

Assignment 4 - Graph

PROGRAM ANALYSIS AND OUTPUTS

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Functions Description:

solution 1:

- `Void dfs()`: perform dfs on graph. It always starts with the lowest valued node in the graph.
- `void traverse_dfs(int v)`: recursive function for dfs on given node v
- `void mark_edge()`: marks edge type(tree,forwad,etc) and print dot code to get graph image.

Solution 2:

- `void tarjan(), void tar_jan(int i);` : Performs tarjan algorithm based on the following pointers:

- `void printComponents()`: Prints all SCCs of graph in image `g_component.png`.

Solution 3:

- `void minGraph()`: Here I created a new graph `gmin` and filled it with edges from original graph, such that it fulfills the 3 conditions of question 3.

First , I filled the edges in every component in a cycle so as to get a SCC with minimal edges. Then, checked in original graph for inter SCC edges and added atmost 1 edge between them, thus the resulting graph `g_min` has minimal edges, has same no of SCC , and has the same component graph.

It creates 3 graph images of graph `g_min`. These are:

1. `gmin_graph.png` : The graph `g_min`. This is graph `g` with minimal edges.

2. `gmin_components.png` : Shows the SCCs of `g_min`.
You can check the no of SCC are same as with `g_components`.
3. `gmin_component_graph.png` : This shows connections between all SCCs. You can check the DAGs of `g_Component_Graph` and `gmin_component_graph` are same. The SCC labels(number) will be changed as its not upto me to keep same label for `g` and `g_min`.
- `bool path_present(int s, int d)`: checks if there is path from `s` to `d` or not. Based on this, all edges are filled in `gmin`.

Solution 4:

The algo used for this is:

Algorithm 7 IS-SEMI-CONNECTED(G)

```
    Compute the component graph of  $G$ , call it  $G'$ 
    Perform a topological sort on  $G'$  to get the ordering of its vertices
     $v_1, v_2, \dots, v_k$ .
    for  $i=1..k-1$  do
        if there is no edge from  $v_i$  to  $v_{i+1}$  then
            return FALSE
        end if
    end for
    return TRUE
```

Working:

I created a new graph g_4 which contains all SCCs of graph ' g ' as nodes and edges between them if any. This forms the component graph of g . This is done in $O(V+E)$.

$V \rightarrow$ nodes, $E \rightarrow$ edges.

Next, topological sort is done on graph g_4 to get the ordering of SCC. T.C : $O(x+e)$.

X = no of SCC \leq no of nodes V . At worst case it can be = no. Of nodes V . Then T.C = $O(V+E)$

Finally, we check if there is any edge between two adjacent nodes in topological sort or not.

If not, then the graph is not semi-connected, else it is semi-connected. T.C $\rightarrow O(V)$ worst case.

So, Overall T.C = $O(V+E)$

Functions used:

- `bool soln4()`: just calls the below function.
- `bool isSemiCon()`: creates component graph.
- `void topologicalSort()`: topological sort on component graph
- `void dfs_topological(int v)`: helper for above
- `bool checklinearorder()`: checks if a linear path present in component graph or not.
- `void printComponentGraph()`: prints component graph dot code.

Solution 5:

`void shortestPath(int src,int dest)` : Finds shortest path from src to dest , using Dijkstra with min priority queue in $O(E \log V)$ time and prints it.

Important Property:

