Extended Kalman Filter – Problem Statement

Implement the following equations in Simulink:

$$\dot{x}_1 = \mu x_1 + u$$
$$\dot{x}_2 = \lambda(x_2 - x_1^2)$$

With $\mu = -0.1$ and $\lambda = -1$. Make the input, u, be a Signal Generator block set to a square wave with amplitude 3 and a 1 Hz frequency.

Use a 20 second simulation with a fixed step solver with a time step of 0.01 seconds and initial conditions of the states to be [5, 0]. Add a process noise before the integrators in the true physics equations (above) with zero mean and variance of 1 with different seeds for each. Implement measurement noise added to the true states (the output of the integrators which include the process noise effects) to create the measurement vector. Set the measurement noise variance to be [0.1, 0.1] (zero mean) for each of the two states. Make sure to give each random number generator a different seed.

Modify the MATLAB blocks you used in Homework 3 to implement the extended Kalman filter, noting that the dynamics are now nonlinear (so you'll have to compute the Jacobian and implement it in the predictor block) but assume the measurement equation is linear and all states are directly observable (i.e. H = eye(2)). Set the initial value of P to be eye(2)*5 and the initial values of the estimated states to be [5, 0], the same as the true states. The variances of the process and measurement noises (Q and R) will be up to you to determine, given the requirements that the error between the true states (with the process noise integrated into them) and the estimated states should be less than 0.5. Adjust the Q and R matrix values to try to get the error to be within the square root of the covariance about 80% of the time or higher and the error magnitude less than 0.5 for each state.

After performing and documenting the above tasks, now set the filter to have an initial state of [0, 0] and rerunning. Document the results and comment on whether the incorrect guess at the filter states made a difference in this case.

Sometimes we don't get the physics model correct but the filter still can do a pretty good job of estimating the states. Change your predictor model (but not the true physics) to have $\mu = -0.01$ and rerun with the initial state [5, 0]. Modify the Q and R values if you must to meet the requirements of both state errors being less than 0.5 in magnitude for more than 80% of the time. While I think this should work, if you can't achieve the goal, document your attempts and note which got you the closest.

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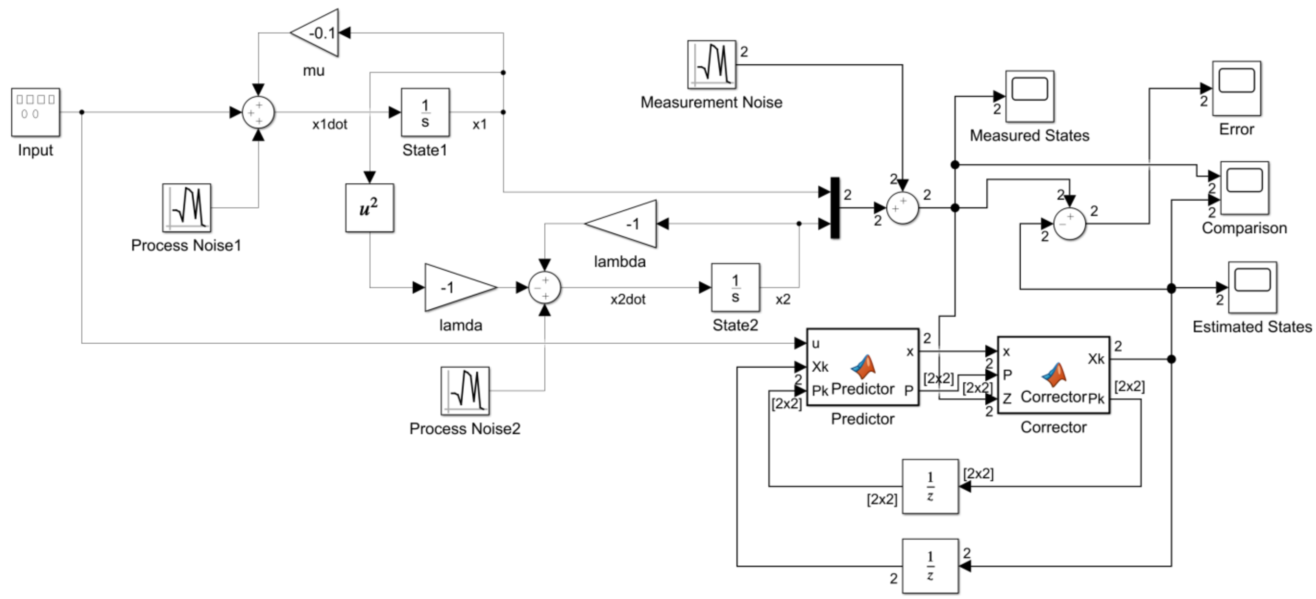
Simulink Diagram



Predictor Code

```
function [x,P] = Predictor(u,Xk,Pk)

mu=-0.1; lam=-1;

A = [mu 0;-2*lam*Xk(1) lam];

B= [1;0];

Ad=expm(A*0.01);

Bd=inv(A)*(Ad-eye(2))*B;

Q=eye(2);

F=[-A Q;zeros(2,2) transpose(A)];

G=expm(F*0.01);

Qd=transpose(G(3:4,3:4))*G(1:2,3:4);

x=Ad*Xk + Bd*u;

P=(Ad*Pk*transpose(Ad)+Qd);

end
```

Corrector Code

```
function [Xk,Pk] = Corrector(x,P,Z)

H=eye(2); R=0.1*eye(2); Rd=R/0.01;

K=P*transpose(H)*inv(H*P*transpose(H)+Rd);

Pk=(eye(2)-K*H)*P*transpose(eye(2)-K*H)+K*Rd*transpose(K);

Xk=x+K*(Z-H*x);

end
```

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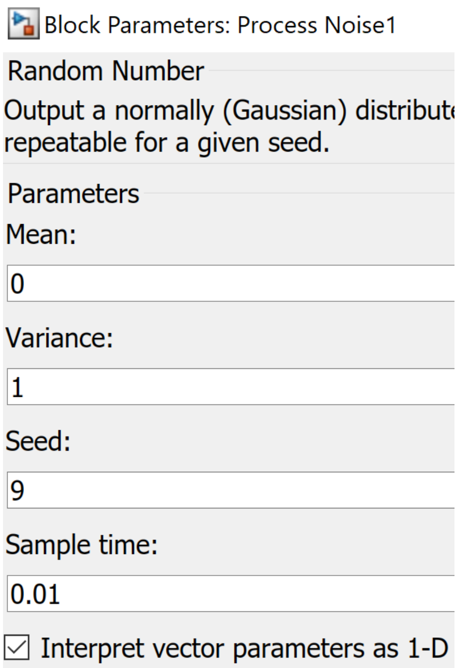
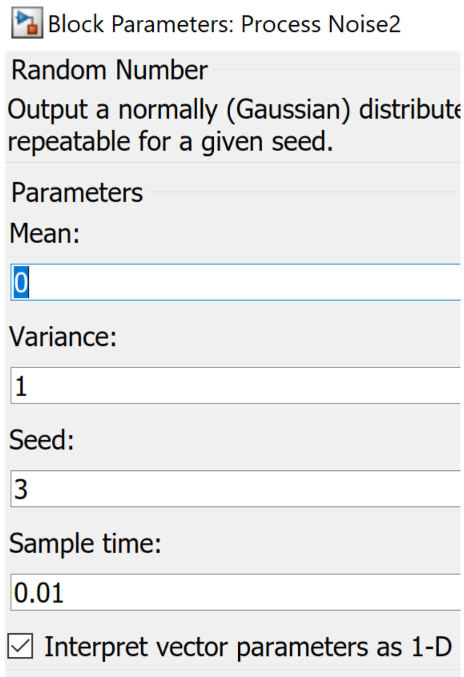
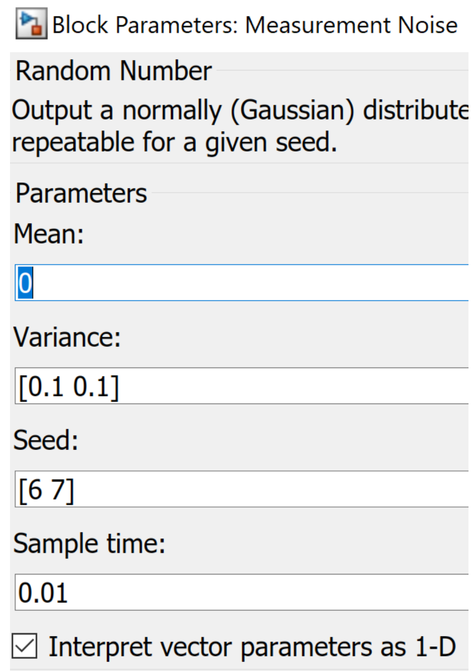
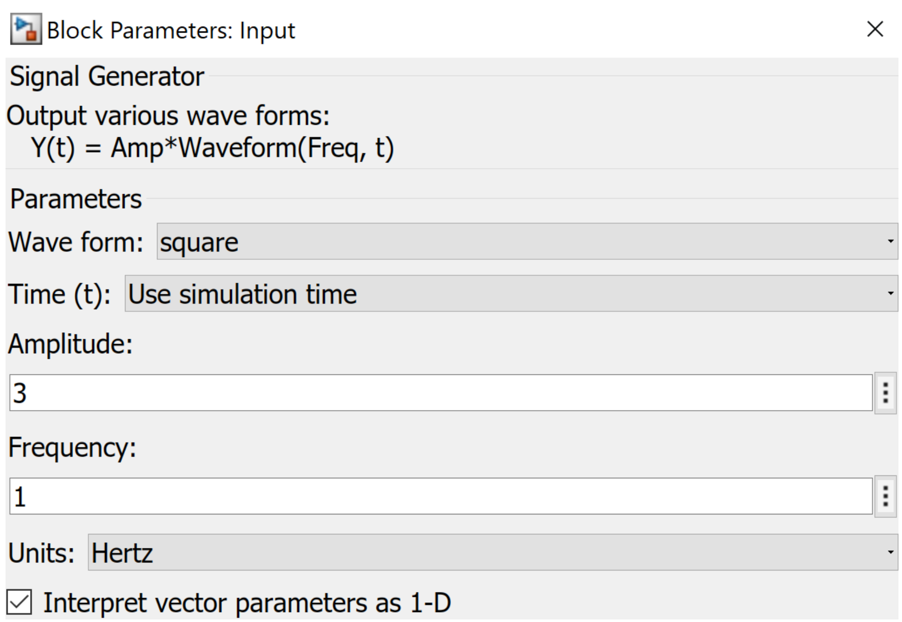
Noise Parameters



