ON THE INEFFICIENCY OF SOCIAL LEARNING

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ABSTRACT. We study whether a social planner can improve the efficiency of learning, measured by the expected total welfare loss, in a sequential decision-making environment. Agents arrive in order and each makes a binary action based on their private signal and the social information they observe. The planner can intervene by jointly designing the social information disclosed to agents and offering monetary transfers contingent on agents' actions. We show that, despite such flexibility, efficient learning cannot be restored with a finite budget: whenever learning is inefficient without intervention, no combination of information disclosure and transfers can achieve efficient learning while keeping total expected transfers finite.

1. INTRODUCTION

Many decisions under uncertainty are made in a social context, in which individuals rely on both their private information and the observed actions of others. Understanding how quickly privately held information is aggregated in a population is central to explaining collective outcomes across diverse domains—including the adoption of new technologies and medical treatments, investment decisions, and political opinion formation. Yet, this aggregation process need not be efficient: actions only provide coarse information about agents' private information and agents may persistently make mistakes. This raises a natural question: can a social planner intervene to accelerate learning and improve its efficiency?

To study this question, we consider the canonical sequential social learning model [Bikhchandani, Hirshleifer, and Welch, 1992, Banerjee, 1992], in which agents arrive in order and each receives a private signal. Unlike the standard model—where each agent observes the actions of all her predecessors—we introduce a social planner who can intervene by jointly designing the social information disclosed to the agents and offering monetary incentives to influence their decisions. Given the planner's intervention, each agent chooses a binary action in order to match an unknown binary state.

We measure the speed of learning using the criterion of *efficient learning*, introduced by Rosenberg and Vieille [2019], which requires that the expected number of agents who choose incorrectly be finite. This criterion captures not only eventual correctness

of behavior—thereby ruling out incorrect herds—but also the welfare losses incurred along the path to convergence. In the absence of planner's intervention, Smith and Sørensen [2000] show that with unbounded signals, the total number of agents who choose incorrectly must be finite, but its expectation may be either finite or infinite. Indeed, a finite expectation is achieved only when agents receive highly informative private signals with sufficiently high probability [Rosenberg and Vieille, 2019]. This is a demanding condition that many signal distributions do not satisfy, so learning is often inefficient.

To illustrate the source of this inefficiency, suppose all private signals were made publicly observable. This is equivalent to a single agent who repeatedly receives private signals and takes an action in each period. As is well known, in this case learning is efficient for any non-trivial signal distribution, including noisy binary signals that are not unbounded.[1] Thus, inefficient learning does not arise from a lack of information per se. Rather, it arises because observed actions are only partially informative about private signals and, more importantly, because these actions reflect agents' strategic behavior.

As we discuss in Section 3, in a benchmark case where the planner does not need to account for agents' strategic incentives, efficient learning can always be achieved. This indicates that coarsening from signals to actions alone is not the primary driver of inefficient learning.[2] We therefore focus on agents' incentives—which depend both on the social information they observe and on monetary rewards—and ask whether a social planner can improve learning efficiency by shaping these incentives. To do so, the planner has two instruments: for each agent, he can (i) design what information about earlier agents' actions is revealed, and (ii) issue a monetary transfer to subsidize one of the two actions contingent on this information.

We focus on a class of unbounded signals with well-behaved tails. This is a mild assumption that rules out distributions with irregular tail behavior and allows efficient learning to be characterized by a single parameter that measures tail thickness. Our main result (Theorem 1) shows that, despite the planner's flexibility, efficient learning cannot be restored with a finite budget: whenever the underlying signal distribution yields inefficient learning in the absence of intervention, no combination of disclosure policy and transfer scheme can achieve efficient learning with a finite expected total transfer. In particular, this result implies that no disclosure policy can, on its own, restore efficient learning (Corollary 2). Compared to the benchmark case, this result thus highlights agents' strategic incentives as the main source of inefficiency.

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[1]More specifically, learning can be viewed as very efficient in this setting since not only does the total number of mistakes have finite expectation, but each agent's mistake probability also converges to zero exponentially fast. See, e.g., Hann-Caruthers, Martynov, and Tamuz [2018], Rosenberg and Vieille [2019], Harel, Mossel, Strack, and Tamuz [2021], Huang, Strack, and Tamuz [2024], and Brandl [2025] for related results on the speed of learning.

[2]As another extreme, Arieli and Mueller-Frank [2017] show that when actions are continuous, private signals are encoded one-to-one into actions and can therefore be inferred from observed behavior. Thus, learning is always efficient.

The mechanism behind Theorem 1 is as follows. From the perspective of information design, the planner faces a fundamental tradeoff between providing precise social information to keep the current agent's mistake probability low, and extracting enough information from the agent's action to inform future agents. Since the agent is strategic, receiving precise social information induces her to rely heavily on it and to largely ignore her private signal. As a result, the more precise the social information, the less informative her action becomes, undermining the planner's ability to learn from it. Conversely, when the social information is imprecise, the agent relies more on her own private signal, making her action more informative to the planner, but at the cost of a higher mistake probability.

