Efficiency in Games with Incomplete Information

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Motivation

– Pareto efficiency is a natural benchmark for social welfare: no individual can be made strictly better off without making another worse off.

– Under incomplete information, players possess private information that could improve collective welfare.

– However, individual incentives may lead them to distort what they know.

– **When is a feasible outcome Pareto efficient? When can strategic behavior lead to efficiency?**

Summary of Results

- Excessive randomization over action profiles leads to inefficiency.
- **Necessary condition for efficiency (generic):**
  \# action profiles across states < # players + # states.
  Two players: only pure or quasi-pure outcomes can be efficient.

Economics Applications

1. Cheap talk
2. Bayesian persuasion
3. Mechanism design without transfers

Finite game with incomplete information

- **Players:** $i \in \{1, \ldots, k\}$.
- **States:** $\omega \in \Omega$ drawn from prior $p \in \mathrm{int}(\Delta\Omega)$.
- **Actions:** $a = (a_1, \ldots, a_k) \in A = \prod_{i=1}^{k} A_i$.
- **Payoffs:** $u_i : \Omega \times A \to \mathbb{R}$.
- **Types:** $t = (t_1, \ldots, t_k) \in T = \prod_{i=1}^{k} T_i$, with $\pi : \Omega \to \Delta T$.

Outcomes and feasible payoffs

- **Outcome:** $\mu : \Omega \to \Delta A$.
- **Induced ex-ante payoff:**

$$u(\mu) = \sum_{\omega \in \Omega} p(\omega) \sum_{a \in A} \mu(a \mid \omega)\, u(\omega, a) \in \mathbb{R}^k.$$

- **Feasible payoffs (given p):**

$$F_p = \{u(\mu) \in \mathbb{R}^k : \ \mu : \Omega \to \Delta A\}.$$

The set F_p is a convex polytope; extreme points = pure outcomes.

Efficiency

- An outcome μ is **efficient** if there is no outcome η with $u(\eta) \geq u(\mu)$ and at least one strict inequality.

- μ is efficient $\iff$ it maximizes a **strictly positive weighted sum** of players' ex-ante payoff:

$$\exists n \in \mathbb{R}^k_{++} \text{ s.t. } u(\mu) \in \arg\max_{x \in F_p} n^\top x.$$

Necessary Condition for Efficiency

Let $|\mu(\omega)|$ denote number of action profiles taken in state ω.

Generically, an outcome μ is efficient only if $\sum_{\omega} |\mu(\omega)| < k + |\Omega|$.

Two players: generically, efficient outcomes are pure or quasi-pure (binary randomization in exactly one state).

This condition does not depend on the prior, which action profiles are used, or the weight of randomization.

Ex-ante and ex-post efficiency: a geometric link

- **Minkowski sum decomposition:**

$$F_p = \sum_{\omega \in \Omega} p(\omega)\, F_\omega, \text{ where } F_\omega = \mathrm{Co}\{u(\omega, a) : a \in A\}.$$

- **Ex-ante efficiency:** $u(\mu)$ lies on the Pareto frontier of F_p.
- **Ex-post efficiency:** $u(\mu \mid \omega)$ lies on the Pareto frontier of F_ω.
- **Key geometric result:**

$$\mu \text{ is efficient } \iff \exists\, n \in \mathbb{R}^k_{++} \text{ s.t. } u(\mu \mid \omega) \in \arg\max_{x \in F_\omega} n^\top x \quad \forall \omega \in \Omega$$

Intuition for the result

- Efficiency requires a **common** positive weight vector $n \in \mathbb{R}^k_{++}$ to support **all** states.
- In state ω, ex-post efficiency means that all actions played maximize the same linear objective $n^\top u(\omega, \cdot)$.
- Generically, each additional action profile imposes an additional independent linear constraint on the admissible weight vectors.
- Summing constraints across states, excessive randomization generically rules out any strictly positive weight vector.

