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PARKINSON'S LAW AND ITS IMPLICATIONS FOR PROJECT MANAGEMENT*

GENARO J. GUTIERREZ AND PANAGIOTIS KOUVELIS
Graduate School of Business, Management Department,
University of Texas at Austin, Austin, Texas 78712

Critical path models concerning project management (i.e. PERT/CPM) fail to account for work force behavioral effects on the expected project completion time. In this paper, we provide a modelling framework for project management activities, that ultimately accounts for expected worker behavior under Parkinson's Law. A stochastic activity completion time model is used to formally state Parkinson's Law. The developed model helps to examine the effects of information release policies on subcontractors of project activities, and to develop managerial policies for setting appropriate deadlines for series or parallel project activities.
(PROJECT MANAGEMENT; PARKINSON'S LAW; STOCHASTIC MODELING)

1. Introduction

It is widely accepted among the community of project management scholars and practitioners that the conventional procedures (i.e. PERT/CPM) usually fail to provide an accurate estimate for large-scale project completion time. It is almost axiomatically stated that planned project schedules, obtained using critical path analysis, will be optimistic (Feiler 1972, Schonberger 1981). An experimental validation of the above statement, via Monte Carlo simulation of project networks, appeared in an early work by Klingel (1966). Some of the pitfalls of critical path analysis about project management have been adequately exposed in research papers (Britney 1976, Schonberger 1981, Hughes 1986) and textbook chapters on project management (Rosenau 1981, Wiest and Levy 1977, Chase and Aquilano 1989).

A deterministic critical path analysis, though appealing in its simplicity, fails to account for activity-time variability. The probabilistic PERT analysis, via beta type activity time distributions, on the one hand, allows for activity time variability, but, on the other, fails to allow for time delays from path interactions at shared nodes. The more paths sharing a specific node in the PERT network, the larger the deviation of the expected project completion time from its expected duration (Schonberger 1981). What's more, the inability to estimate correctly activity duration times, the improper focus on the concept of a "critical path" versus the more relevant concept of a "critical activity," and the failure to revise initial estimates are contributing to the underestimation of project duration (Chase and Aquilano 1989). To add to the above list of pitfalls, the critical path analysis ignores work force behavioral issues, and fails to model behavioral implications about critical and noncritical activity durations, that subsequently may cause project delays.

Ignoring behavioral issues in modelling project activity durations is equivalent to assuming that there is no relationship between the actual amount of work to be done, the deadline set for the worker to finish that work, and the actual completion time of the work. But that contradicts a widely accepted behavioral law, Parkinson's Law. This law, as stated in its original source (Parkinson 1957), manifests that "work expands so as to fill the time available for its completion." The law is widely accepted, and expressed in popular proverbial phrases ("It is the busiest man who has time to spare"). This relationship between allocation of effort and goals (in our context the goal is to finish within

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the preset deadline) has been extensively studied by behavioral scientists. A fundamental result is that there is a direct relationship between the level of performance and the goals set; the higher the goal, the higher the level of performance (Locke 1966, 1967). This result interpreted in a project management context is equivalent to Parkinson's Law. A loose deadline (i.e. lowering the goal) leads, thus, to a decline of the worker's performance and to a delay of the activity. The above goal setting result has been shown to generalize from laboratory to field settings (Locke et al. 1981). Moreover, this basic relationship also holds if the goals are set participatively or unilaterally (Latham et al. 1982), as well as for individuals with different levels of self-efficacy and experience (Locke et al. 1984).

Krakowski (1974) pointed out that the slack of project activities in PERT/CPM networks may eventually be absorbed by Parkinson's Law effects. Unfortunately, behavioral scientists and operations management researchers have not studied in further detail the wider implications of Parkinson's Law in the field of project management. The elasticity of work in its demand on time (especially for activities requiring human involvement) is in many cases, as most project managers have experienced, a major cause for project delays.

In this paper, we provide a modelling framework designed for project management activities, that incorporates worker and/or subcontractor behavioral issues as depicted in Parkinson's Law. We develop a descriptive model of the expected completion time of an activity. This model expresses completion time as a function of the time allocated for that activity, i.e. the deadline set by the project manager, and the actual amount of work required by the specific project activity. The developed model is of an analytic nature, and is used to enhance our understanding of activity management. Project managers obtain valuable insights in the management of individual activities and answers to important questions such as: What type of information release policies to subcontractors could have harmful effects on the project completion time? What are the preferable managerial policies for setting deadlines for serial activities (for parallel activities)?

In §2 we introduce a simple stochastic model of activity completion time, and use it to formally state Parkinson's Law. The above model is extended into a more general modelling framework for project activities in §3. The effects of information release policies to subcontractors are examined in §4. Managerial policies on setting deadlines for series or parallel project activities are discussed in §5. Finally, §6 summarizes our results and points future research issues.