A similar tension arises when the planner uses monetary incentives. Intuitively, subsidizing the contrarian action—that is, the action opposite to the one favored by social information—encourages agents to act on their private signals. This accelerates learning as it allows the planner to infer more from their actions. Yet, if agents nonetheless continue to choose the action favored by social information despite such subsidies, it would strengthen future agents' inferences about that action being correct, forcing the planner to offer even larger transfers over time. Thus, while transfers can in principle alleviate this tension and restore efficient learning, doing so requires an infinite amount.

In sum, we show that the aforementioned tensions are insurmountable: any disclosure policy or transfer scheme that provides agents with the appropriate incentives to achieve efficient learning necessarily either leaves the planner with too little information to sustain precise social beliefs or requires transfers so large that their expected total amount is infinite.

We contribute to the literature of social learning in three aspects. First, previous work has largely focused on full (social) information disclosure, showing that insufficiently informative signal structures are the key determinant of inefficient learning. By contrast, we consider general information disclosure policies and show that inefficiency stems from agents' strategic incentives, rather than from limitations of the signal structure alone. Second, while different information structures have been shown to improve learning outcomes, our main result suggests that such improvements are fundamentally limited: they cannot restore efficient learning, even with the help of finite transfers. Third, our proof techniques directly use the induced distribution of agents' beliefs, rather than analyzing trajectories of beliefs and actions across periods. These techniques could be useful in other social learning settings or, more generally, in multi-period learning environments.

Related Literature. A closely related paper is Rosenberg and Vieille [2019], which characterizes the conditions for efficient learning in a sequential social learning environment with a general signal distribution. Moreover, they show that this condition holds regardless of whether agents observe the entire history of past actions or only the action of their immediate predecessor. This leaves open the question of whether there exists

a social information disclosure policy under which learning is efficient whenever signals are unbounded. As suggested by our main result, the answer is negative, at least for tail-regular signals.

Similar negative results have been documented in the recent literature on social learning with misspecification, which highlights a distinct mechanism through which learning can fail. For example, Bohren [2016] shows that when agents have misspecified beliefs about the correlation in others' actions, a large degree of misspecification can harm learning.[3] In a broader framework, Frick, Iijima, and Ishii [2023] demonstrate that even mild misspecification can lead to extreme learning failure. Relatedly, in a repeated setting, Chen [2024] identifies the phenomenon of group irrationality as the cause of belief nonconvergence.

At the same time, this literature has also identified some forms of misspecification that can improve learning. With binary signals, Bernardo and Welch [2001] demonstrate mostly numerically that the presence of occasionally overconfident agents has a positive effect on learning. Indeed, in a setting close to ours, Arieli, Babichenko, Müller, Pourbabaee, and Tamuz [2025] show that within the same class of unbounded signals that we consider, a mild form of overconfidence can restore efficient learning in cases where learning would be inefficient under correct specification. By contrast, in our setting, agents are fully rational and strategically respond to incentives; consequently, the positive learning effects from misspecification do not apply.

Our insight that agents' strategic incentives can undermine learning outcomes is not entirely new. Such an observation already appears in Rosenberg and Vieille [2017]—a working paper version of Rosenberg and Vieille [2019]—which circumvents strategic incentives by considering a planner who can dictate agents' strategies. They provide a sequence of cutoff strategies under which learning is efficient for a uniform signal distribution. Similarly, Smith, Sørensen, and Tian [2021] study a setting in which a planner can also dictate agents' strategies, but instead aims to maximize discounted social welfare. With the same objective, Martynov [2020] studies the optimal pricing policy of a planner who can subsidize and tax goods in every period. Even for such planners, characterizing the social optimum is known to be technically challenging.[4] We therefore ask whether a social planner who respects agents' strategic incentives can ever improve learning efficiency.

Our paper is also related to the literature on sequential social learning with different observational structures. With bounded signals, Acemoglu, Dahleh, Lobel, and Ozdaglar [2011] provide a sufficient condition on a stochastic, independent network structure under which the probability that agents choose incorrectly converges to zero.[5] Peres, Rácz,

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[3]See Bohren and Hauser [2021] for a more general treatment on different types of misspecifications.
[4]To the best of our knowledge, this remains an open question.
[5]Relatedly, Lobel and Sadler [2015] study learning in networks where the set of observed neighbors may be correlated across agents, highlighting how departures from independence affect learning outcomes.

Sly, and Stuhl [2020] later characterize the optimal rate at which a stream of "sacrificial lambs"—that is, agents who observe nothing except their private signals—should be injected to accelerate the rate at which the mistake probability converges to zero. More recently, Xu [2025] considers a setting in which agents arrive in cohorts and observe coarse signals of past cohorts' actions, and provides necessary and sufficient conditions for learning in probability.[6] Arieli, Babichenko, and Hann-Caruthers [2024] study a complementary problem in which a regulator can garble agents' private signals, rather than the social information, to maximize the probability that agents herd on the correct action. For comprehensive surveys on recent developments in social learning, see, e.g., Golub and Sadler [2016], Bikhchandani et al. [2024].

