Normal Distribution

Video

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- Hello, and welcome back.

So next we would like to discuss Gaussian Distributions,

or, commonly known as normal distributions.

You've noticed that some, many of the people

we've talked about are quite famous,

they've appeared on things like statues and stamps,

but Gauss actually appeared on a deutschemark.

5:00 / 0:00 1.0x

9.5_Gaussian_Distribution

POLL

If you fix the mean but increase the variance of a normal distribution, its pdf will

- **move to the left**
- **move to the right**
- **become taller and narrower**
- **become shorter and flatter**

Submit

1 Highest probability

0 points possible (ungraded)

Let $X \sim \mathcal{N}(\mu, \sigma)$ be a normal random variable, then the maximum value of its pdf is

- 1

- $\frac{1}{\sqrt{2\pi}}$
- $\frac{1}{\sqrt{2\pi\sigma}}$
- $\frac{1}{\sqrt{2\pi\sigma^2}}$ ✔

Explanation

A Gaussian pdf achieves its maximum value at its mean $f_X(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2 Linear transformations

0 points possible (ungraded)

The linear transformation of a normal random variable is also a normal random variable.

- True ✔
- False

Explanation

For any $X \sim \mathcal{N}(\mu, \sigma^2)$, we have $aX + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

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Answers are displayed within the problem

3

0 points possible (ungraded)

Given $X \sim U_{[a,b]}$ with $E[X] = 2$ and $V(X) = 3$, find a and b.

- a

5 ✖ **Answer:** -1

5

- b

✖ **Answer:** 5

Explanation

The mean and variance of $X \sim U_{[a,b]}$ are given by $E[X] = (a+b)/2$, $V(X) = (b-a)^2/12$ respectively. Thus $(a+b)/2 = 2$ or $a = 4 - b$. Further $(b-a)^2/12 = 3$, and substituting $a = 4 - b$, we have $(b-2)^2 = 9$. Solving this and keeping the solution with $b \geq a$ gives $b = 5,\ a = -1$.

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Answers are displayed within the problem

4

0 points possible (ungraded)

If X, Y are two independent random variable with $X \sim \mathcal{N}(1, 16)$ and $Y \sim \mathcal{N}(1, 9)$, then find $\mathrm{Var}(XY)$.

10 ✖ **Answer:** 169

10

Explanation

Noted that $E(X^2) = V(X) + E^2(X) = 17, E(Y^2) = V(Y) + E^2(Y) = 10$, we have
$V(XY) = E(X^2Y^2) - E^2(XY) = E(X^2)E(Y^2) - E^2(X)E^2(Y) = 170 - 1 = 169$

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Answers are displayed within the problem

5

0 points possible (ungraded)

Suppose X is a Gaussian random variable with mean 2 and variance 4. Find $E\left(e^{\frac{X}{2}}\right)$.

10 ✖ **Answer:** 4.4816

10

Explanation

$$E(e^{\frac{x}{2}}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi 2^2}} e^{-\frac{(x-2)^2}{2 \cdot 2^2}} e^{\frac{x}{2}} \, dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi 2^2}} e^{-\frac{x^2-8x+4}{2 \cdot 2^2}} \, dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi 2^2}} e^{-\frac{(x-4)^2}{2 \cdot 2^2}} e^{\frac{3}{2}} \, dx = e^{\frac{3}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi 2^2}} e^{-\frac{(x-4)^2}{2 \cdot 2^2}} \, dx = e^{\frac{3}{2}}$$

.

Or using the MGF (taught in Topic 10), which is $e^{\mu t + \sigma^2 t^2/2}$, and plugging in $t = 1/2$, we get $e^{3/2}$.

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Answers are displayed within the problem

6

0 points possible (ungraded)

If $x \sim \mathcal{N}(0, 1)$, find $E\left(e^{-X^2}\right)$.

00 ✖ **Answer:** 0.577

00

Explanation

$$E\left(e^{-X^2}\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} e^{-x^2} \, dx = \frac{1}{\sqrt{3}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \frac{1}{3}}} e^{-\frac{x^2}{2 \cdot \frac{1}{3}}} \, dx = 1/\sqrt{3}$$

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Answers are displayed within the problem

7

0.0/3.0 points (graded)

Let X be distributed according to the pdf ke^{-x^2-7x}. Find $E\left(X^2\right)$.

0

Answer: 51/4

0

Explanation

Notice that $f_X\left(x\right)=ke^{-x^2-7x}=\left(ke^{49/4}\right)\cdot e^{-\frac{(x+7/2)^2}{2\cdot 0.5}}=c\cdot e^{-\frac{(x+7/2)^2}{2\cdot 0.5}}$, where c is a constant. Therefore X is normally distributed with $\mu=-7/2$ and $\sigma^2=0.5$ and thus $E\left(X^2\right)=V\left(X\right)+E(X)^2=1/2+49/4=51/4.$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

3.0/3.0 points (graded)

Let $X\sim N\left(0,9\right)$ have mean 0 and variance 9. Find the expected value of $X^2\left(X+1\right)$.

9

Answer: 9

9

Explanation

Notice that since the Guassian distribution is symmetric around its mean 0, $E\left(X^3\right)=\int_{-\infty}^{\infty}z^3\frac{1}{\sqrt{2\pi 9}}e^{-\frac{z^2}{2}}dz=0$. Further since $E\left(X^2\right)=Var\left(X\right)+E(X)^2=9$ the answer follows.

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Answers are displayed within the problem

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