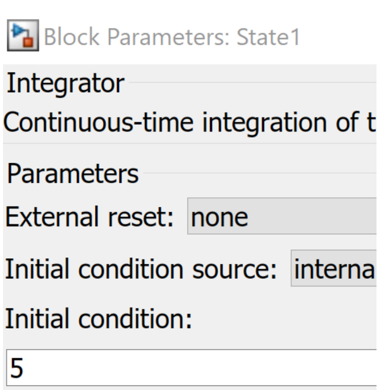


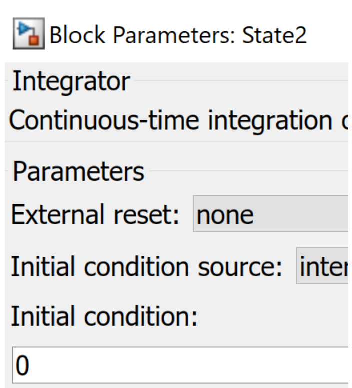


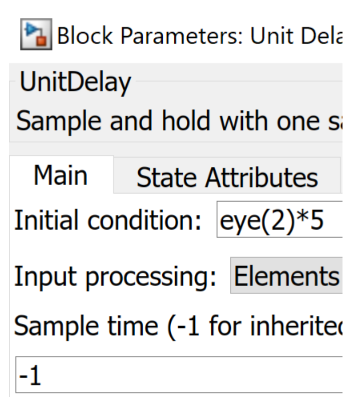
Input Parameters

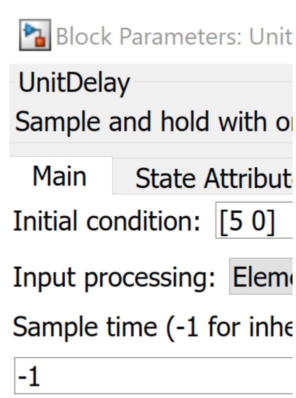


Initial State Conditions

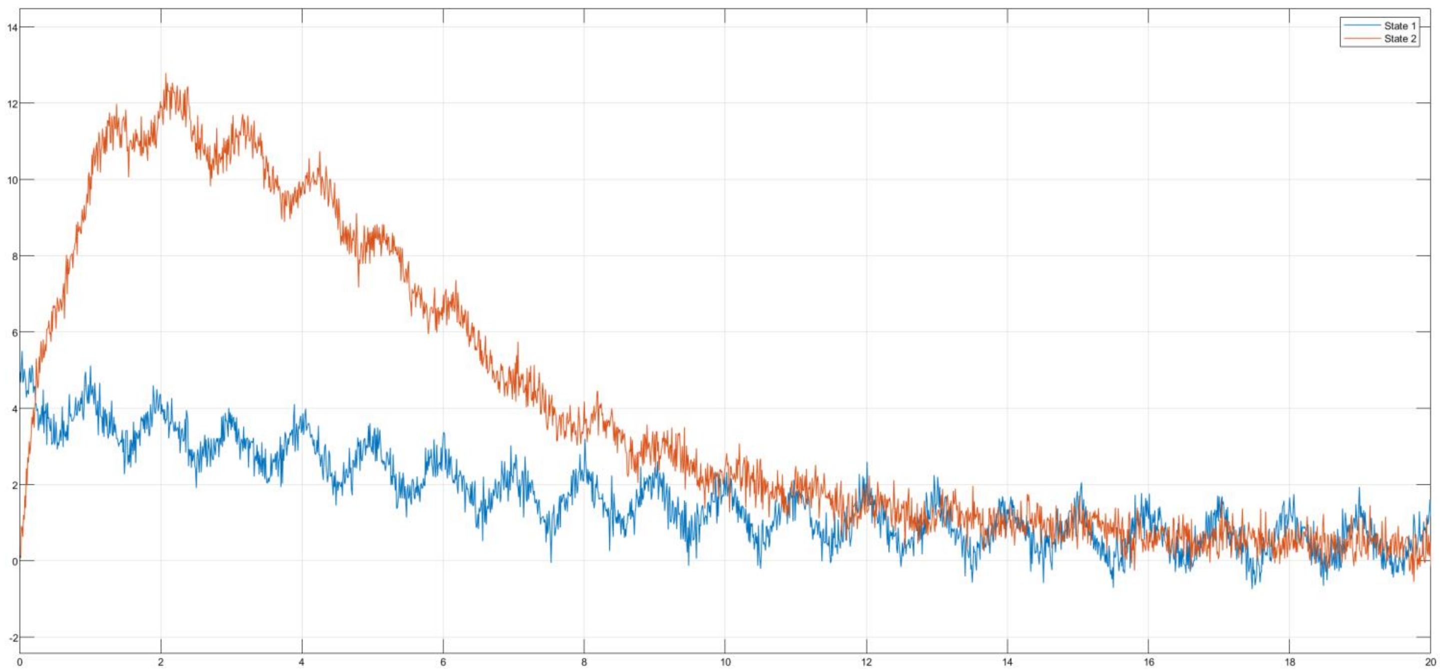




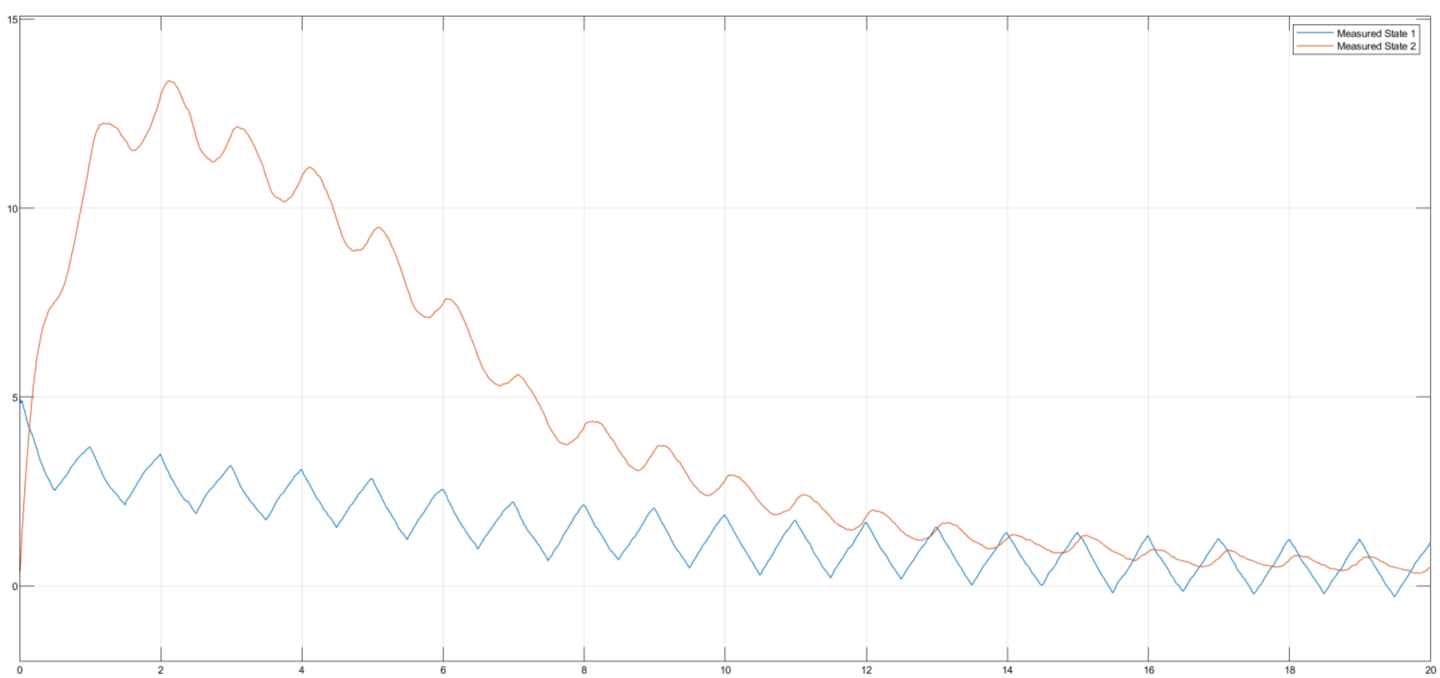




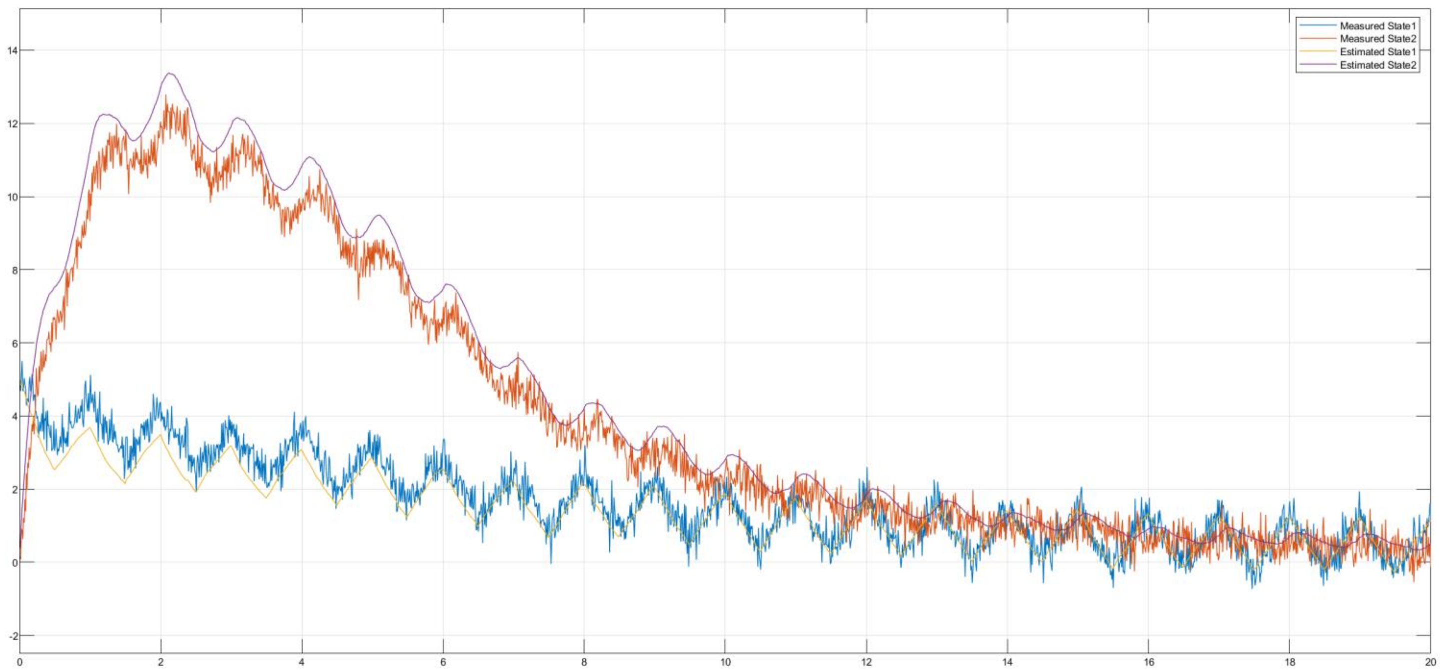
Measured States



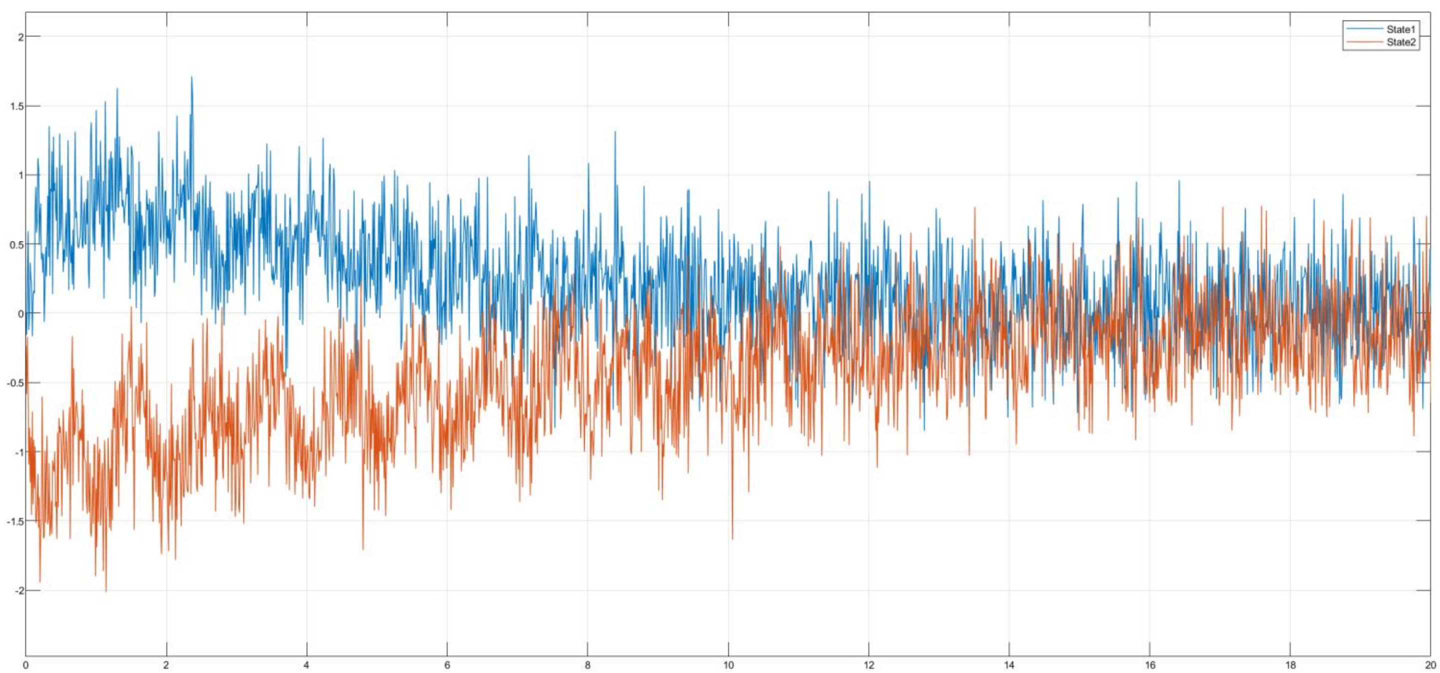