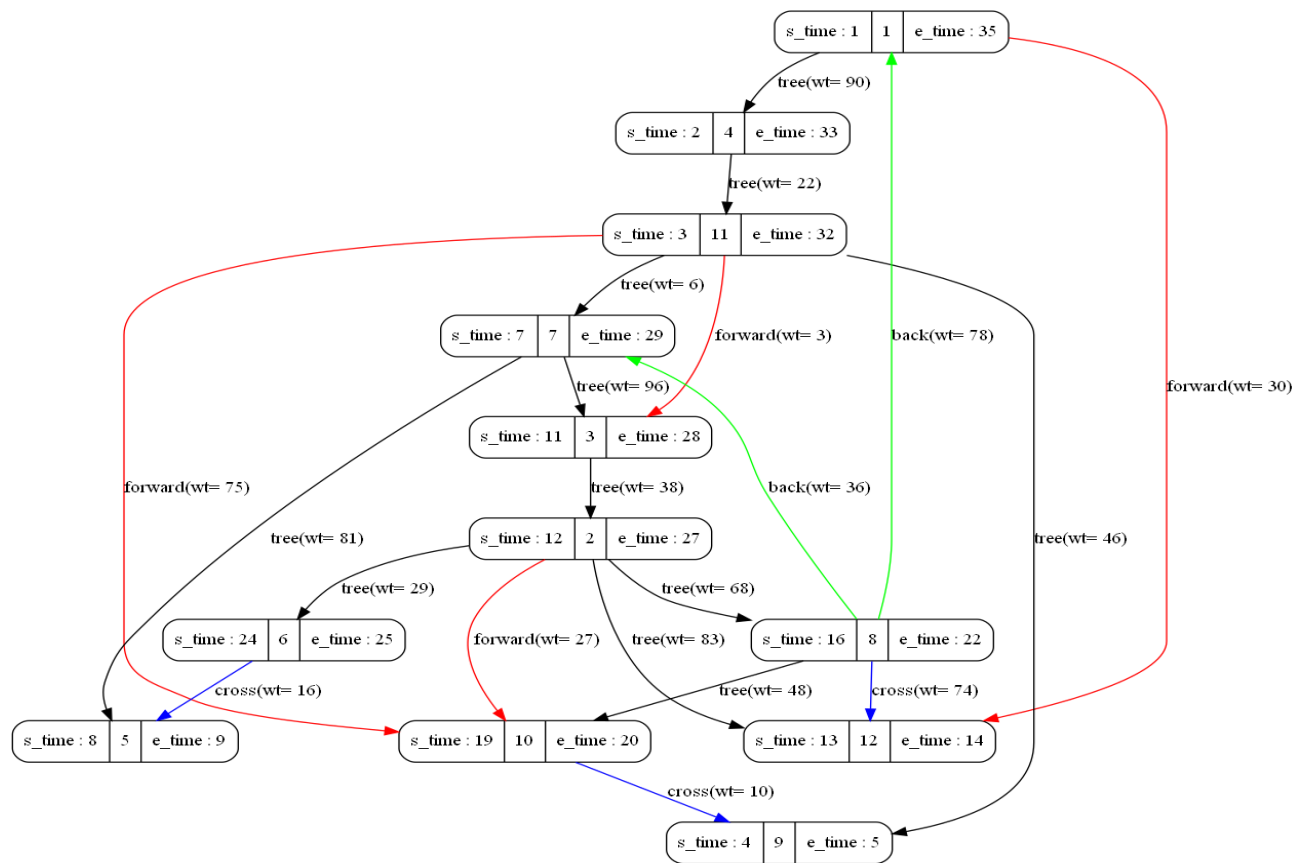
- a. Whenever distance of a vertex is reduced, we add one more instance of vertex in priority_queue. Even if there are multiple instances, we only consider the instance with minimum distance and ignore other instances.
- b. The time complexity remains $O(E \log V)$ as there will be at most $O(E)$ vertices in priority queue and $O(\log E)$ is same as $O(\log V)$

Graph images generated for my given “input.txt” :

From Soln1:

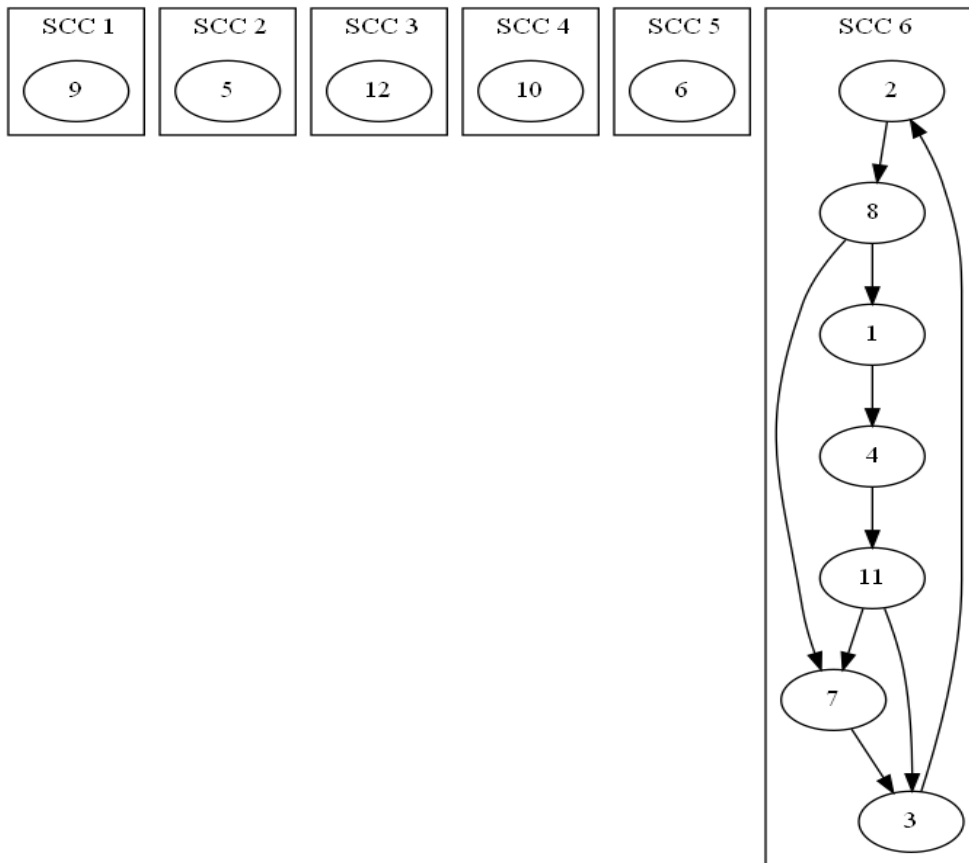
g_Graph.png - DFS graph of graph g.

