Fuzzy Systems - Problem Statement

Implement the following system in Simulink and assumed a damping ratio (zeta) of 0.5, a fixed step integration with a time step of 0.01 sec, and a unit step input occurring at 1 second, with a simulation stop time of 15 seconds. Generate a training dataset of (time, input, omega) as inputs and y as the output for values of omega 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5 rad/sec.

$$y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} u(s)$$

Problem 1) Fuzzy Inference System Estimator – No Noise (60 points)

a) Use the neuroFuzzyDesigner tool to fit a few fuzzy inference systems. First, let's see which one has the best fit to the training data. Try the following making note of the final error after 3 epochs of training (to save time):
    1) 3-3-3 with *triangular* membership functions and *linear* output.
    2) 5-5-5 with *triangular* membership functions and *linear* output
    3) 3-3-3 with *generalized bell* membership functions and *linear* output.
    4) 5-5-5 with *generalized bell* membership functions and *linear* output.

b) Export each FIS model to the workspace and include in the Simulink simulation using a Fuzzy Logic Controller block. Generate comparisons of the FIS model with the training data. Make any observations about the type of FIS and the quality of fit it produces.

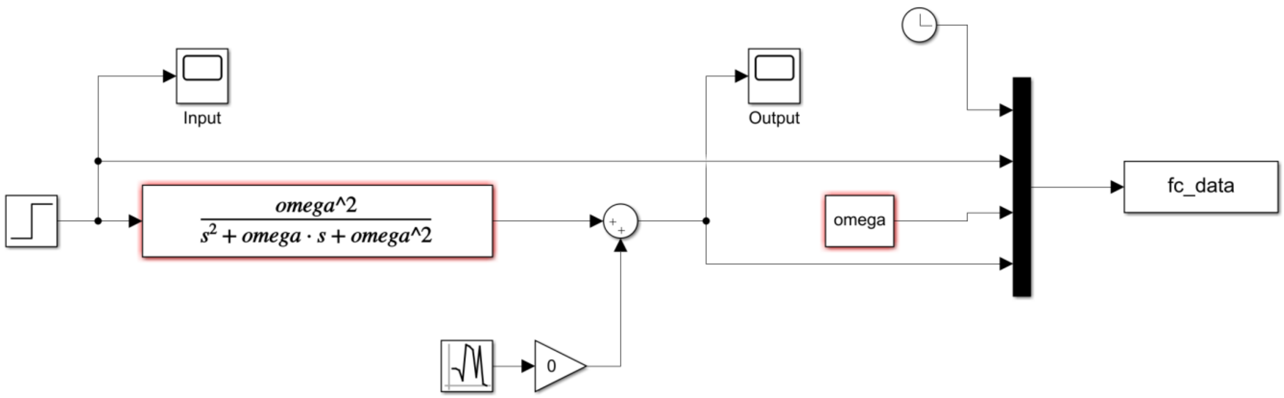
c) Now test the FIS by comparing its output with the true system output for omega values 0.75, 1.25, 1.75, 2.25, 2.75, and 3.25 rad/sec. Make any observations about the type of FIS and the quality of fit it produces for this test data.

d) Finally, comment on the overall ability of the FIS to work or not work for this application. **Note: the model produced by this neural-fuzzy system will likely not be nearly as good as the neural network produced.**

Problem 2) Fuzzy Inference System as Estimator with Noisy Data (40 points)

For Problem 2, repeat the steps of Problem 1 but add noise with variance 0.001 to the output data used for training. Comment on the effects of noise on the ability of the fuzzy system to model these data.

Problem – 1



a.

Simulink model to generate training data

MATLAB commands for generating the training data set

omega=0.5

fc_trdata=fc_data;

omega=1

fc_trdata=[fc_trdata;fc_data]

omega=1.5

fc_trdata=[fc_trdata;fc_data]

omega=2

fc_trdata=[fc_trdata;fc_data]

omega=2.5

fc_trdata=[fc_trdata;fc_data]

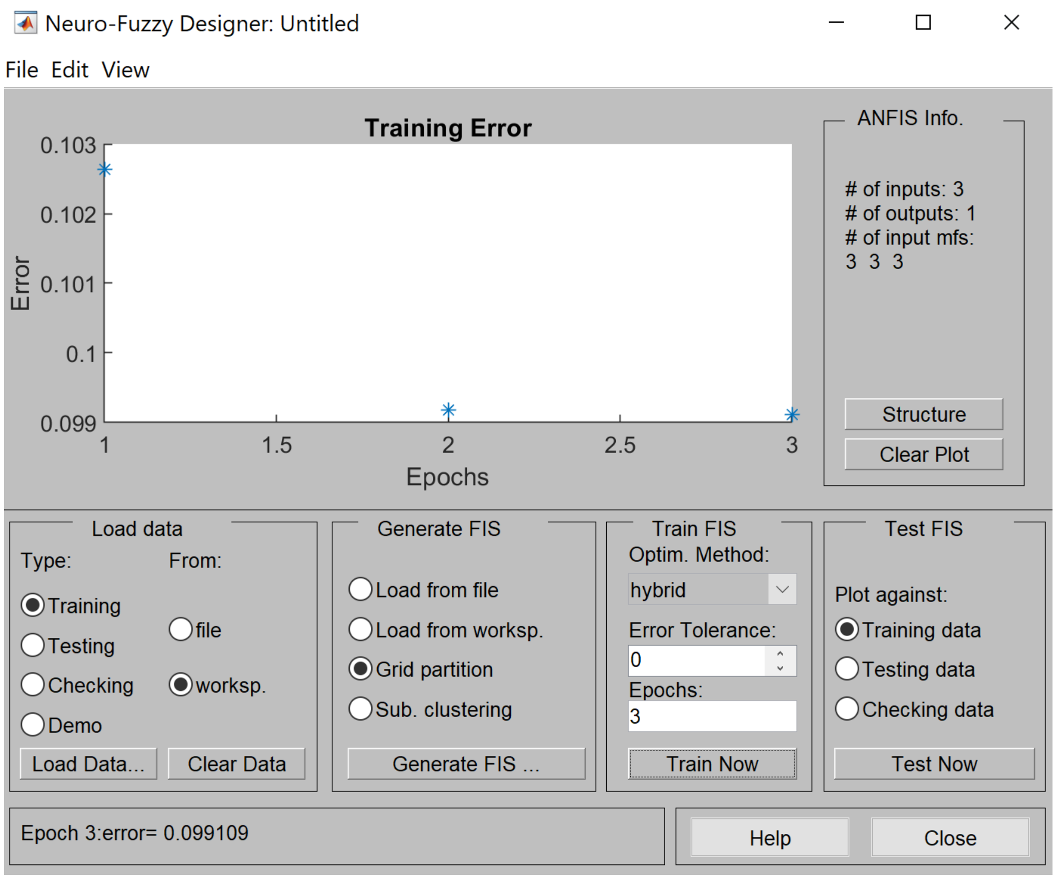
omega=3

fc_trdata=[fc_trdata;fc_data]

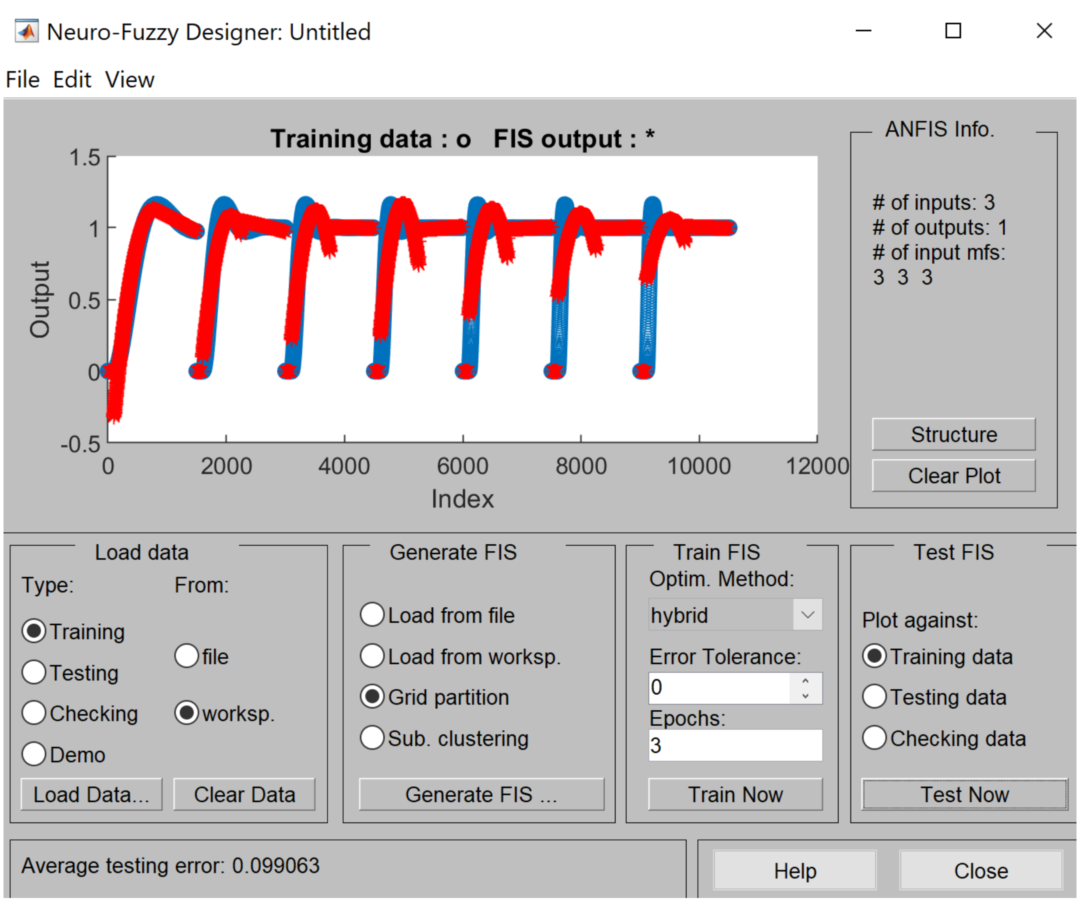
omega=3.5

fc_trdata=[fc_trdata;fc_data]

TMF 333

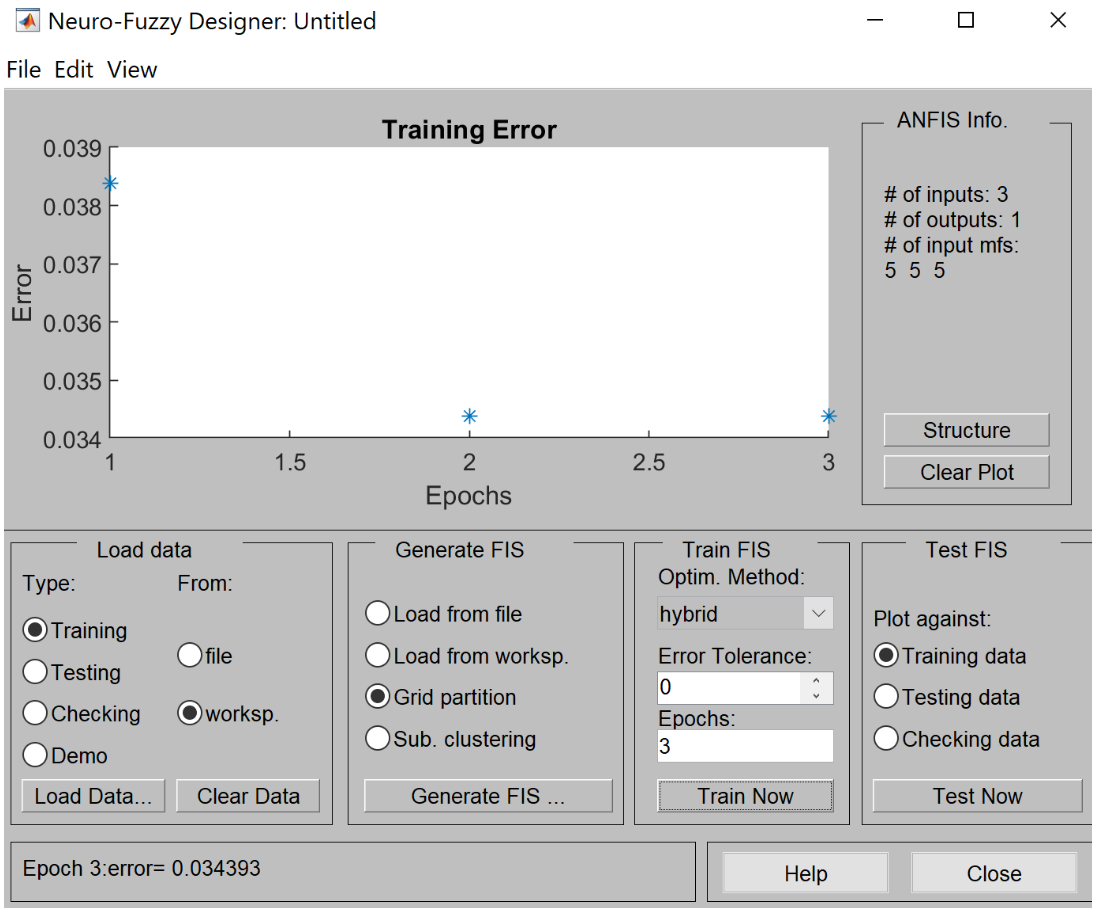


Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
Error
Epochs
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Epoch 3:error= 0.099109
Help
Close

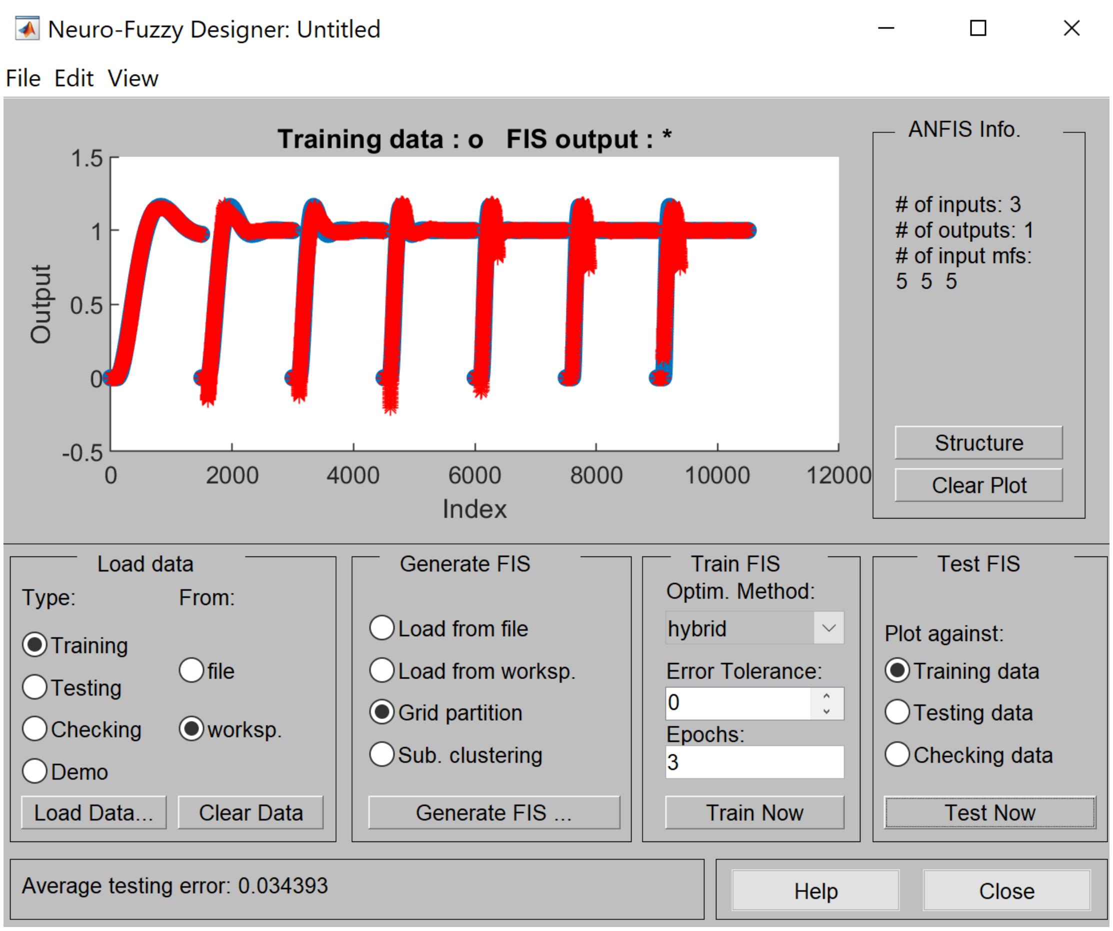


Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
Output
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.099063
Help
Close

TMF 555

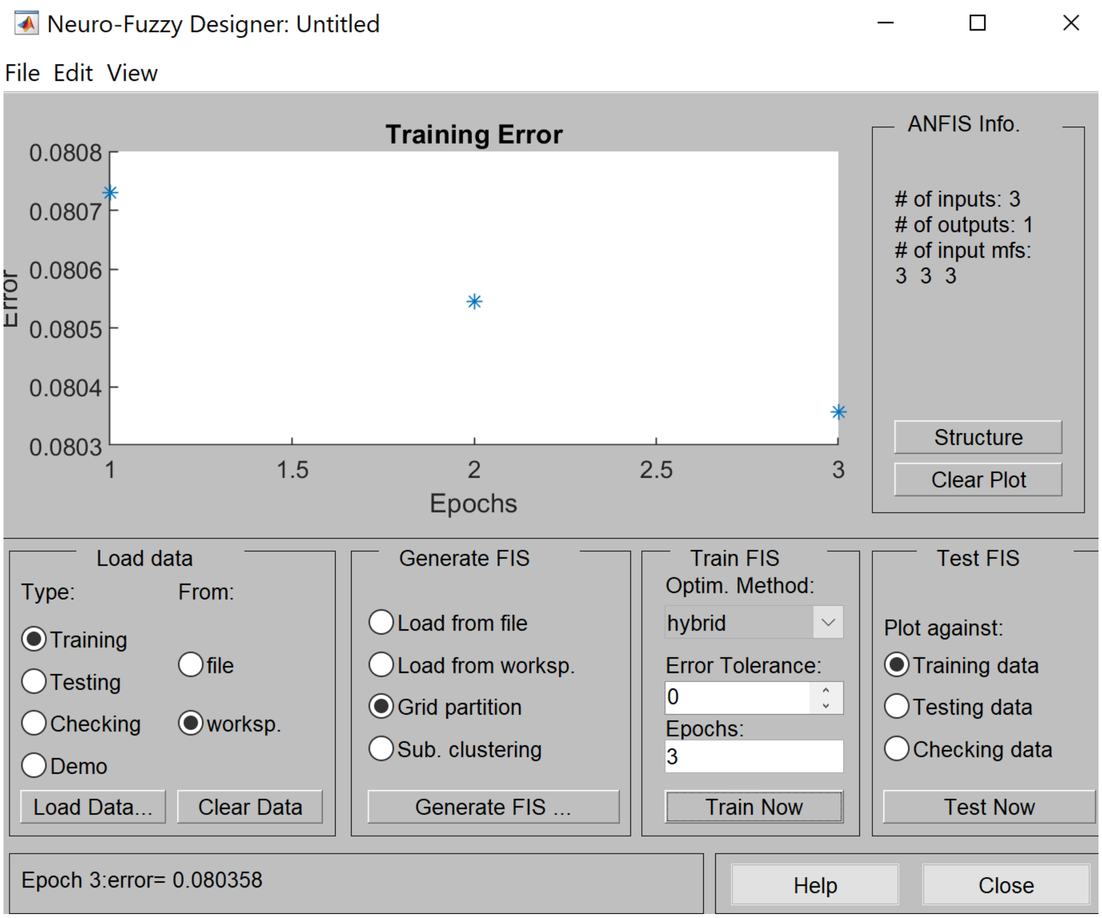


Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
Error
Epochs
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
5 5 5
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Epoch 3:error= 0.034393
Help
Close



Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
Output
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
5 5 5
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.034393
Help
Close