2. Parkinson's Law and a Simple Stochastic Model of Activity Completion Time

Consider a project activity A, whose completion time is stochastic. Let us assume that the activity includes, or can be thought as consisting of, two subtasks A_1 and A_2, with subtask A_1 preceding A_2. Let T_i, $i = 1, 2$ denote the random variable representing the duration of A_i. Ignoring Parkinson's Law, the completion time of activity A, call it T, is a random variable independent of the amount of time allocated for that activity and holds that $T = T_1 + T_2$. But Parkinson's Law suggests that such activity modelling is inappropriate, and the completion time of an activity should be treated as a function of the allocated time. Denote the preset deadline as d (assuming without loss of generality that the activity starts at time 0), then the completion time is a function of d, i.e., $T = T(d)$.

In order to develop a model for activity completion time that is consistent with Parkinson's Law, we introduce a work expansion term, w, that accounts for leisure time or unnecessary expansion introduced. This refinement or delay, added by the worker or the subcontractor, depends on the actual progress of the subtasks and time remaining until the due date. If we are modelling subcontractor behavior, the work expansion term accounts for delays introduced due to shifting of resources to other projects to which the subcontractor is committed. For the rest of the paper we will use the term "worker" to refer to "worker/subcontractor" unless stated otherwise. According to Parkinson's Law, the work expansion w is a function of the time allocated to the activity, i.e. $w = w(d)$. Then the completion time of activity A, $T(d)$, is a random variable given by:

$$T(d) = T_1 + T_2 + w(d). \tag{1}$$

If the first task is completed at T_1, the expected duration of activity A is $T_1 + ET_2$, and the slack perceived by the worker will be $d - T_1 - ET_2$. To define $w(d)$, assume that if the perceived slack is positive, work will expand by that amount, otherwise no work expansion will take place thus:

$$w(d) = (d - T_1 - ET_2)^+. \tag{2}$$

From (1) and (2) we can easily derive the expected completion time of activity A as

$$ET(d) = ET_1 + ET_2 + E(d - T_1 - ET_2)^+. \tag{3}$$

Observing that $E(d - T_1 - ET_2)^+ \geq d - ET_1 - ET_2$ leads to

$$ET(d) \geq d. \tag{4}$$

Relationship (4) states the simple fact that due to the above modelled worker behavior, as implied by Parkinson's Law, the expected completion time of the activity is going to be on average larger than the time allocated to the activity by the project manager, regardless of what that amount of time might be. In other words, the worker will take longer than originally planned to finish the activity, even if the set deadline is loose.

Observe from (2) that $w(d)$ is nondecreasing and nonnegative, thus $ET(d)$ is also nondecreasing and $ET(d) \geq ET_1 + ET_2$. This implies that the project manager would want to set d as small as possible. However, unreasonably small values of d, i.e. $d < ET_1 + ET_2$, should not be used, since they are unfair to the worker, and in the long run such unfair treatment will be understood by the workers, with harmful consequences. In an ideal situation, the project manager has accurate information on the value of $ET_1 + ET_2$, and he will allocate to the activity time equal to the "true" amount of work involved, i.e. $d = ET_1 + ET_2$.

The worker behavior pattern discussed thus far will be referred to as the "expanding worker" behavior. It is characterized by an immediate involvement in the activity (i.e. start at time 0) and with work expansion behavior described in (2) and (3). Consider a different worker behavior pattern; we will refer to it as the "busy worker" pattern. The "busy worker" will postpone the start of activity A, until there is barely enough time to finish it. However, once he starts working on the activity, he does not expand on the required work. In that case, the expected completion time of activity A as a function of the set deadline is given by

$$ET'(d) = ET_1 + ET_2 + (d - ET_1 - ET_2)^+ = \max\{d, ET_1 + ET_2\}. \tag{5}$$

Relationship (3), the convexity of $w(d)$ and Jensen's inequality imply that $ET(d) \geq ET'(d) \geq d$.

In other words, the "busy worker," who postpones the activity initially, is going to finish on average earlier than the expanding worker. The obvious message about productivity management, from this analysis, is that, by keeping the workers reasonably "busy," you improve their overall productivity, without necessarily harming the progress of the project.

Summarizing our discussion so far, we present three statements of *Parkinson's Law for Project Management Activities*, that are appropriate accompanied with relevant proverbial phrases:

$$
(\mathrm{PL})\begin{cases}
(\mathrm{PL1}) & ET(d) \geq d: \quad \text{"Expect all activities to be late,"} \\
(\mathrm{PL2}) & ET(d_1) > ET(d_2) \quad \text{for} \quad d_1 > d_2: \quad \text{"Work is elastic in its demand of time,"} \\
(\mathrm{PL3}) & ET'(d) \leq ET(d): \quad \text{"It is the busiest man who has time to spare."}
\end{cases}
$$

3. Stochastic Modelling of Project Activities

In this section, our discussion concentrates on the development of a more general stochastic model of an activity completion time that demonstrates, in a more general framework, the importance of Parkinson's Law in project management. Consider an activity A that includes, or can be thought as consisting of, n subtasks A_i, $i = 1, \ldots, n$. Denote by T_i, $i = 1, \ldots, n$, the random variable representing the duration of subtask A_i. For the subtask indexes k, j, with $k > j$, define $\zeta_{j,k} = \sum_{i=j}^{k} T_i$.

