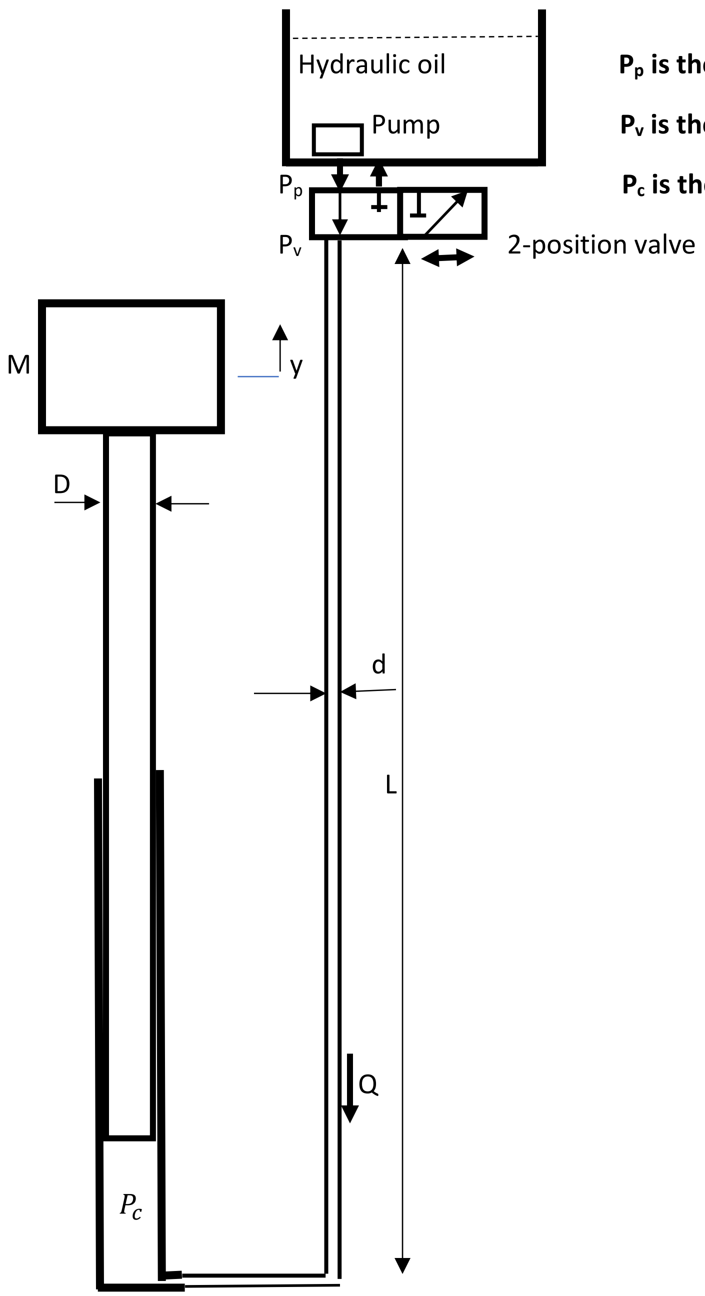
This problem concerns the operation of a hydraulic elevator system. A schematic of the system is shown below. The system consists of a single acting hydraulic cylinder with diameter D. The fluid reservoir contains a submersible pump and is located in a room above the top floor. A signal to raise the elevator activates the pump and shifts a two-position valve with area A_v so that fluid is pumped with flow rate Q out of the reservoir through the valve into a hydraulic line of length L and diameter d down to the bottom of the cylinder into a chamber at the bottom of the cylinder. The elevator has mass M which includes the mass of the piston and the passengers. M=1500 kg, Cd=0.6, D=0.15 m, L=9 m, g=9.81 m/s², $\mu = 0.02\ Ns/m^2, \rho = 1000\ kg/m^3$ and $d = 0.015\ m$.

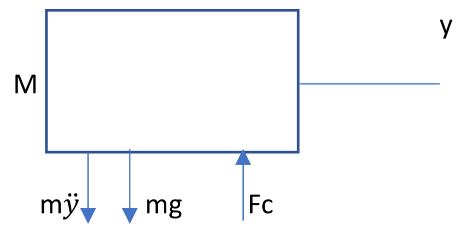


P_p is the exit pressure from the pump.

P_v is the exit pressure from the valve.

P_c is the pressure below the piston in the cylinder.

(a) Using the free body diagram for the mass, write the differential equation for the displacement y of the mass.



$$Fc = Pc * \pi * \frac{D^2}{4}$$

$$m\ddot{y} + mg - Fc = 0$$

$$\ddot{y} = \frac{Fc}{m} - g$$

(b) Using the fluid capacitance for the chamber at the bottom of the cylinder, write the differential equation for the pressure P_c. Use $\beta_e = 4x10^8\ N/m^2$ and include the volume of the line in the equation for the volume of the chamber, i.e. $V_c = \frac{\pi D^2}{4} y + \frac{\pi d^2}{4} L$

$$\dot{Pc} = \frac{\beta_e \left(Q - \frac{\pi D^2}{4} \dot{y}\right)}{V_c}$$

(c) Since the flow rate Q suddenly increases from zero, show that the differential pressure associated with accelerating the fluid in the line is $(P_v - P_c)_i = I\dot{Q}$ $\quad I = ?$ Use 1000 Kg/m^3 for the density ρ of the fluid in the line

$$I = \frac{\rho L}{\frac{\pi d^2}{4}}$$

(d) The differential pressure associated with flow resistance in the line is conditional depending on the Reynolds number, $R_n = \frac{4\rho Q}{\pi \mu d}$, i.e.