GBELL 333


Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
Error
Epochs
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
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Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
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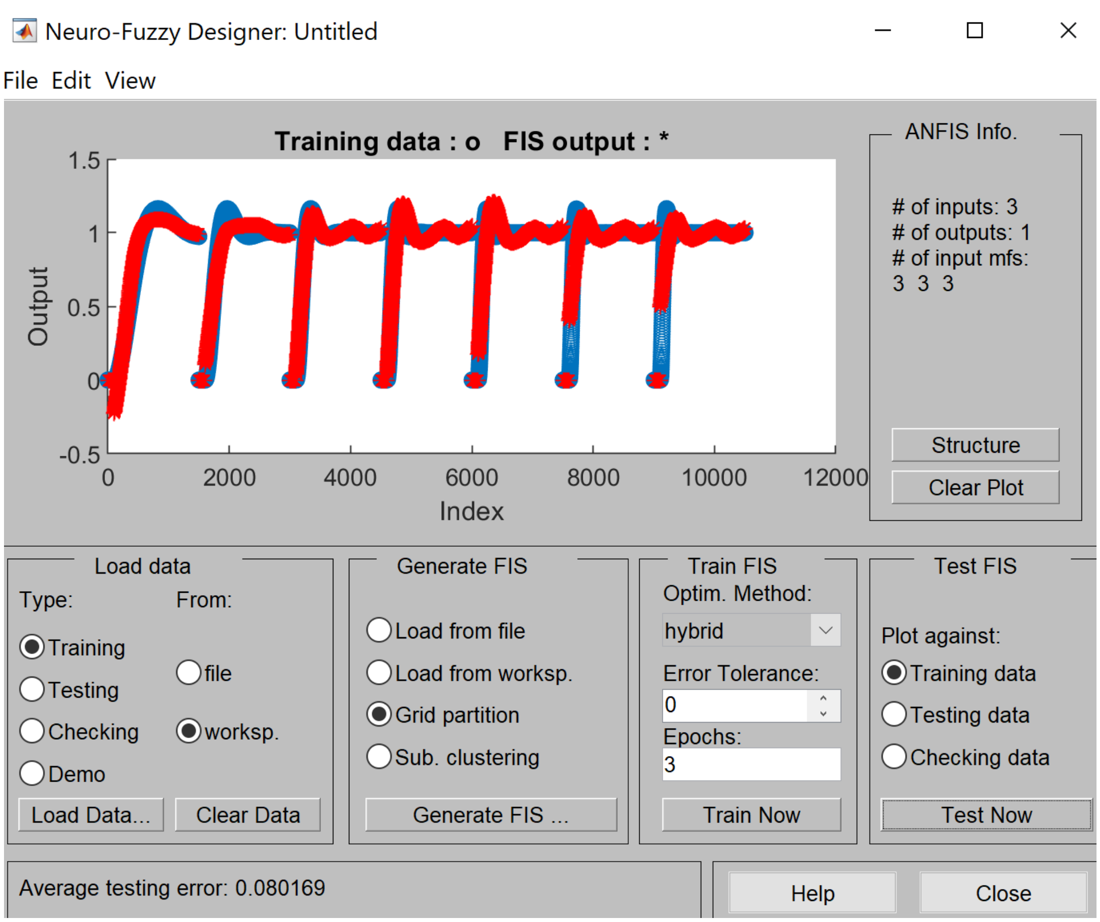
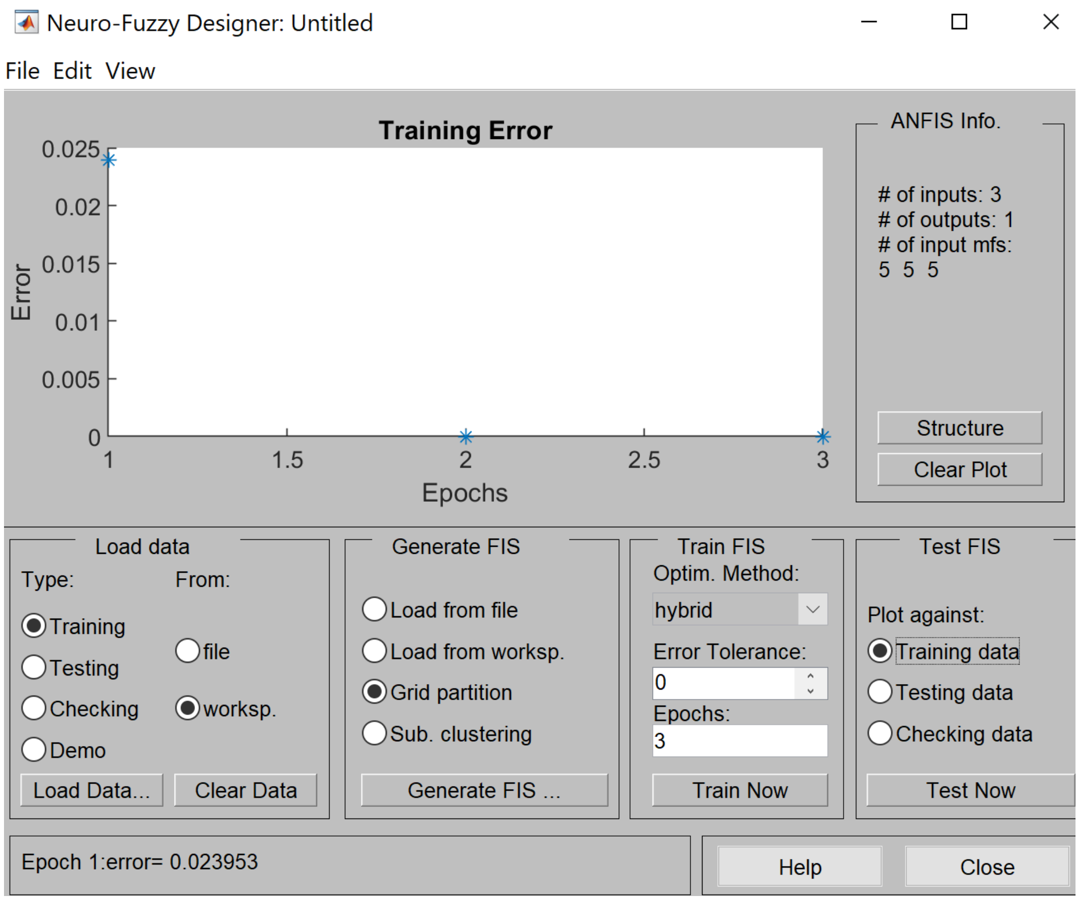
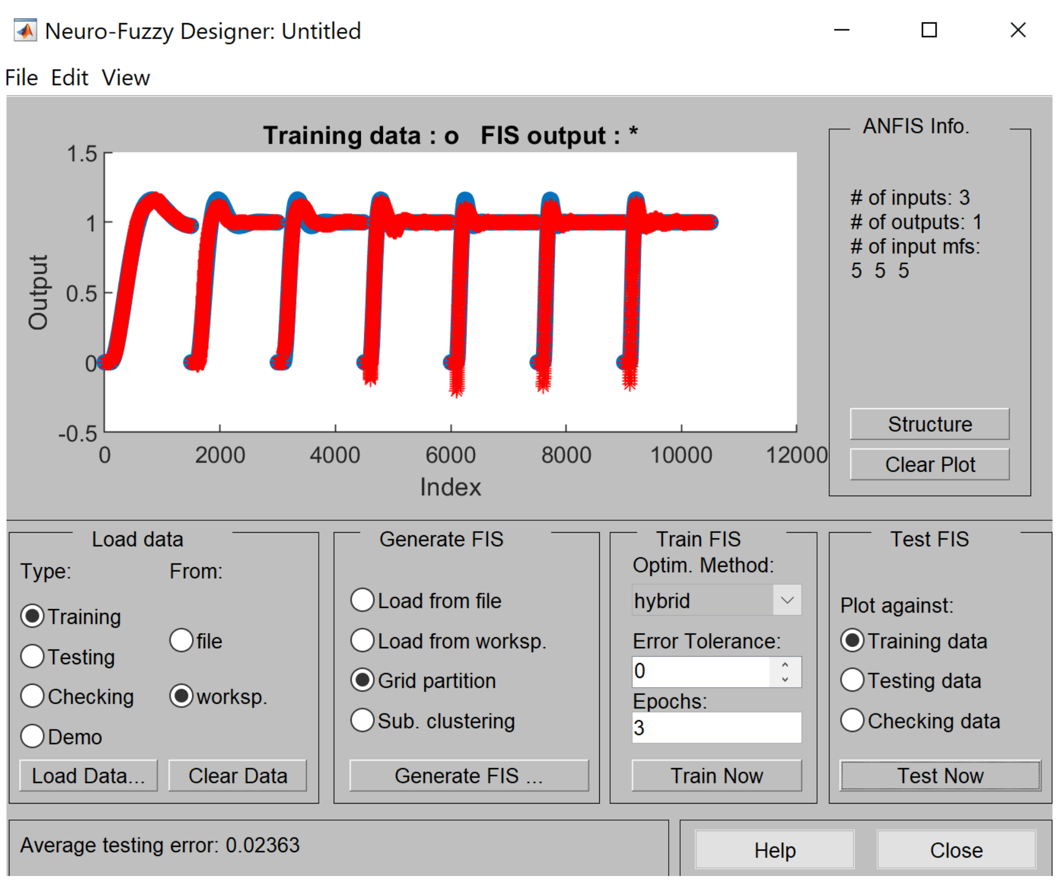
Help
Close


Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
Output
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.080169
Help
Close

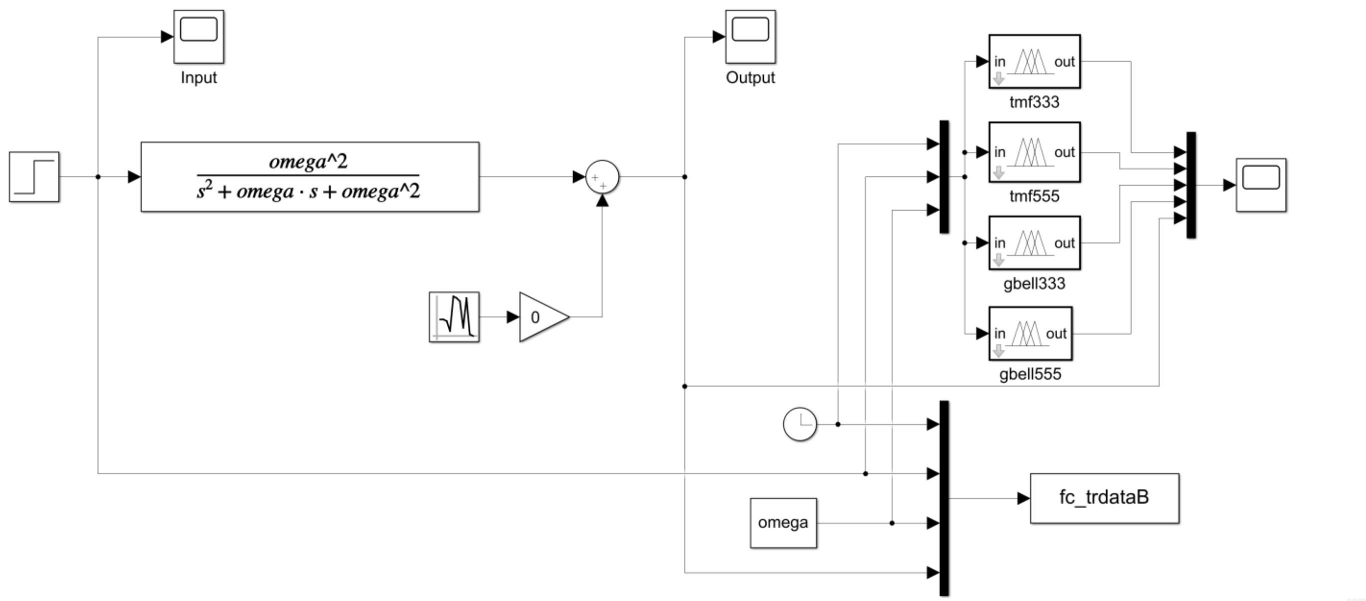
GBELL 555





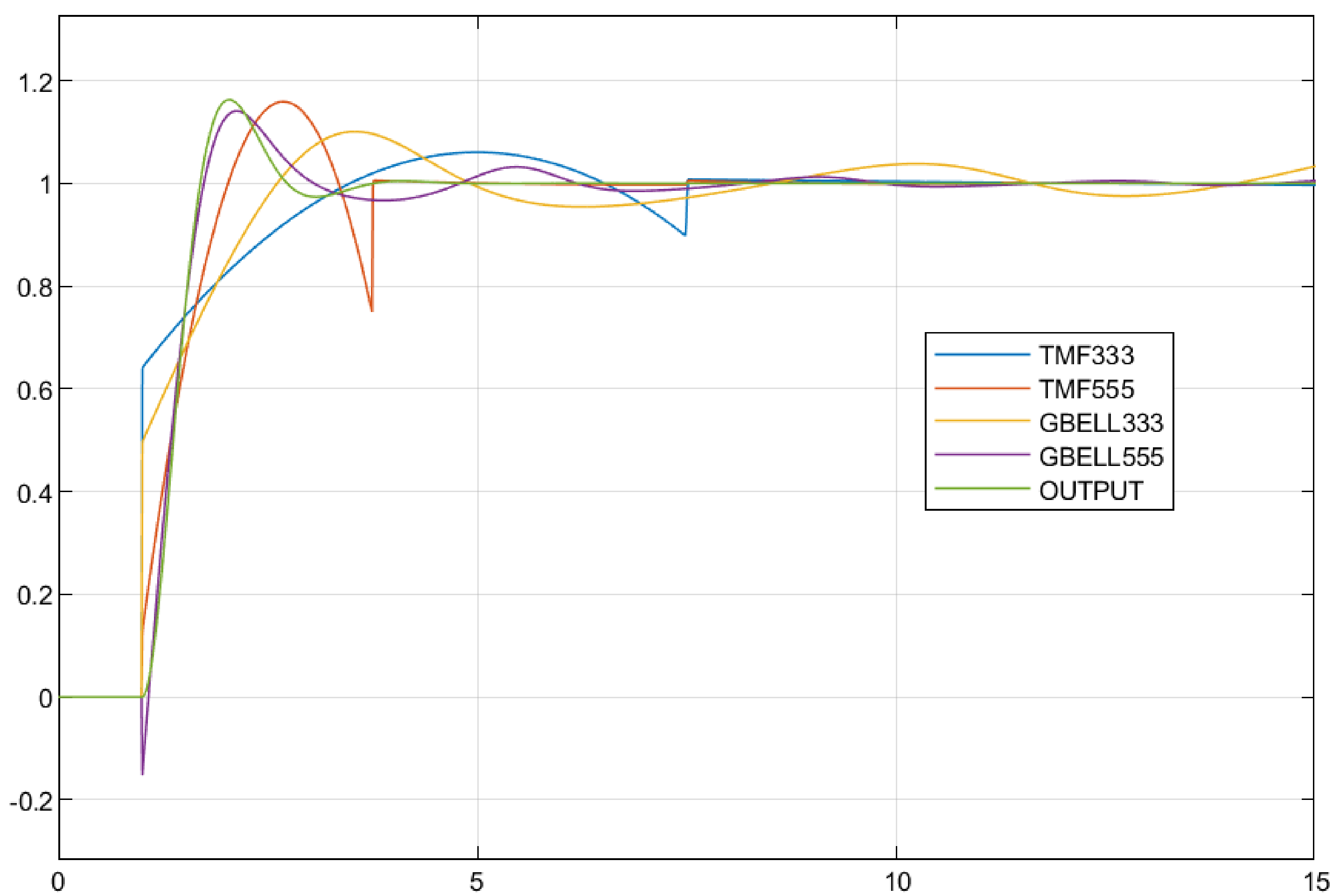
Conclusion – Using average testing error values, **GBELL 555 produces the best fit to training data followed by TMF 555**