Whenever a subtask is completed, the worker might decide to expand work. At that time the worker compares the remaining time for completing activity A, as he perceives it, with the available time until the set deadline d. If the available time is larger than the remaining time required for completion of the activity, the worker expands the work by an amount of time equal to their difference. Otherwise no work expansion occurs. Let $w_i(d)$ be the cumulative work expansion, measured in time units, just before starting subtask $i + 1$, considering it a function of the set deadline d for the activity. Then the completion time of activity A, $T(d)$, is a random variable given by:

$$T(d) = \sum_{i=1}^{n} T_i + w_{n-1}(d) = \zeta_{1,n} + w_{n-1}(d). \tag{6}$$

The total work expansion term $w_{n-1}(d)$ is defined recursively through the following relationships:

$$w_i(d) = w_{i-1}(d) + (d - \zeta_{1,i} - w_{i-1}(d) - E\zeta_{i+1,n})^+ \qquad \text{for} \qquad i = 2, \ldots, n-1 \tag{7}$$

and

$$w_1(d) = (d - T_1 - E\zeta_{2,n})^+. \tag{8}$$

Some interesting properties of $w_i(d)$, $i = 1, \ldots, n-1$, are discussed next. Since their methodological proofs do not provide additional managerial insights, we provide them in an Appendix for the interested reader.

PROPOSITION 1. *The cumulative work expansion terms $w_i(d)$, $i = 1, \ldots, n-1$, and their expectations $Ew_i(d)$ are convex and nondecreasing functions in d.*

PROPOSITION 2. *The right-hand side derivative with respect to the set deadline d of the expectation of the cumulative work expansion term $w_i(d)$, $i = 1, \ldots, n-1$, denoted by $D_d^+ Ew_i(d)$, satisfies $0 \leq D_d^+ Ew_i(d) \leq 1$.*

Propositions 1 and 2 characterize $ET(d)$ as a convex nondecreasing function with $ET(0) = E\zeta_{1,n}$, and $D_d^+ ET(d) \leq 1$ for all d. The functional form of $ET(d)$ is shown in Figure 1. Observing that $Ew_{n-1}(d) \geq d - E\zeta_{1,n}$, in conjunction with (6), leads to $ET(d) \geq d$, i.e. the first statement of Parkinson's Law (PL1). Proposition 1 together with (6) prove the validity of (PL2), i.e. $ET(d_1) > ET(d_2)$ for $d_1 > d_2$, in this more general activity modelling framework. The validity of (PL3) can, also, be easily established following logic similar to the one presented in §2, and by repeated use of Proposition 1 and Jensen's inequality.

The behavior of the "expanding worker" is accurately described through relationships

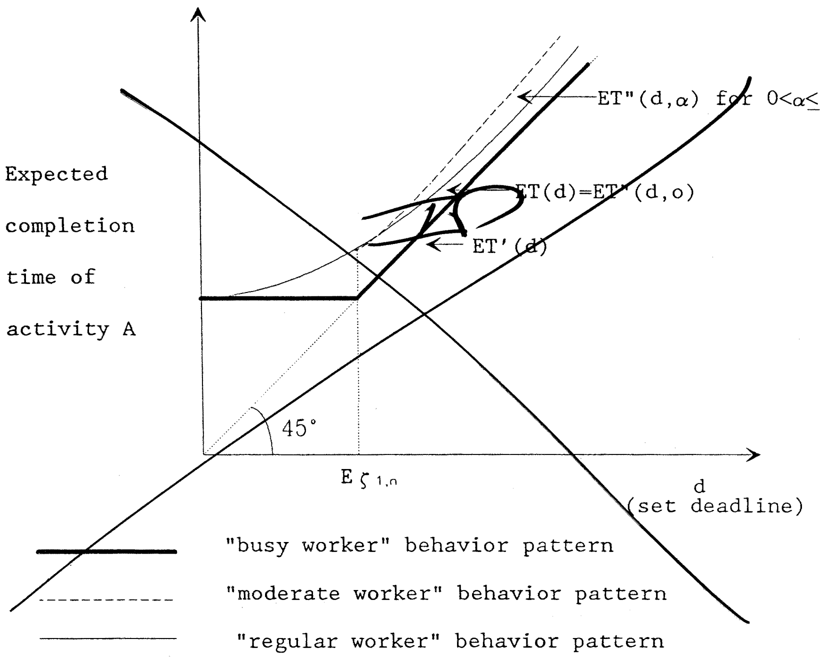

FIGURE 1. Expected completion time of activity A for different behavior patterns, as a function of the allocated amount of time for performing the activity (set deadline d if activity starts at time 0).