2. MODEL

2.1. **Baseline.** Time is discrete, and the horizon is infinite, i.e., $t \in \mathbb{N} = \{1, 2, \ldots\}$. There is a binary state $\theta \in \{h, \ell\}$ with a uniform prior. A sequence of agents indexed by time t arrive in order, each acting once by choosing a binary action $a_t \in \{h, \ell\}$. Agents have a normalized base utility: a correct action yields a payoff of one and an incorrect action yields zero. Before acting, agent t observes the actions of all predecessors, $H_t = (a_1, \ldots, a_{t-1})$.

In addition, agent t receives a private signal $s_t \in S$ where S is a measurable set of signal realizations. Conditional on θ, private signals are independent across agents. Let $q_t = \mathbb{P}[\theta = h \mid s_t]$ be the agent's posterior belief conditional on the private signal. Since q_t is a sufficient statistic for θ given s_t, we assume that $s_t = q_t$. Denote by F_h and F_ℓ the CDFs of q_t conditioned on $\theta = h$ and $\theta = \ell$, respectively. The unconditional CDF is thus $F = \frac{1}{2}(F_h + F_\ell)$. We assume that F is symmetric about $1/2$, i.e., $F(q) + F(1-q) = 1$ for each q. Equivalently, the pair of conditional CDFs (F_h, F_ℓ) is symmetric about $1/2$, i.e., $F_h(q) + F_\ell(1-q) = 1$. We further assume that F_h and F_ℓ are continuous, so that agents are almost surely not indifferent between actions. We use $\mathbb{P}_h$ to denote $\mathbb{P}[\cdot|\theta = h]$, the probability measure conditional on $\theta = h$, and use $\mathbb{P}_\ell$ analogously.

We say that signals are unbounded if the support of q_t contains 0 and 1 [Smith and Sørensen, 2000]. Following Rosenberg and Vieille [2019] and Arieli et al. [2025], we say that learning is *efficient* if the expected number of incorrect choices is finite,

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(2.1) \qquad \sum_{t=1}^{\infty} \mathbb{P}[a_t \neq \theta] < \infty,
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and *inefficient* otherwise. Efficient learning implies that the number of incorrect choices is finite.

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[6]A similar cohort setup also appears in Section 2.6 of Rosenberg and Vieille [2019].

In the above setting, Rosenberg and Vieille [2019] show that learning is efficient if and only if the following condition for private signals holds:

$$(2.2) \qquad \int_0^1 \frac{1}{F(x)}\, dx < \infty.$$

Intuitively, efficient learning requires not only the existence of highly informative signals, but also that such signals occur with sufficient frequency so that incorrect herds can be overturned quickly.

For tractability, we impose a regularity condition on the signal structure that restricts only the tail behavior of the distribution. Specifically, the symmetric unconditional distribution F is *tail-regular* if there exists $\alpha > 0$ such that

$$0 < \liminf_{q \to 0} \frac{F(q)}{q^\alpha} \leq \limsup_{q \to 0} \frac{F(q)}{q^\alpha} < \infty,$$

in which case we write $F(q) = \Theta(q^\alpha)$. In other words, $F(q)$ behaves like q^α near $q = 0$. Hence, a larger value of α corresponds to a lower likelihood of receiving very informative signals. Clearly, the uniformly distributed private beliefs, i.e., $F(q) = q$, are tail-regular with $\alpha = 1$. On the other hand, Gaussian signals with state-dependent means are not tail-regular.

Note that all tail-regular signals are unbounded. Moreover, under tail-regular signals, the criterion for efficient learning is precisely captured by the single parameter α: a lower value of α corresponds to a higher likelihood of receiving very informative signals. It follows from (2.2) that learning is efficient if $\alpha < 1$, and becomes inefficient once $\alpha \geq 1$.

2.2. **Planner's Intervention.** We now introduce a social planner who seeks to accelerate learning—that is, to achieve efficient learning by speeding up the convergence of the mistake probability to zero—through interventions in agents' decisions. The planner observes the agents' actions, but not their private signals or the state θ. That is, at each time t, he knows the entire past action history $H_t = (a_1, \ldots, a_{t-1})$. The planner has two instruments at his disposal, information disclosure and transfers, which we describe in detail below.

Information Disclosure. Recall that in the baseline model, the social information available to each agent t consists of the full past action history H_t. Under intervention, the planner instead determines what social information is disclosed to each agent. Formally, given a history H_t, the planner first forms the Bayesian belief

$$\pi_t := \mathbb{P}[\theta = h \mid H_t]$$

about the state being high. Then, at each time $t \geq 1$, conditional on π_t, the planner selects—possibly at random—a signal to disclose to the agent, which induces a social belief ν_t for agent t. As is well-known, any distribution that is a mean-preserving contraction of the distribution of π_t can be induced as the distribution of ν_t, via some signal based on

π_t. We interpret ν_t directly as the signal revealed to the agent, so that the information available to her before choosing an action is (q_t, ν_t)—her private signal plus the social information revealed by the planner.[7]