b. Comparison

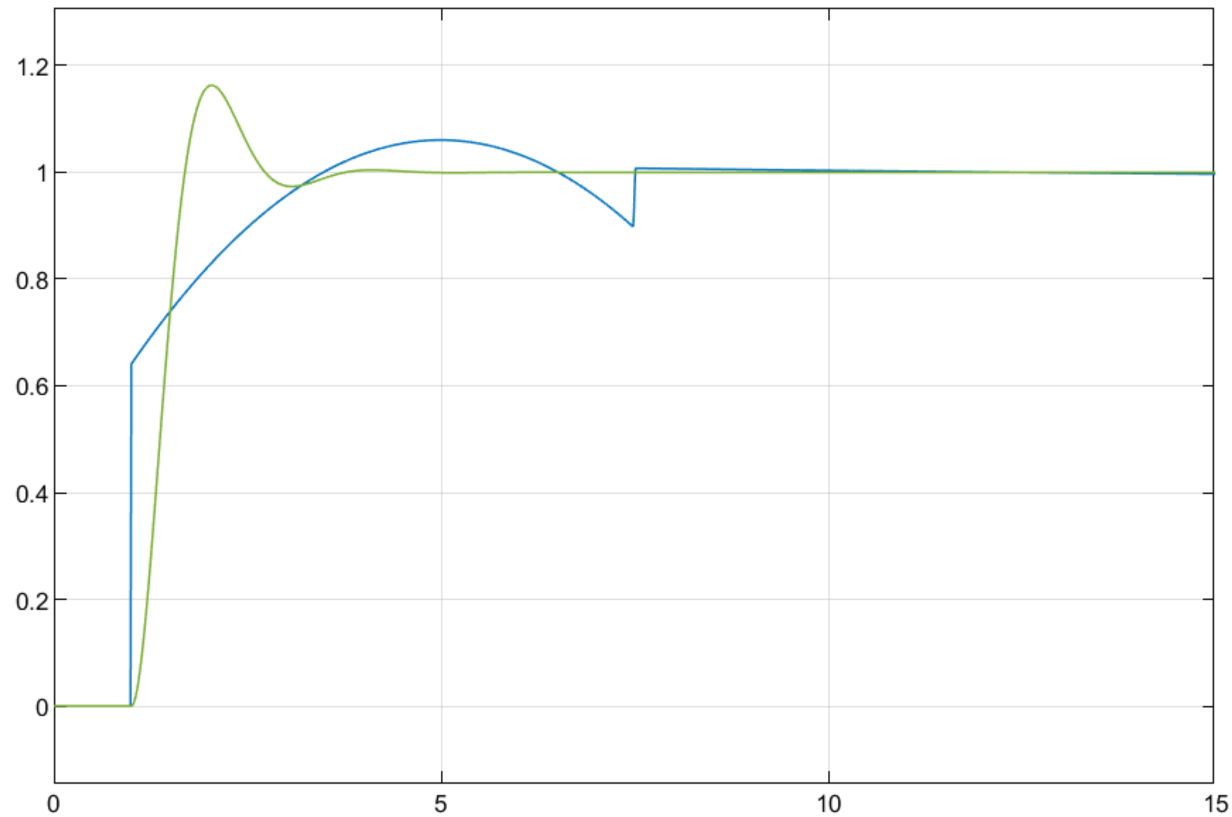


Simulink Diagram for comparing all the outputs

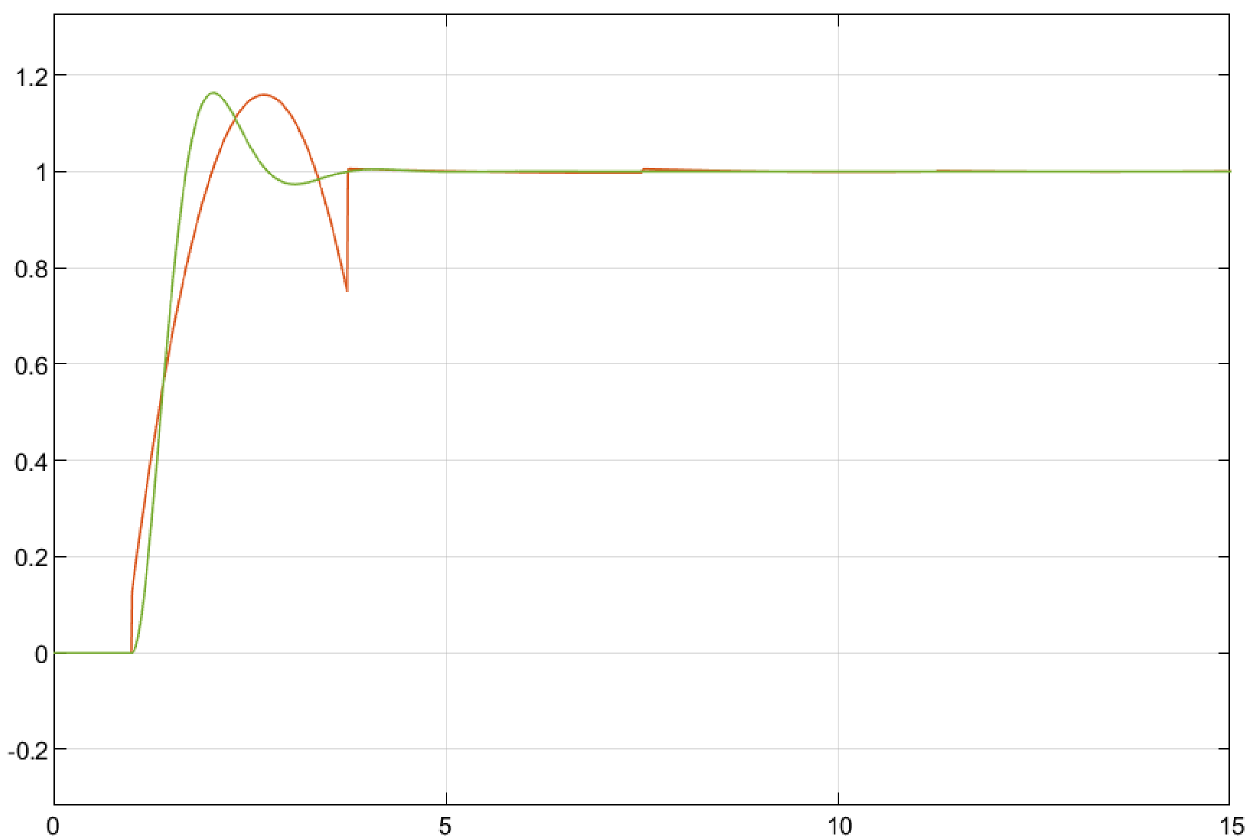
At Omega = 3.5



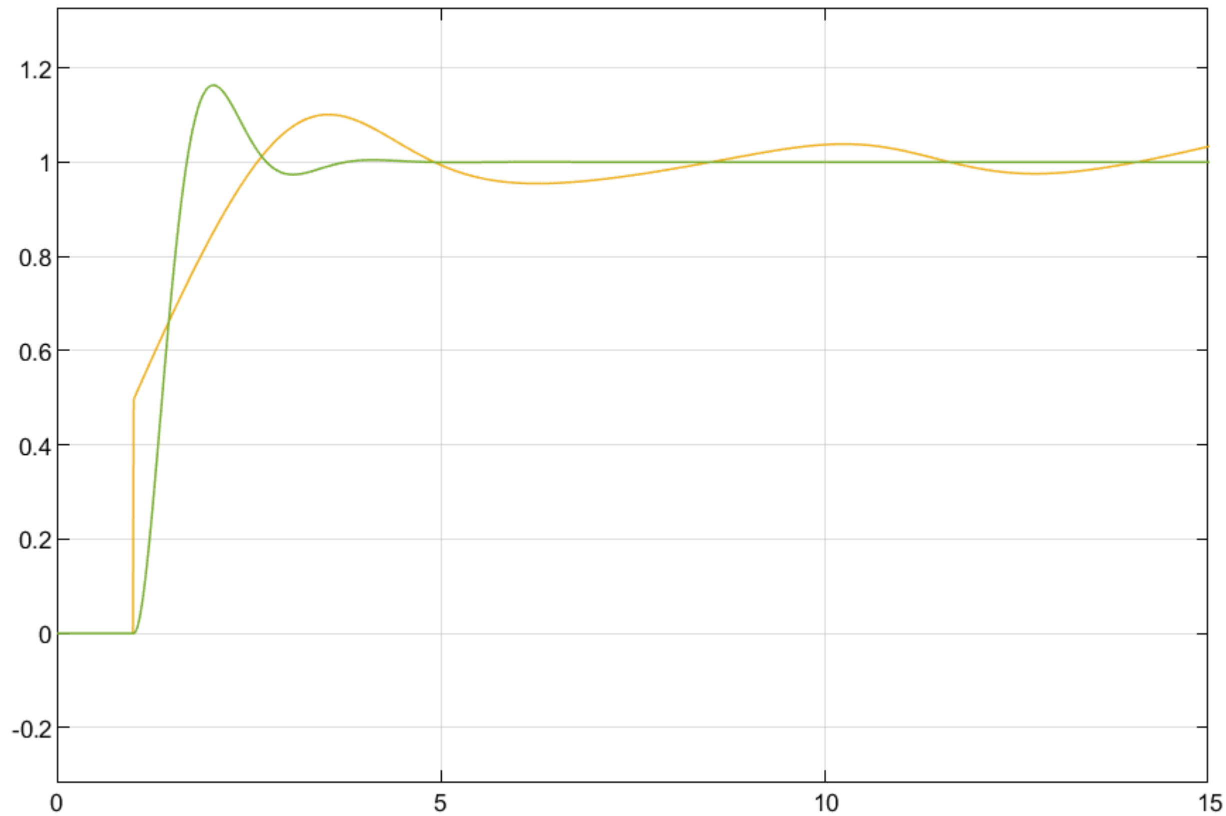
TMF 333



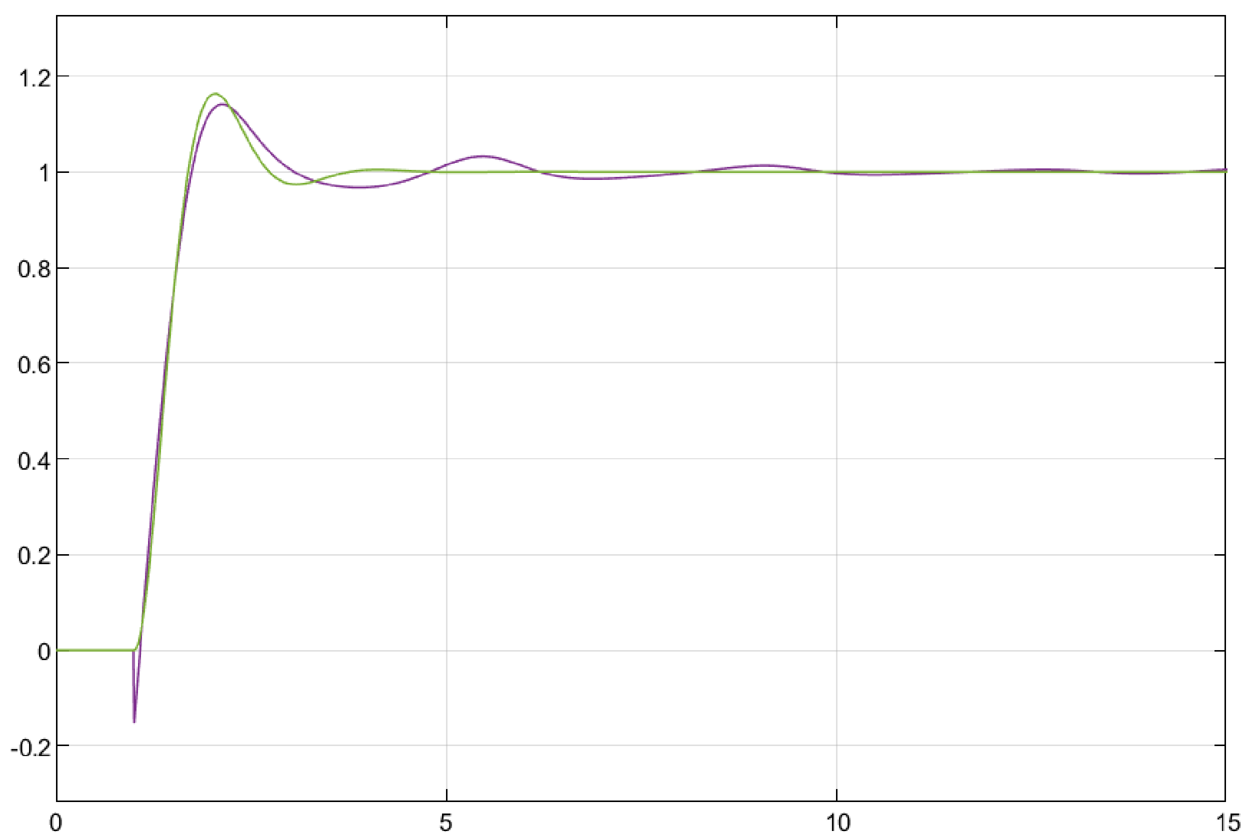
TMF 555



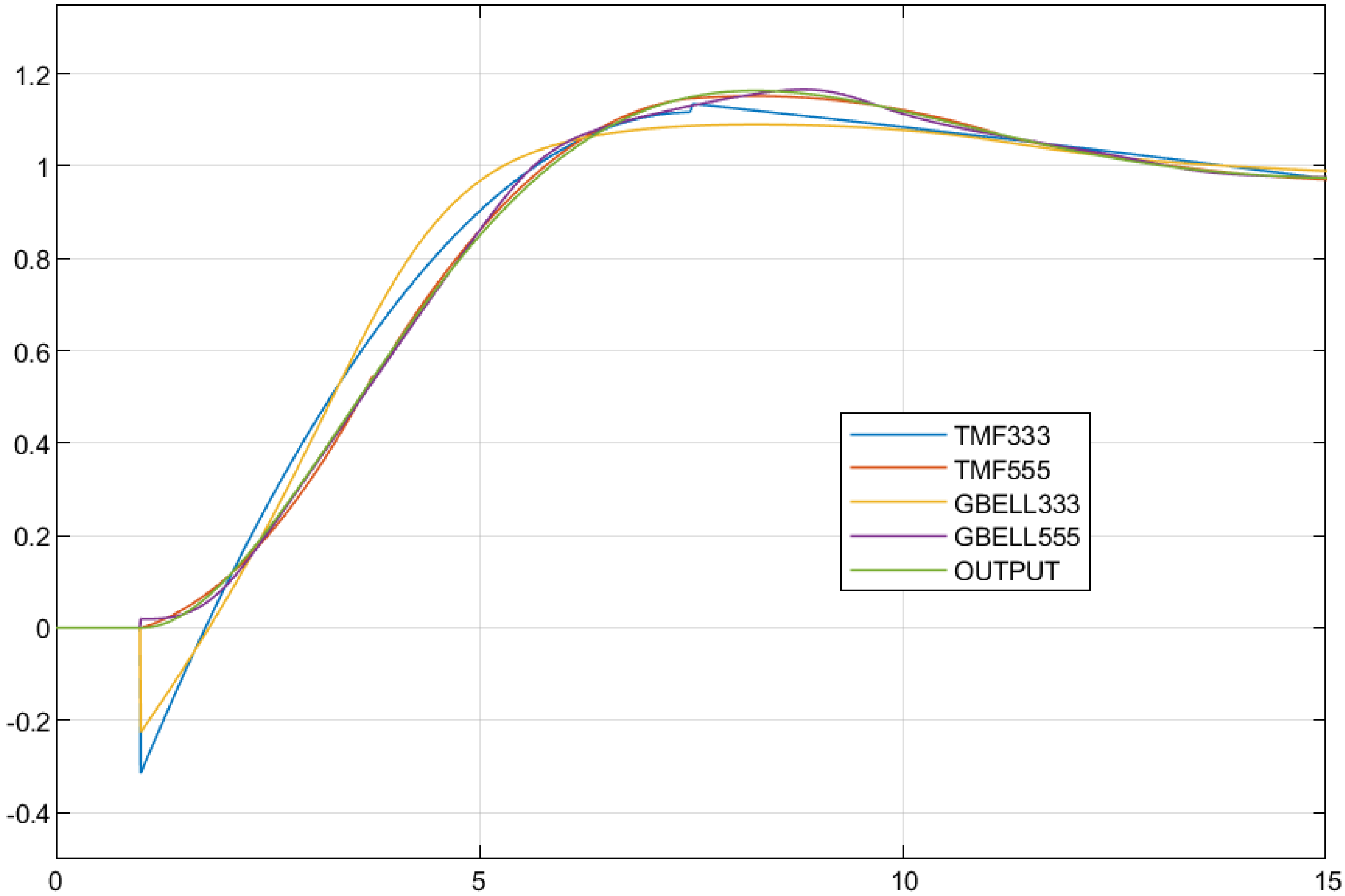
GBELL 333



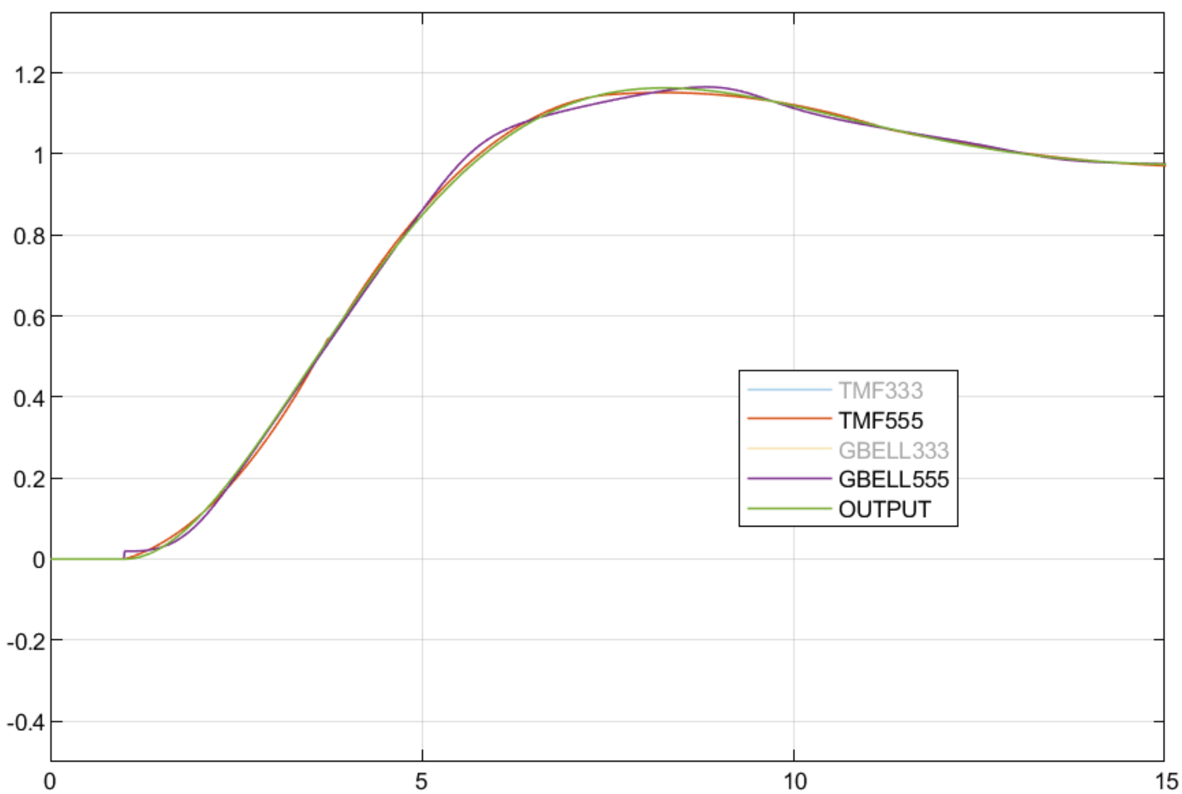
GBELL 555



At Omega = 0.5



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
TMF333
TMF555
GBELL333
GBELL555
OUTPUT



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
TMF333
TMF555
GBELL333
GBELL555
OUTPUT

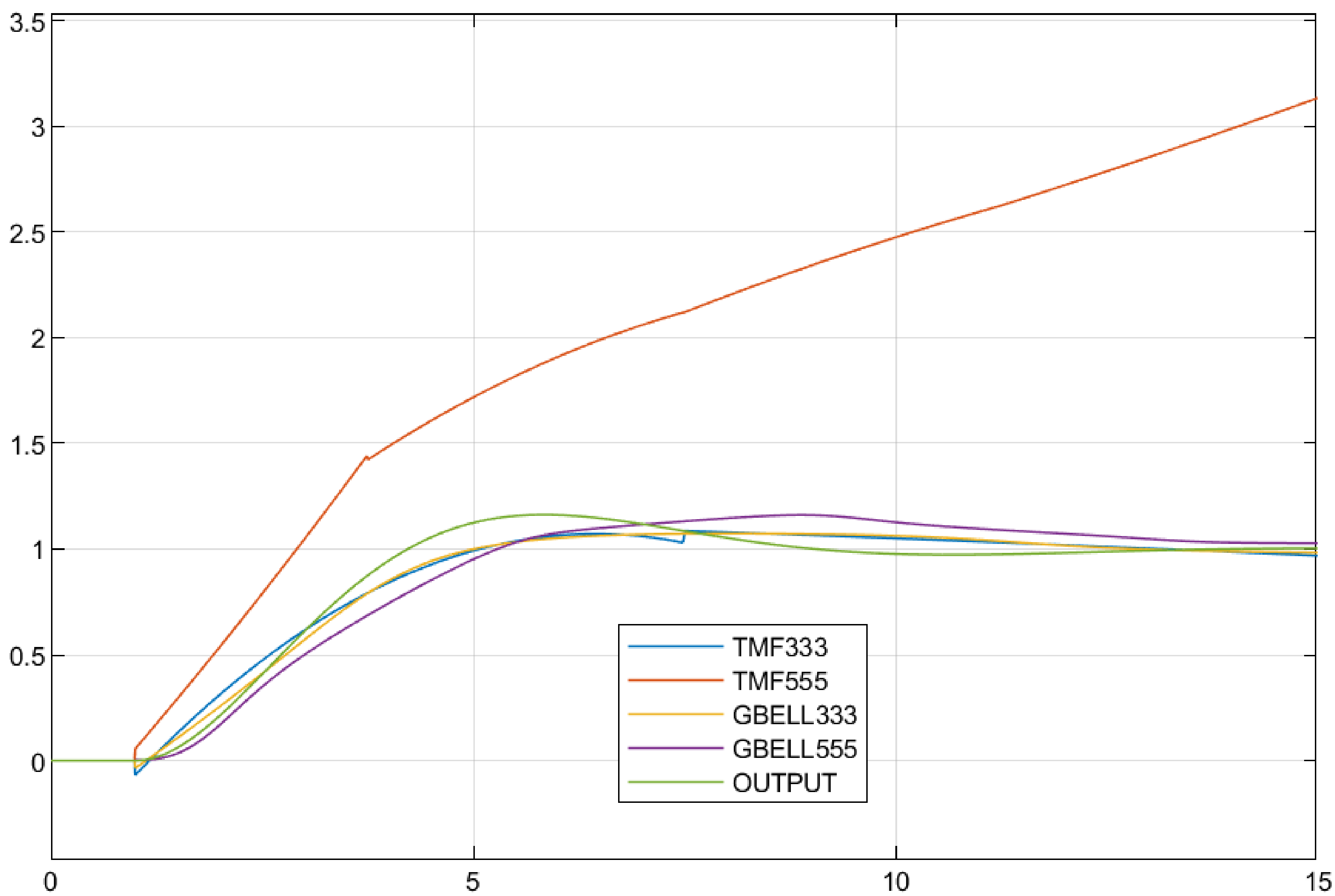
Conclusion – From the above plots it is observed that all fuzzy systems do a good job of matching the actual output at low omega values. As expected from the fit data in part a, the GBELL 555 and TMF 555 produce the best fits at omega = 0.5. On increasing the values of omega, the damping produces quicker transition to the steady state. This leads to divergence from the output but the GBELL 555 still produces a similar output to the actual output although with more oscillations. **It is seen that FIS with more membership functions is better at estimating the output.**

The **triangular membership function has sharp edges** due to the straight triangular edges used for defining the output. This is not seen in gaussian membership functions as they have smooth curves. The above values are for omega=3.5. As it can be seen, for lower omega values almost all fuzzy identification systems work well with the best ones being TMF555 and GBELL555 due to the higher number of functions.

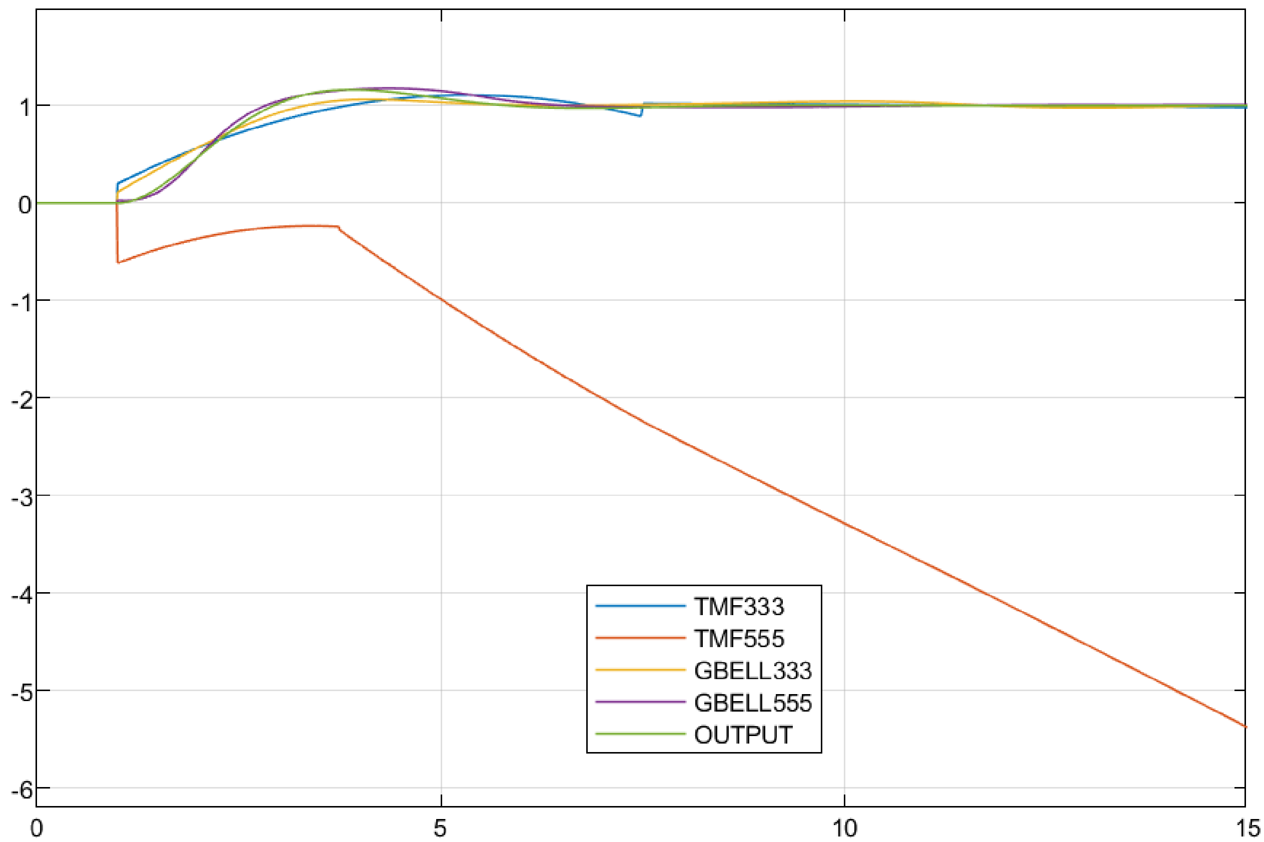
Other than that, it is also noted that while the GBELL 555 FIS produces the best fit, it gives a sharp dip before the input at higher omega values.

