Regularized Autoencoders

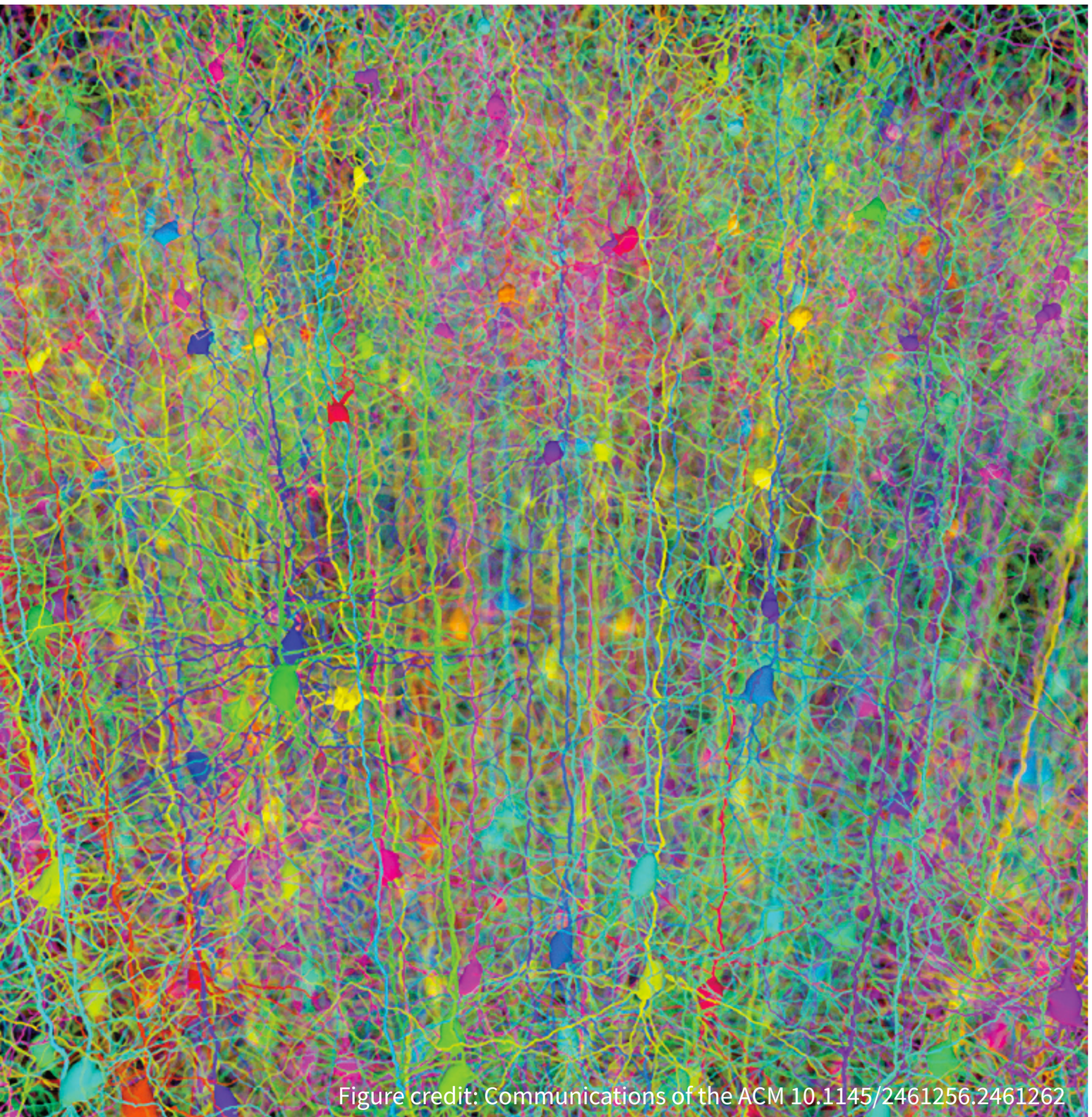

Figure credit: Communications of the ACM 10.1145/2461256.2461262

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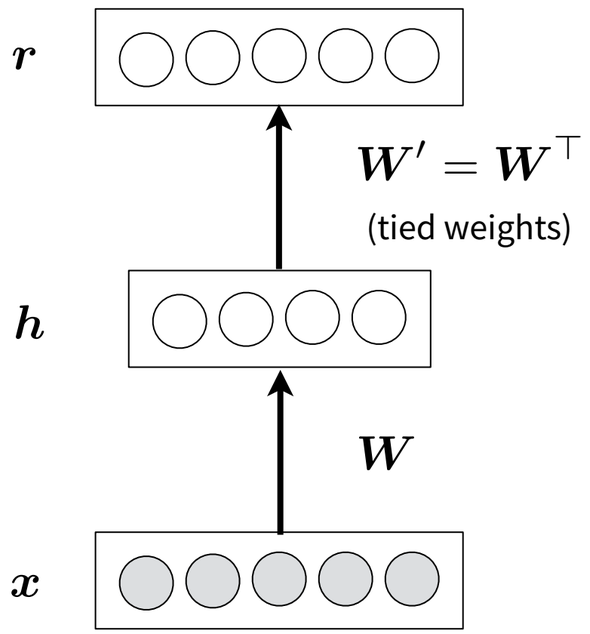
Undercomplete Hidden Layer