$$(P_v - P_c)_r = RQ \qquad R_n \leq 1187.6 \quad laminar\ flow$$

or

$$(P_v - P_c)_r = bQ|Q|^{0.75} \qquad R_n > 1187.6 \quad turbulent\ flow$$

R = ? b = ?

$$R = \frac{128 \mu L}{\pi D^4}; \qquad b = \frac{0.2414 \rho^{0.75} \mu^{0.25} L}{D^{4.75}}$$

(e) Considering that the differential pressure in the line due to the height is $(P_v - P_c)_h = -\rho g L$, write the conditional equation for the total pressure drop in the line

$$P_v - P_c = (P_v - P_c)_h + (P_v - P_c)_r + (P_v - P_c)_i = ?$$

$if\ R > 1187.6,$

$$P_v - P_c = -\rho g L + bQ|Q|^{0.75} + I\dot{Q}$$

Else

$$P_v - P_c = -\rho g L + RQ + I\dot{Q}$$

(f) In addition, the pressure differential across the valve is $P_p - P_v = \frac{\rho}{2C_d^2 A_v^2}|Q|Q$. Explain why the orifice equation for the valve has been formulated solving for the pressure difference instead of solving for the flow Q. Note, when the valve is shifted to the position shown in the schematic above, the area A_v of the passage through the valve is equal to the area of the line, i.e. $A_v = \frac{\pi d^2}{4}$.

Q has a differential equation as shown above and is taken as a state variable. This makes Q known at all points, thus the orifice equation solves for pressure difference which is unknown.

(g) The pump equation is supplied by the manufacturer, i.e.

$$P_p = (Q_m - Q)\frac{P_m}{Q_m} \quad \text{where} \quad P_m = 4.25x10^7 \ N/m^2 \qquad Q_m = 0.0177 \ m^3/s$$

Including this equation, list your equations and the unknowns and confirm an ode45 simulation can be performed.

$$P_c = x1; \quad Q = x2; \quad y = x3; \quad \dot{y} = x4$$

$$P_v = P_p - \frac{\rho}{2C_d^2 A_v^2}|Q|Q$$

$$R_n = \frac{4\rho Q}{\pi u d}$$

$$\dot{x1} = \frac{\beta_e\left(x2 - \frac{\pi D^2}{4}x4\right)}{V_c}$$

$$if\ R_n > 1187.6,$$

$$\dot{x2} = \frac{(P_v - x1) + \rho g L - b * x2 * |x2|^{0.75}}{I}$$

$$else, \quad \dot{x2} = \frac{(P_v - x1) + \rho g L - R * x2}{I}$$

$$\dot{x3} = x4$$

$$\dot{x4} = \left(\frac{x1 * \frac{\pi D^2}{4}}{M}\right) - g$$

(h) Perform an ode45 simulation of your equations; run the simulation for only 1 s. Assume the following initial conditions:

$$P_c(0^-) = \frac{Mg}{A_c}, \quad Q(0^-) = 0, \quad y(0^-) = 0, \quad \dot{y}(0^-) = 0.$$

Plot $Q(t),\ y(t),\ \dot{y}(t),\ and\ \frac{P_c(t)}{Mg/A_c}$

m-file code:

```
function hw5parth
M = 1500; D=0.15; Cd=0.6; L=9; g=9.81; u=0.02; rho=1000;
d=0.015; be=4*(10^8); Pm=4.25*(10^7); Qm=0.0177;
Ac=pi*0.25*(D^2);
Pc0=(M*g)/Ac;
Ap=pi*0.25*(d^2);
[t,x]=ode45 (@eqns, [0 1], [Pc0 0 0 0]);
Pc=x(:,1); Q=x(:,2); y=x(:,3); dy=x(:,4);
figure(1)
plot(t,Q,'r','linewidth',2)
xlabel('time, s','fontsize',18)
ylabel('flow rate Q, m^3/s','fontsize',18)
figure(2)
plot(t,y,'r','linewidth',2)
```

```
xlabel('time, s','fontsize',18)
ylabel('elevator displacement y, m','fontsize',18)
figure(3)
plot(t,dy,'r','linewidth',2)
xlabel('time, s','fontsize',18)
ylabel('elevator velocity y'', m/s','fontsize',18)
figure(4)
plot(t,Pc/(M*g/Ac),'r','linewidth',2)
xlabel('time, s','fontsize',18)
ylabel('cylinder pressure P_c, N/m^2','fontsize',18)

    function dx=eqns(t,x)
        dx=zeros(4,1);
        Pc=x(1); Q=x(2); y=x(3); dy=x(4);
        Vc=(pi*0.25)*(((D^2)*y)+((d^2)*L));
        Pp=(Qm-Q)*(Pm/Qm);
        Rn=(4*rho*Q)/(pi*u*d);
        Pv=Pp-((rho*abs(Q)*Q)/(2*(Cd^2)*(Ap^2)));
        dx(1)=(be*(Q-Ac*dy))/Vc;
        if Rn>1187.6
            Res=((0.2414*(rho^0.75)*(u^0.25)*L)/(d^4.75))*Q*abs(Q)^0.75;
        else
            Res=((128*u*L)/(pi*(d^4)))*Q;
        end
        dx(2)=(Ap*((Pv-Pc)+(rho*g*L)-Res))/(rho*L);
        dx(3)=x(4);
        dx(4)=((Pc*Ac)/M)-g;
        Pc
    end
end
```

(i) We know that once the elevator starts ascending, the flow rate Q and velocity $\dot{y}$ will become constant (steady state conditions). Using the results of your simulation in (h), what is the steady state velocity? ~6.5 m/s
What is the steady state flow rate? 0.01136
What is the steady state Reynolds number? 4.8254e+01500*9.814
Is the steady state flow laminar or turbulent? Turbulent
What is the steady state pressure; does it equal the correct value of Mg/A_c? Yes.