c. For different omega values

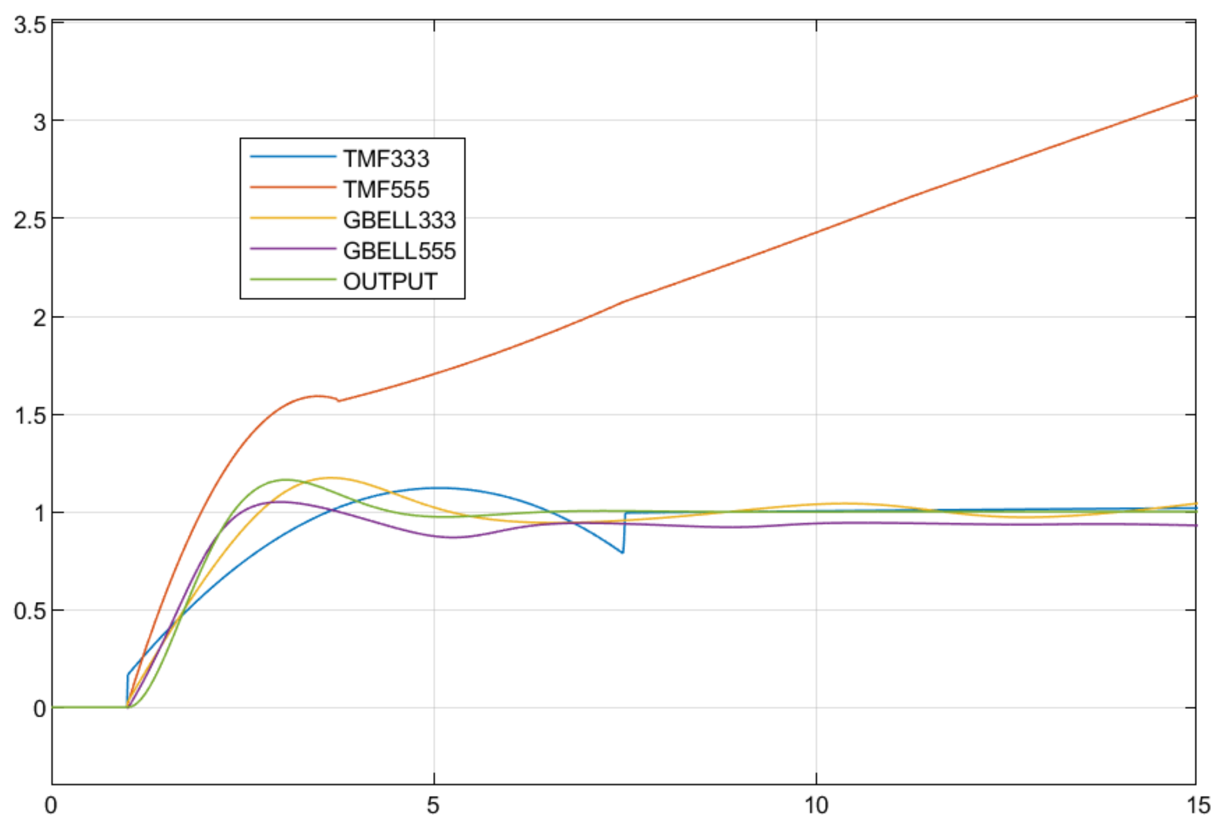
Omega 0.75



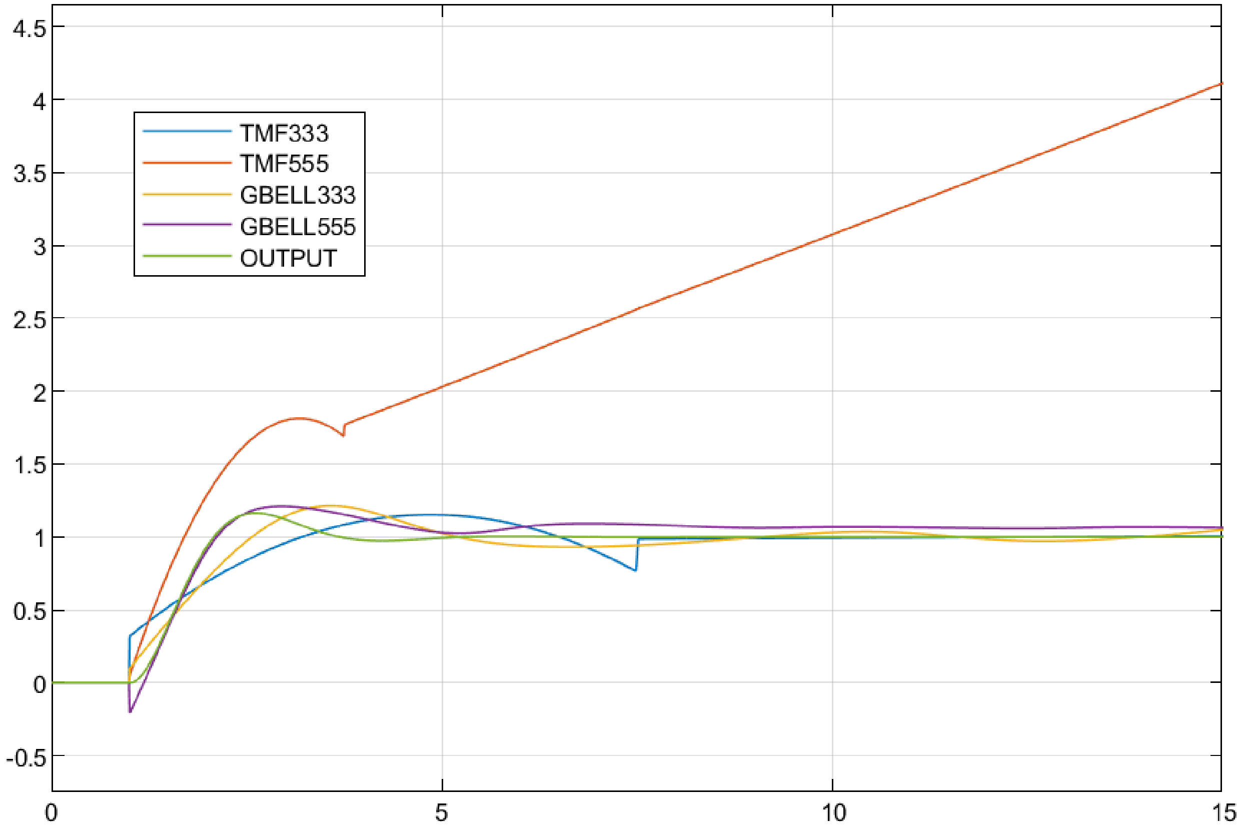
Omega = 1.25



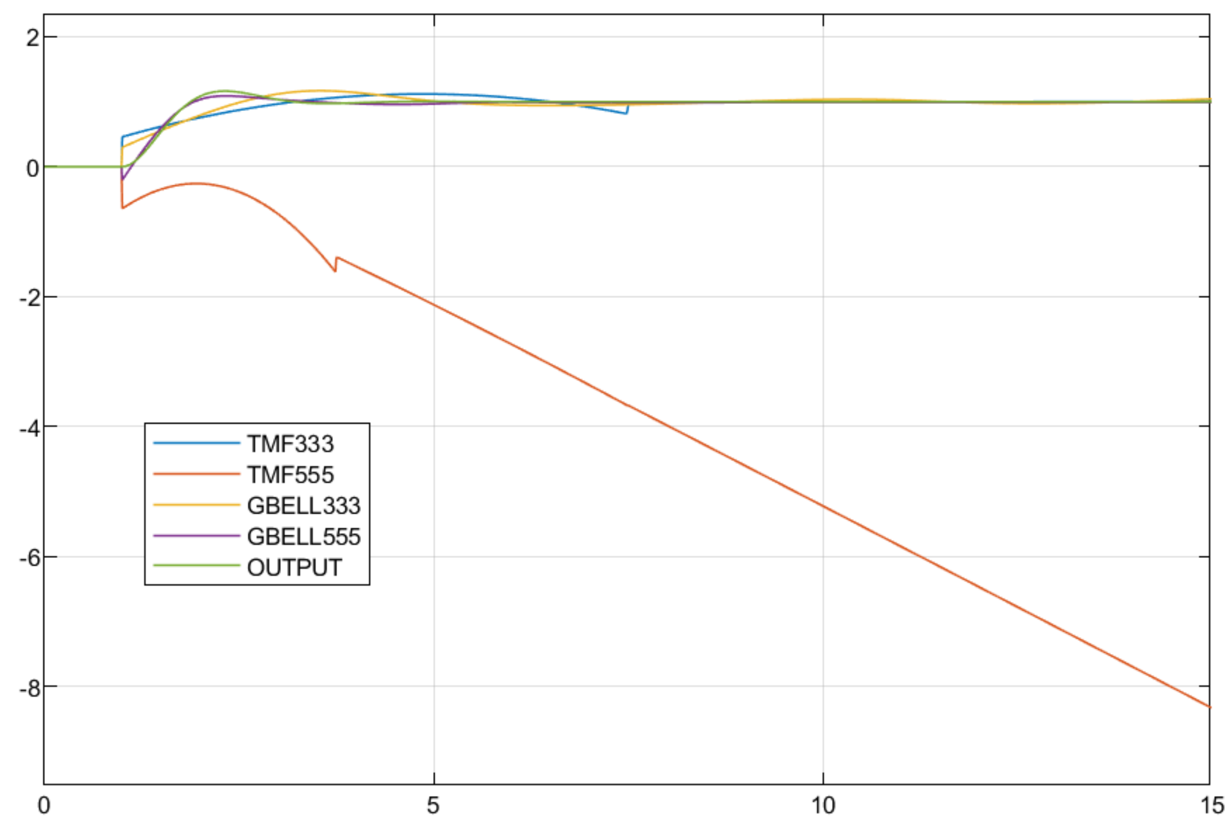
Omega 1.75



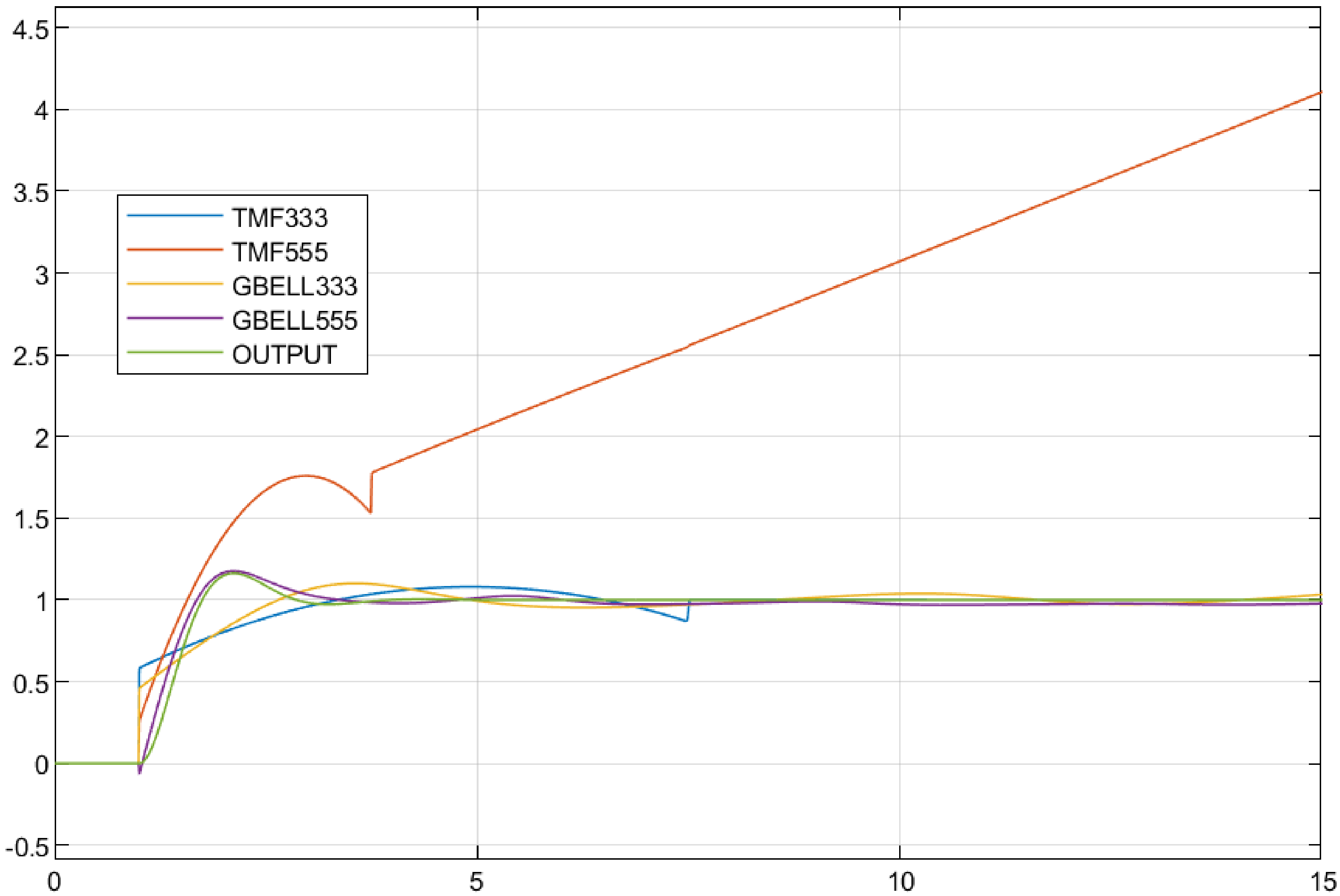
Omega = 2.25



Omega = 2.75



Omega = 3.25

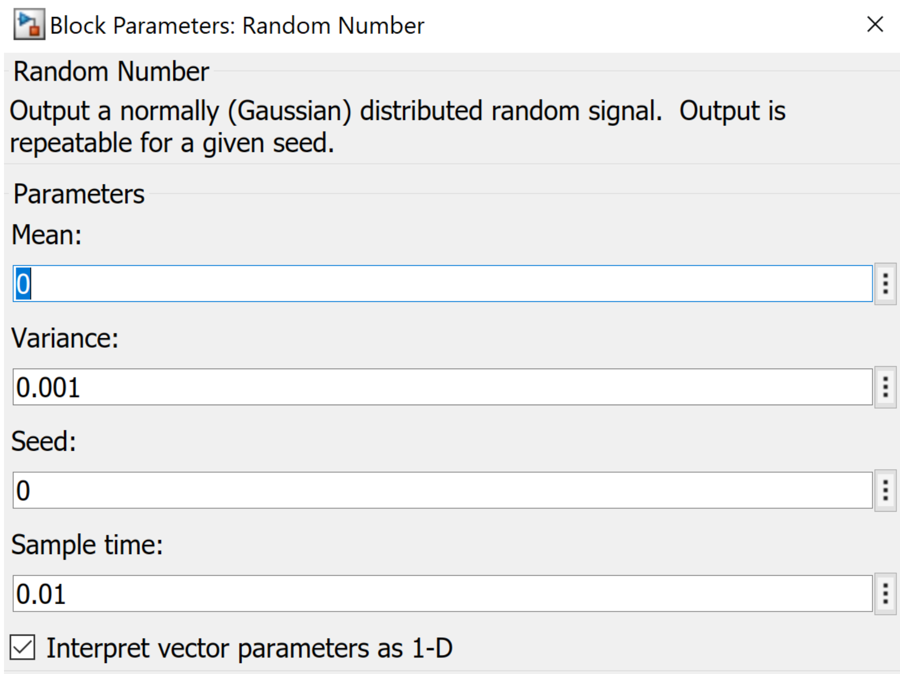


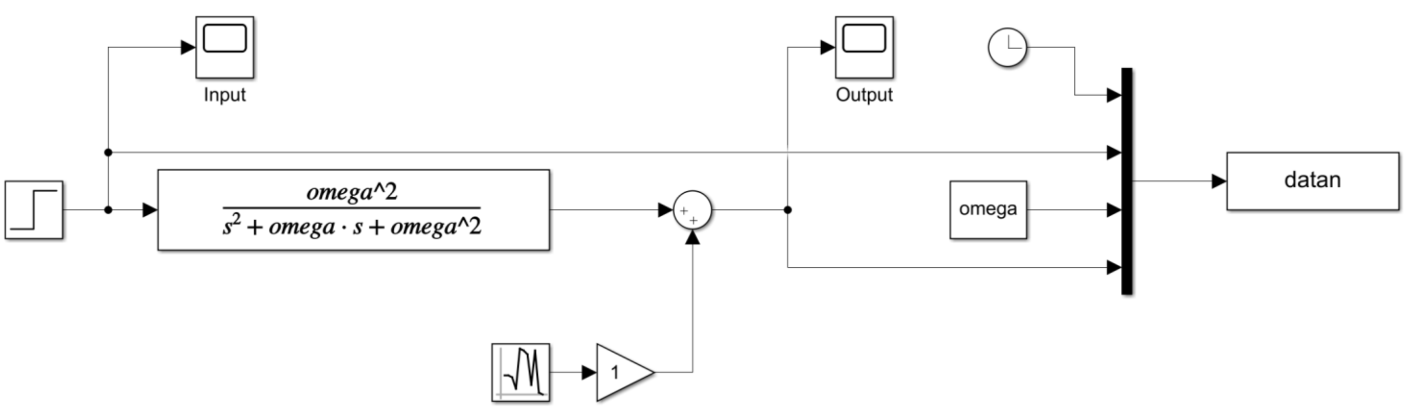
Conclusion – It can be observed that TMF555 is not able to provide output when values are interpolated. It might be because the triangular membership function tries to fit more information with 5 membership functions but can only use rigid triangles to do so. This leads to it overfitting incorrectly. **The best fit for all interpolated values in the transition state is provided by GBELL 555 FIS** but it often leaves a steady state error which may not be desirable. **In steady state, best fit is provided by TMF 333 FIS.** This may be because the function is simple and dependent on linear triangles so it only fits some data which is easy to interpolate.

d. The **GBELL 555 FIS** is a good alternative if the steady state error can be removed by a different method while interpolating. If only operating on the values on which it has been trained, the **GBELL 555 and TMF 555** both do a good job of producing desired outputs. If the system stabilizes quickly and it's performance during transition state can be neglected then **TMF 555** is the best choice. Overall, depending on the acceptable errors and desired output, both GBELL 555 and TMF 555 can work for this system.

Problem – 2:

a. Fitting

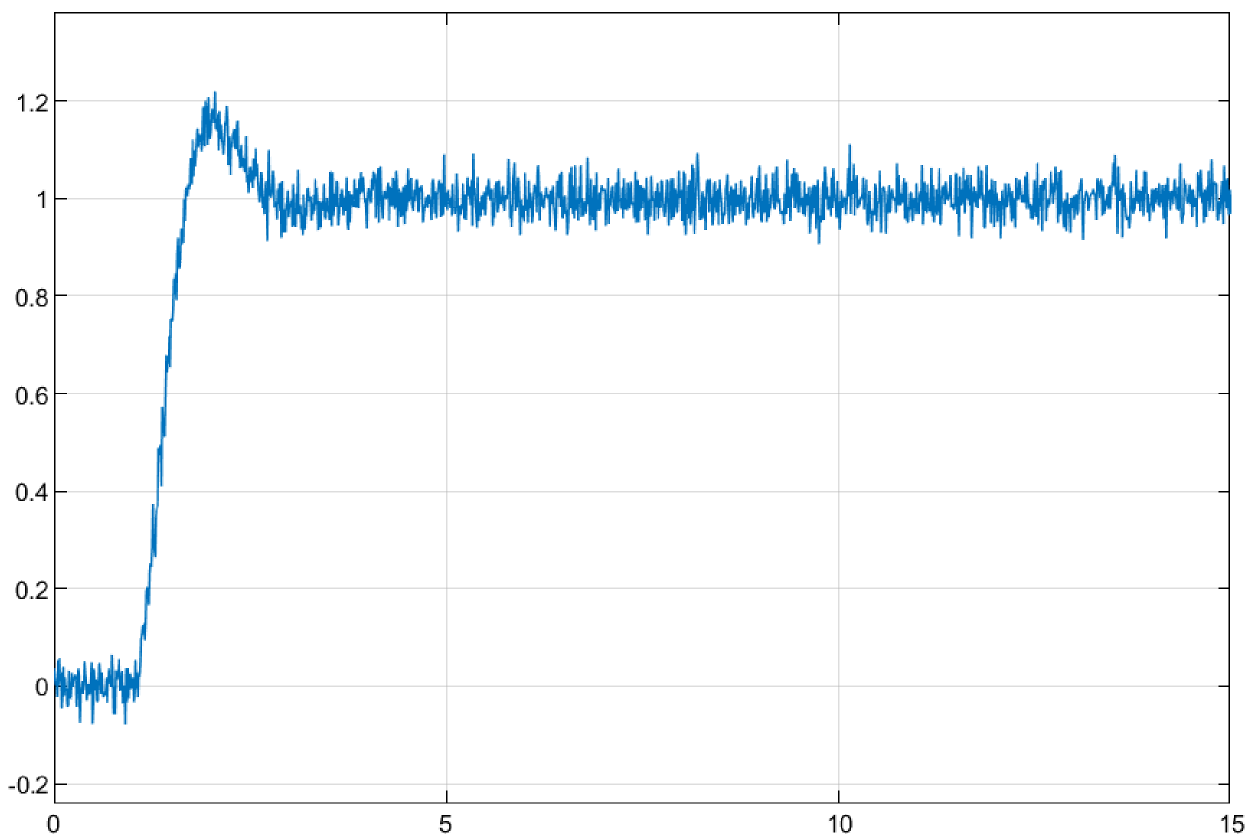




Simulink Diagram for generating noisy data

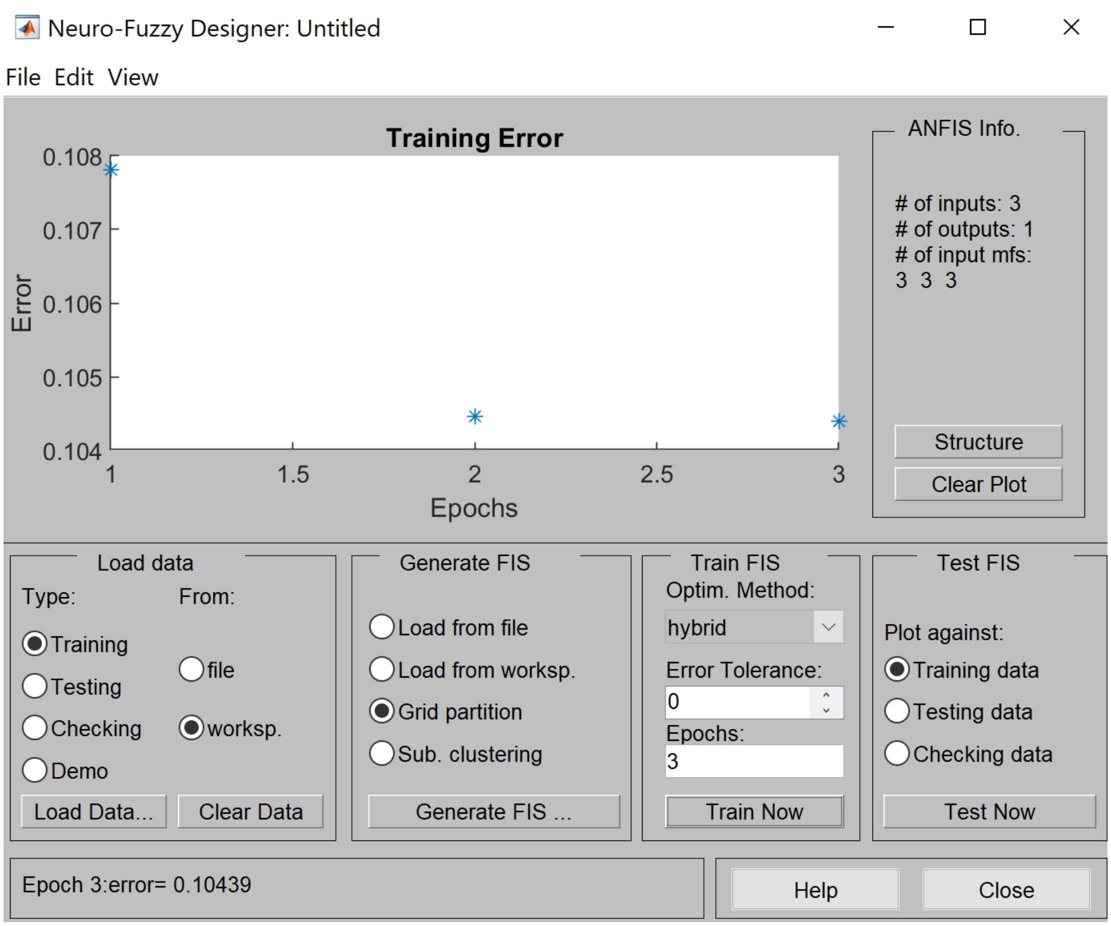
MATLAB Commands to generate the training data

```
omega=0.5 %run the Simulink diagram each time after defining the omega value

fc_trdatan=datan

omega=1

fc_trdatan=[fc_trdatan;datan]

omega=1.5

fc_trdatan=[fc_trdatan;datan]

omega=2.0

fc_trdatan=[fc_trdatan;datan]

omega=2.5

fc_trdatan=[fc_trdatan;datan]

omega=3.0

fc_trdatan=[fc_trdatan;datan]

omega=3.5

fc_trdatan=[fc_trdatan;datan]
```



System output with noise for omega = 3.5

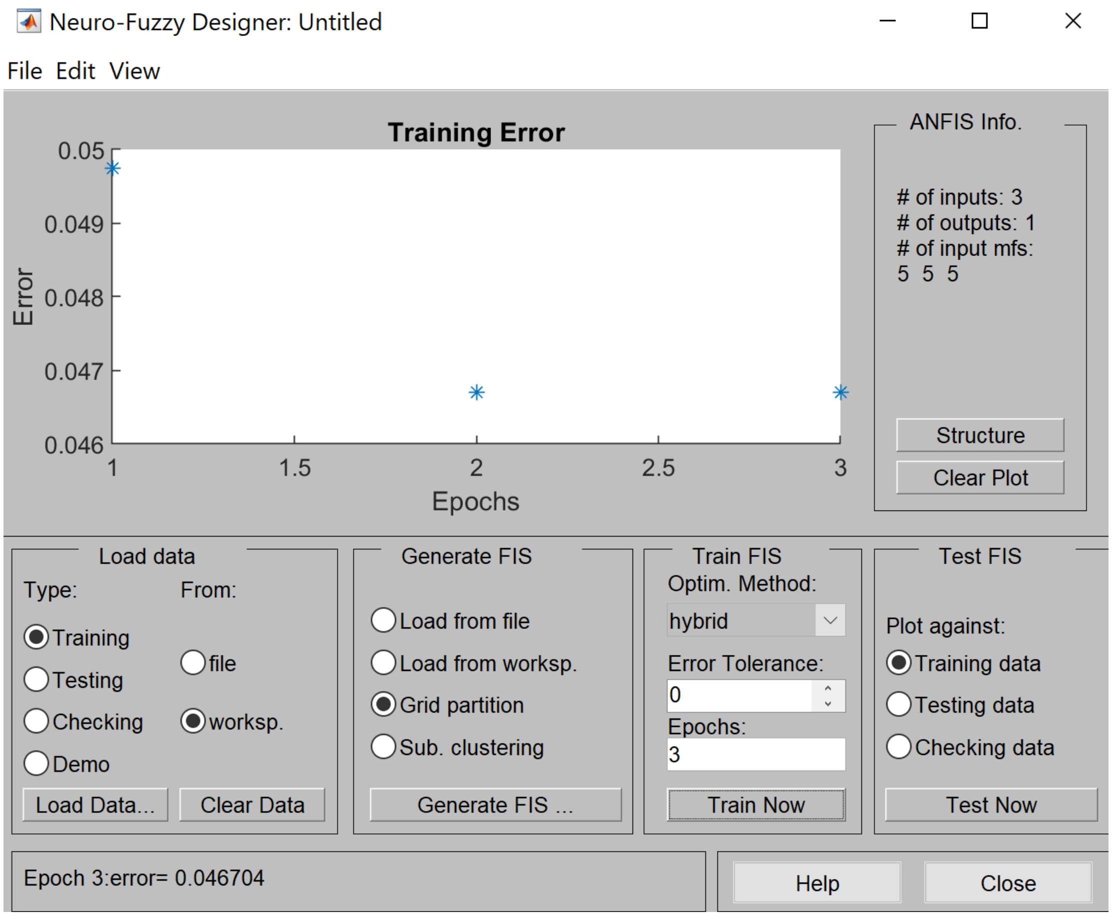
TMF 333



Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
Error
Epochs
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Epoch 3:error= 0.10439
Help
Close

Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
Output
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.10435
Help
Close

TMF 555



Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
Error
Epochs
ANFIS Info.
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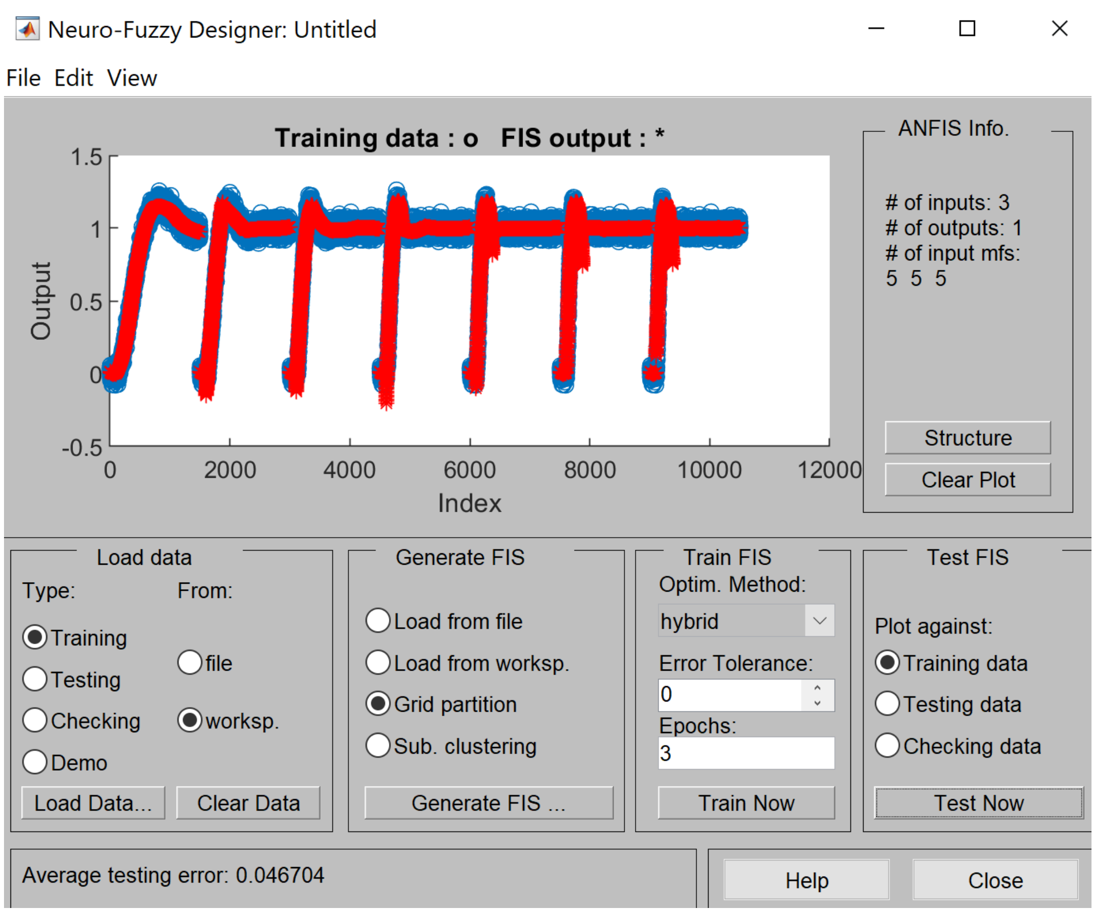
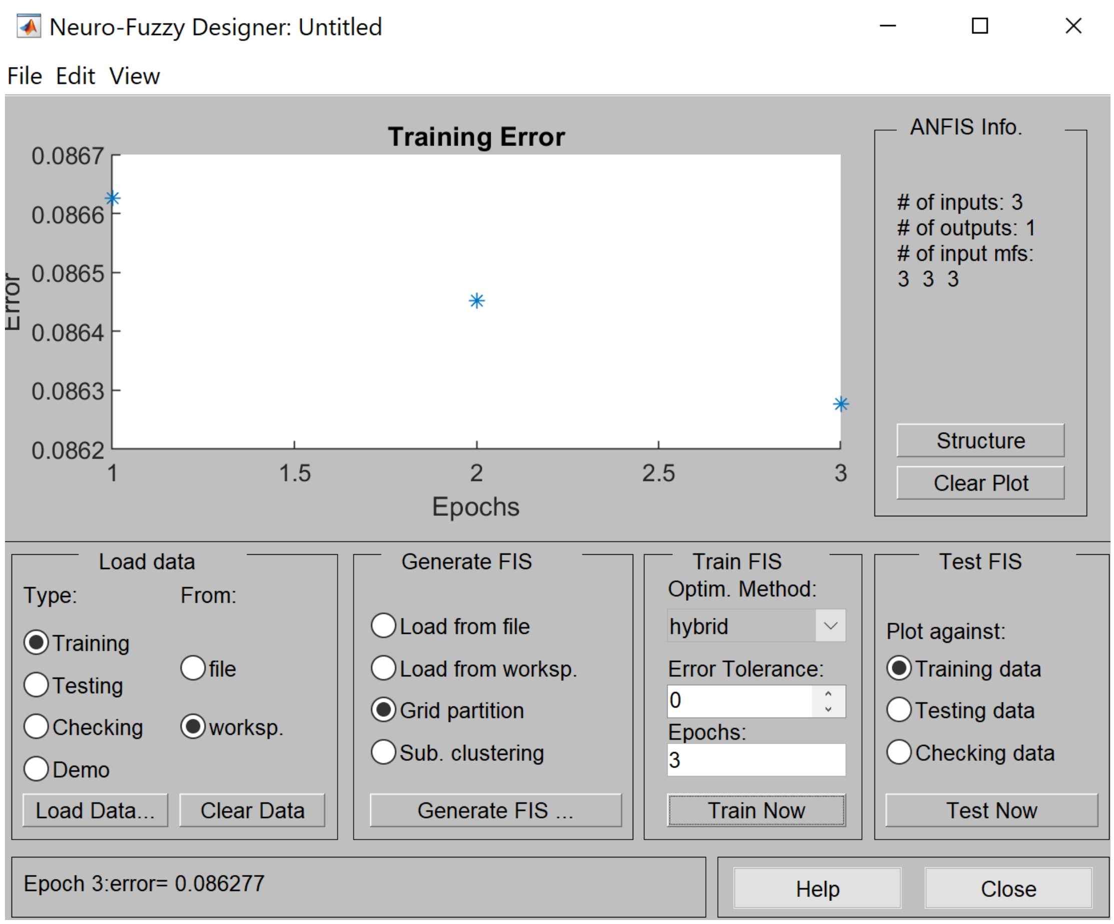
# of outputs: 1
# of input mfs:
5 5 5
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Epoch 3:error= 0.046704
Help
Close

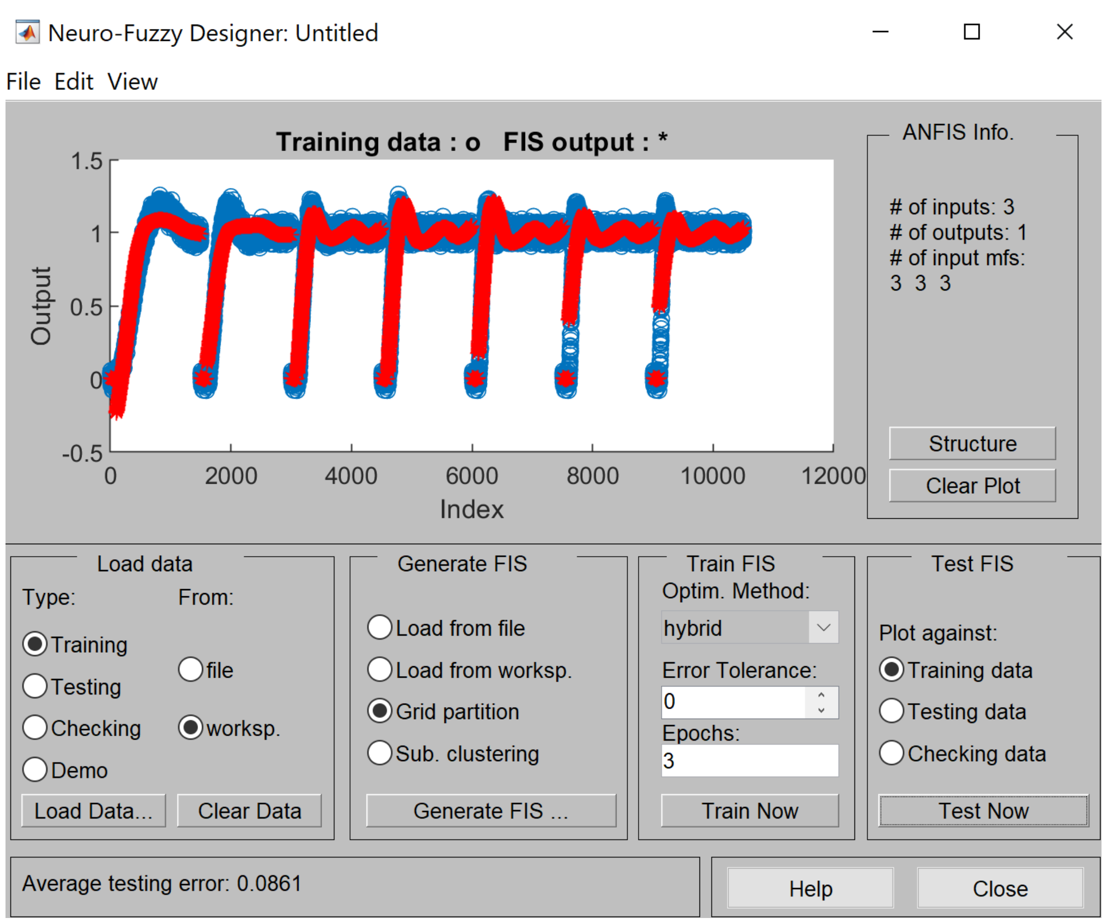


Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
Output
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
5 5 5
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.046704
Help
Close

GBELL 333

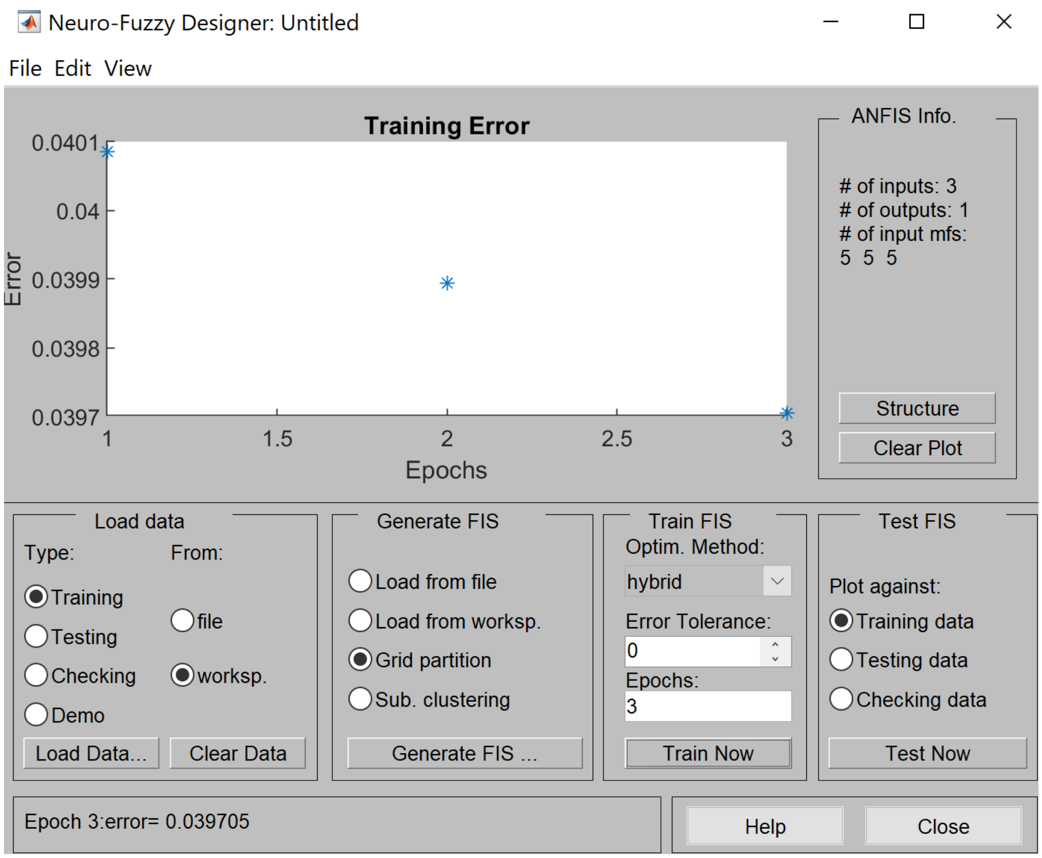


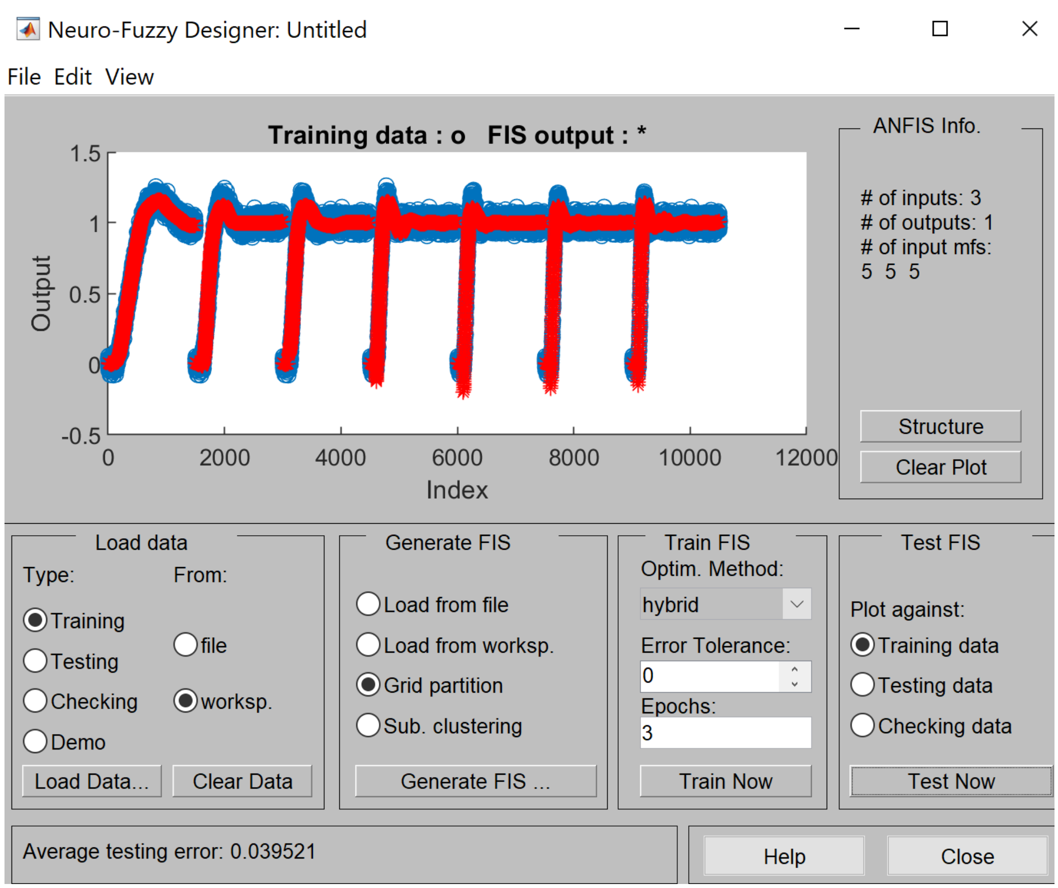
Neuro-Fuzzy Designer: Untitled
File Edit View
Training Error
0.0867
0.0866
0.0865
0.0864
0.0863
0.0862
Error
1
1.5
2
2.5
3
Epochs
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Epoch 3:error= 0.086277
Help
Close