Hidden layer is undercomplete if smaller than the input layer

- Hidden layer "compresses" the input
- Will compress well only for the training distribution



Hidden units will be:

- good features for the training distribution

- but bad for other types of input

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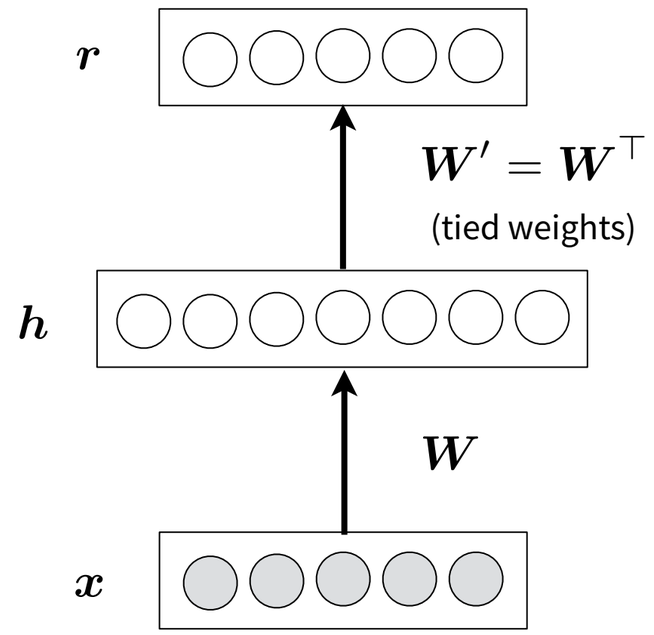
Overcomplete Hidden Layer

Hidden layer is undercomplete if greater than the input layer

- Hidden layer "expands" the input
- Each hidden unit could copy a different component

No guarantee that the hidden units will extract meaningful structure



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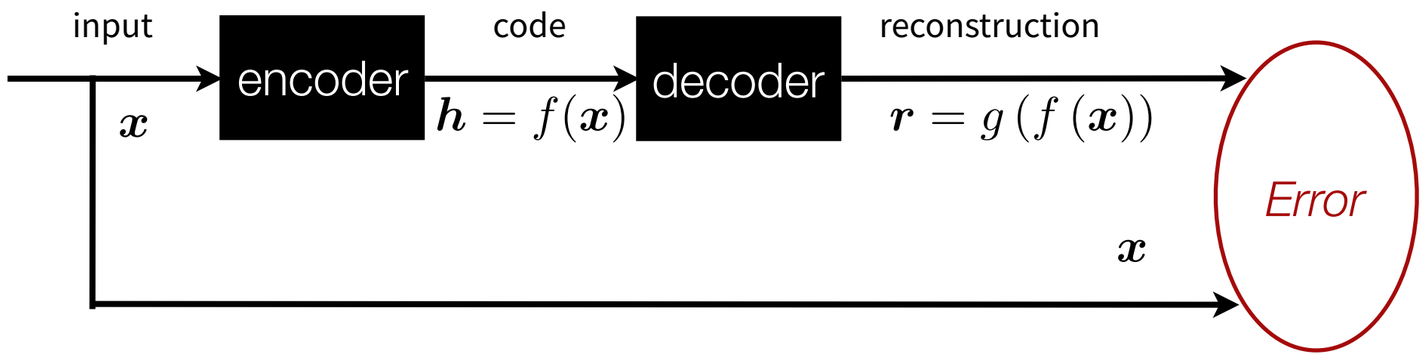
Regularized Auto-encoders



- Permit code to be higher-dimensional than the input

- Capture structure of the training distribution due to predictive opposition b/w reconstruction distribution and regularizer

- Regularizer tries to make enc/dec as simple as possible

Simple?

