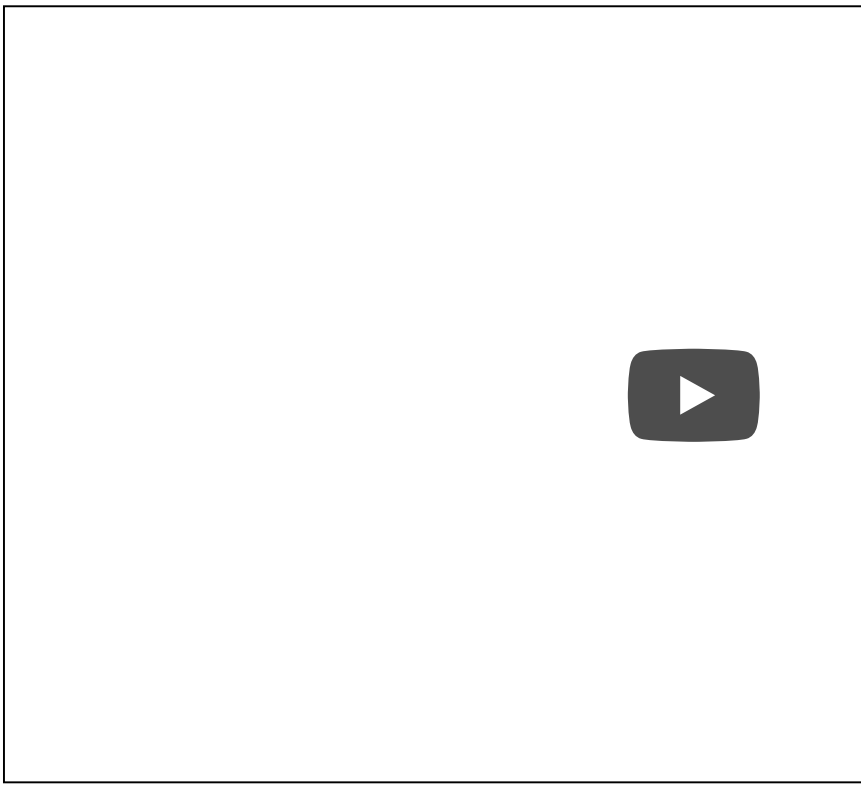
Linearity of Expectations Video



minus twice the expected value of x

times the expected value of y.

Now, the first two terms are the variance of x,

and the next two terms are the variance of y,

and what we're left with is

twice the expected value of xy

minus the expected value of x

times the expected value of y.

So, the answer as to whether the variance of x plus y

is equal to the variance of x plus the variance of y

depends on whether the expected value of xy

equals the expected value of

7.8_Linearity_of_Expectation

POLL
Which of the following always holds?

RESULTS

Both	**76%**
E[X+Y]=E[X]+E[Y]	**16%**
None	**6%**
E[X-Y]=E[X]-E[Y]	**3%**

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Results gathered from 200 respondents.

FEEDBACK
Both of them hold.

1

0 points possible (ungraded)
Let X be number of heads you get by flipping a fair coin 100 times. Then what is $E\left(X\right)$?

- $E\left[X\right]=25$
- $E\left[X\right]=50$ ✔
- $E\left[X\right]=75$
- None of the above

Explanation
Let X_i be the random variable for the i-th flip, with 1 representing heads and 0 representing tails. Then $E\left(X_i\right)=\frac{1}{2}$.
It is obvious that $X=\sum_{i=1}^{100}X_i$. Its expectation
$E\left(X\right)=E\left(\sum_{i=1}^{100}X_i\right)=\sum_{i=1}^{100}E\left(X_i\right)=100\times\frac{1}{2}=50$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

3.0/3.0 points (graded)

Starting with 10 blue balls, in each of 10 sequential rounds, we remove a random ball and replace it with a new red ball. For example, after the first round we have 9 blue balls and one red ball, after the second round, with probability $9/10$ we have 8 blue balls and 2 red balls, and with probability $1/10$ we have 9 blue balls and one red ball, etc.

What is the probability that the ball we remove at the 11th round is blue?

0.348678 ✔ **Answer:** 0.349

0.348678

Explanation

Imagine that the balls are placed in 10 locations 1 to 10. Let B_i be the event that at the final (11th) round, the ball in location i is blue. B_i occurs iff the ball in location i was not discarded in any of the previous 10 rounds, hence $P(B_i) = (1-1/10)^{10} = (9/10)^{10}$. Let B be the event that the final ball, picked at the 11th round, is blue. By the rule of total probability, $P(B) = \sum_{i=1}^{10} \frac{1}{10} P(B_i) = 10 \cdot \frac{1}{10} (\frac{9}{10})^{10} = (\frac{9}{10})^{10} = 0.3486$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

3

2.0/2.0 points (graded)

$\mathbb{E}(X) = 2$ and $\mathbb{E}(X(X-1)) = 5$ Find $V(X)$.

3 ✔ **Answer:** 3

3

Explanation

$5 = \mathbb{E}(X(X-1))$
$= \mathbb{E}(X^2 - X)$
$= \mathbb{E}(X^2) - \mathbb{E}(X)$
$= \mathbb{E}(X^2) - 2$
$\rightarrow \mathbb{E}(X^2) = 5 + 2 = 7$

$$V(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 7 - 4 = 3$$

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

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