Large Numbers

Add a Post

Show all posts ▼ by recent activity ▼

Problem 4 What is the Xi distribution in question 4?	2
Problem 3 Questions and comments regarding problem 3. Staff	5
General Comments Questions and comments regarding this section. Staff	1
Problem 1 Questions and comments regarding problem 1. Staff	1
Problem 2 Questions and comments regarding problem 2. Staff	1