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- Reconstruct the input from the code and make the code compact
  (PCA, auto-encoder with bottleneck)

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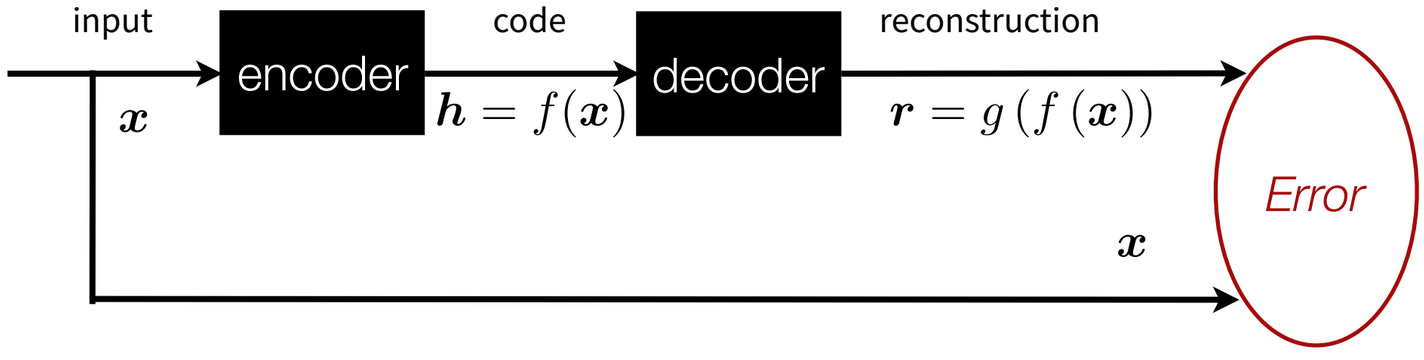
- Reconstruct the input from the code and make the code sparse
  (sparse auto-encoders)
- Add noise to the input or code and reconstruct the cleaned-up version
  (denoising auto-encoders)
- Reconstruct the input from the code and make the code insensitive to the input (contractive auto-encoders)

Sparse Auto-encoders



$$J = L(\boldsymbol{x}, \boldsymbol{r}) + \lambda \sum_j \mathrm{KL}\left(\rho || \hat{\rho}_j\right)$$

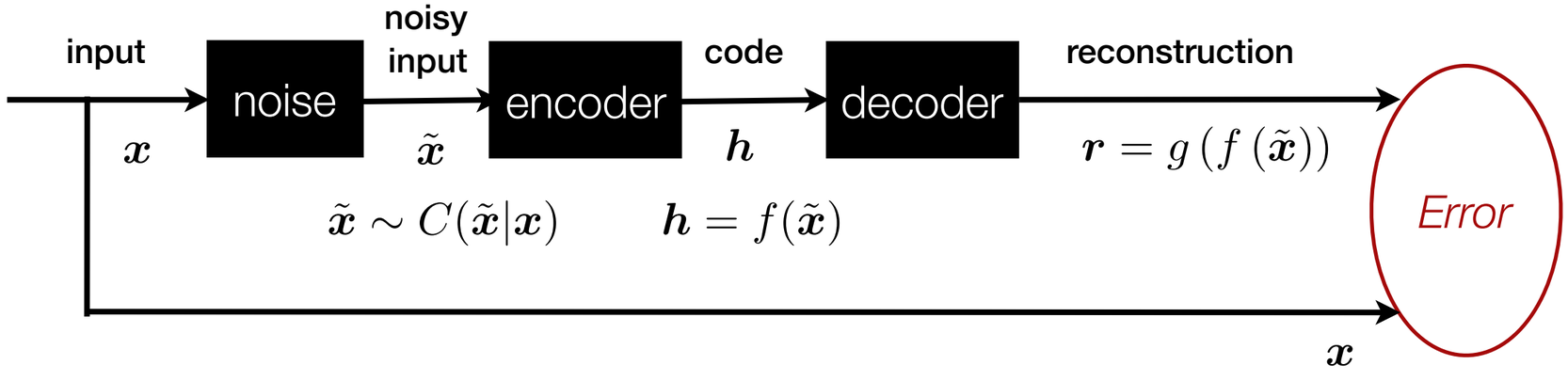
$$\hat{\rho}_j = \frac{1}{m} \sum_{i=1}^{m} h_j(\boldsymbol{x}^{(i)}) : \text{mean activation}$$

ρ : target activation (small)

- Apply a sparsity penalty to the hidden activations
- Also see Predictive Sparse Decomposition (Kavukcuoglu et al. 2008)

(Vincent et al. 2008)

