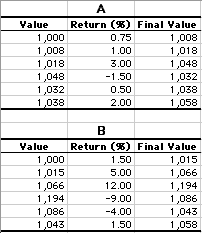
**Standard Deviation as a Measure of Risk**

The standard deviation is often used by investors to measure the risk of a stock or a stock portfolio. The basic idea is that the standard deviation is a measure of volatility: the more a stock's returns vary from the stock's average return, the more volatile the stock. Consider the following two stock portfolios and their respective returns (in per cent) over the last six months. Both portfolios end up increasing in value from $1,000 to $1,058. However, they clearly differ in volatility. Portfolio A's monthly returns range from -1.5% to 3% whereas Portfolio B's range from -9% to 12%. The standard deviation of the returns is a better measure of volatility than the range because it takes all the values into account. The standard deviation of the six returns for Portfolio A is 1.52; for Portfolio B it is 7.24.   
  


[Expected return](https://www.investopedia.com/terms/e/expectedreturn.asp) is calculated as the weighted average of the likely profits of the assets in the portfolio, weighted by the likely profits of each asset class. Expected return is calculated by using the following formula:

|  |
| --- |
| [https://i.investopedia.com/inv/articles/site/Corpfin80.jpg](https://www.investopedia.com/terms/e/expectedreturn.asp) |

Written another way, the same formula is as follows: E(R) = w1R1 + w2Rq + ...+ wnRn

***Example: Expected Return***  
For a simple portfolio of two [mutual funds](https://www.investopedia.com/terms/m/mutualfund.asp), one investing in [stocks](https://www.investopedia.com/terms/s/stock.asp) and the other in [bonds](https://www.investopedia.com/terms/b/bond.asp), if we expect the stock fund to return 10% and the bond fund to return 6% and our allocation is 50% to each asset class, we have the following:

Expected return (portfolio) = (0.1)\*(0.5) + (0.06)\*(0.5) = 0.08, or 8%

Expected return is by no means a guaranteed rate of return. However, it can be used to forecast the future value of a portfolio, and it also provides a guide from which to measure actual returns.

Let's look at another example. Assume an [investment manager](https://www.investopedia.com/terms/i/investment-manager.asp) has created a portfolio with Stock A and Stock B. Stock A has an expected return of 20% and a weight of 30% in the portfolio. Stock B has an expected return of 15% and a weight of 70%. What is the expected return of the portfolio?

E(R) = (0.30)(0.20) + (0.70)(0.15)  
= 6% + 10.5% = 16.5%

The expected return of the portfolio is 16.5%.

Now, let's build on our knowledge of expected returns with the concept of variance.

**Variance**  
[**Variance**](https://www.investopedia.com/terms/v/variance.asp)(σ2) is a measure of the dispersion of a set of data points around their mean value. In other words, variance is a mathematical expectation of the average squared deviations from the mean. It is computed by finding the probability-weighted average of squared deviations from the expected value. Variance measures the variability from an average ([volatility](https://www.investopedia.com/terms/v/volatility.asp)). Volatility is a measure of risk, so this statistic can help determine the risk an investor might take on when purchasing a specific security.

***Example: Variance***  
Assume that an analyst writes a report on a company and, based on the research, assigns the following probabilities to next year's sales:

|  |  |  |
| --- | --- | --- |
| **Scenario** | **Probability** | **Sales ($ Millions)** |
| 1 | **0.10** | $16 |
| 2 | 0.30 | $15 |
| 3 | 0.30 | $14 |
| 3 | 0.30 | $13 |

The analyst's expected value for next year's sales is (0.1)\*(16.0) + (0.3)\*(15.0) + (0.3)\*(14.0) + (0.3)\*(13.0) = $14.2 million.

Calculating variance starts by computing the difference in each potential sales outcome from $14.2 million, then squaring:

|  |  |  |  |
| --- | --- | --- | --- |
| **Scenario** | **Probability** | **Deviation from Expected Value** | **Squared** |
| 1 | 0.1 | (16.0 - 14.2) = 1.8 | 3.24 |
| 2 | 0.30 | (15.0 - 14.2) = 0.8 | 0.64 |
| 3 | 0.30 | (14.0 - 14.2) = - 0.2 | 0.04 |
| 4 | 0.30 | (13.0 - 14.2) = - 1.2 | 1.44 |

Variance then weights each squared deviation by its probability, giving us the following calculation:

(0.1)\*(3.24) + (0.3)\*(0.64) + (0.3)\*(0.04) + (0.3)\*(1.44) = 0.96

**Portfolio Variance**

Now that we've gone over a simple example of how to calculate variance, let's look at [portfolio variance](https://www.investopedia.com/terms/p/portfolio-variance.asp).