Filtered States



Comparison between measured and estimated states

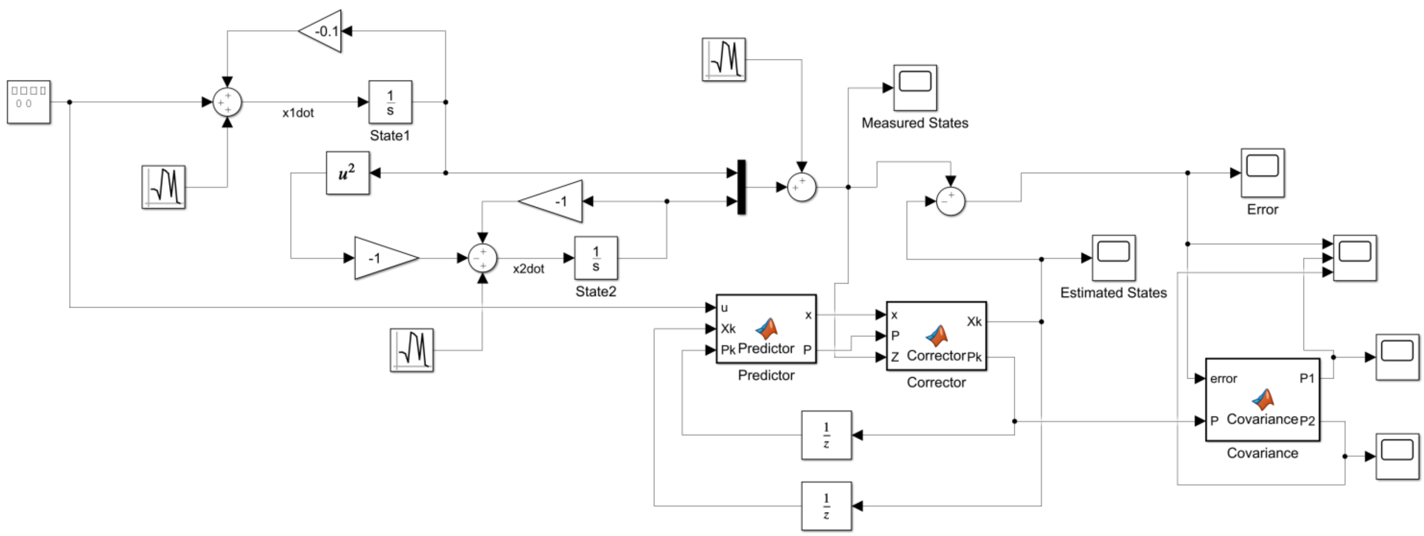


Error Plot



As observed from the comparison and error plots, the Kalman Filter is slightly inaccurate in the initial stages but it converges quickly within 10 seconds and brings the error values within the 0.5 range.

Simulink Diagram



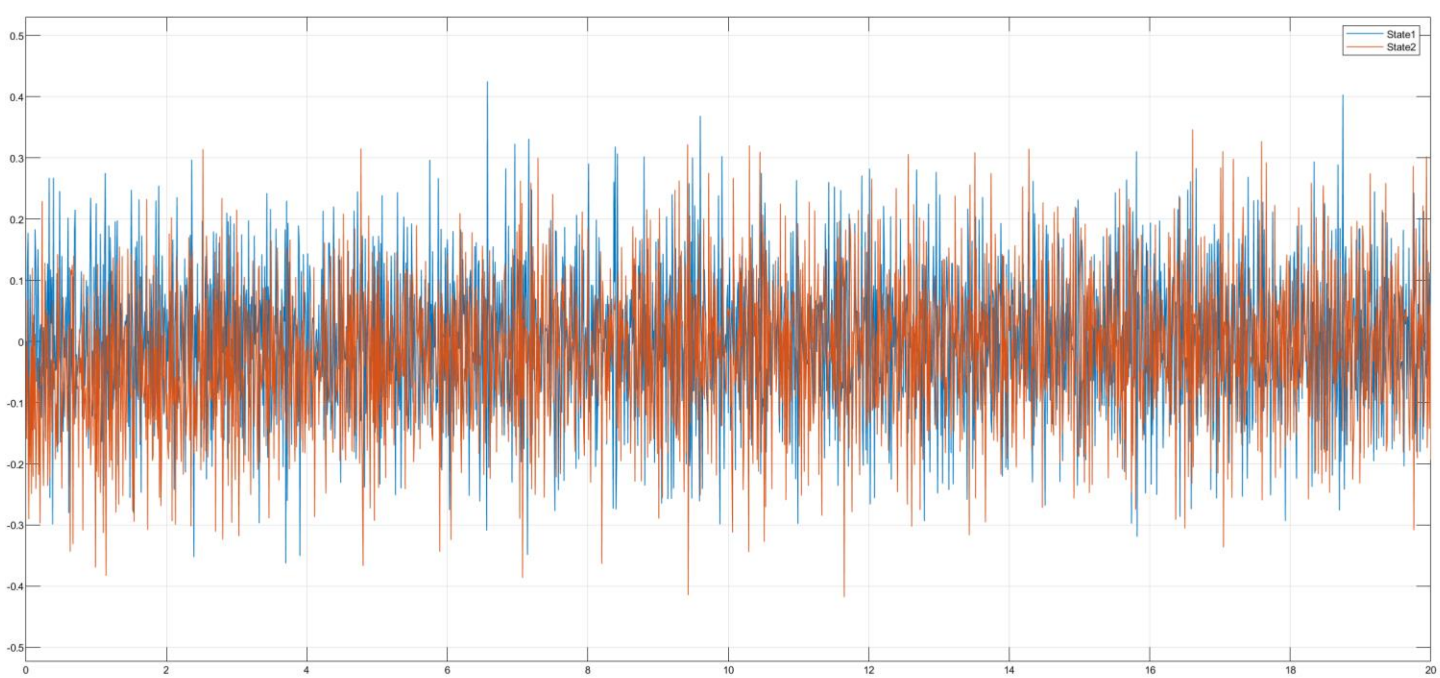