Denoising Auto-encoders



$$J = L(\boldsymbol{x}, g\left(f\left(\tilde{\boldsymbol{x}}\right)\right))$$

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{\epsilon}$$

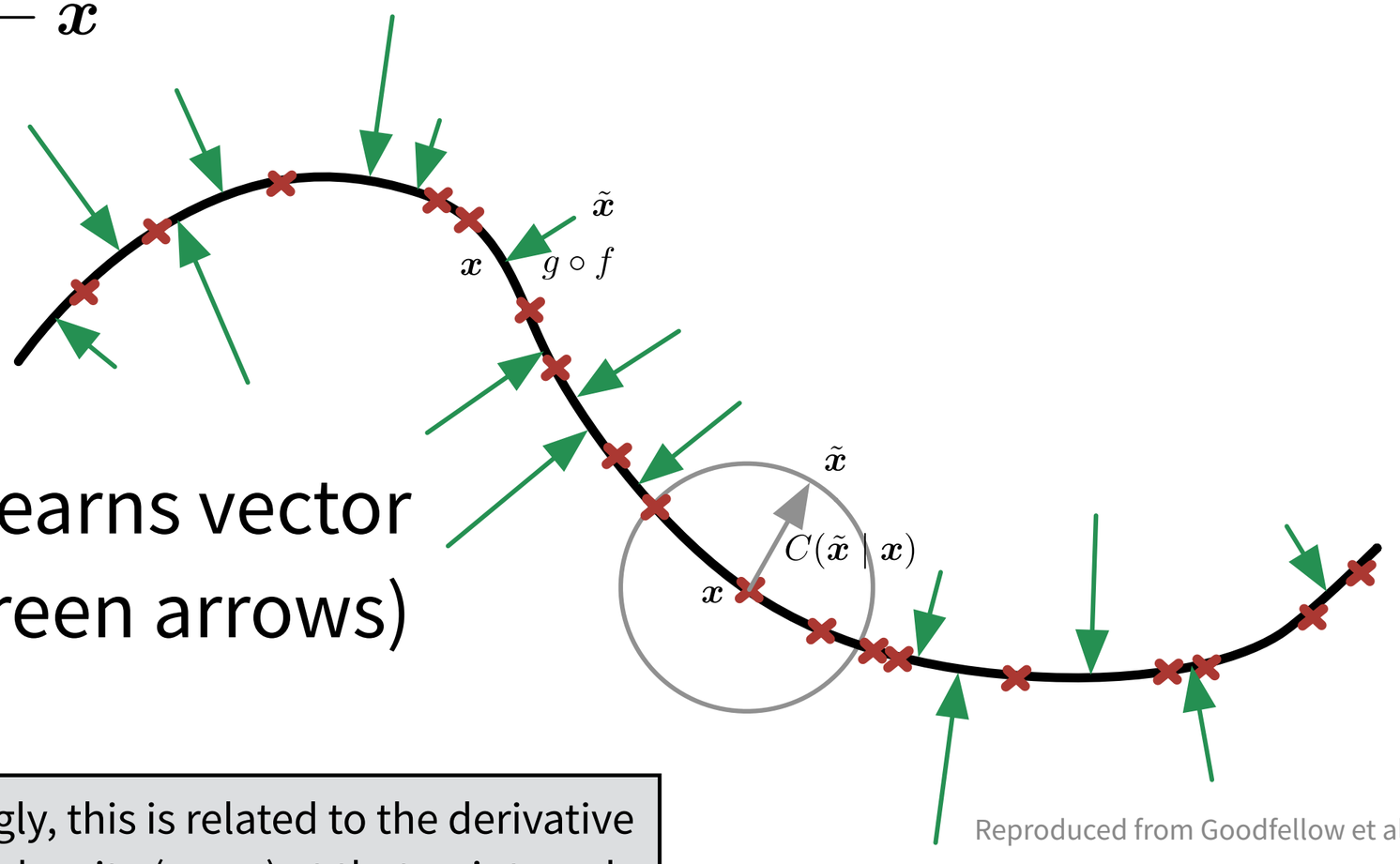
$$\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 I)$$

only one possible choice of noise model

- The code can be viewed as a lossy compression of the input
- Learning drives it to be a good compressor for training examples (and hopefully others as well) but not arbitrary inputs

What a Denoising Autoencoder Learns

- Trained to map a corrupted data point back to the original data point
- Grey arrow demonstrates how one data point is corrupted
- The vector $g(f(\tilde{\boldsymbol{x}})) - \tilde{\boldsymbol{x}}$ points toward the nearest point on the manifold
- This, autoencoder learns vector field $g(f(\boldsymbol{x})) - \boldsymbol{x}$ (green arrows)