The disclosure mechanisms described above encompass a broad range of informational policies available to the planner. At one extreme, the planner may choose a *full disclosure* policy, under which he fully reveals his belief so that $\nu_t = \pi_t$ for each t. This is equivalent to the baseline model where the entire history of past actions is revealed to the agent. At the other extreme, the planner may adopt a *no disclosure* policy, under which the agent receives no social information—i.e., observes no past action history—so that $\nu_t = 1/2$. As an intermediate case, the planner may adopt a *partial disclosure* policy, under which only the k most recent actions are revealed. That is,

$$\nu_t = \mathbb{P}[\theta = h \mid a_{t-k}, \ldots, a_{t-1}],$$

so that the agent observes only a truncated history. As another example, the planner may employ a *stochastic disclosure* policy: with probability $\varepsilon_t(\pi_t)$, he fully reveals his belief π_t to the agent, and with the complementary probability $1 - \varepsilon_t(\pi_t)$, he discloses no information. The agents who receive no social information under such a policy are known as "sacrificial lambs."

Transfers. Beyond shaping agents' social information, the planner can also influence behavior through monetary incentives. Specifically, the planner may offer transfers to each agent prior to her action. These transfers can thus depend on the agent's action and on the information disclosure policy applied to the agent.

Formally, a transfer scheme $\tau = (\tau_t)_t$ consists of a sequence of functions where each $\tau_t \colon [0,1] \to \mathbb{R}$. At time t, if agent t takes the high action with induced social belief ν_t, she receives additional transfer $\tau_t(\nu_t)$ that depends only on ν_t. It is without loss of generality to restrict attention to transfer schemes that depend only on the current induced social belief, rather than on the full action history. This is because any history-dependent transfer scheme can be replicated by a planner using a transfer that conditions only on ν_t, as an agent's incentive to take an action is determined by the pair (ν_t, τ_t). Moreover, note that transfers do not depend on the planner's belief and therefore do not convey additional information about the state beyond what is already contained in the agent's induced social belief.[8]

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[7]Note that it is without loss of generality to assume that the distribution of ν_t depends only on π_t, rather than also on $\nu_1, \ldots, \nu_{t-1}$. This is because any such policy induces a joint distribution of (π_t, ν_t), which the planner can replicate without conditioning on past induced social beliefs, since the evolution of beliefs depends only on (π_t, ν_t).

[8]Allowing transfers to depend on the realized state does not affect agents' incentives. Since agents are expected utility maximizers, any state-contingent transfer scheme induces, in expectation, the same incentives as a scheme that depends only on the agent's action and induced social belief, and is therefore behaviorally equivalent.

2.3. **Agents' Decision.** Given an information disclosure policy and a transfer scheme, the agent's decision problem can be decomposed into two parts: belief formation and action choice. At time t, the agent first forms a posterior belief,

$$p_t := \mathbb{P}[\theta = h \mid \nu_t, q_t],$$

by combining her private belief q_t with the induced social belief ν_t. Conditional on this belief, the agent then chooses an action while taking into account the current transfer $\tau_t(\nu_t)$. Recall that the base payoff from taking the correct action is normalized to one. Thus, choosing the high action maximizes agent t's expected payoff if

$$(2.3) \qquad\qquad p_t \geq \frac{1}{2}(1 - \tau_t(\nu_t)).$$

Observe that when $\tau = 0$, this cutoff reduces to the case without transfers. From (2.3), we see that a positive transfer $\tau > 0$ lowers the cutoff for taking the high action and thus subsidizes the high action, whereas a negative transfer $\tau < 0$ raises the cutoff and subsidizes the low action. Throughout, we restrict attention to transfer schemes satisfying $|\tau| < 1$, since any transfer scheme outside this range would make one action strictly dominant regardless of the agent's belief.

3. The First-Best Benchmark

As a benchmark, we briefly discuss the case in which the social planner can directly dictate agents' strategies. This is a stronger intervention regime than those considered above, as the planner no longer needs to account for agents' strategic incentives. We ask whether, under this benchmark, the planner can restore efficiency in environments where learning is otherwise inefficient.

Rosenberg and Vieille [2017] show that for some tail-regular signals, the answer is positive. In particular, under a uniform signal distribution, i.e., $F(p) = p$ for all $p \in [0,1]$, they construct a sequence of cutoff strategies that achieves efficient learning and show that these cutoffs differ significantly from those chosen by rational agents.[9] More generally, as shown in Arieli et al. [2025], learning is efficient for all tail-regular signals if agents exhibit a mild form of misspecification—namely, mild condescension—under which they slightly underestimate the precision of others' signals relative to their own, despite all signals having identical precision.[10] The resulting cutoff rules used by these behavioral agents therefore also deviate from those used by rational agents. We note that a planner can replicate the behavior induced by such misspecification by instructing agents to follow these cutoff rules. Consequently, for any tail-regular signal distribution,

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[9]Proposition 6 of Rosenberg and Vieille [2017] shows that with these cutoffs, the probability that agent t chooses the wrong action converges to zero exponentially in t. Up to a change in the constant, this is the same rate of convergence as if all signals were public.

[10]That is, agents correctly observe their own signals which are α-tail-regular but incorrectly believe that all others' private signals are β-tail-regular. The condition for mild condescension requires $\beta \in (\alpha, \alpha+1)$.

these results imply that a planner who can dictate agents' strategies can always restore efficient learning.