Structure is : start_time | Node | end_time



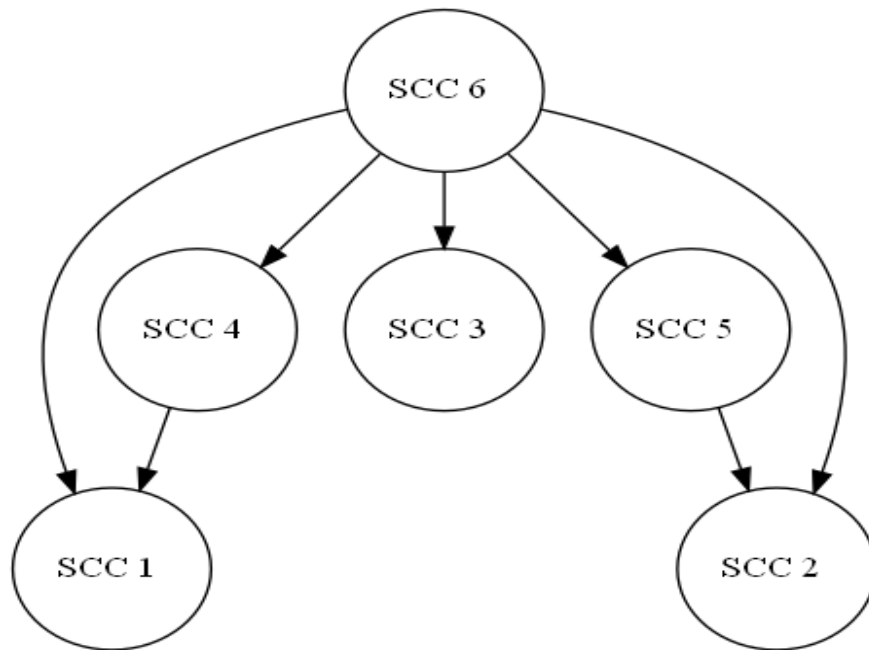
From Soln2:

`g_components.png` - SCComponents of `g`.



From soln 4:

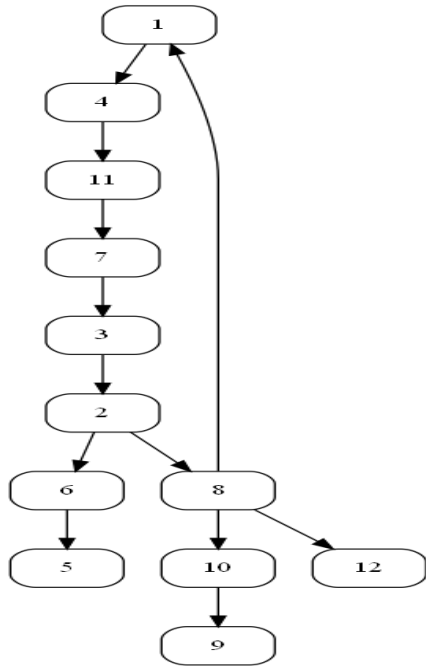
`g_Component_Graph` – of graph `g`.