Covariance Function Code

```
function [P1,P2]  = Covariance(error,P)

P1=[+sqrt(P(1,1)) -sqrt(P(1,1)) error(1)];

P2=[+sqrt(P(2,2)) -sqrt(P(2,2)) error(2)];

end
```

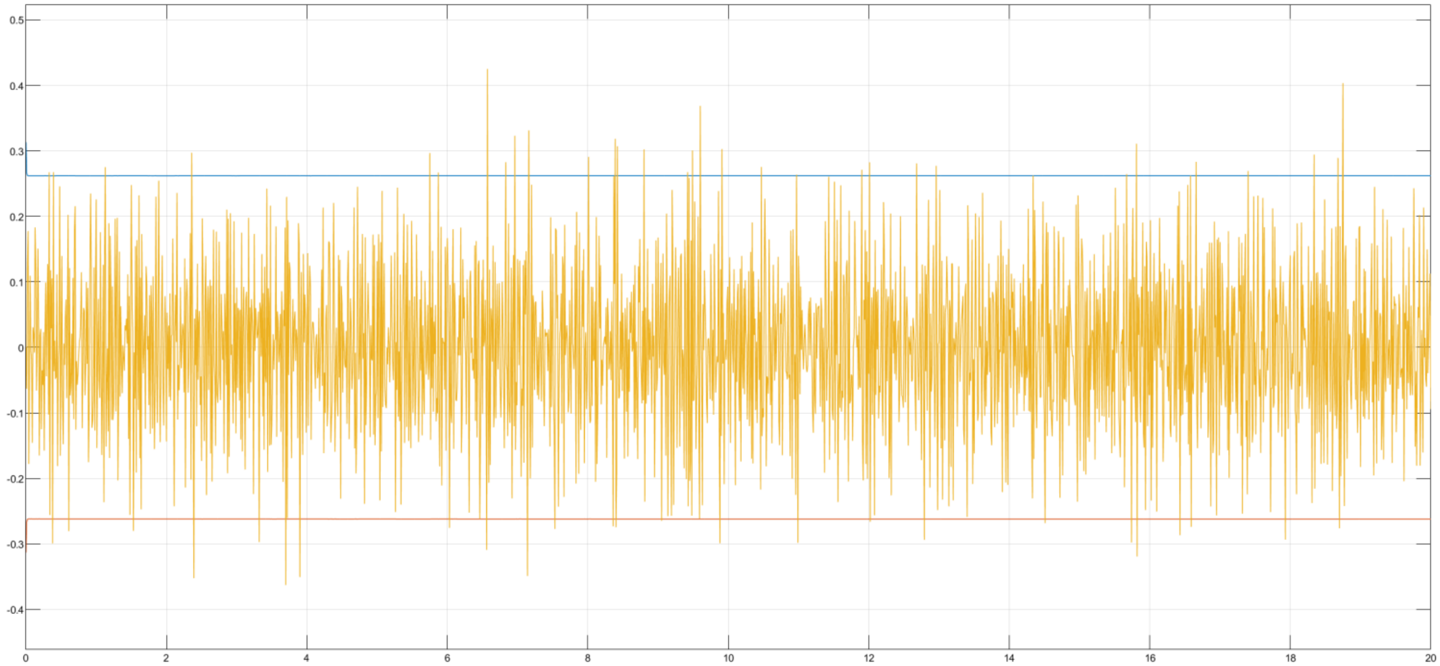
The following Q and R matrices have been used to get error within 0.5 and the error within the covariance about 80% of the time

```
Q=15*eye(2);
R=0.001*eye(2);
```