4. RESULTS

We now state our main result. In contrast to the benchmark, we show that once agents' strategic incentives are taken into account, even a powerful planner, who can jointly design social information disclosure and transfers, cannot restore efficient learning with a finite budget. Recall that in the baseline model (under full disclosure and no transfers) learning is inefficient for tail-regular signals if and only if $\alpha \geq 1$.

Theorem 1. *Suppose $\alpha \geq 1$. Then, for any information disclosure policy and any transfer scheme, either learning is inefficient or the expected sum of absolute transfers is infinite.*

This result implies that efficient learning cannot be restored by any combination of disclosure policy and transfer scheme with finite expected transfers. In particular, a planner with a finite budget cannot restore efficiency. Compared to the first-best benchmark, Theorem 1 thus highlights agents' strategic behavior as an important source of inefficiency. An immediate consequence of Theorem 1 is the following.

Corollary 2. *Suppose $\alpha \geq 1$ and the planner imposes no transfers. Then, for any information disclosure policy, learning is inefficient.*

This result shows that, using information disclosure alone, the planner cannot fine-tune agents' social information to overturn the incorrect herds quickly enough for learning to become efficient. Note that this result does not imply that full disclosure is always socially optimal. Indeed, limited disclosure can sometimes improve overall outcomes by delaying herding and facilitating additional information aggregation, at the cost of a higher mistake probability for some early agents.[11] Nevertheless, our result shows that such overall improvements are limited in that they cannot restore efficient learning.

The idea behind Theorem 1 is as follows. Because agents are strategic, a planner who aims to accelerate learning for all agents faces a fundamental tradeoff in designing both information disclosure and transfers. On the one hand, the planner must provide the current agent with sufficiently precise social information to keep her mistake probability low. On the other hand, he must extract enough information from the agent's action to

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[11]For example, consider a signal structure with four signals: for some small $\varepsilon > 0$, agents receive one of two perfectly revealing signals (one for each state), each occurring with probability $\varepsilon/2$, and otherwise receive a noisy signal that induces a posterior of $2/3$ or $1/3$ with equal probability. Under full disclosure, all agents follow agent 1 unless a contradicting perfectly revealing signal arrives. If instead the planner withholds agent 1's action from agent 2, the first two agents act solely on their private signals, thereby generating two conditionally independent observations rather than one. This reduces the probability that an incorrect herd forms from $1/3$ to $7/27$, and consequently lowers the expected number of mistakes. Although this signal distribution is not tail-regular, it can be approximated by a tail-regular distribution, for which the same conclusion holds.

generate more precise information for future agents. A similar tension arises with transfers. Intuitively, to accelerate learning, the planner should subsidize contrarian actions so that an incorrect herd can end quickly. However, once such actions are subsidized, future agents who observe the herd action being taken despite these subsidies will infer more from it, strengthening their incentive to follow the herd. As a result, the planner must offer even larger transfers to future agents to offset this additional inference.

These tradeoffs arise precisely because rational agents optimally respond to social information and incentives. When an agent receives very precise social information, she tends to ignore her private signal unless it is extremely strong, which occurs with small probability. As a consequence, the more precise the social information is, the less informative the agent's action becomes, undermining the planner's ability to learn. Conversely, when the planner provides imprecise social information, the agent relies more heavily on her private signal, allowing the planner to learn from her action, but at the cost of a higher mistake probability. Similarly, the planner could use transfers to further elicit more information from agents' actions, but doing so would require a large amount of transfers.

In sum, we show that the aforementioned tensions are insurmountable: any intervention—jointly designing information disclosure and transfers—that provides agents with the appropriate incentives to achieve fast learning, necessarily either leaves the planner with too little information to sustain precise social beliefs or requires consistently large transfers whose expected total amount is infinite.

5. ANALYSIS

In this section, we provide a detailed analysis of the dynamics of the planner's belief and its relationship to agents' mistake probabilities. This will lead to a proof sketch for Theorem 1 at the end of the section.

5.1. **Planner's Belief Dynamics.** Recall that the planner's belief that the state is high at time t is $\pi_t = \mathbb{P}[\theta = h \mid a_1, \ldots, a_{t-1}]$, and that agent t's private belief is $q_t = \mathbb{P}[\theta = h \mid s_t]$. Starting from π_t, the planner chooses an information disclosure policy that induces a social belief ν_t for agent t and offers a transfer $\tau_t(\nu_t)$. After the planner's intervention, agent t chooses her action based on her posterior belief

$$p_t = \mathbb{P}[\theta = h \mid q_t, \nu_t].$$

From (2.3), agent t chooses the high action if and only if

$$p_t \geq \tfrac{1}{2}(1 - \tau_t(\nu_t)).$$

By Bayes' rule, this condition can be written as

$$\frac{p_t}{1 - p_t} = \frac{q_t}{1 - q_t} \times \frac{\nu_t}{1 - \nu_t} \geq \frac{1 - \tau_t(\nu_t)}{1 + \tau_t(\nu_t)}.$$