Example: Sender–Receiver

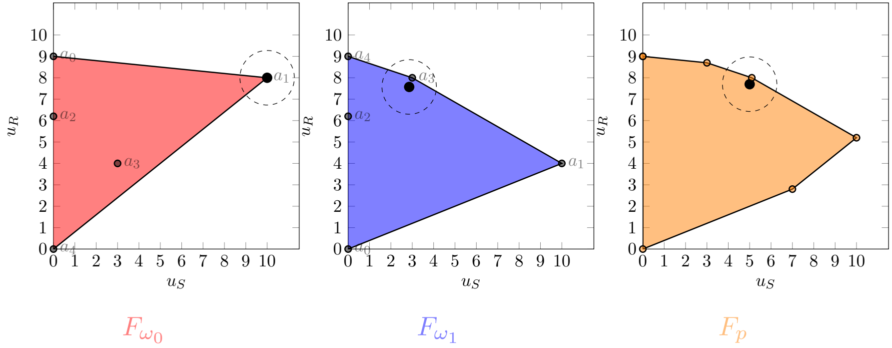
Let $\Omega = \{\omega_0, \omega_1\}$ and $A = \{a_0, a_1, a_2, a_3, a_4\}$. Sender–receiver payoffs (u_S, u_R):

	a_0	a_1	a_2	a_3	a_4
ω_0	$(0, 9)$	$(10, 8)$	$(0, 6.2)$	$(3, 4)$	$(0, 0)$
ω_1	$(0, 0)$	$(10, 4)$	$(0, 6.2)$	$(3, 8)$	$(0, 9)$

In this environment, we illustrate the efficiency properties of different outcomes.

Example 1: Ex-post inefficient

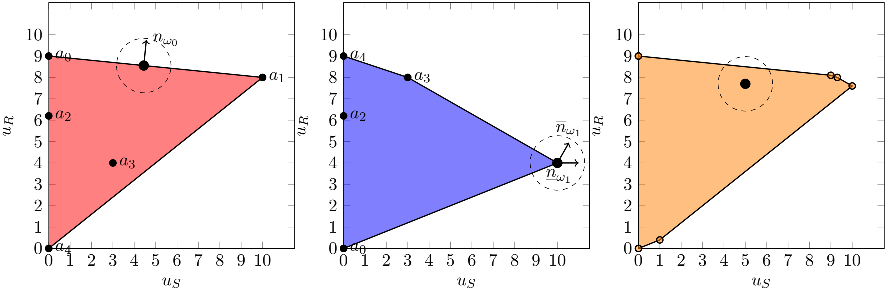
1. Outcome where action a_1 is taken in ω_0, and actions a_1 and a_4 are taken in ω_1



Ex-post inefficient in state $\omega_1 \Rightarrow$ Inefficient

Example 2: Ex-post efficient but ex-ante inefficient

2. Outcome where actions a_0 and a_1 are taken in ω_0 and action a_1 is taken in ω_1

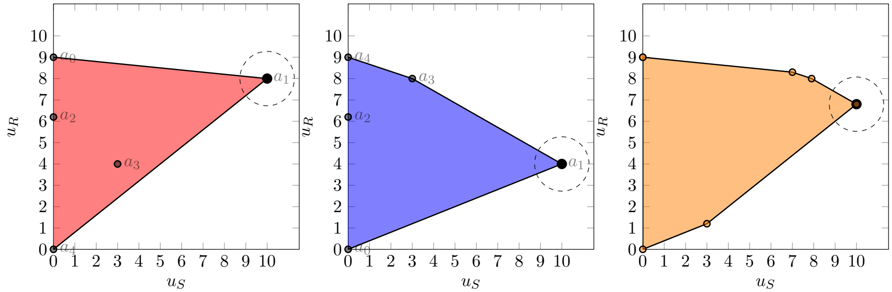


F_{ω_0} F_{ω_1} F_p

Ex-post efficient in both states but no common positive weight vector ($n_{\omega_0} \notin \text{cone}\{\underline{n}_{\omega_1}, \overline{n}_{\omega_1}\}$) $\Rightarrow$ Inefficient

Example 3: Ex-ante efficient

3. Outcome where only action a_1 is taken in both states



F_{ω_0} F_{ω_1} F_p

Outcome is ex-post efficient in both states and a common positive weight vector exists $\Rightarrow$ Efficient

Cheap Talk

Cheap Talk: Model

- State $\omega \in \Omega$ drawn according to prior $p \in \text{int}(\Delta\Omega)$.
- Sender observes ω and chooses a message $m \in M$ to send.
- Upon seeing m, receiver chooses an action $a \in A$.
- Results in payoffs $u_S(\omega, a)$ and $u_R(\omega, a)$.

When is a cheap talk outcome efficient?

Generically, a cheap talk outcome is efficient only if it is pure.

- Suppose a cheap talk equilibrium induces a stochastic outcome.
- Generically, efficiency implies that in some state ω^* no more than two actions a_1, a_2 are played.

$$\textbf{Sender's equilibrium:} \quad u_S(\omega^*, a_1) = u_S(\omega^*, a_2)$$

$$\textbf{Ex-post efficiency:} \quad u_R(\omega^*, a_1) = u_R(\omega^*, a_2)$$

- Non-generic condition! Any small perturbation breaks this knife-edge indifferences.