(6)–(8). For the "busy worker," in this more general activity modelling framework, it holds that

$$ET'(d) = \max\{d, E\zeta_{1,n}\}. \tag{9}$$

Another interesting worker behavior pattern, that we call the "moderate worker," is the one in which the start of the activity is delayed, but the delay is smaller than that of the busy worker, and work expansion takes place as the work progresses. This worker behavior pattern is described by

$$ET''(d, \alpha) = \alpha(d - E\zeta_{1,n})^+ + E\zeta_{1,n} + Ew''_{n-1}(d, \alpha), \qquad \text{where} \tag{10}$$

$$w''_i(d, \alpha) = w_i(d - \alpha(d - E\zeta_{1,n})^+) \qquad \text{for} \qquad 0 \le \alpha \le 1, \qquad 2 \le i \le n-1. \tag{11}$$

For the "moderate worker" the following results can be stated:

PROPOSITION 3. *The expected completion time*, $ET''(d, \alpha)$, *is increasing in* α.

It follows from Proposition 3 that

$$ET'(d) \le ET(d) = ET''(d, 0) \le ET''(d, \alpha) \le ET''(d, 1). \tag{12}$$

The above functional relationships are schematically depicted in Figure 1.

There are two observations we can make from Figure 1. First, when the expanding behavior is introduced, delays in the start of the activity, introduced by the moderate worker, translate into delays in the completion of the activity, i.e., $ET''(d, \alpha) \ge ET''(d, 0)$. Second, it follows from (10)–(12) and Proposition 2 that increases in completion time of the moderate worker are smaller than or equal to initial delays of the activity, i.e., $ET''(d, \alpha) - ET(d, 0) \le \alpha(d - E\xi_{1,n})^+$. A corollary of this observation is that if the moderate worker is engaged in a useful activity during the initial delay, the moderate worker will be more productive than the regular worker even though both have similar expanding behavior.

4. Parkinson's Law for Parallel Project Activities and Its Implications for Subcontractor Management

A usual practice in project management is to ask periodically for subcontractors' progress reports on the subcontracted activity. Some project managers advocate the free access of such information to other subcontractors participating in the project. In some cases, particularly governmental agencies, well-established procedures that prescribe organized ways (i.e. work progress reports, information bulletins) do exist for the release of such information. In other cases, such information is indirectly available, since most or all of the subcontracting activities take place on the same physical location. Experienced project managers have repeatedly communicated to us some annoying implications of such information release policies. Even more, subcontractors who have fallen behind schedule use such information to avoid the blame for project delays by pointing out the delay of other subcontractors on parallel activities. While it is a shared feeling among many managers that free access information policies to subcontractors—particularly those working on parallel activities—might have harmful effects on the project completion time, currently available project management models fail to account for them. In this section, using the activity modelling framework that we have developed, we provide an organized approach to account for such effects, and provide valuable insights for subcontractor management on parallel activities.

Let us consider the segment of a PERT network for a large-scale project depicted in Figure 2. The segment consists of K parallel activities (i.e. activities that merge into the same PERT node) P_k, $k = 1, \ldots, K$, that all precede the critical activity A. As is the usual case for large-scale projects, each activity, that in some cases might be a small project by itself, is assigned to a different subcontractor. All subcontractors of the activities P_k, $k = 1, \ldots, K$, face the same deadline d, since delaying any of them will result in delaying the starting point of the critical activity A and consequently of the project. Let activity P_k, $k = 1, \ldots, K$, be composed of n_k subtasks, and t_k be the starting time of this activity.

We use a superscript k to denote the completion times and the expansion terms pertaining to parallel activity k (e.g., $T^k(d)$, ζ^k_{1,n_k} and $w^k_i(d)$). Relationships (6)–(8) are used to define them. The expected start time of activity A, call it $ET(d)$, is the same as the expected time at which all K parallel activities are completed, i.e.

$$ET(d) = E\{\max_k T^k(d)\}. \tag{13}$$

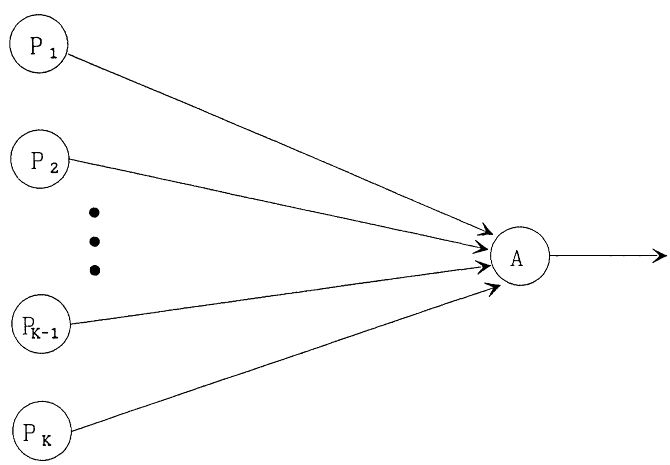

FIGURE 2. Segment of a PERT Network with K Parallel Activities.