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We define agent t's private-belief cutoff as

$$(5.1) \qquad c(\nu_t, \tau_t(\nu_t)) = \frac{(1 - \nu_t)(1 - \tau_t(\nu_t))}{\nu_t(1 + \tau_t(\nu_t)) + (1 - \nu_t)(1 - \tau_t(\nu_t))}.$$

It follows that agent t chooses the high action if her private belief satisfies

$$q_t \geq c_t(\nu_t, \tau_t(\nu_t)),$$

and chooses the low action otherwise. Since the transfer τ_t is determined by ν_t, for brevity, we write the cutoff in (5.1) as $c_t(\nu_t) = c(\nu_t, \tau_t(\nu_t))$. Therefore, conditional on θ, the probability that agent t chooses the high action is $1 - F_\theta(c_t(\nu_t))$, while the probability that agent t chooses the low action is $F_\theta(c_t(\nu_t))$. This implies that if agent t chooses the high action, then the planner's belief π_{t+1} evolves as

$$(5.2) \qquad \frac{\pi_{t+1}}{1 - \pi_{t+1}} = \frac{\pi_t}{1 - \pi_t} \times \frac{1 - F_h(c_t(\nu_t))}{1 - F_\ell(c_t(\nu_t))}.$$

If agent t chooses the low action, then π_{t+1} evolves as

$$(5.3) \qquad \frac{\pi_{t+1}}{1 - \pi_{t+1}} = \frac{\pi_t}{1 - \pi_t} \times \frac{F_h(c_t(\nu_t))}{F_\ell(c_t(\nu_t))}.$$

The second terms in the above products capture the information conveyed by agent t's action to the planner's belief. Since F_h first-order stochastically dominates F_ℓ, the likelihood ratio in (5.2) is always greater than one. Consequently, when the planner observes a high action, his belief that the state is high increases. Likewise, the likelihood ratio in (5.3) is always less than one, so observing a low action decreases the planner's belief that the state is high.

5.2. **Agent's Mistake Probability.** Note that from (5.1), we have $c(\nu_t, 0) = 1 - \nu_t$, so $1 - \nu_t$ is the private-belief cutoff used by agent t with induced social belief ν_t under zero transfers. We first observe that any transfer can shift an agent's private-belief cutoff by at most its magnitude.

Lemma 1. *For any t, any induced social belief ν_t and the associated transfer $\tau_t(\nu_t)$, the private-belief cutoff $c(\nu_t, \tau_t(\nu_t))$ satisfies:*

$$|c(\nu_t, \tau_t(\nu_t)) - (1 - \nu_t)| \leq |\tau_t(\nu_t)|.$$

This lemma suggests that the planner cannot substantially shift agents' cutoffs using small monetary incentives. Next, for a given induced social belief and transfer, we compare the mistake probabilities of an agent before and after receiving her private signal. Intuitively, the former would perform worse than the latter since the latter has more information. Nevertheless, we show that the private signal improves the mistake probability by at most a constant factor, which depends only on the private signal distribution. We denote by b_t the action chosen by agent t prior to receiving her private signal.

Lemma 2. *There exists a constant $\varepsilon_F > 0$ such that for any time t and any induced social belief ν_t,*

$$\mathbb{P}[a_t \neq \theta \mid \nu_t] \geq \varepsilon_F \cdot \mathbb{P}[b_t \neq \theta \mid \nu_t].$$

The proof of Lemma 2 uses a simple idea: if an agent who relies only on social information chooses the incorrect action, she will continue to do so after receiving any private signal that reinforces the incorrect action, and those signals occur with constant probability. We will use this result to relate agents' mistake probabilities to the rate at which the planner's belief can grow.

5.3. **Expected Belief Growth.** To analyze the growth of the planner's belief, we examine the expected incremental changes in its log-likelihood ratio. As the planner observes more actions from agents, his belief π_t converges almost surely, since it is a bounded martingale. In other words, the planner becomes more certain about the state. We denote the corresponding log-likelihood ratio at time t by

$$\ell_t := \log \frac{\pi_t}{1 - \pi_t}.$$

Taking logarithms on both sides of (5.2) and (5.3), the evolution of the planner's log-likelihood ratio ℓ_t can now be written as

$$(5.4) \qquad\qquad \ell_{t+1} = \ell_t + U_t^a(\nu_t) \quad \text{if } a_t = a \in \{h, \ell\},$$

where we define

$$U_t^h(\nu) := \log \frac{1 - F_h(c_t(\nu))}{1 - F_\ell(c_t(\nu))}, \quad \text{and} \quad U_t^\ell(\nu) := \log \frac{F_h(c_t(\nu))}{F_\ell(c_t(\nu))}.$$

When F_h and F_ℓ are tail-regular with exponent α, we can approximate $F_h(q)$ and $F_\ell(q)$ for small q by $q^{\alpha+1}$ and q^α, respectively, up to multiplicative constants (see Lemma 4 in the appendix). As a result, $F_h(q)$ is much smaller than $F_\ell(q)$ when q is close to zero. Thus, receiving a private signal below a small threshold q is much less likely in the high state than in the low state, and so agents are much more likely to take the low action in the low state than in the high state. In consequence, when agents hold a high induced social belief, their private-belief cutoffs fall in this region. In that case, observing an agent take the low action is highly informative for the planner, since it is largely unexpected, whereas observing the high action conveys relatively little information.