Cheap talk with state-independent sender payoff

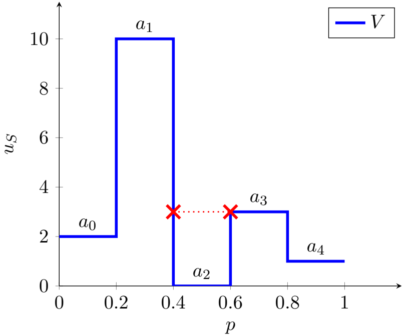
- Let a^* denote the sender's most preferred action among receiver's best responses.

> Suppose $u_S(a)$. A cheap talk outcome is efficient $\iff$ a^* is induced with certainty.

- Any non-trivial cheap talk outcome, where communication affects the receiver's action, is inefficient.

$u_S(a)$: Influential equilibrium

Assume $p = 0.5$ and posteriors $q_1 = 0.4$ and $q_2 = 0.6$ are induced.



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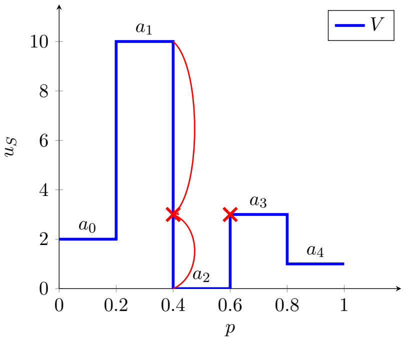
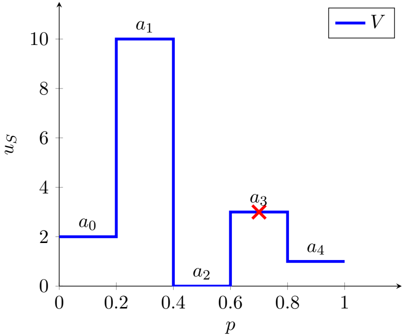


Given posterior $q_1 = 0.4$, the sender must randomize between a_1 and a_2 to satisfy sender's indifference condition.

But both players strictly prefer a_1 over a_2 in state $\omega_0 \Rightarrow$ inefficiency.

$u_S(a)$: Babbling equilibrium

Suppose $p = 0.7$ where the action $a_3 \neq a^*$ is played with certainty.



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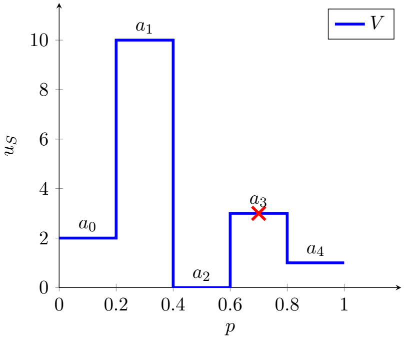
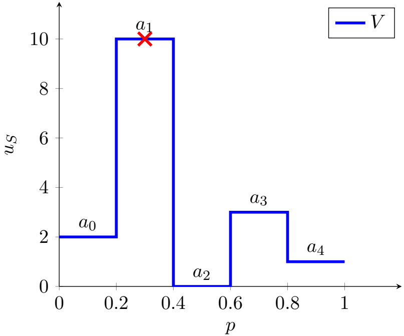


Again both players strictly prefer $a^* = a_1$ over a_3 in state ω_0

$u_S(a)$: Babbling equilibrium

Suppose $p = 0.3$ where the action $a_1 = a^*$ is played with certainty.



This babbling equilibrium is efficient, as it results in the sender's first-best outcome.

Relation to previous literature

- **Pareto efficiency: Rudov, Sandomirskiy, and Yariv, 2025**, Arieli and Babichenko, 2012, Pradelski and Young, 2012, Marden, Young, and Pao, 2014, Jindani, 2022,…
- **Strategic communication:** Ichihashi, 2019, Doval and Smolin, 2024, Crawford and Sobel, 1982, Kamenica and Gentzkow, 2011, …

Summary

- We analyzed Pareto efficiency in finite games with incomplete information.
- Generically, excessive randomization across action profiles leads to inefficiency.
- Necessary condition (generic):

$$\#\text{ action profiles across states } < \#\text{ players } + \#\text{ states.}$$

- Incentive constraints often prevent efficiency in natural strategic environments.

Thank you! Any questions?