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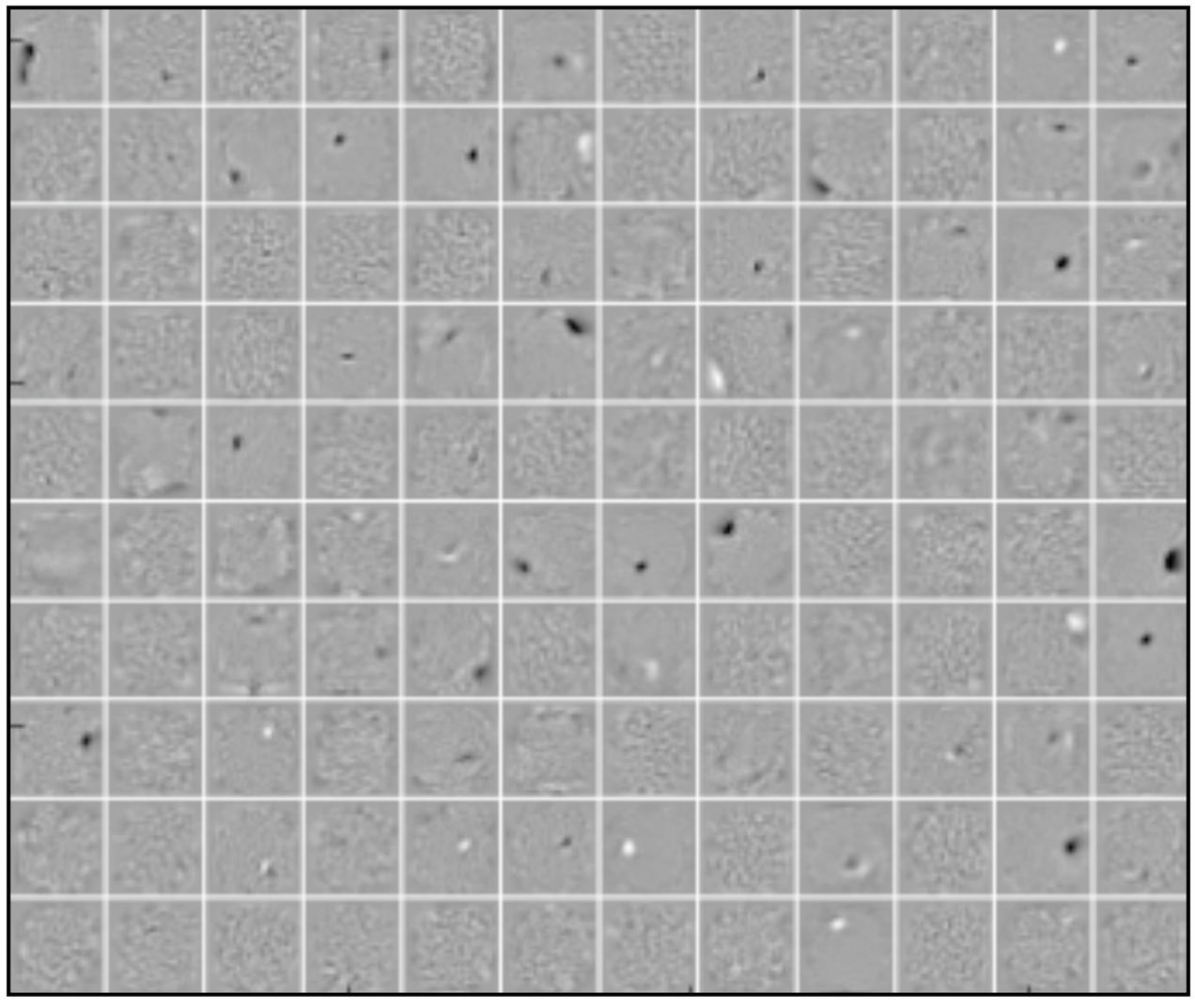
Interestingly, this is related to the derivative of the log-density (score) at that point, and also to the curvature at that point

Learned Filters: Noiseless

- No corruption, cross-entropy loss



(Vincent et al. 2008)