Neuro-Fuzzy Designer: Untitled
File Edit View
Training data : o FIS output : *
1.5
1
0.5
0
-0.5
Output
0
2000
4000
6000
8000
10000
12000
Index
ANFIS Info.
# of inputs: 3
# of outputs: 1
# of input mfs:
3 3 3
Structure
Clear Plot
Load data
Type:
From:
Training
Testing
Checking
Demo
file
worksp.
Load Data...
Clear Data
Generate FIS
Load from file
Load from worksp.
Grid partition
Sub. clustering
Generate FIS ...
Train FIS
Optim. Method:
hybrid
Error Tolerance:
0
Epochs:
3
Train Now
Test FIS
Plot against:
Training data
Testing data
Checking data
Test Now
Average testing error: 0.0861
Help
Close

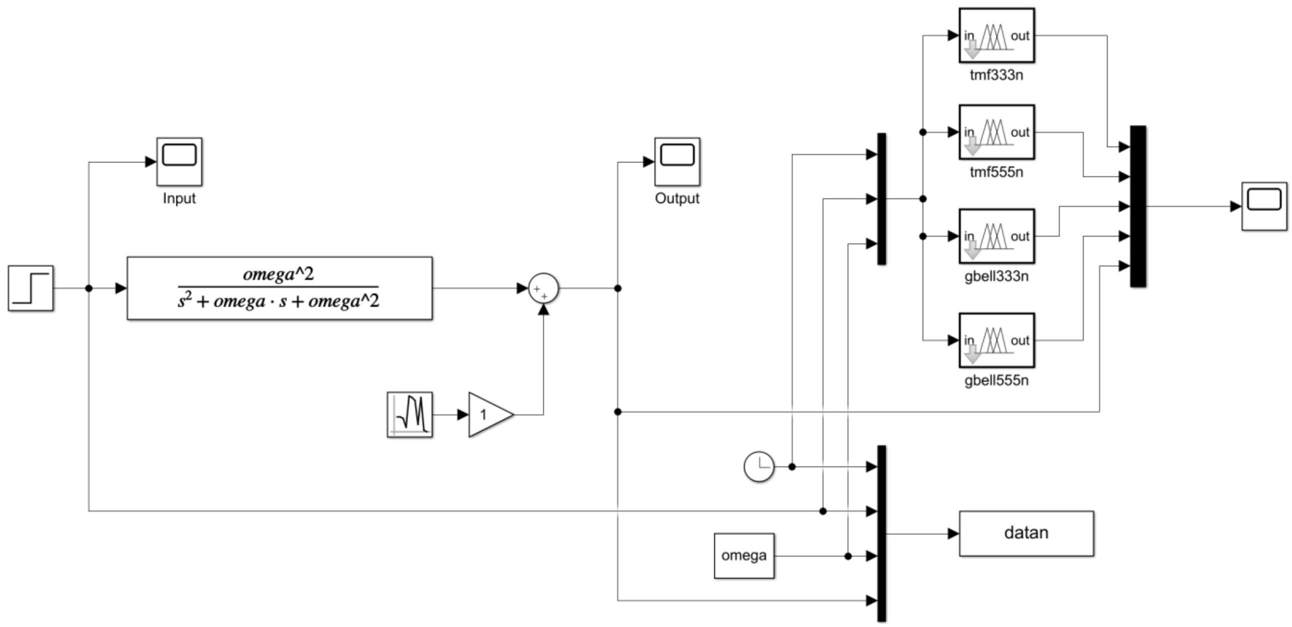
GBELL 555





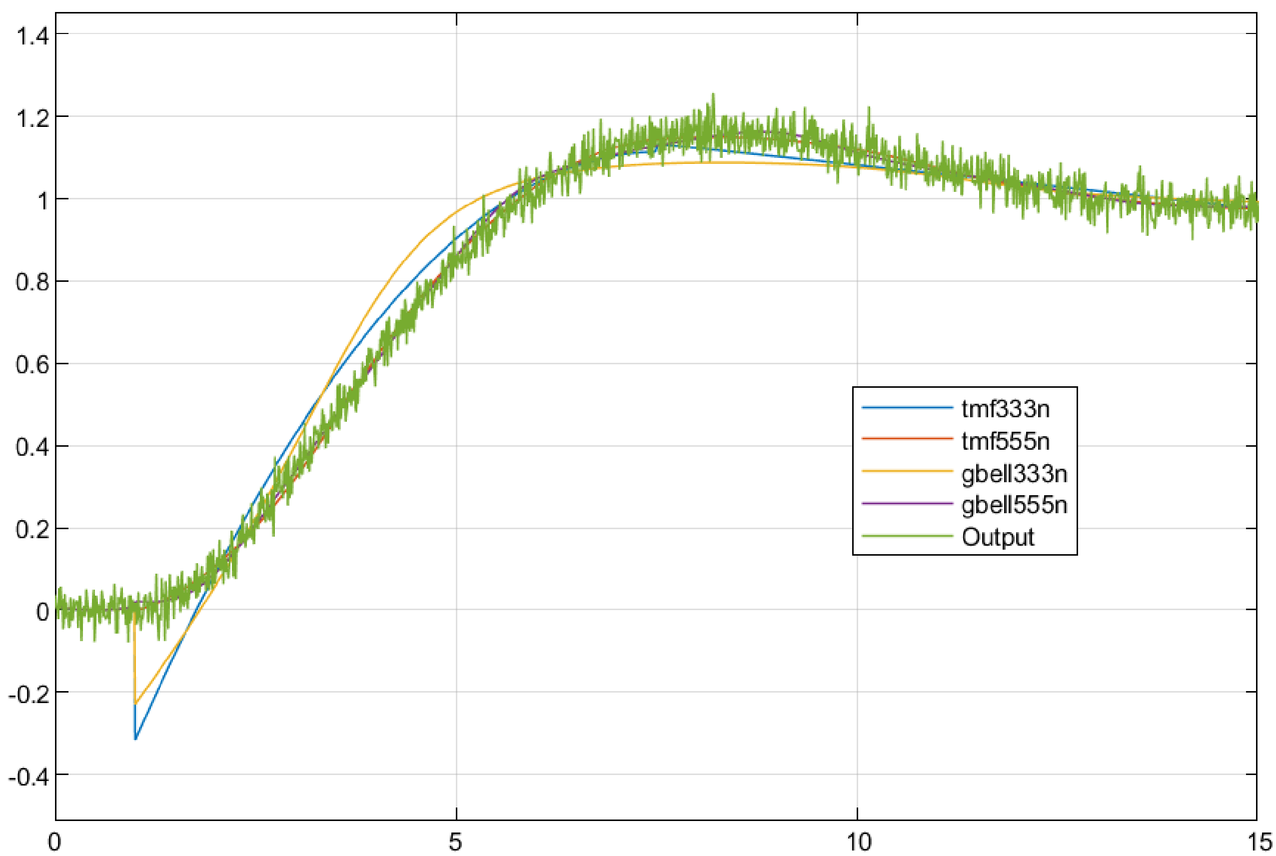
Conclusion – Using average testing error values, **GBELL 555 produces the best fit to training data followed by TMF 555**