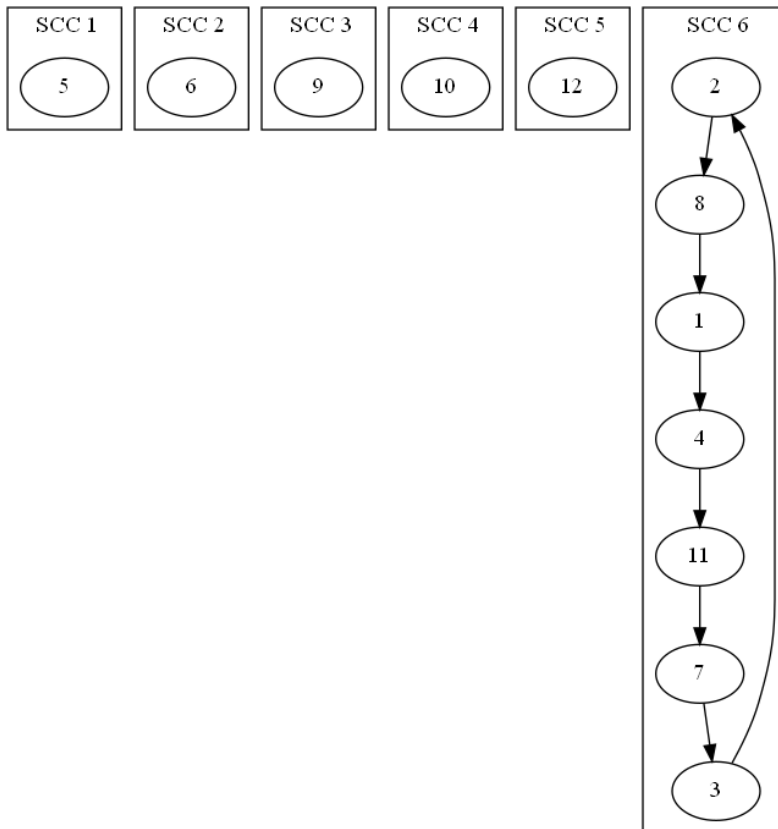
From soln 3:

3 images of graph g_{\min} . As explained above, it can be used to compare with above 3 images of graph g .

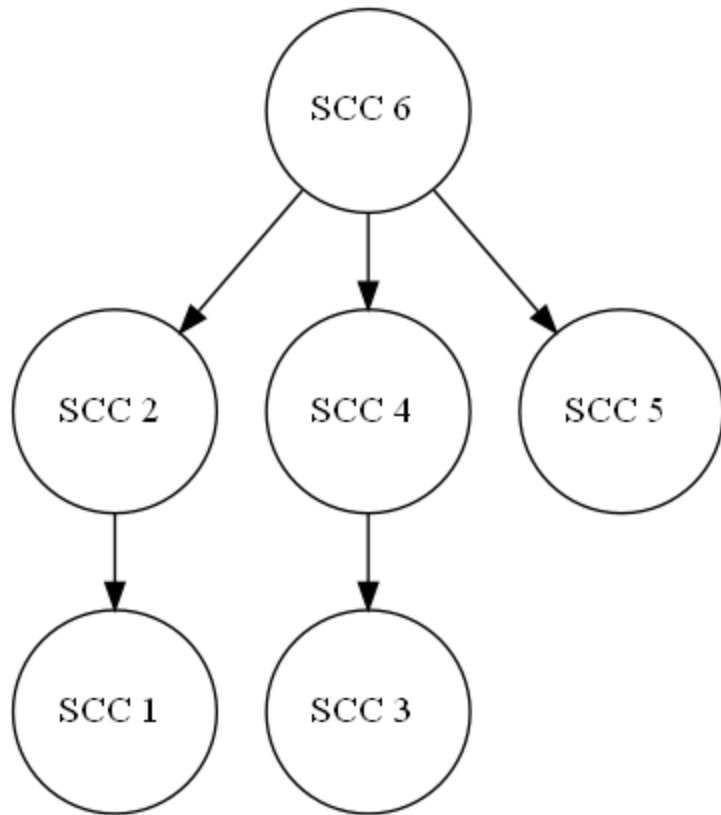
gmin_Graph:



gmin_components:



gmin_Component_Graph:



Note : The Component_graph of g and g_{\min} will be same but component label(numbering) will be different as it is not upto me to fix the labels of SCC.

You can check the difference in component label from `g_components.png` and `gmin_components.png`

END OF ANALYSIS !!!