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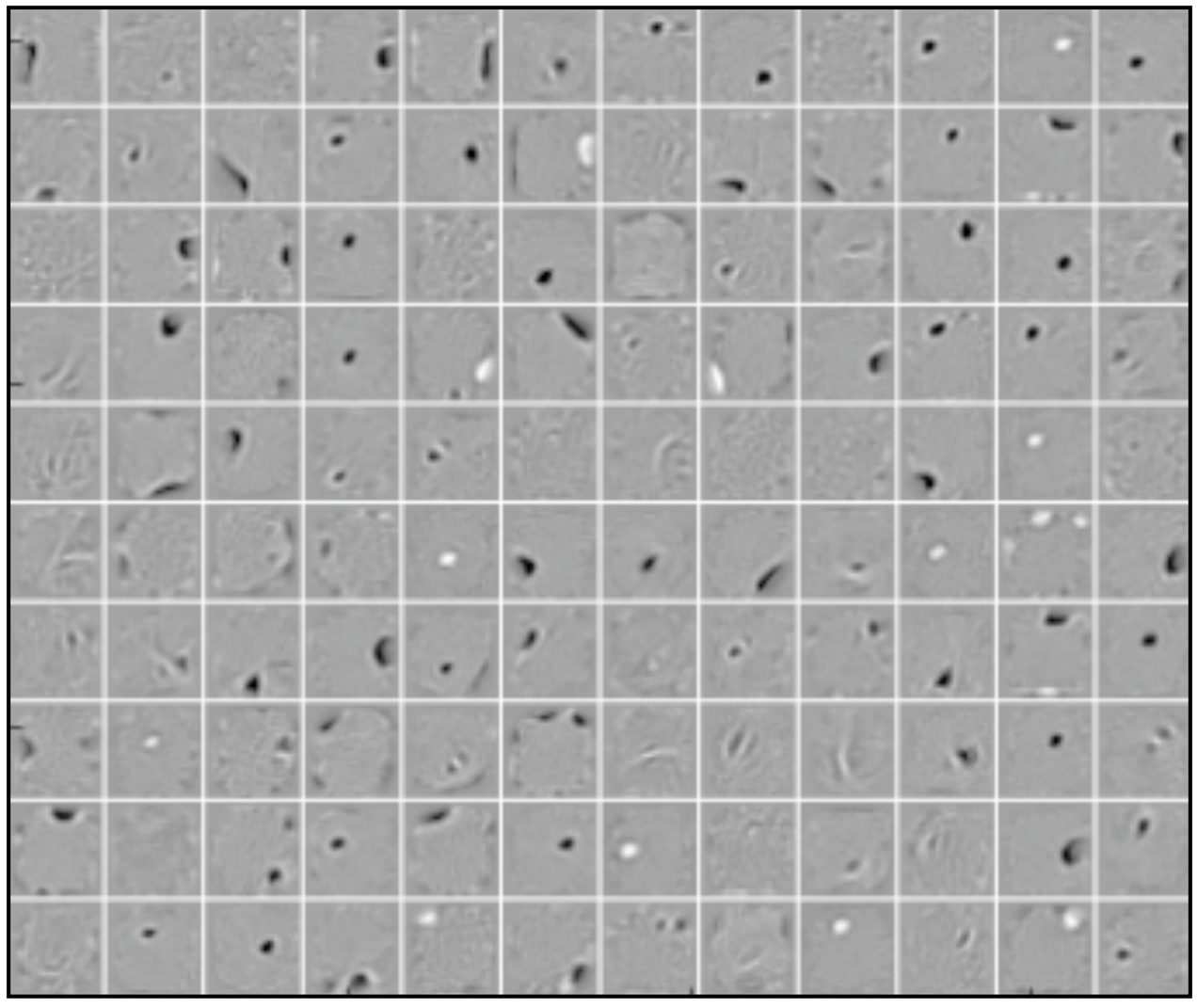
Learned Filters: Light Corruption

- 25% corruption



(Vincent et al. 2008)

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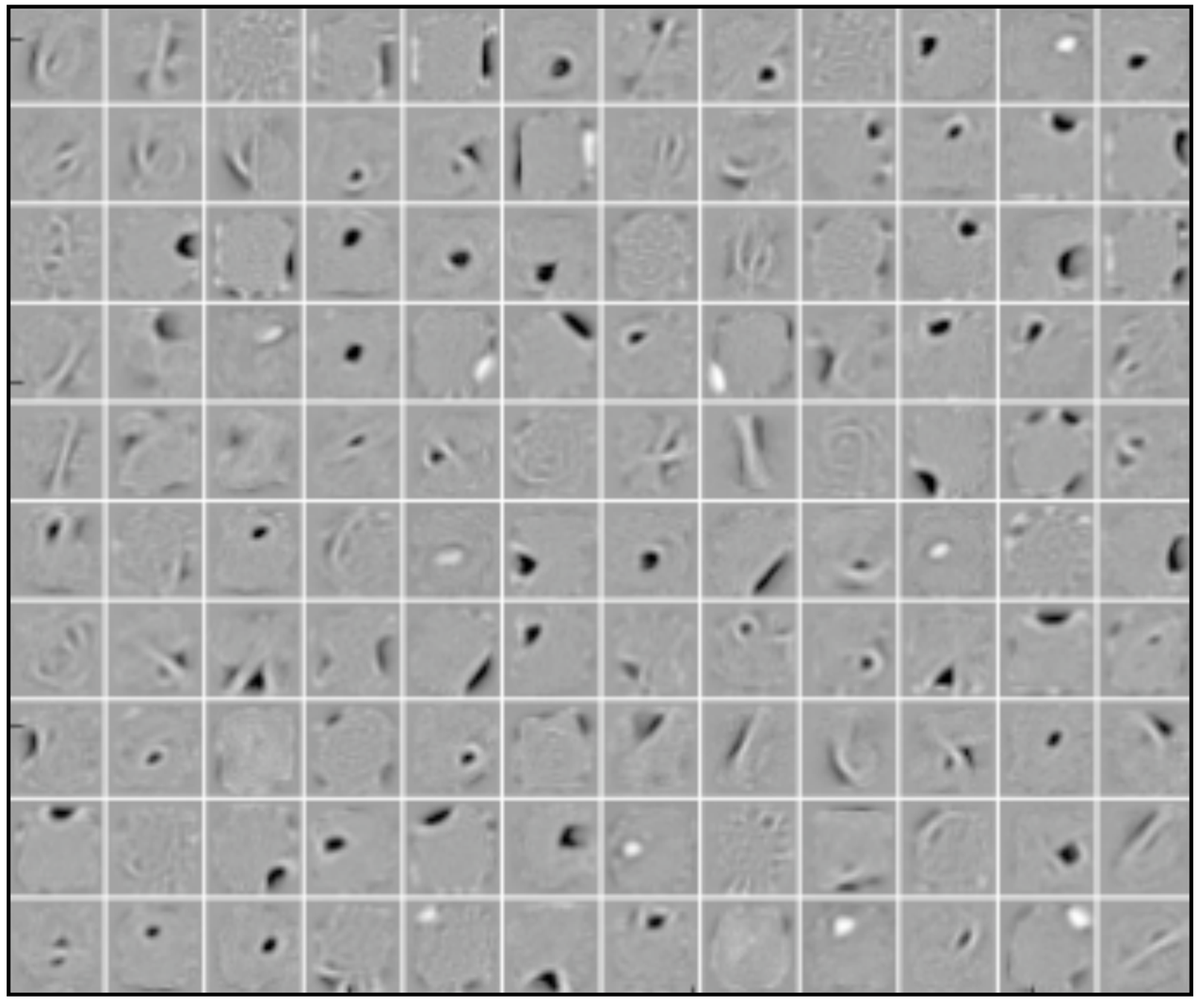
Learned Filters: Strong Corruption