b. Comparison

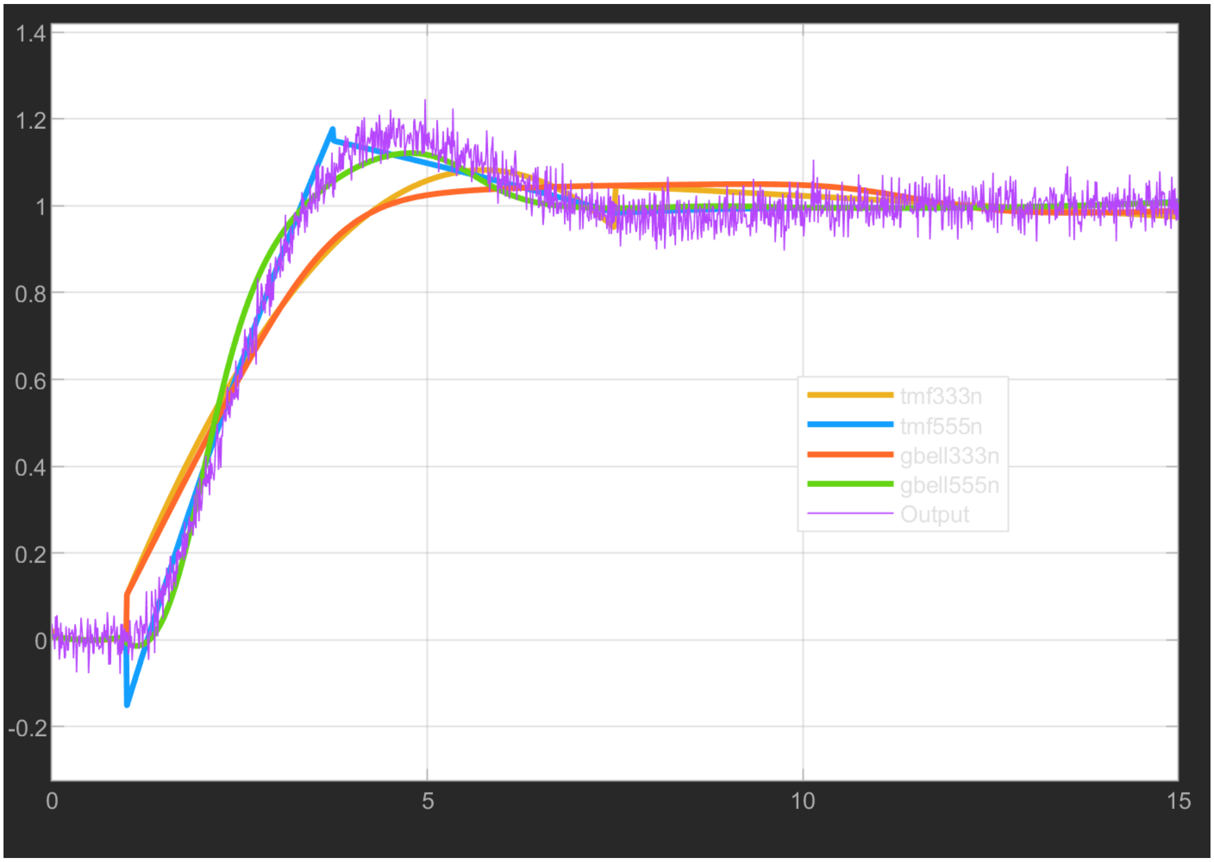


Simulink Diagram for comparing the outputs

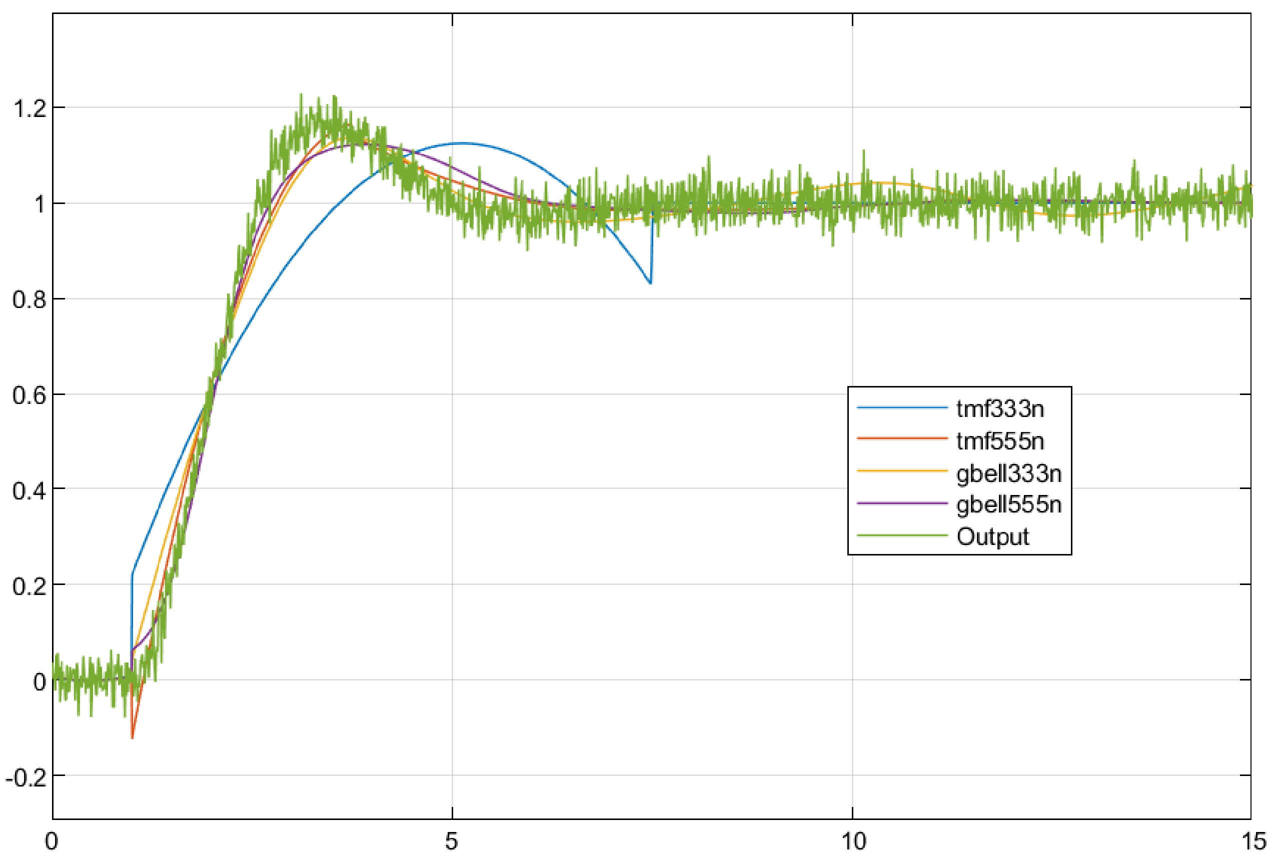
Omega = 0.5



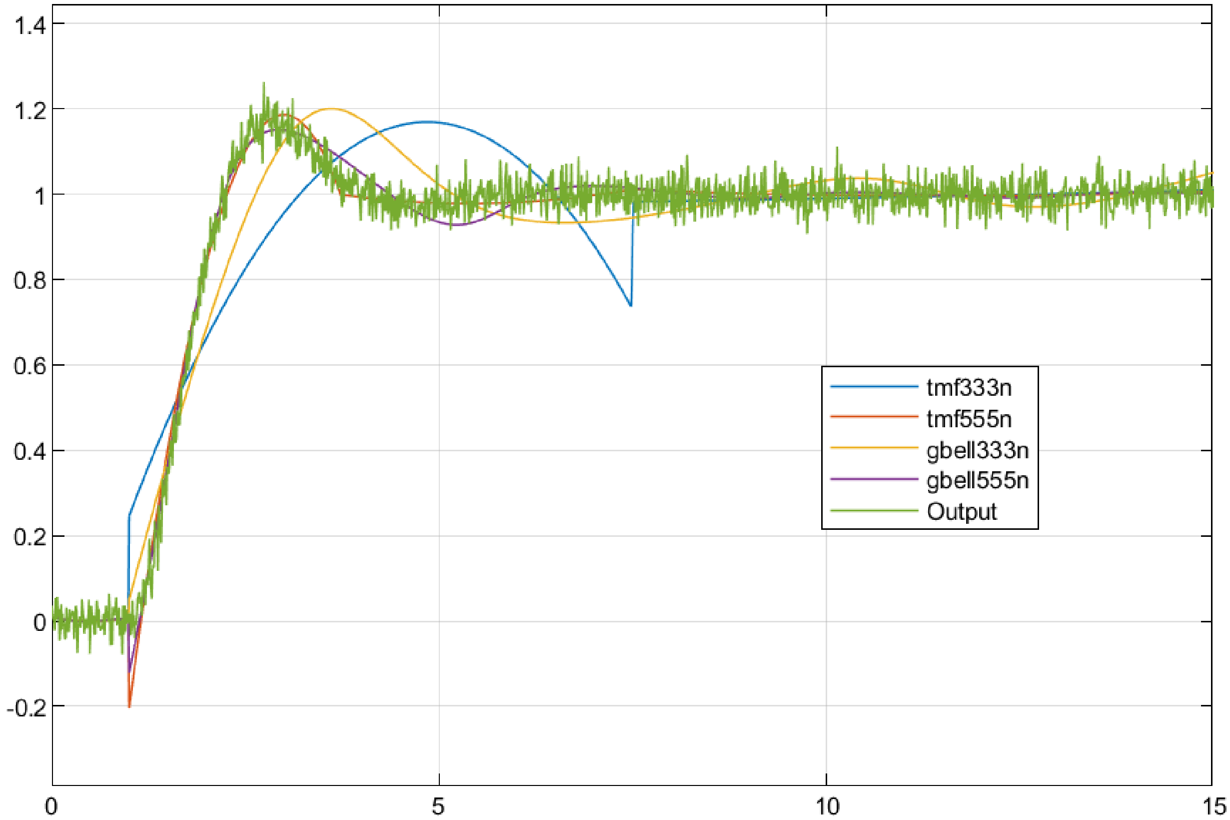
Omega = 1



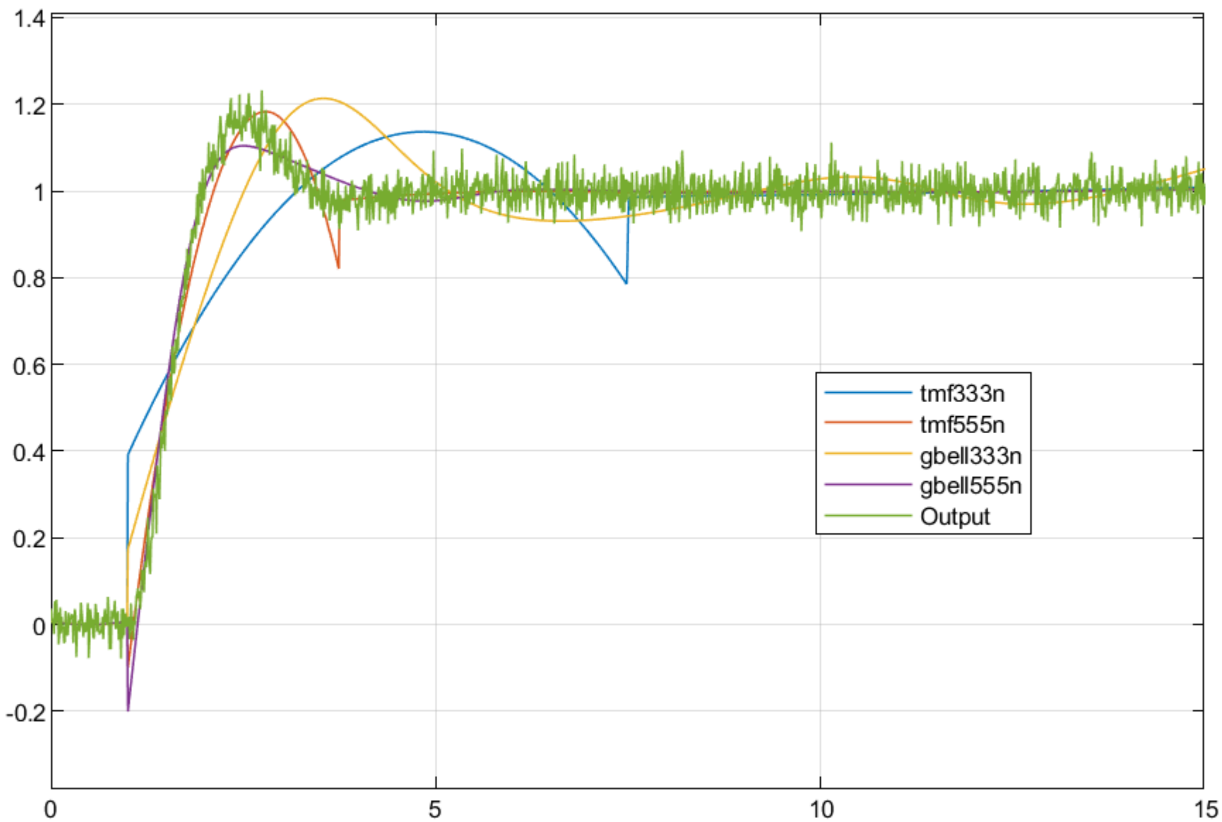
Omega = 1.5



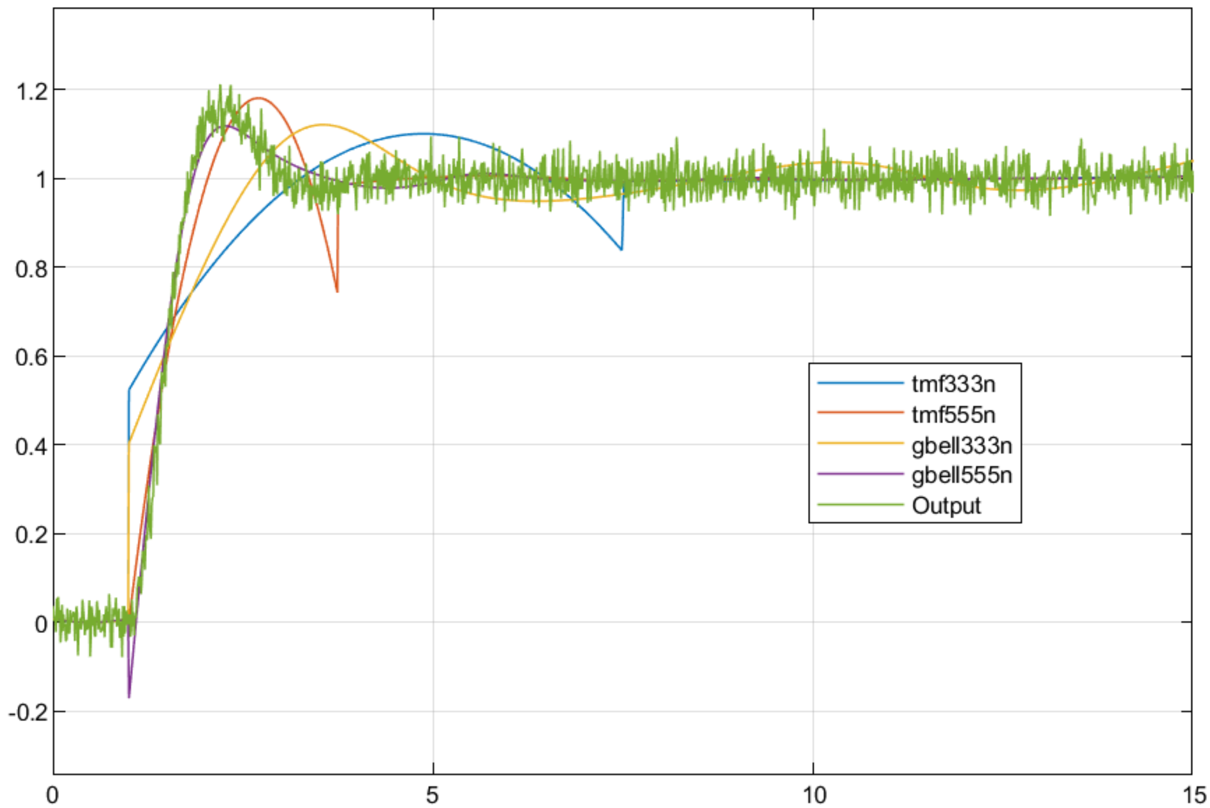
Omega = 2.0



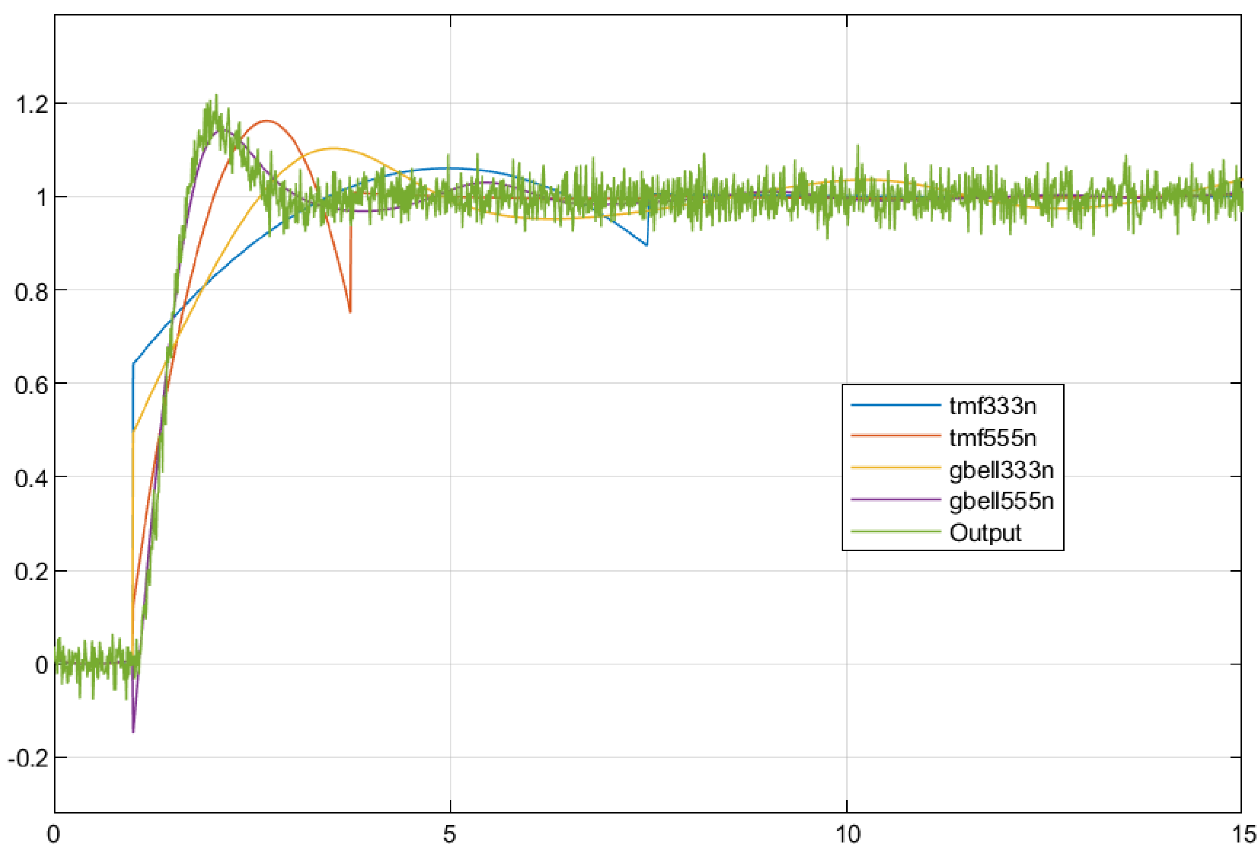
Omega = 2.5



Omega = 3.0

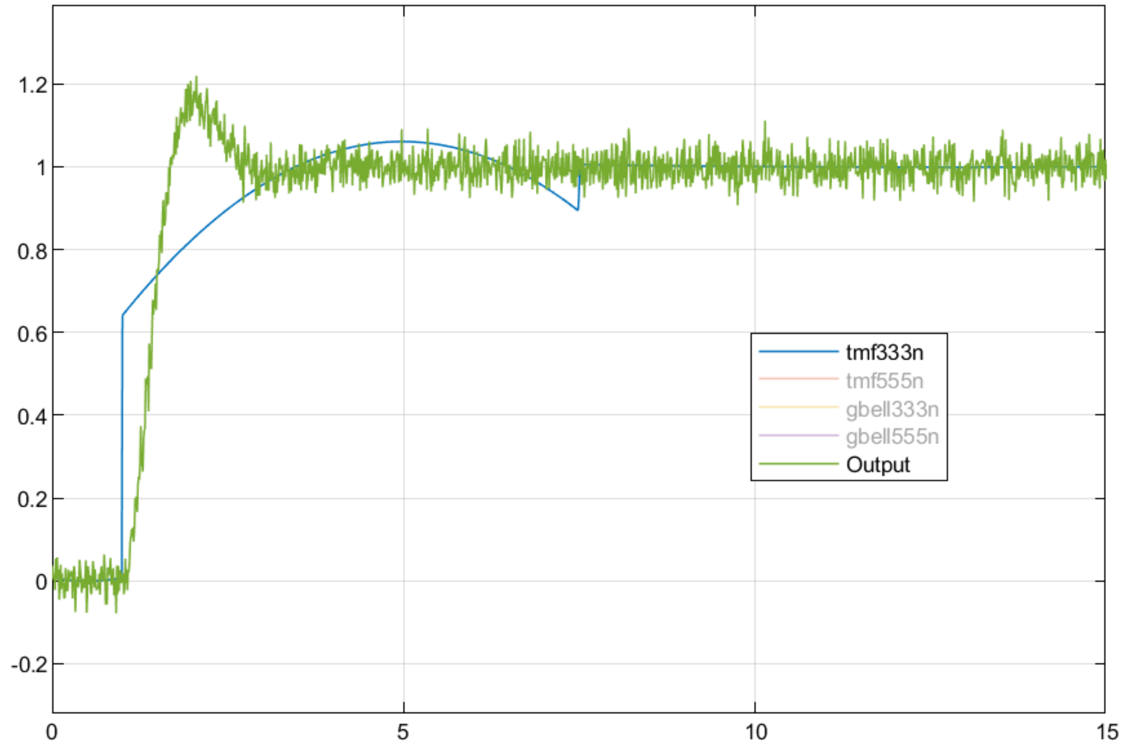


Omega = 3.5

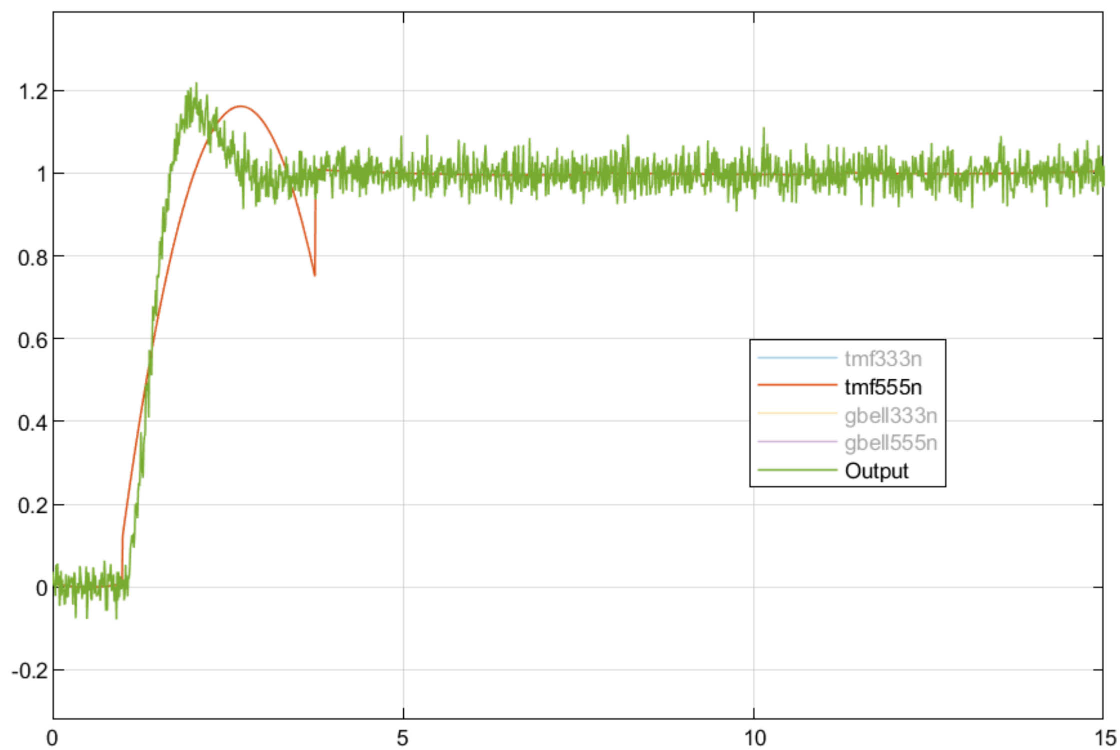


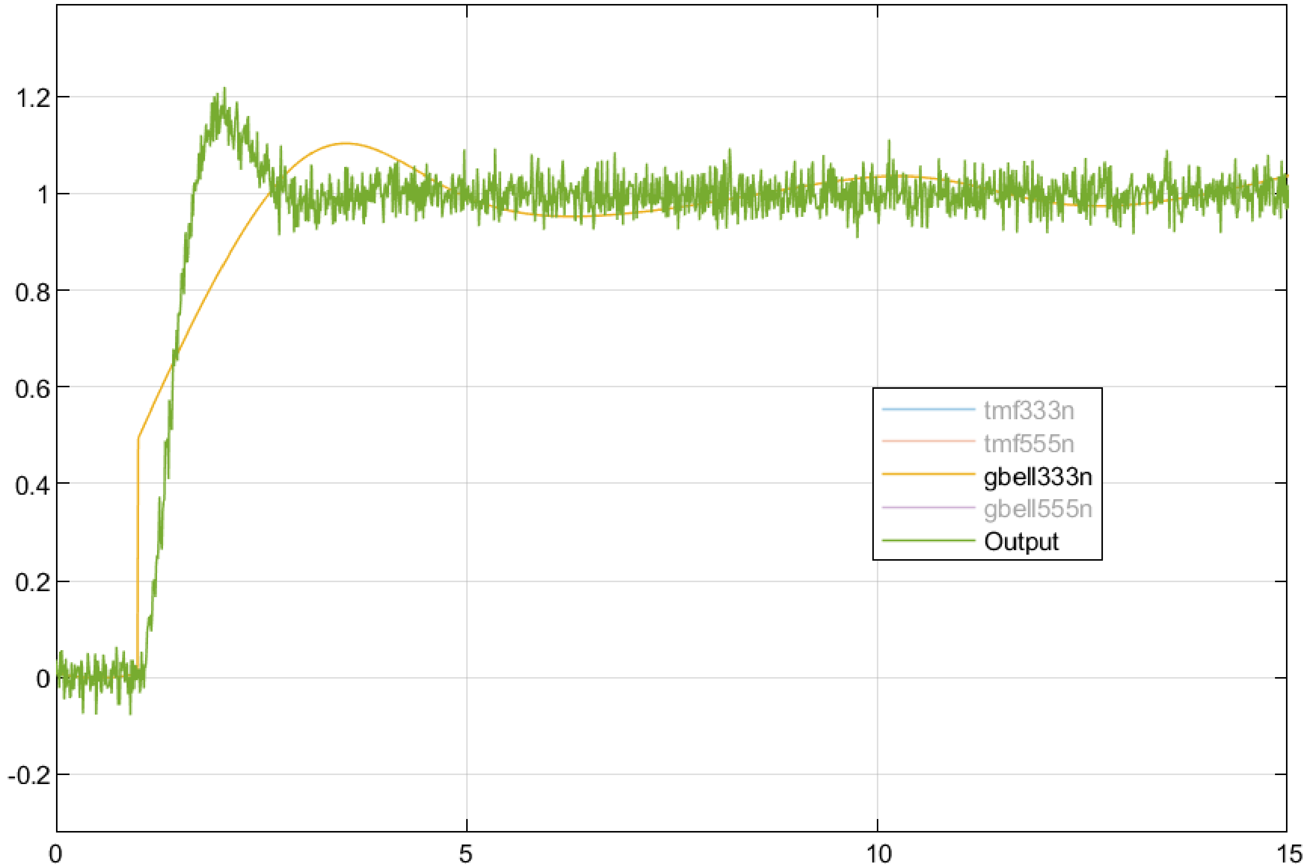
One by One Comparison

It is observed that the maximum divergence from output values occurs at omega = 3.5. Thus, instead of comparing 28 different graphs to determine the best fit, only 4 graphs at omega = 3.5 are compared. This is also done for part b in the previous problem.

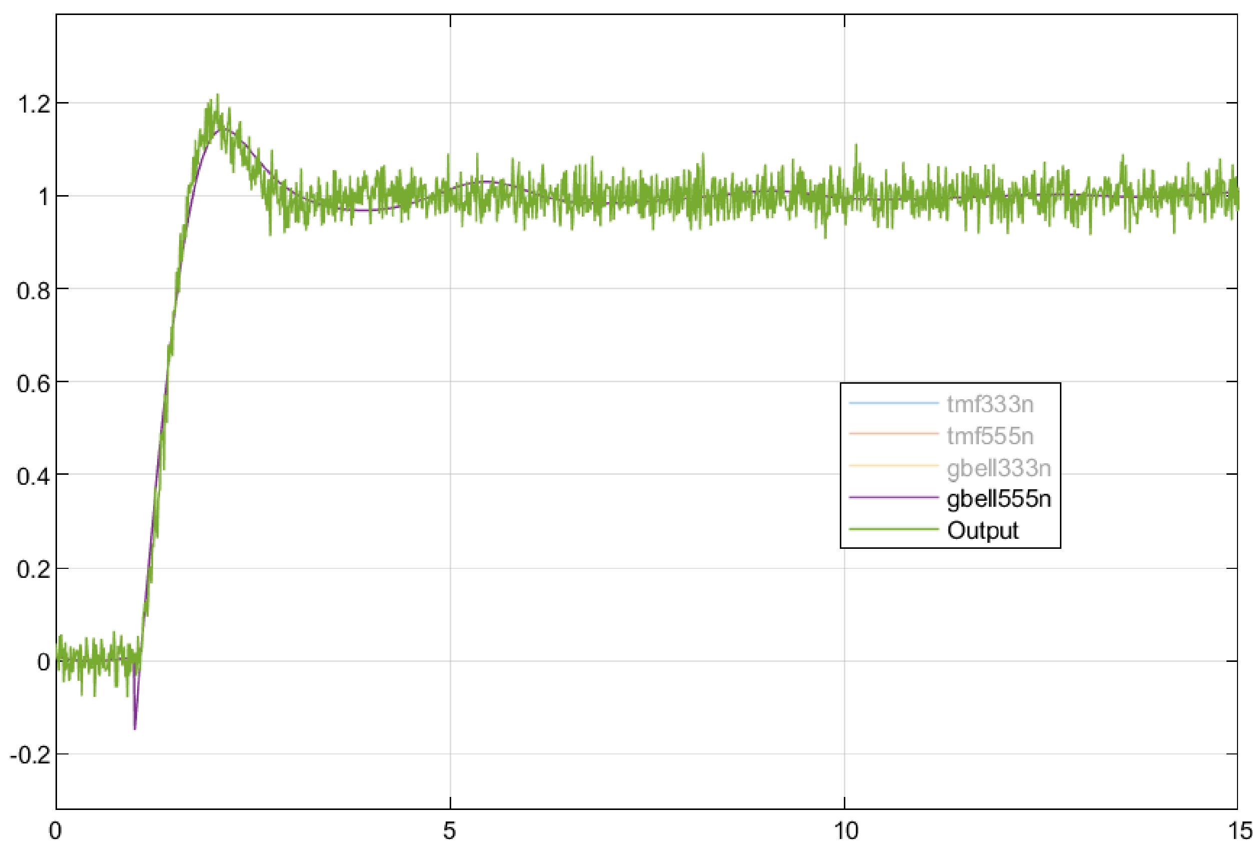


Plot above, TMF 333 **Plot below – TMF 555**





Plot above – GBELL 333 **Plot below – GBELL 555**



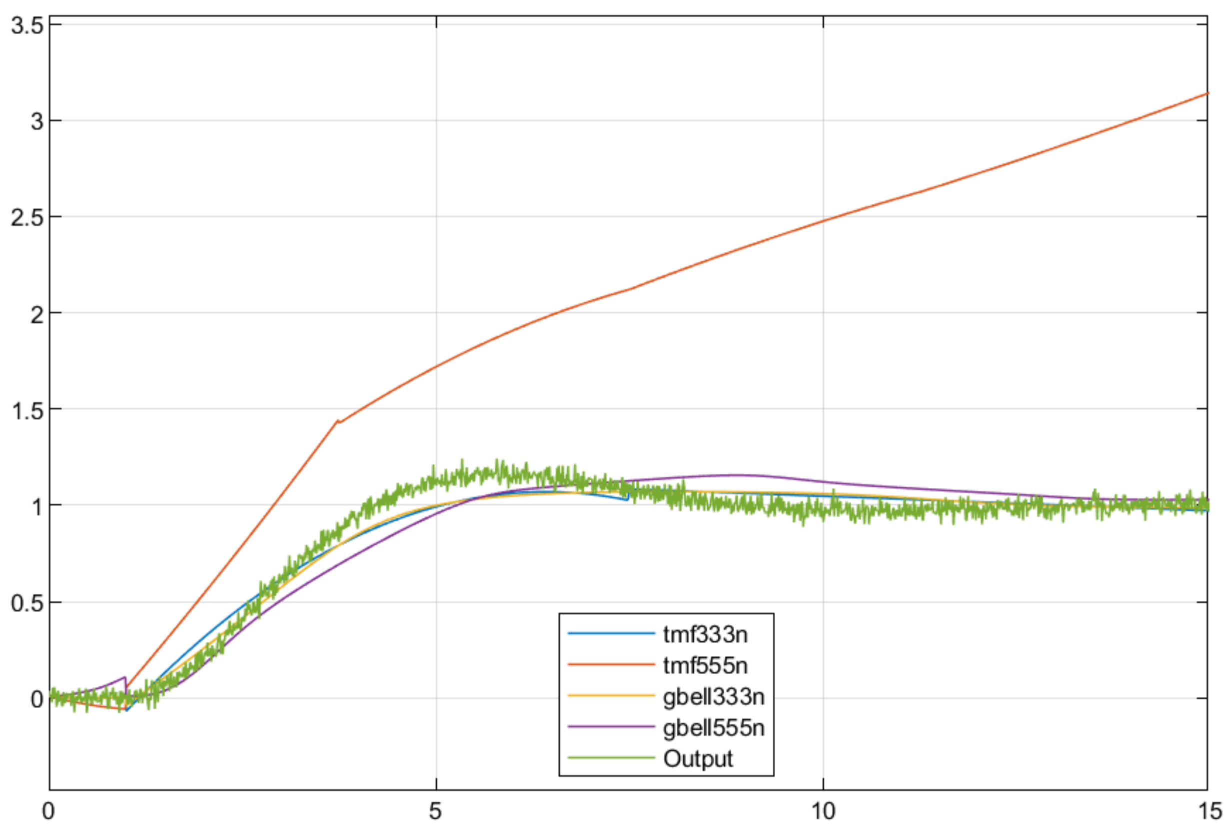
Conclusion – From the above plots it is observed that all fuzzy systems provide less accurate outputs in the presence of noise. As expected from the fit data in part a, **the GBELL 555 produces the best fits at all omega values**. Although the fits are close at omega =0.5, they quickly diverge from the actual output as omega values are increases. **It is seen that FIS with more membership functions is better at estimating the output.**

The **triangular membership function has sharp edges** due to the straight triangular edges used for defining the output. This is not seen in gaussian membership functions as they have smooth curves. As it can be seen, for omega=0.5 almost all fuzzy identification systems work well with the best ones being TMF555 and GBELL555 due to the higher number of functions.