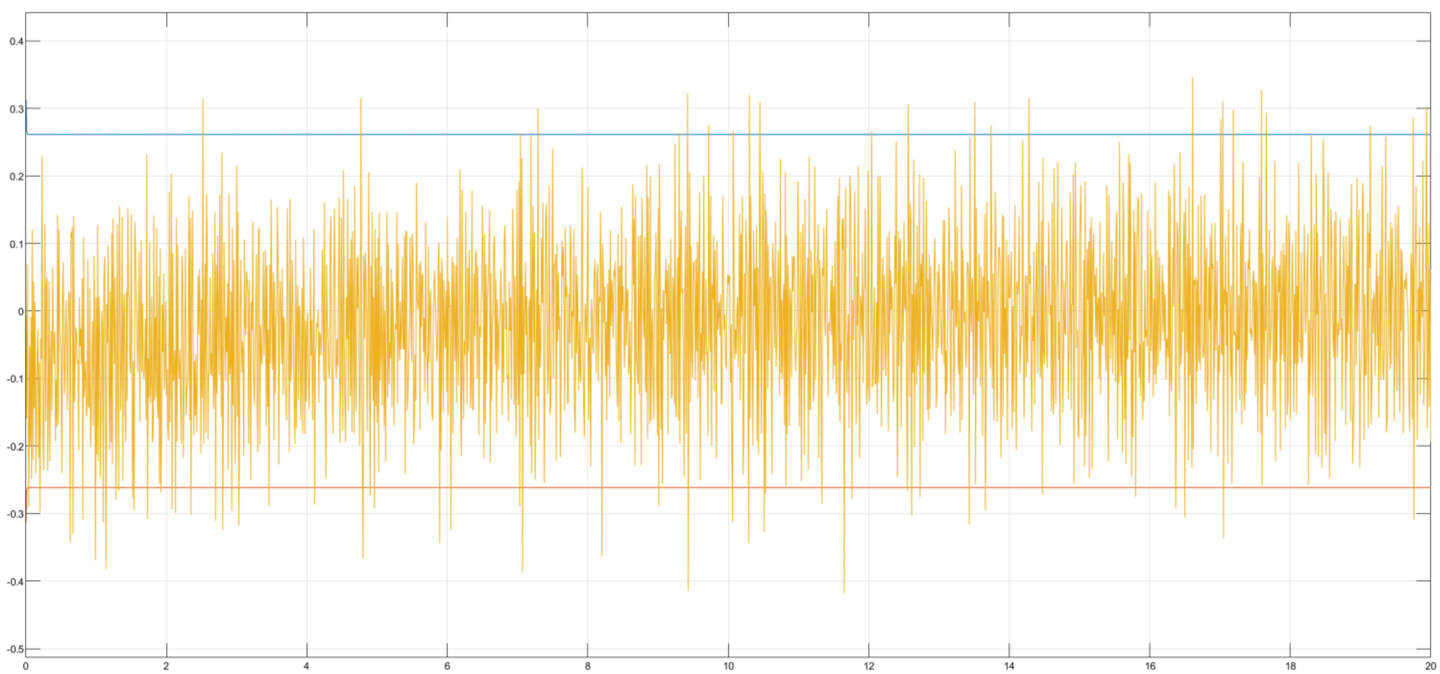
Error Plot (within 0.5)



Covariance Plot for State 1

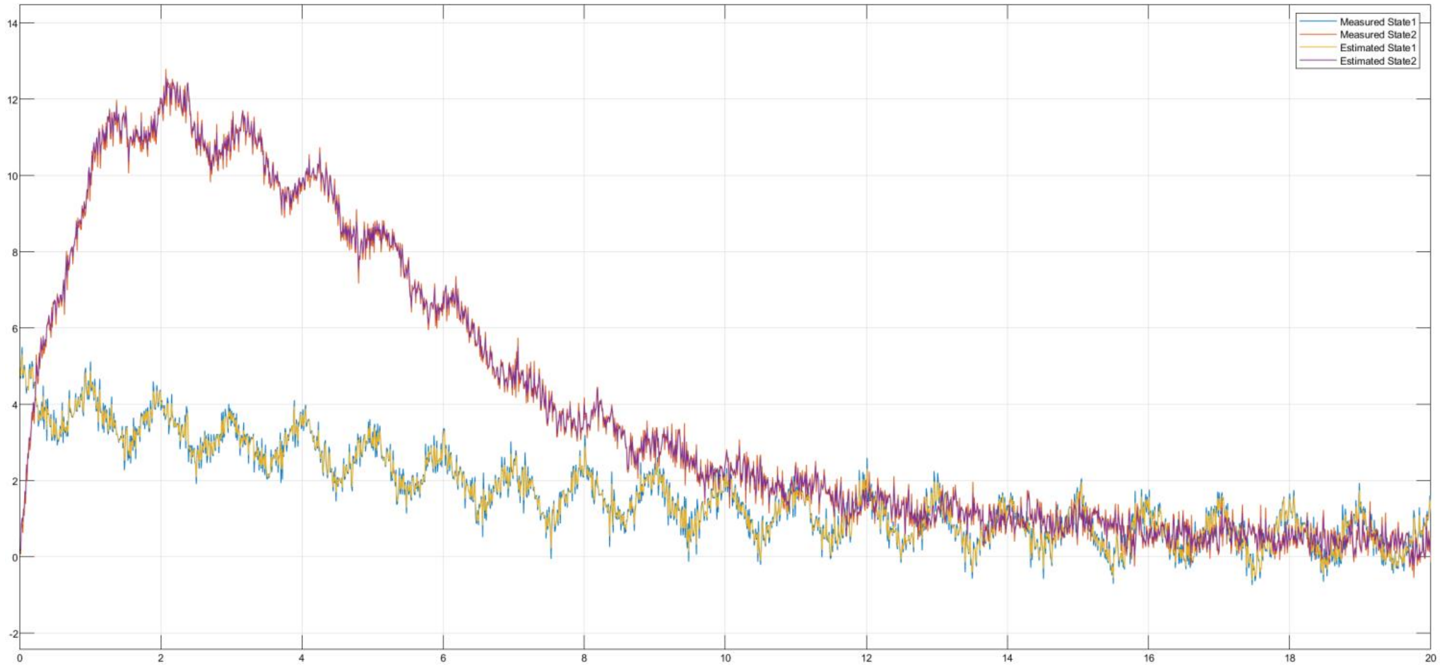


Covariance Plot for State 2



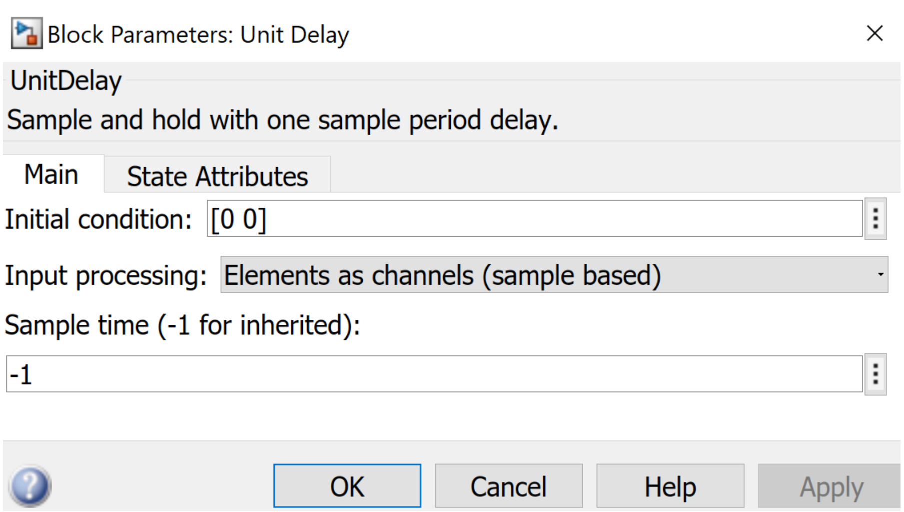
Note: The magnitudes cross the covariance lines multiple times. However, the error is within the covariance limits about 80% of the time. So, the average time of those peaks is around 20% of the total time.