Now, let us model the periodic information release policies of the project manager to the K subcontractors through a time dependent matrix $I_t = \{I_t^k, k = 1, \ldots, K\}$, where I_t^k is the vector of information that subcontractor k receives regarding the work progress on the other subcontracted activities, without excluding the possibility of no information for some activities (null elements of I_t^k). What we assume in our further discussion is that the information vector I_t^k affects the perception of the effective due date of the subcontracted activity for contractor k. Let ed_t^k denote the effective due date perceived by subcontractor k at time t. We model the information transformation process used by subcontractor k to determine the effective due date as

$$ed_t^k = \max\{d, g^k(I_t^k)\} \tag{14}$$

where $g^k(I_t^k)$ is an information transformation function which outputs an artificial deadline, that subcontractor k perceives as the minimum potentially achievable completion time by the other subcontractors based on the information vector I_t^k. Relationship (14) implies that the effective due date perceived by subcontractor k at time t is no earlier than either the preset deadline d, or the earliest possible completion time, according to subcontractor k's perception of the other parallel activities. The work expanding behavior of subcontractor k will be guided by the effective due date ed_t^k and not by d. As a consequence of Proposition 1 and relationship (14), we can conclude that

$$T^k(ed_t^k) \geq T^k(d). \tag{15}$$

This implies that the expected start time of activity A in the presence of the information release policies, call it $E\hat{T}(d)$, is going to be

$$E\hat{T}(d) = E\{\max_k T^k(ed_t^k)\} \geq E\{\max_k T^k(d)\} = ET(d). \tag{16}$$

Relationship (16) states that any work progress information release policy by project management to subcontractors of the parallel activities, that could affect their perception of the effective due date according to (14), can only have harmful effects on the completion time of the project. Observe that (16) is valid for any information transformation function g^k. The most usual cases in project management are:

(a) $g^k(I_t^k) = \max_{j \neq k}\{E^k(T_j(d) \mid I_t^k)\}$,
i.e. the kth subcontractor generates subjective estimates $E^k(T_j(d) \mid I_t^k)$ given the available information, and uses the maximum of those estimates to update the effective due date. The reasoning behind such behavior is summarized in the proverbial phrase "I do not want to be the last to finish."

(b) $g^k(I_t^k) = \min_{j \neq k}\{E^k(T_j(d) \mid I_t^k)\}$,
i.e. the kth subcontractor uses the minimum of the subjective estimates $E^k(T_j(d) \mid I_t^k)$ to update the effective due date. The reasoning behind such behavior is summarized in the phrase "I will be late only if everybody is late."

5. Managerial Policies for Series and Parallel Project Activities Implied by Parkinson's Law

Most of our discussion to now has concentrated on the importance of setting appropriate deadlines to minimize the completion delays of project activities. An appropriate managerial policy to minimize the effects of Parkinson's Law on project activities is to set "reasonably tight" deadlines (i.e. in our notation $d = E\zeta_{1,n}$), although in general we assume that $d \geq E\zeta_{1,n}$. However, in large-scale projects activities do not usually stand alone but are part of one of the two common network structures of the project's PERT network, i.e. series activities or parallel activities. In this section, we examine the effects of different managerial policies for setting deadlines for activities in series or in parallel on the completion time of the project segment of which these activities are part.

Consider a segment of the PERT network of a large-scale project consisting of two activities in series to be performed by the same worker or contractor. Denote by d_1 and d_2 the (estimated by the project manager) true amount of work involved in activities 1 and 2 respectively. First, assume activity 1 precedes 2 and, then, consider the following three managerial policies in setting deadlines for the activities of that network segment:

Policy 1. Set d_1 as the deadline for completion of activity 1. Wait until the completion time of activity 1 (denoted by $T(d_1)$). Then set $T(d_1) + d_2$ as the deadline for completion of activity 2.

Policy 2. Set d_1 as the deadline for completion of activity 1, and at the same time set $d_1 + d_2$ as the deadline for completion of the second activity.

Policy 3. Set $d_1 + d_2$ as the deadline for completion of both activities, with no intermediate deadline for activity 1 (i.e. activities 1 and 2 can be treated as a single combined activity).