- 50% corrupted inputs



(Vincent et al. 2008)

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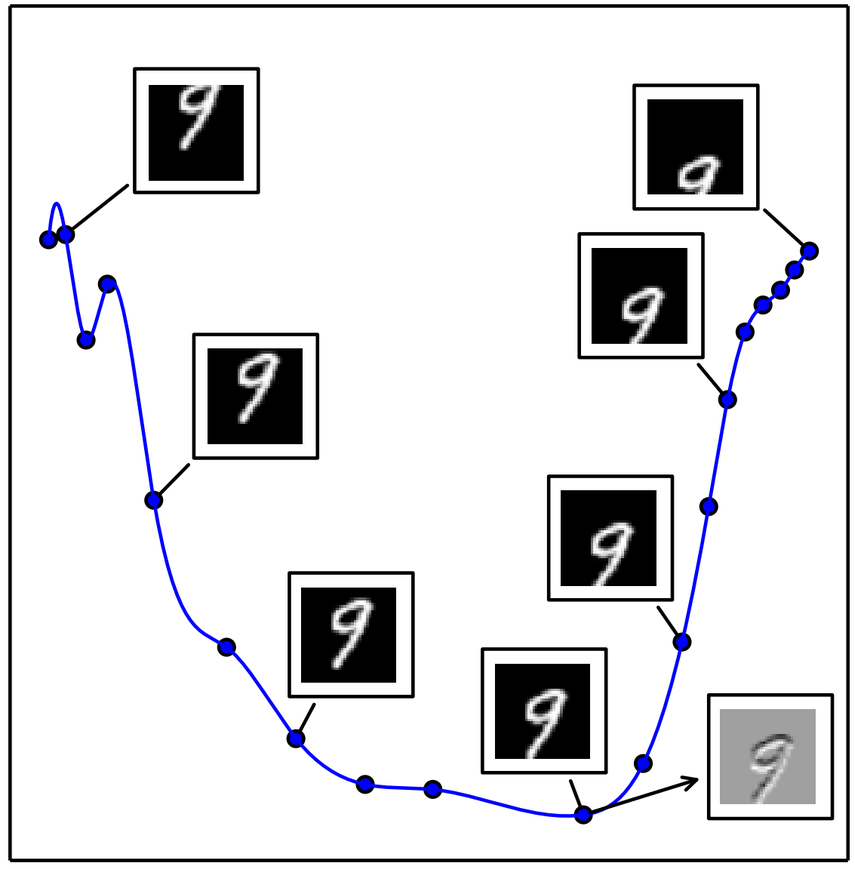
Autoencoders and Manifolds