Comparison Plots with new Q and R matrices



Now everything in the Simulink diagram is kept same, only the initial states of the filter are changed to [0,0] for observing the effects on estimated values

Defining the initial states in Filter



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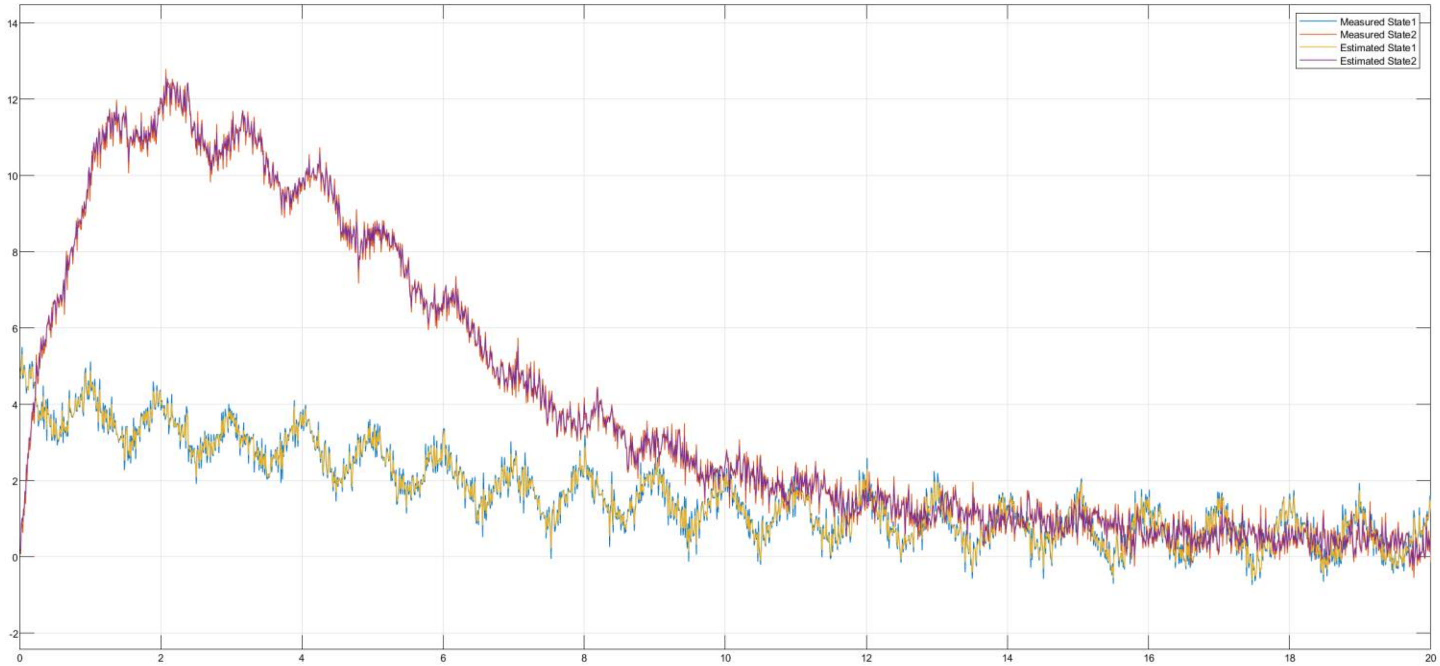
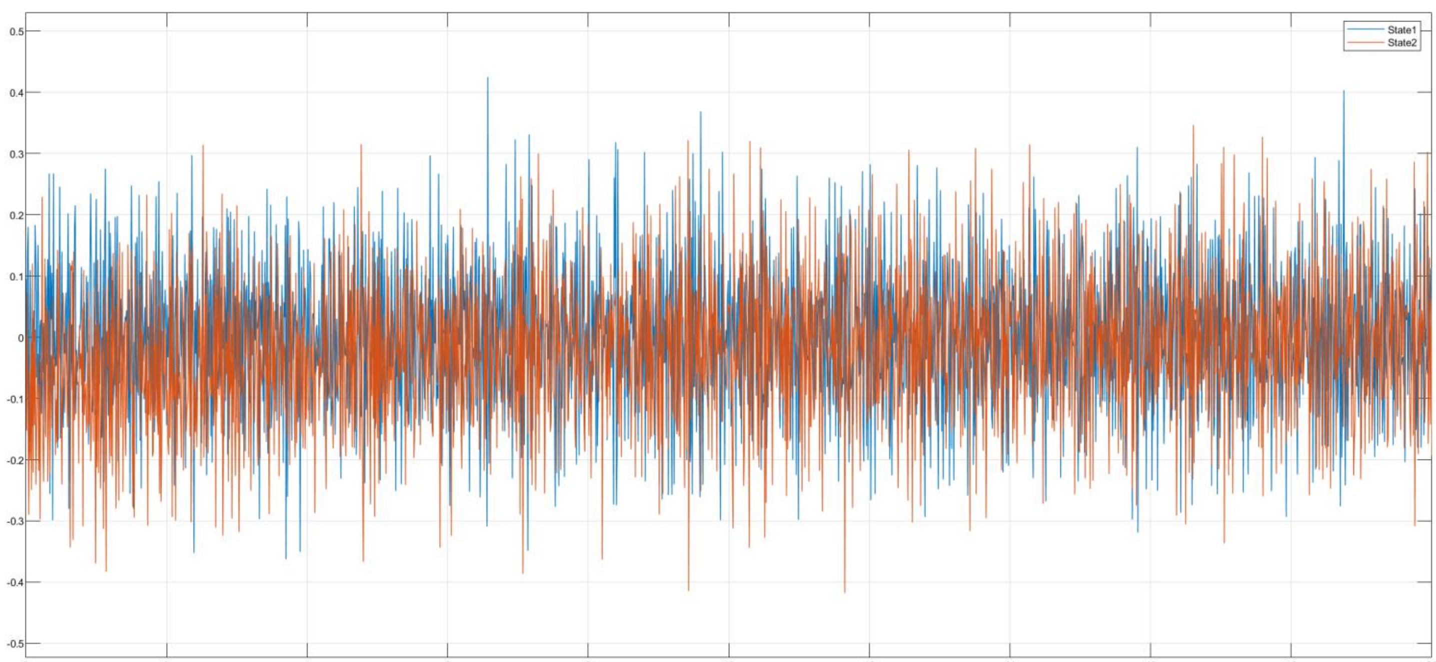
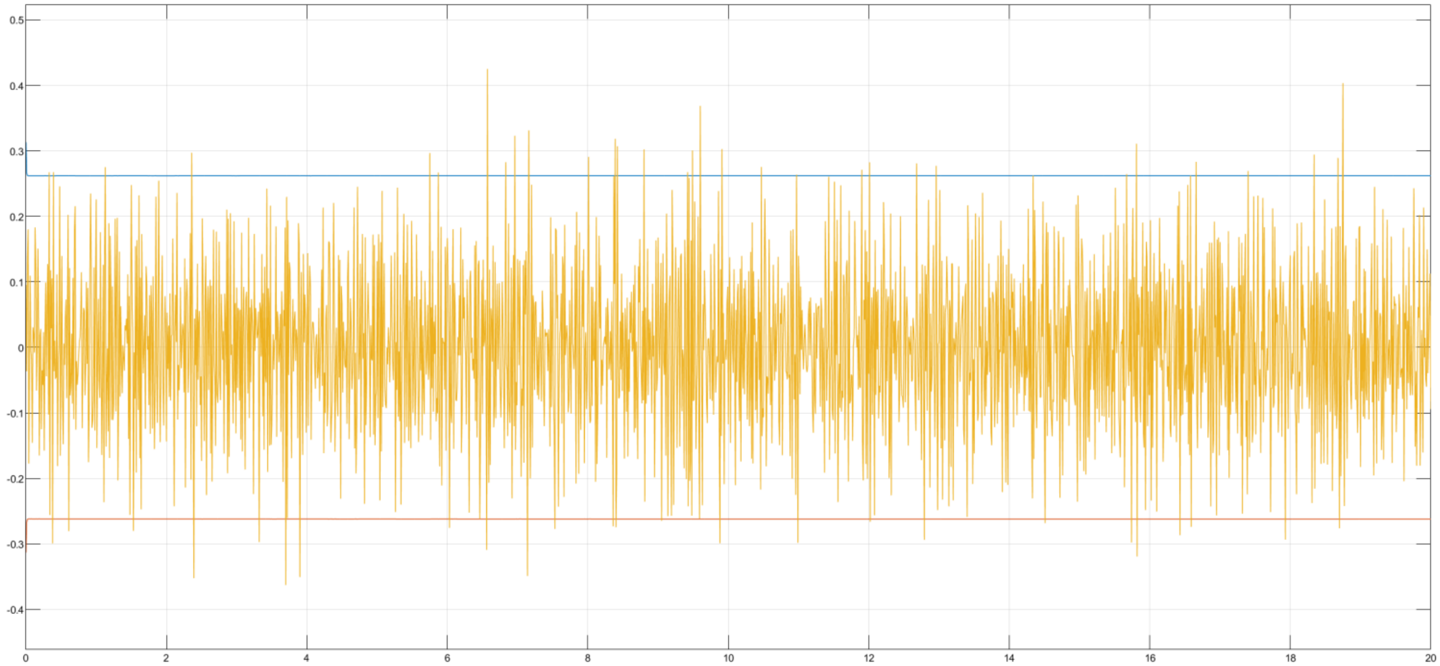
Comparison Plots for Measured and Estimated States