We say that a managerial policy A for setting activity deadlines in a PERT network segment dominates another policy B, if the expected completion time of the PERT network segment under policy A is always less than or equal to that under policy B. Then:

PROPOSITION 4. *Policy* 2 *dominates Policy* 3 *for the case of two serial activities.*

The above proposition can be extended for any number of serial activities (the proof in the Appendix is given for the general case). Proposition 4, in general, states that a managerial policy using more deadlines, and consequently exerting stricter control, should be preferred for the management of series activities to the single deadline policy. Furthermore, Policy 3, by setting a single deadline, exerts less control. This could be interpreted as providing a higher level of flexibility to the worker. But this is a misconception. The serial structure of the precedence activity diagram has already eliminated any flexibility that the worker could advantageously use. On the other hand, exerting less managerial control, according to our model of Parkinson's Law, can only have harmful effects on the completion time of that network segment.

A comparison between Policies 1 and 2 is not a straightforward matter. Under the expanding behavior implied by Parkinson's Law, neither of these policies dominates the other. Examples that favor either of the two policies can be constructed easily (refer to Gutierrez and Kouvelis 1989).

In our discussion, so far, we have used the usual statement of Parkinson's Law, i.e. $ET(d) \geq d$ (PL1). For many project activities, a stronger form of Parkinson's Law may be applicable:

(SPL) $T(d) \geq d$ with probability 1.

While (PL1) states that on average the activity will be late, (SPL) makes the stronger statement that the activity will "never" be completed before its deadline, with the word "never" having the probabilistic interpretation that every measurable realization of the random variable $T(d)$ will be at least equal to the set deadline. Such a statement is naturally motivated by cases in which there is no incentive for the worker or subcontractor to report the end of an activity and the project manager does not have any way to detect completion of the activity. But in order to make the motivation of the statement stronger, we simply need to observe that, under the general modelling framework of §3, the existence of a deterministic subtask of any length at the end of an activity leads to (SPL). In other words, as far as the above case is concerned (PL1) implies (SPL). Fortunately, under the strong form of Parkinson's Law (SPL) comparison between the two managerial Policies 1 and 2 can be performed unambiguously.

PROPOSITION 5. *When the strong form of Parkinson's Law (SPL) describes the worker's behavior in the case of two serial activities, the managerial Policy 2 dominates Policy 1.*

The above proposition can be stated for any number of serial activities (the proof in the Appendix is stated in the general case). Policy 1 sets the deadline for the next activity, when the previous activity is completed. Proposition 5, in general, states that the above policy for setting deadlines should not be used in the management of series activities in the presence of worker behavior described by (SPL). While Policy 1 sets the same number of deadlines as Policy 2, it exerts less pressure and control over the worker.

Consider now a segment of a PERT network consisting of two activities in parallel to be performed by the same worker or subcontractor, with again d_1 and d_2 denoting the estimated amount of work involved in activities 1 and 2 respectively. Denote by $T^i(d)$ the completion time of activity i, when this activity is performed independently, $i = 1$, 2. Consider the following three managerial policies in setting deadlines for the activities of that network segment:

Policy 1. Sequence the activities (for convenience assume activity 1 before 2). Set d_1 as the deadline for completion of activity 1. Then wait until $T^1(d_1)$, i.e. until activity 1 is completed, and set $T^1(d_1) + d_2$ as the deadline for completion of activity 2.

Policy 2. Sequence the activities (assume activity 1 before 2). Set d_1 as the deadline for completion of activity 1, and $d_1 + d_2$ as the deadline for completion of both activities.

Policy 3. Do not sequence the activities, but only set $d_1 + d_2$ as the deadline for completion of that network segment.

Using the result obtained for the series case, we can claim that, for a given activity, Policy 2 dominates Policy 1 for the cases in which (SPL) holds. A worker who allocates all of his effort to an activity will complete it in T time units. If the same worker allocates only a fraction α of his effort to the same activity, he will complete it in T/α. The following proposition shows how the work expansion terms, denoted as $w_i'(d)$, are determined as a function of $w_i(d)$ and α.

PROPOSITION 6. *If a worker allocates a fraction α of his effort to an activity, the work expansion terms are given by*

$$w_i'(d) = \frac{1}{\alpha} w_i(\alpha d).$$

Observe that a static allocation of worker effort to the two parallel activities, with a fraction α_1 allocated to activity 1 and α_2 to activity 2 ($\alpha_1 + \alpha_2 = 1$), results in an expected completion time of the network segment of:

$$ET(d) = \frac{1}{\alpha_1} Ew(\alpha_1 d) + E\zeta_{1,n}^1 + \frac{1}{\alpha_2} Ew(\alpha_2 d) + E\zeta_{1,m}^2,$$

where activity 1 has n subtasks and activity 2 has m. Then $ET(d) = ET^1(\alpha_1 d) + ET^2(\alpha_2 d)$. Thus any deadline assignment under Policy 1, according to the above relationship, can be thought as equivalent to a static allocation of worker effort, with $\alpha_1 = d_1/d$, $\alpha_2 = d_2/d$ and $d = d_1 + d_2$.