More precisely, suppose agents have a large induced social belief ν, so that their private-belief cutoff $c_t(\nu)$ is close to zero. We show in Lemma 5 in the appendix that, for large ν, the log-likelihood ratio increments $U_t^h(\nu)$ and $U_t^\ell(\nu)$ can be well approximated by

$$U_t^h(\nu) \approx (c_t(\nu))^\alpha \quad \text{and} \quad U_t^\ell(\nu) \approx \log(c_t(\nu)).$$

Hence, when ν is large, taking the high action is the likely outcome, so observing it increases the planner's belief by a small amount of $(c_t(\nu))^\alpha$. In contrast, taking the low action is unlikely, and observing it decreases the planner's belief by a large amount,

i.e., $\log(c_t(\nu))$. This asymmetry in the planner's belief updating is balanced by the probabilities of these events, implying that belief updates are typically small, with large updates occurring only rarely. As a result, although the planner's belief converges almost surely, its expected growth over time is gradual.

To quantify this gradual learning, we measure the strength of the planner's belief by the absolute log-likelihood ratio $|\ell_t|$. In the following lemma, we establish an upper bound on the incremental change in its expected value, which is crucial for proving our main theorem. Recall that b_t is the action that agent t would take without receiving her private signal—that is, the action favored by social information alone, given the transfer.

Lemma 3. *For any $\alpha \geq 1$, there exists some constant $M < \infty$ such that for any time t,*

$$0 \leq \mathbb{E}[|\ell_{t+1}| - |\ell_t|] \leq M \cdot (\mathbb{P}[b_t \neq \theta] + \mathbb{E}[|\tau_t|]).$$

This result shows that, although the strength of the planner's belief increases on average, learning cannot proceed too quickly: at any given time, the per-period growth of the planner's belief is bounded above, up to a constant, by the agent's mistake probability prior to receiving her private signal and the expected transfer.

The proof of Lemma 3 relies on the observation that, on average, the planner's belief evolves through a combination of frequent small updates and rare large updates. Intuitively, the planner learns the most about the state when agents act against the action favored by their induced social beliefs. Such deviations arise either because the existing social information points in the wrong direction, or because transfers shift the agent's private-belief cutoff away from that implied by social information. The former events are rare since signals are tail-regular with $\alpha \geq 1$, while the latter cutoff shifts are bounded by the size of the transfer (Lemma 1). Thus, their expected contribution to the planner's belief growth is bounded by the mistake probability under social information and the expected transfer.

5.4. **Proof Sketch of Theorem 1.** We end this section by providing a proof sketch of Theorem 1. Fix a tail-regular signal distribution with $\alpha \geq 1$. Suppose, toward a contradiction, that there exist an information disclosure policy and a transfer scheme that achieve efficient learning with a finite expected total transfer. Then the sum of agents' mistake probabilities, as well as the expected total transfers, must be finite. By Lemma 2, the planner cannot do much worse than the agents, and therefore the sum of the planner's mistake probabilities is also finite. Meanwhile, Lemma 3 implies that, for $\alpha \geq 1$, the expected strength of the planner's belief is uniformly bounded over time. This, in turn, implies that the planner's mistake probability is bounded away from zero. Since agents cannot outperform the planner by more than a constant factor, their mistake probabilities are likewise bounded away from zero, a contradiction to efficient learning. Therefore, we conclude that no information disclosure policy and transfer scheme can restore efficient learning with a finite expected total transfer.

6. Conclusions

In this paper, we show that a social planner cannot improve the efficiency of learning with a finite budget, despite having flexible information design choices. This stands in contrast to a benchmark setting in which efficient learning can always be achieved when the planner is able to disregard agents' incentives and directly dictate their strategies. Our result thus underscores that inefficiency in social learning is not only a consequence of informational constraints or coarseness of observed actions, but also arises fundamentally from agents' strategic behavior. When agents optimally respond to both social information and incentives, we show that any attempt to accelerate learning either deprives the planner of sufficiently informative actions to sustain precise social beliefs over time or requires an infinite amount of expected transfers.

One promising direction for future research is to study the optimal design of policy interventions in environments where efficient learning is feasible. In such settings, different disclosure policies may all lead to eventual learning, yet differ substantially in the rate at which learning occurs. Characterizing the disclosure policy that minimizes the total expected number of mistakes is an open question. More broadly, the limits of information disclosure as a tool for improving learning outcomes are not yet well understood. For example, it remains open whether full disclosure can always be improved upon, or how to characterize the class of disclosure policies that are optimal for a given private signal distribution.