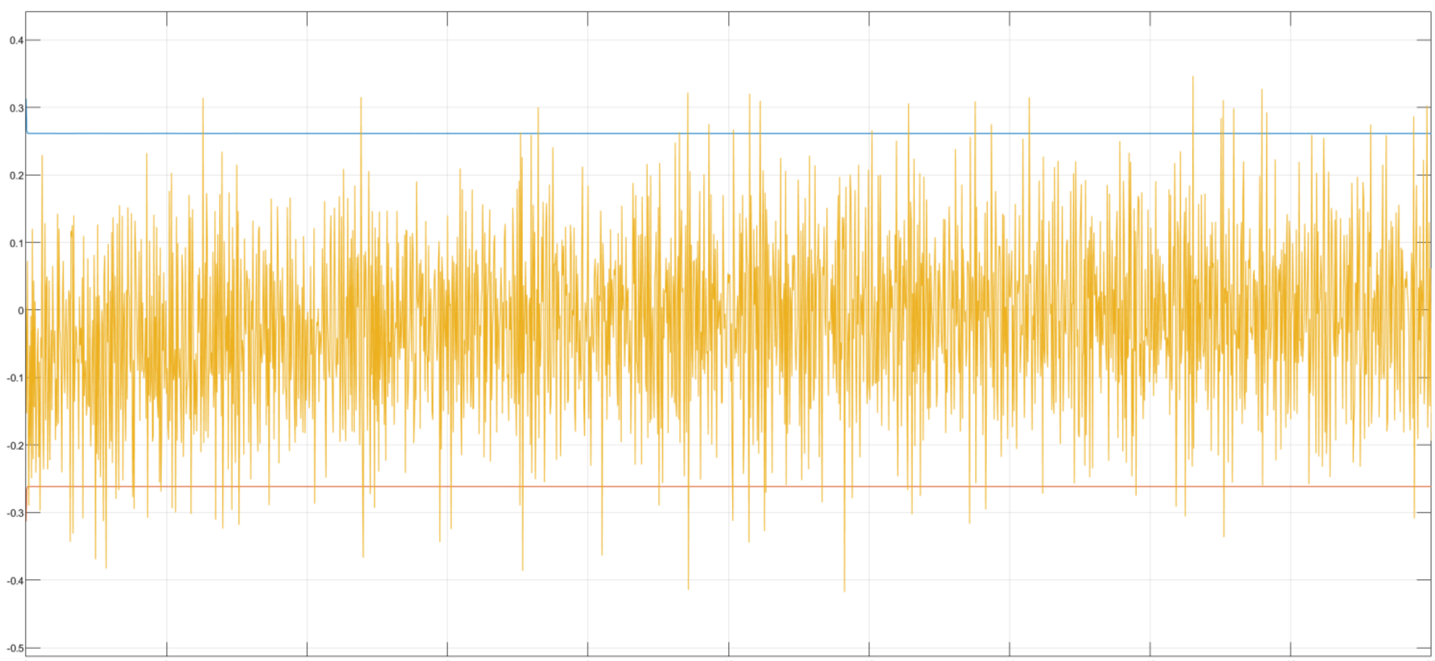
Error Plot



Covariance Plot for State 1



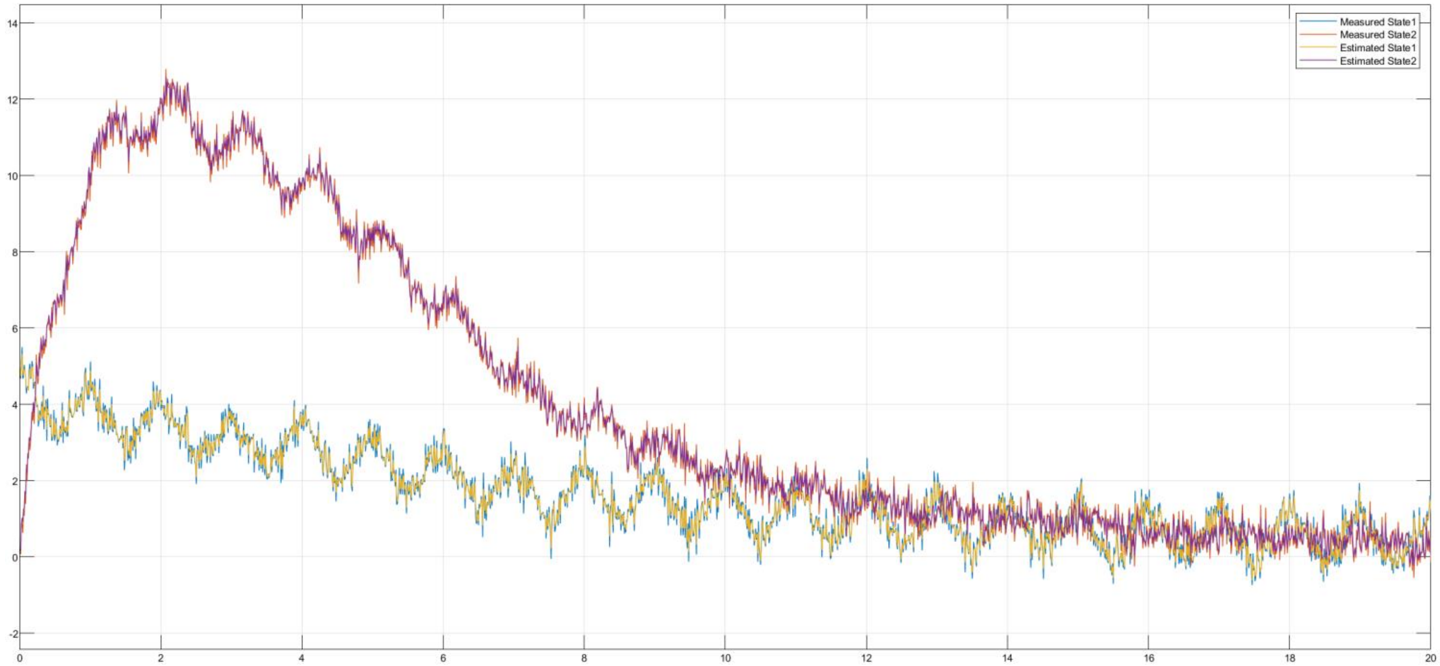
Covariance Plot for State 2



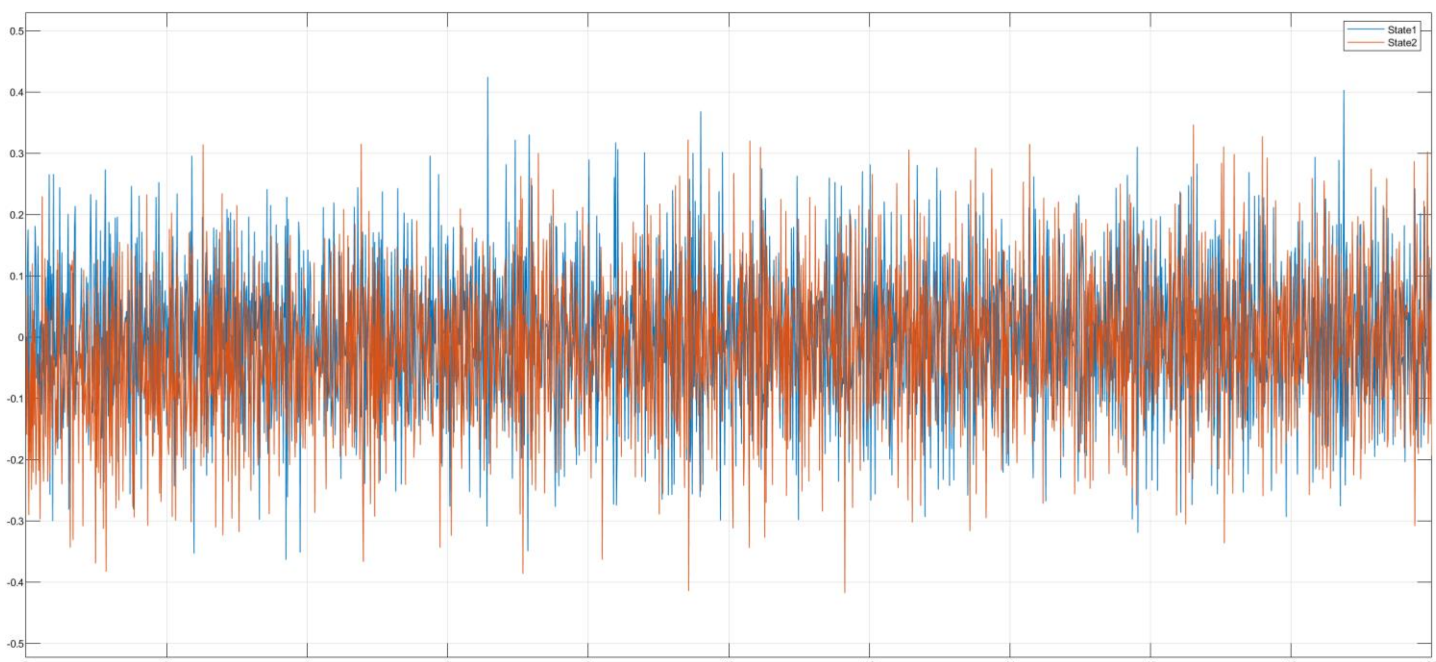
For this particular case/system, <u>the change in initial value of first state from 5 to 0 has no noticeable effect on the output of the filter</u>. For many systems, a significant change in the initial values of the filter may throw it off into another direction.