However, under Policy 3 the worker is allowed the flexibility to allocate effort in a dynamic way between the two activities. For example, for a time interval t he may assign α_t^1, α_t^2 ($\alpha_t^1 + \alpha_t^2 = 100$) effort to activities 1, 2 respectively. The interval t can be thought as the time elapsed between the start and completion of a subset of the activity's subtasks. Later, the worker may reevaluate the effort allocation policy, and, based on the work progress, he may decide to shift effort between the two activities. A static allocation of effort over the complete duration of the activity, including the static allocation implied by Policy 1, is also an alternative for the worker under Policy 3. The above reasoning indicates that Policy 3 dominates Policy 1 in the case of parallel activities. The above argument is valid for any number of parallel activities. The following proposition, finally, summarizes our discussion.

PROPOSITION 7. *In the management of parallel activities the managerial Policy* 3, *that sets a single deadline for all parallel activities, should be preferred to Policy* 1, *that sequences the parallel activities and dynamically sets deadlines for them.*

The comparison of Policies 2 and 3 is considerably more complex. In this case, no technological constraints prescribe the sequence in which the two activities should be performed. However, Policy 2, by setting intermediate due dates, implicitly prescribes a sequence. It is not clear how the activities should be sequenced, if the behavior described by Parkinson's Law is considered. This is an issue in which further research is needed. The performance of Policy 3, on the other hand, is also affected by the way in which the worker/subcontractor allocates his effort to the different subtasks. To illustrate this point, we present the following example consisting of two identical activities without precedence constraints. In this case, the sequencing of the activities is irrelevant for Policy 2.

EXAMPLE. Consider two parallel activities. Each activity consists of two subtasks. The completion time distribution for each subtask is given below:

Activity 1			Activity 2		
Subtask	Completion Time	Prob.	Subtask	Completion Time	Prob.
1	1	0.5	1	1	0.5
	5	0.5		5	0.5
2	1	1	2	1	1

For the above activities $d_1 = d_2 = 5$. Under Policy 2, we would set the due dates at 5 and 10 time units respectively for activities 1 and 2. Using the modelling framework of §2, this results in an expected completion time of 10.75 time units. Now, consider Policy 3. From our discussion of the management of series activities, we can see that if the worker performs the activities sequentially, Policy 3 is dominated by Policy 2. However, this is not true if the worker allocates his effort dynamically among both activities. As a simple allocation-of-effort rule, assume that the worker initially allocates 50% of his effort (resources) to each activity. When one subtask is completed he shifts all effort (resources) to the other activity. This will result in an expected completion time of 10.5 time units for Policy 3.

It cannot be implied from this example that Policy 3 (always) dominates Policy 2. What the example shows is that the "cost" of the reduced flexibility can more than offset the benefits of the increased control implied by Policy 2. Further study is required to gain more insight into the cases in which one policy will dominate the other. However, when parallel activities are performed by a single worker, it is important that the project manager encourages and provides the required infrastructure, so that the worker performs these activities simultaneously.

6. Conclusions

In this paper we developed a modelling framework for project activities that incorporate the behavioral issues described as Parkinson's Law. Rather than proving or disproving Parkinson's Law, our analysis focuses on the managerial implications of this behavior. Out of these models, the project manager gets insights into the effect of deadlines on the duration of project activities. The project activity models provide an organized framework for analyzing the effects of different managerial policies for setting deadlines for parallel and series project activities. The effects of work progress information release policies on subcontractors can be also accounted for by such a framework.

Further research effort is deemed necessary to expand the activity modelling framework to account for the potential dependency of the work expansion behavior on worker's/subcontractor's risk attitudes. The implications of such a behavior for deadline setting policies need to be examined.[1]

[1] The authors wish to thank George P. Huber and Charles C. Holt for reviewing a previous version of this paper. Their remarks substantially improved our paper. We also gratefully acknowledge the constructive criticism of two anonymous referees that helped us to improve substantially the clarity of exposition of our results.

Appendix

PROOF OF PROPOSITION 1. Follows immediately by induction on i.

PROOF OF PROPOSITION 2. Since $Ew_i(d)$ is nondecreasing, $D_d^+ Ew_i(d) \geq 0$. The convexity of $w_i(d)$ and the monotone convergence theorem justify that $D_d^+ Ew_i(d) = ED_d^+ w_i(d)$. Thus, it is sufficient to show that $D_d^+ w_i(d) \leq 1$ for any realization of the random variable $\zeta_{1,i}$. We use an inductive proof. From §2, $D_d^+ w_i(d) \leq 1$. Assume $D_d^+ w_{i-1}(d) \leq 1$ and observe from (7) and (8) that

$$D_d^+ w_i(d) = \begin{cases} D_d^+ w_{i-1}(d) & \text{if} \quad d \leq \zeta_{1,i} + w_{i-1}(d) + E\zeta_{i+1,n}, \\ 1 & \text{if} \quad d \geq \zeta_{1,i} + w_{i-1}(d) + E\zeta_{i+1,n}. \end{cases} \qquad \blacksquare$$

PROOF OF PROPOSITION 3. We examine two cases.