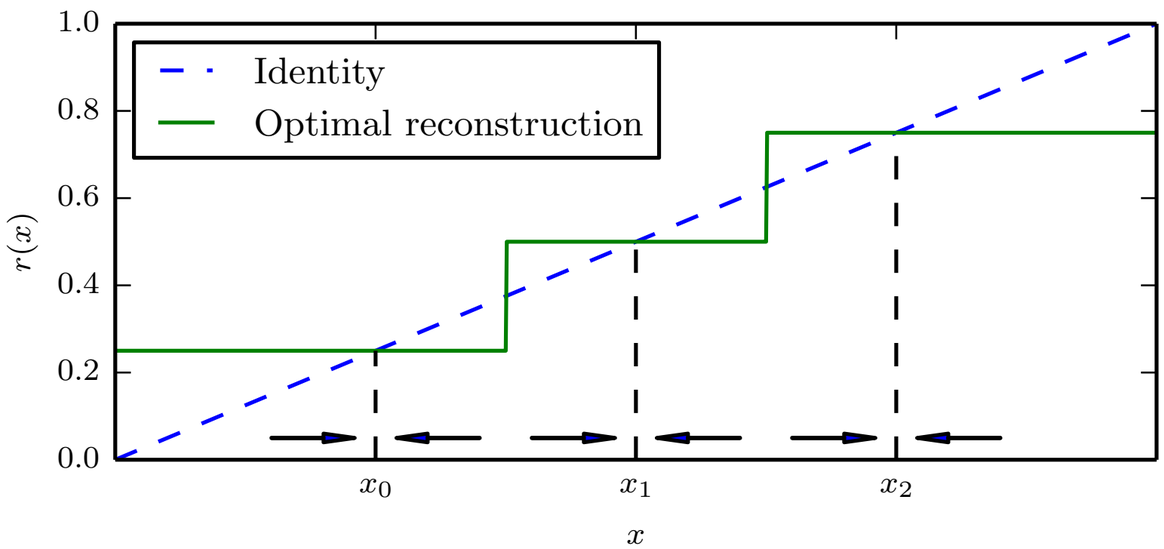
Regularized autoencoder training schemes involve a “battle” between two forces:

1. Learning a representation of a training example such that it can be reconstructed

2. Satisfying the constraint or regularization penalty

    - These techniques typically prefer solutions that are less sensitive to the input

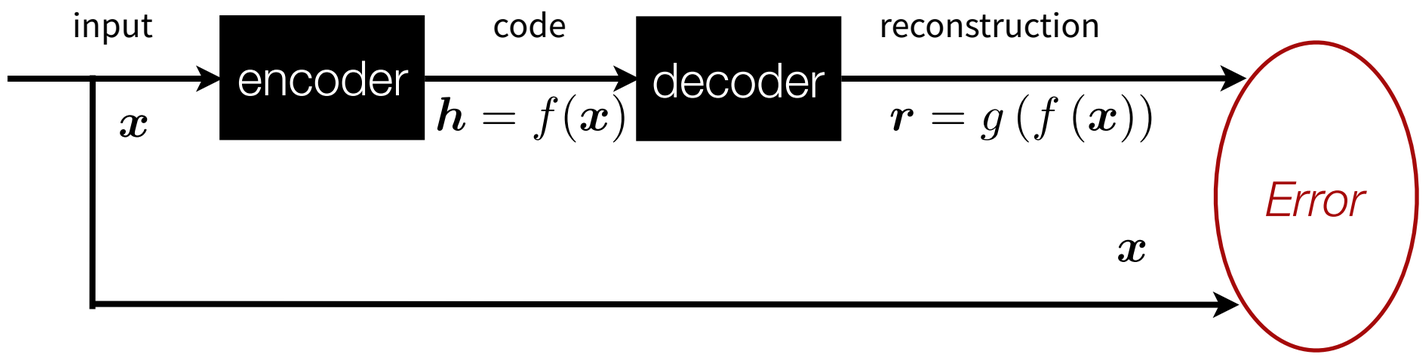




Reproduced from Goodfellow et al.

(Rifai et al. 2011)

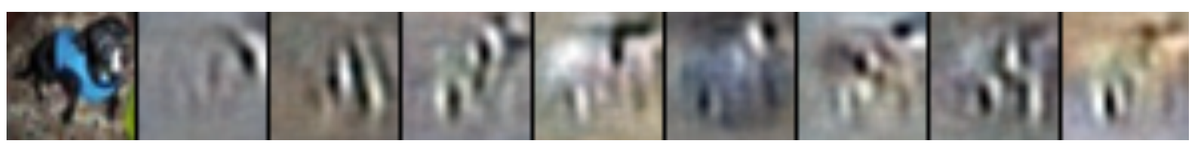
Contractive Auto-encoders



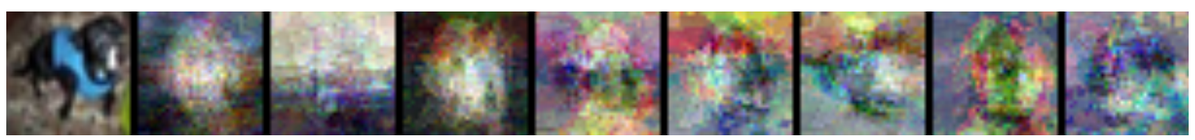
$$J = L(\boldsymbol{x}, \boldsymbol{r}) + \lambda \left|\left| \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} \right|\right|_F^2$$

Tangent vectors on (CIFAR-10) manifold estimated by:

CAE



Local PCA



location on manifold determined by dog image

- Learn good models of high-dimensional data (Bengio et al. 2013)
- Can obtain good representations for classification
- Can produce good quality samples by a random walk near the manifold of high density (Rifai et al. 2012)

via Rifai et al. 2012