Now, only the value of mu is changed to -0.01 and the initial states are reverted back to [5,0] and the simulation is rerun.

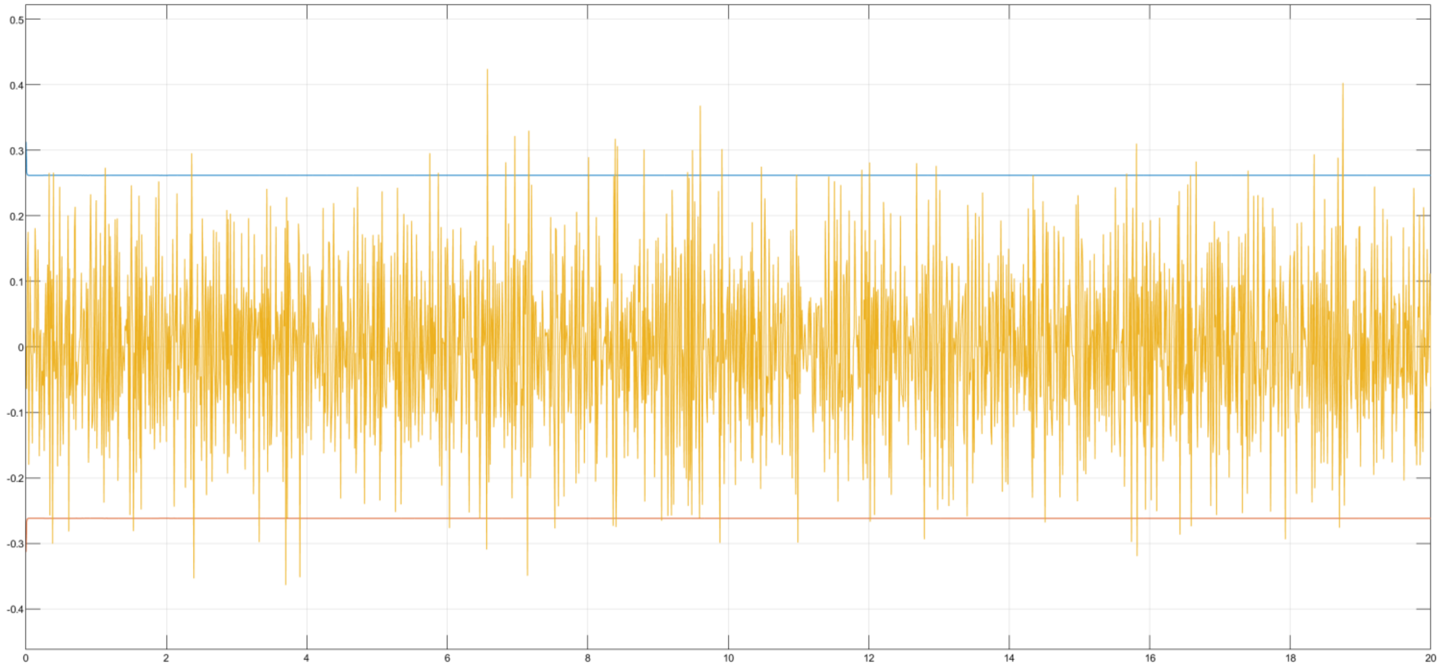
Comparison Plots for measured and estimated values



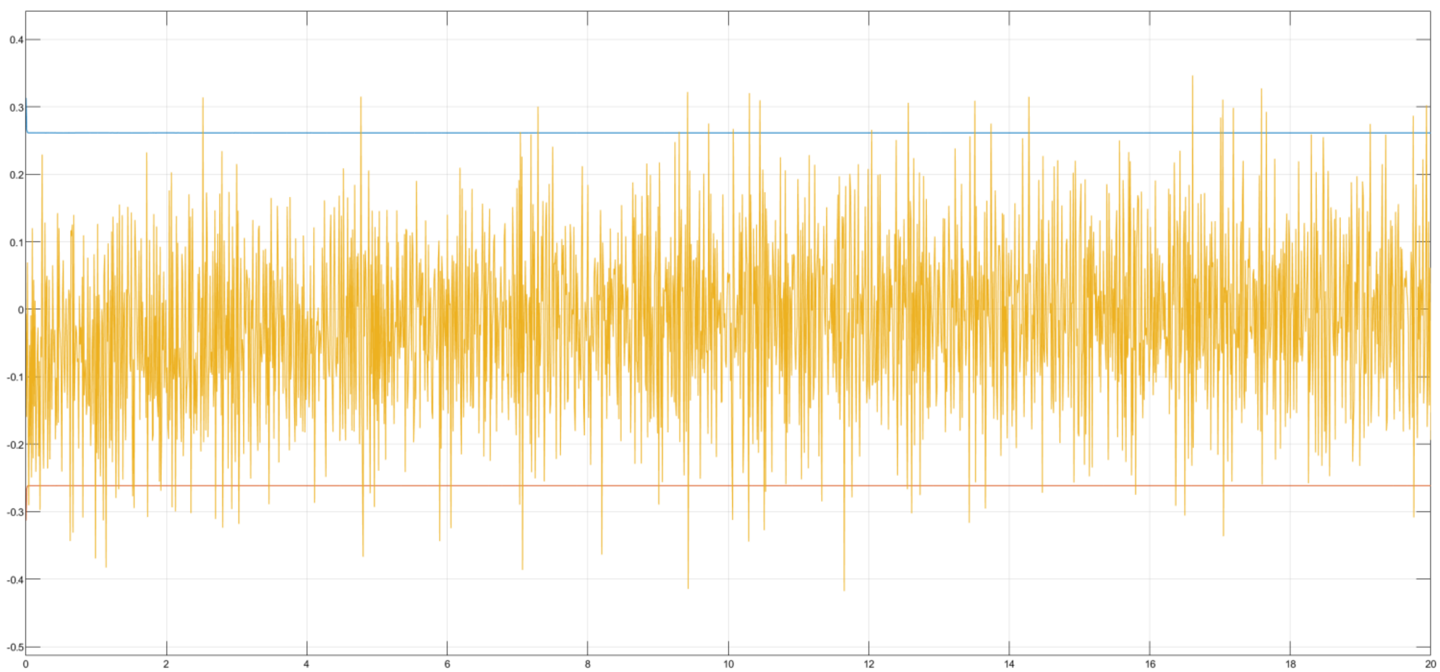
Error Plot



Covariance Plot for State 1



Covariance Plot for State 2



As observed from the above plots, there is no noticeable change in the output of the filter when the value of mu is changed from -0.1 to -0.01 in the predictor block and the initial states are reversed to the original values of [5,0].

This means that for this particular case, the filter does a good job even when the physics specified in the predictor block is slightly incorrect.

References

1. System Identification and Estimation – Dr. Michael A. Niestroy (Faculty, Electrical Engineering, The University of Texas at Arlington)
2. Intelligent Control Systems – Dr. Frank L. Lewis (Professor, Electrical Engineering, The University of Texas at Arlington)
3. Dynamic Systems Modeling and Simulation – Dr. David A. Hullender (Professor, Mechanical and Aerospace Engineering, The University of Texas at Arlington)