Case 1. Assume $d < E\zeta_{1,n}$. In that case, $ET''(d, \alpha) = E\zeta_{1,n} + w_{n-1}(d)$. Thus

$$ET''(d, \alpha) = ET(d) \quad \text{for all} \quad 0 \leq \alpha \leq 1, \quad \text{and} \quad D_\alpha^+ ET''(d, \alpha) = 0.$$

Case 2. Assume $d > E\zeta_{1,n}$. In that case,

$$ET''(d, \alpha) = \alpha(d - E\zeta_{1,n}) + E\zeta_{1,n} + w_{n-1}((1 - \alpha)d + \alpha E\zeta_{1,n}).$$

Since $0 \leq D^+ w_i(\cdot) \leq 1$, after taking the right derivative with respect to α we have:

$$D_\alpha^+ T''(d, \alpha) = (d - E\zeta_{1,n})(1 - D^+ w_{n-1}((1 - \alpha)d + \alpha E\zeta_{1,n})) \geq 0. \qquad \blacksquare$$

PROOF OF PROPOSITION 4. We prove Proposition 4 for the more general case in which we have N activities in series. Renumber the subtasks of all activities in a sequence from $1, \ldots, n_1, \ldots, n_N$, where n_k is the last subtask of the kth activity in the segment. For consistency in our comparisons, we assume that, under either policy, work is not expanded when an activity is completed. Let d_k be the estimated duration of activity k, and define $d^1 = d_1$, $d^k = d^{k-1} + d_k$, and $d \equiv d^N$. Denote by $w_j^{(i)}$ the work expansion under Policy i before subtask $j + 1$ is started. To prove the proposition we must show that $W_{n_N-1}^{(3)} \geq w_{n_N-1}^{(2)}$.

First observe that for any $k \leq N$, and for any $n_{k-1} \leq j \leq n_k$

$$(d - \zeta_{1,j} - E\zeta_{j+1,n_N})^+ \geq (d^k - \zeta_{i,j} - E\zeta_{j+1,n_k})^+.$$

Thus we have immediately that

$$w_1^{(3)} = (d - \zeta_{1,1} - E\zeta_{2,n_k})^+ \geq (d^1 - \zeta_{1,1} - E\zeta_{2,n_1})^+ = w_1^{(2)}.$$

Now using induction assume that $w_{j-1}^{(3)} \geq w_{j-1}^{(2)}$ for $j < n_N - 1$, and consider two cases:

Case (a) ($j = n_k$ *for some* k). By assumption $w_j^{(i)} = w_{j-1}^{(i)}$, thus $w_j^{(3)} \geq w_j^{(2)}$.

Case (b) ($j \neq n_k$ *for all* k). From the observation above, it follows that

$$w_j^{(3)} = \max\{w_{j-1}^{(3)}, (d - \zeta_{i,j} - E\zeta_{j+1,n_N})^+\} \geq \max\{w_{j-1}^{(2)}, (d^k - \zeta_{i,j} - E\zeta_{j+1,n_k})^+\} = w_j^{(2)}$$

by choosing k such that $n_{k-1} < j < n_k$. $\blacksquare$

PROOF OF PROPOSITION 5. As for Proposition 4, we prove the claim for the general case in which there are N activities in series. Define $d^1 \equiv d_1$, and $d^k = d^{k-1} + d_k$ for $k = 2, \ldots, N$. Let $T_k^{(i)}$ denote the completion time of activity k under policy i. It is sufficient to show that $T_k^{(2)} \geq T_k^{(1)}$ for $k = 1, \ldots, N$. Clearly $T_1^{(2)} = T_1^{(1)} = T(d_1)$, and for $k \geq 2$ we have that

$$T_k^{(2)} = T\left(d^k - \sum_{i=1}^{k-1} T_i^{(2)}\right) \leq T\left(d^k - \sum_{i=1}^{k-1} d_i\right) = T(d_k) = T_k^{(1)}.$$

The inequality above follows from (SPL) and the fact that T is nondecreasing. $\blacksquare$

PROOF OF PROPOSITION 6. Let w' denote the work expansion terms when the worker allocates α of the effort. Then

$$w'_1(d) = \left(d - \frac{1}{\alpha}T_1 - \frac{1}{\alpha}E\zeta_{2,n}\right)^+ = \frac{1}{\alpha}(\alpha d - T_1 - E\zeta_{2,n})^+ \qquad \text{and} \qquad w'_1(d) = \frac{1}{\alpha}w_1(\alpha d).$$

By induction on i, using (7), we obtain

$$w'_i(d) = \frac{1}{\alpha}w_i(\alpha d) \qquad \text{for} \qquad 1 < i \le n-1. \qquad \blacksquare$$

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