The variance of a portfolio's return is a function of the variance of the component assets as well as the covariance between each of them. Covariance is a measure of the degree to which returns on two risky assets move in tandem. A positive covariance means that asset returns move together. A negative covariance means returns move inversely. Covariance is closely related to "[correlation](https://www.investopedia.com/terms/c/correlation.asp)," wherein the difference between the two is that the latter factors in the standard deviation.

[Modern portfolio theory](https://www.investopedia.com/terms/m/modernportfoliotheory.asp) says that portfolio variance can be reduced by choosing asset classes with a low or negative covariance, such as stocks and bonds. This type of diversification is used to reduce risk.

Portfolio variance looks at the covariance or correlation coefficient for the securities in the portfolio. Portfolio variance is calculated by multiplying the squared weight of each security by its corresponding variance and adding two times the weighted average weight multiplied by the covariance of all individual security pairs. Thus, we get the following formula to calculate portfolio variance in a simple two-asset portfolio:

(weight(1)^2\*variance(1) + weight(2)^2\*variance(2) + 2\*weight(1)\*weight(2)\*covariance(1,2)

Here is the formula stated another way:

|  |
| --- |
| Portfolio Variance = w2A\*σ2(RA) + w2B\*σ2(RB) + 2\*(wA)\*(wB)\*Cov(RA, RB) *Where: wA and wBare portfolio weights, σ2(RA) and σ2(RB) are variances and* *Cov(RA, RB) is the covariance* |

***Example: Portfolio Variance***  
Data on both variance and covariance may be displayed in a covariance matrix. Assume the following covariance matrix for our two-asset case:

|  |  |
| --- | --- |
| **Stock** | **Bond** |
| Stock | 350 | 80 |
| Bond | 150 |  |

From this matrix, we know that the variance on stocks is 350 (the covariance of any asset to itself equals its variance), the variance on bonds is 150 and the covariance between stocks and bonds is 80. Given our portfolio weights of 0.5 for both stocks and bonds, we have all the terms needed to solve for portfolio variance.

Portfolio variance = w2A\*σ2(RA) + w2B\*σ2(RB) + 2\*(wA)\*(wB)\*Cov(RA, RB) =(0.5)2\*(350) + (0.5)2\*(150) + 2\*(0.5)\*(0.5)\*(80) = 87.5 + 37.5 + 40 = 165.

**Standard Deviation**  
Standard deviation can be defined in two ways:

1. A measure of the dispersion of a set of data from its mean. The more spread apart the data, the higher the deviation. Standard deviation is calculated as the square root of variance.

2. In finance, standard deviation is applied to the annual rate of return of an investment to measure the investment's volatility. Standard deviation is also known as [historical volatility](https://www.investopedia.com/terms/h/historicalvolatility.asp) and is used by investors as a gauge for the amount of expected volatility.

Standard deviation is a statistical measurement that sheds light on historical volatility. For example, a volatile stock will have a high standard deviation while a stable [blue chip](https://www.investopedia.com/terms/b/bluechip.asp) stock will have a lower standard deviation. A large dispersion tells us how much the fund's return is deviating from the expected normal returns.

***Example: Standard Deviation***  
Standard deviation (σ) is found by taking the square root of variance:

(165)1/2= 12.85%.

We used a two-asset portfolio to illustrate this principle, but most portfolios contain far more than two assets. The formula for variance becomes more complicated for multi-asset portfolios. All terms in a covariance matrix need to be added to the calculation.

Let's look at a second example that puts the concepts of variance and standard deviation together.

***Example: Variance and Standard Deviation of an Investment***  
Given the following data for Newco's stock, calculate the stock's variance and standard deviation. The expected return based on the data is 14%.

|  |  |  |  |
| --- | --- | --- | --- |
| **Scenario** | **Probability** | **Return** | **Expected Return** |
| Worst Case | 10% | 10% | 0.01 |
| Base Case | 80% | 14% | 0.112 |
| Best Case | 10% | 18% | 0.018 |

***Answer:***  
σ2= (0.10)(0.10 - 0.14)2 + (0.80)(0.14 - 0.14)2+ (0.10)(0.18 - 0.14)2  
= 0.00032

The variance for Newco's stock is 0.00032.

Given that the standard deviation of Newco's stock is simply the square root of the variance, the standard deviation is 0.0179, or 1.79%.