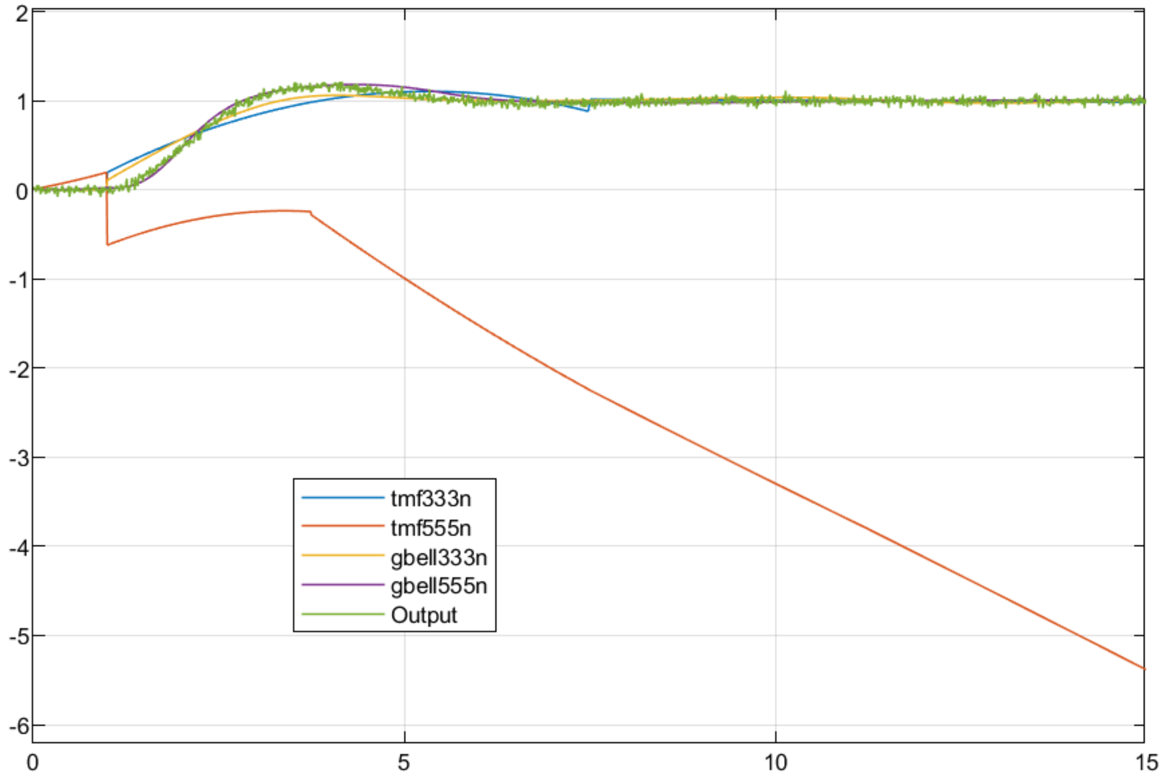
Other than that, it is also noted that while the GBELL 555 FIS produces the best fit, it gives a sharp dip before the input at higher omega values.

c. Interpolating omega values

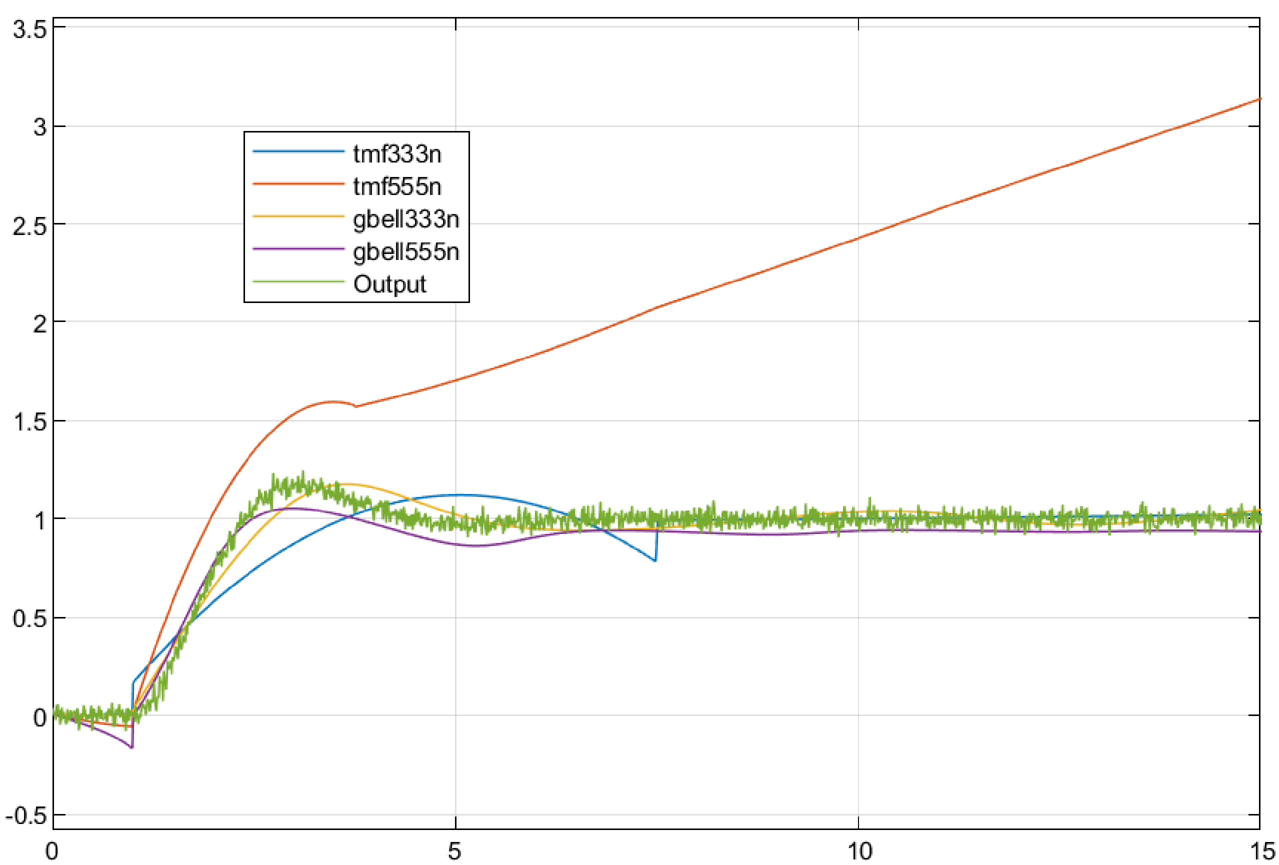
Omega = 0.75



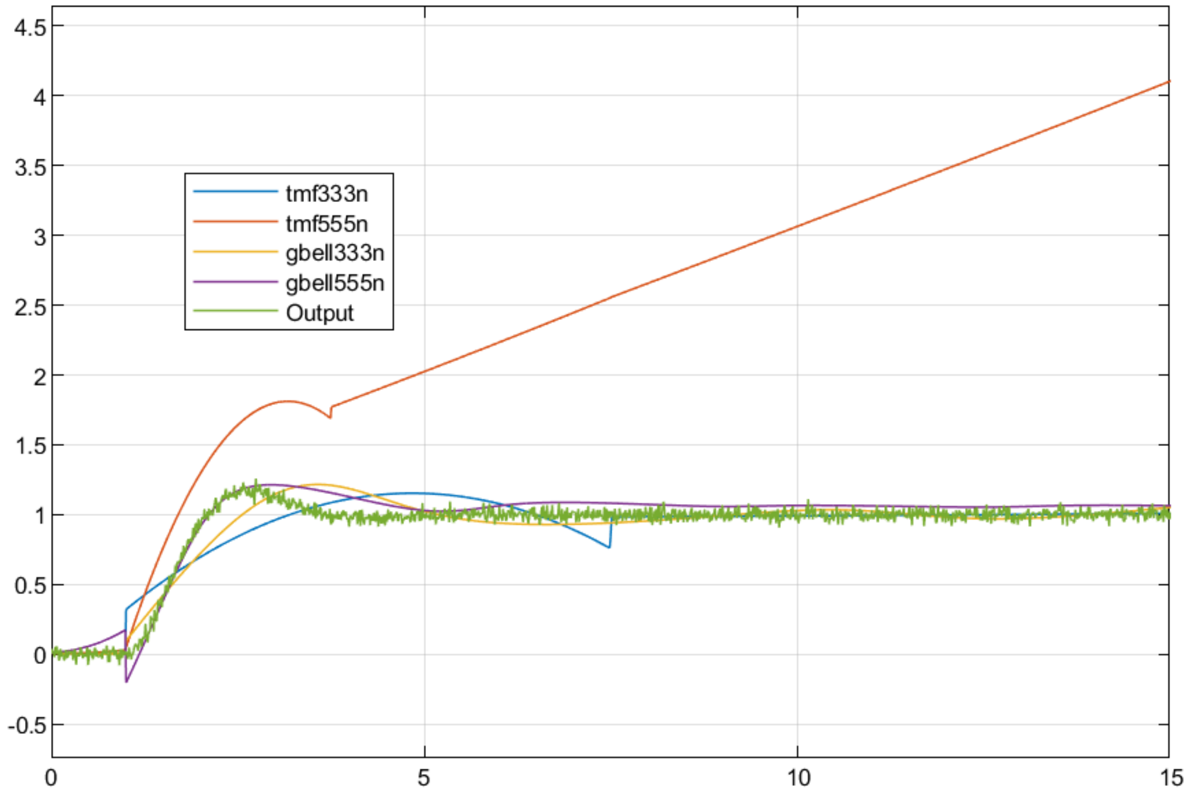
Omega = 1.25



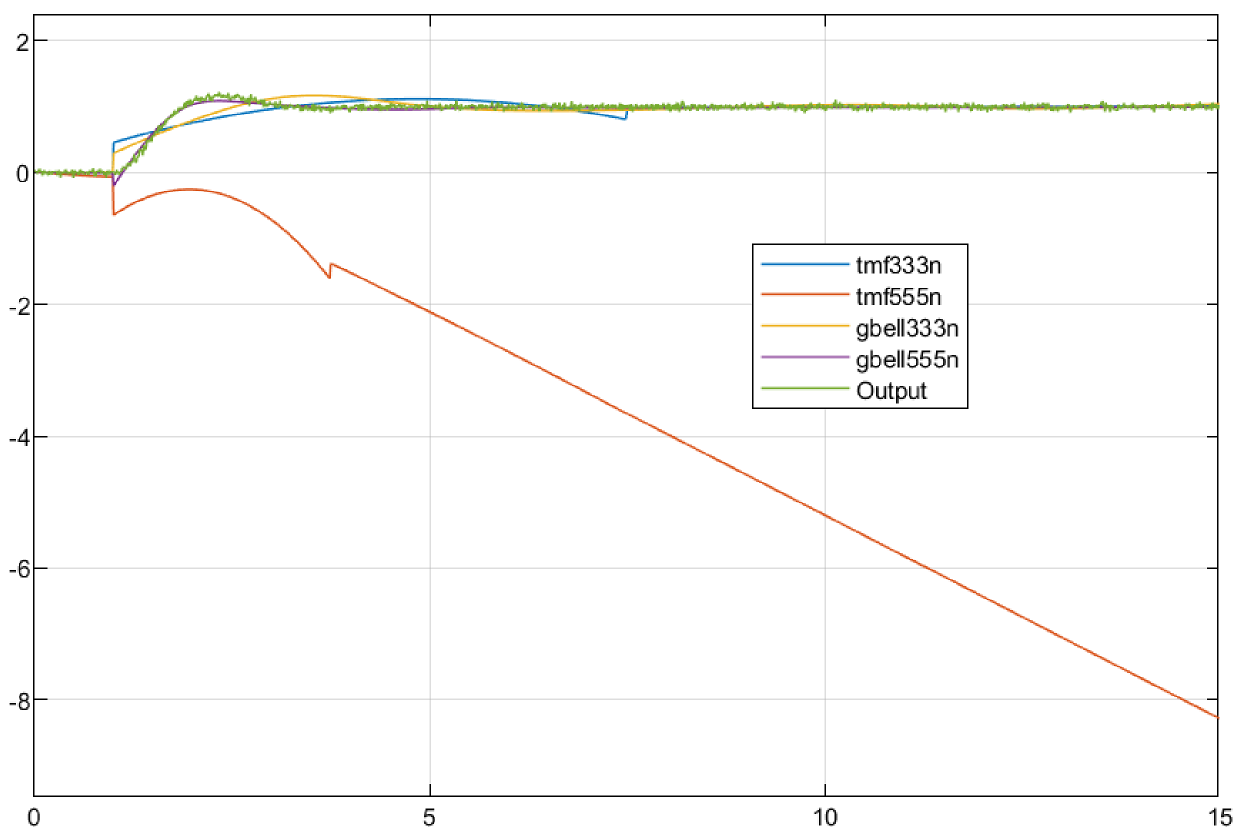
Omega = 1.75



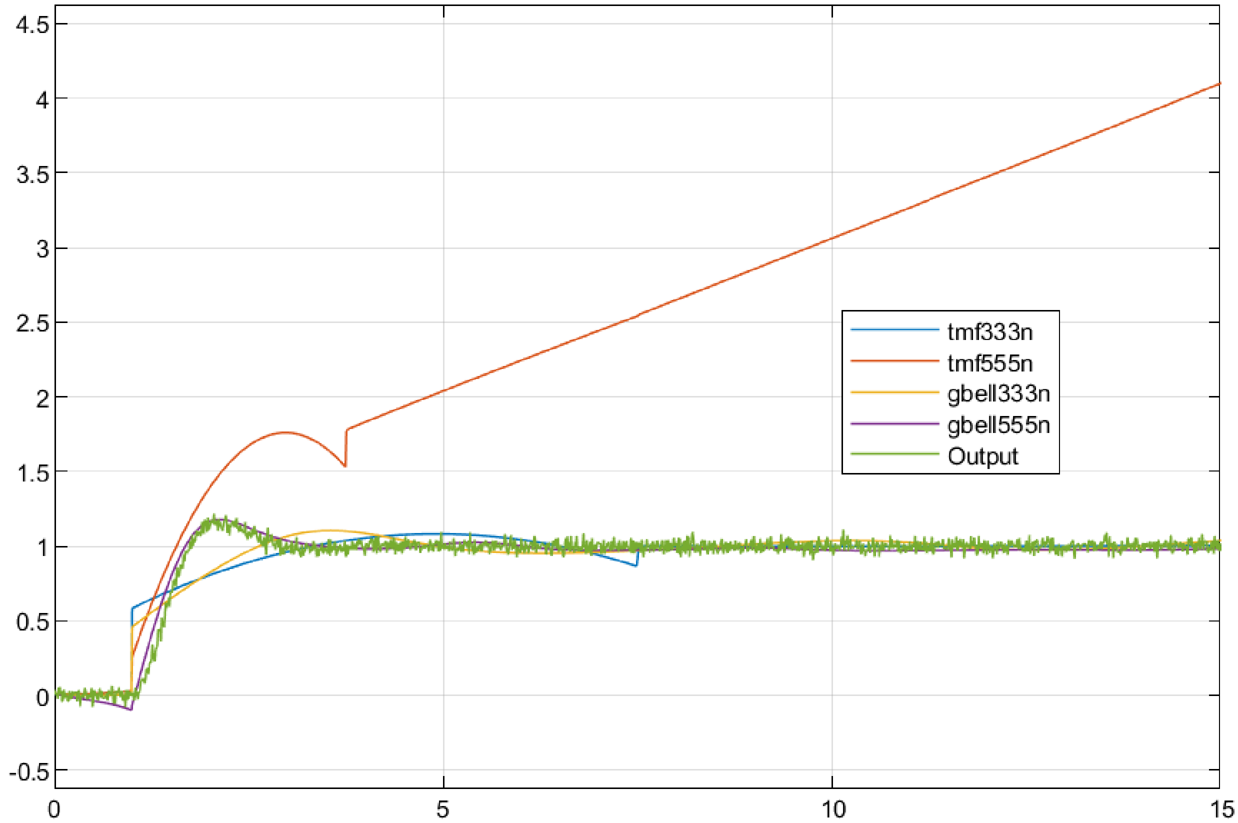
Omega = 2.25



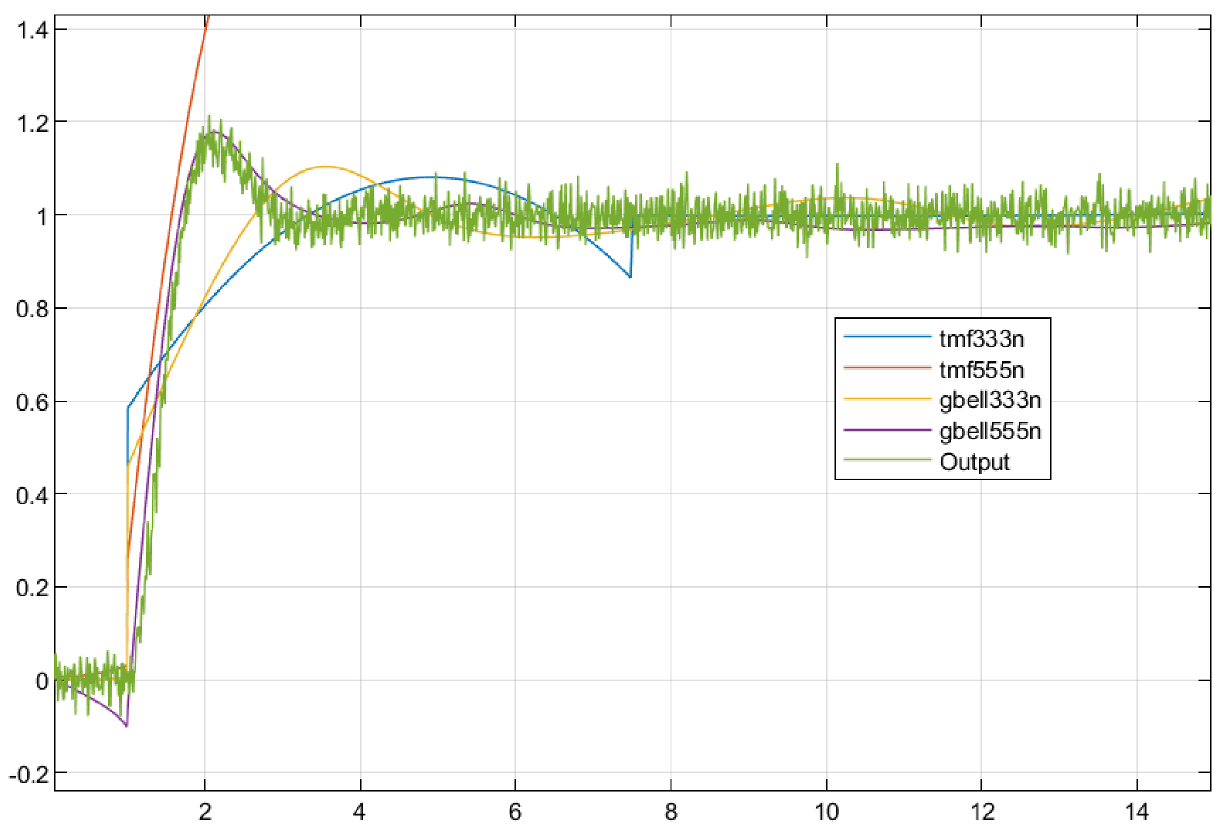
Omega = 2.75



Omega = 3.25



Improving resolution by ignoring tmf555 data,



Conclusion – None of the fuzzy systems are good at interpolation except for the Gaussian Bell with 5 membership functions. The TMF 555 behaves in a strange manner probably due to overfitting data through more information using 5 triangular membership functions which results in incorrect outputs when interpolating.

The best fit for all interpolated values in the transition state is provided by GBELL 555 FIS but it often leaves a steady state error which may not be desirable. **In steady state, best fit is provided by TMF 333 FIS.** This may be because the function is simple and dependent on linear triangles so it only fits some data which is easy to interpolate.

d. The **GBELL 555 FIS** is a good alternative if the steady state error can be removed by a different method when omega values are interpolated. If the system stabilizes quickly and its performance during transition state can be neglected then **TMF 333** is the best choice. Overall, depending on the acceptable errors and desired output, both GBELL 555 and TMF 333 can work for this system in the presence of noise. **Overall, the presence of noise reduces the ability of the fuzzy system to model this data.** For comparison, the average testing error value for the best FIS i.e., GBELL 555 increases by 70 percent when fitting data in the presence of noise.