Our model can be extended in several directions. It is natural to consider an alternative notion of efficient learning, based on the discounted expected number of incorrect choices $\sum_{t \geq 1} \delta^t \mathbb{P}[a_t \neq \theta]$ for $\delta \in (0, 1)$. This criterion is always finite and therefore allows for a finer comparison across interventions. We conjecture that a conclusion similar to our main result continues to hold under this discounted notion of efficient learning, as discounting seems to favor the full disclosure policy.[12] Another direction is to expand the set of tools available to the designer. For example, consider a designer who can directly compensate agents for revealing their private signals (or, more generally, garblings of their signals) rather than inferring the signals from actions. Such mechanisms would allow the planner to extract more information without increasing the probability of incorrect choices, thus alleviating a core tension in our model. A related positive effect of garbling agents' private signals has been shown in settings with bounded signals [Arieli et al., 2024]. Finally, an interesting technical question is whether our results can be extended beyond tail-regular signals. We leave these questions for future research.

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[12]Intuitively, relative to full disclosure, coarser disclosure policies shift accuracy from early to later agents: withholding information from early agents makes their actions more informative, allowing more precise information to be revealed later on. This tends to reduce incorrect choices in later periods, but at the expense of more mistakes early on. Because discounting places greater weight on earlier mistakes, the discounted criterion should shift the comparison in favor of the full disclosure policy.

Appendix A. Omitted Proofs from the Main text

Proof of Lemma 1. For ease of notation, we write $\tau_t = \tau_t(\nu_t)$, and so, agent t's private-belief cutoff is $c(\nu_t, \tau_t)$. A straightforward calculation shows that

$$
\begin{aligned}
c(\nu_t, \tau_t) - (1 - \nu_t) &= \frac{(1 - \nu_t) - \tau_t(1 - \nu_t) - (1 - \nu_t)[1 - \tau_t(1 - 2\nu_t)]}{1 - \tau_t(1 - 2\nu_t)} \\
&= \frac{(1 - \nu_t)[1 - \tau_t - 1 + \tau_t - 2\tau_t\nu_t]}{1 - \tau_t(1 - 2\nu_t)} \\
&= -\tau_t \cdot \frac{2\nu_t(1 - \nu_t)}{1 - \tau_t(1 - 2\nu_t)}.
\end{aligned}
$$

Since $\nu_t \in [0, 1]$ and $|\tau_t| < 1$, the denominator $1 - \tau_t(1 - 2\nu_t)$ is always positive. Thus, it suffices to show that $2\nu_t(1 - \nu_t) \leq 1 - \tau_t(1 - 2\nu_t)$, which holds for any $|\tau_t| < 1$. $\square$

Proof of Lemma 2. Recall that q_t is agent t's belief conditional on the private signal, and we use $\mathbb{P}_h$ and $\mathbb{P}_\ell$ to denote the conditional probabilities. Let

$$
\varepsilon_F = \min\{\mathbb{P}_\ell[q_t \geq 1/2], \mathbb{P}_h[q_t < 1/2]\}.
$$

Since the conditional CDFs F_h and F_ℓ are continuous, it follows that $\varepsilon_F > 0$. For ease of notation, let $\tau_t = \tau_t(\nu_t)$. By Bayes' rule, $b_t = h$ if and only if

$$
\frac{\nu_t}{1 - \nu_t} \geq \frac{1 - \tau_t}{1 + \tau_t}.
$$

Recall that the privately informed agent chooses $a_t = h$ if and only if $q_t \geq c_t(\nu_t)$, where, by (5.1), the cutoff $c_t(\nu_t)$ satisfies

$$
\frac{c_t(\nu_t)}{1 - c_t(\nu_t)} = \frac{1 - \tau_t}{1 + \tau_t} \times \frac{1 - \nu_t}{\nu_t}.
$$

Now, suppose $b_t = h$. This implies $\frac{\nu_t}{1 - \nu_t} \geq \frac{1 - \tau_t}{1 + \tau_t}$, and so we have

$$
c_t(\nu_t) \leq 1/2.
$$

If $\theta = h$, then the inequality holds trivially since $\mathbb{P}[b_t \neq \theta] = 0$. If $\theta = \ell$, then $\mathbb{P}[b_t \neq \theta] = 1$. However, for agent t who observes both q_t and ν_t, she chooses $a_t = h$ if $q_t \geq c_t(\nu_t)$. Since $c_t(\nu_t) \leq 1/2$, we have

$$
\mathbb{P}_\ell[a_t = h \mid \nu_t, \tau_t] = \mathbb{P}_\ell[q_t \geq c_t(\nu_t)] \geq \mathbb{P}_\ell[q_t \geq 1/2] \geq \varepsilon_F.
$$

Hence, $\mathbb{P}_\ell[a_t = h] \geq \varepsilon_F \cdot 1 = \varepsilon_F \cdot \mathbb{P}_\ell[b_t \neq \theta]$. The case where $b_t = \ell$ follows from an analogous argument. Combining all cases, we have established that for any ν_t and its associated τ_t:

$$
\mathbb{P}[a_t \neq \theta \mid \nu_t] \geq \varepsilon_F \cdot \mathbb{P}[b_t \neq \theta \mid \nu_t].
$$

$\square$

The following lemma connects the exponent α with the tail behavior of F_h and F_ℓ. It can be derived from a standard result (see, e.g., Hann-Caruthers et al. [2018], Rosenberg and Vieille [2019] and Arieli et al. [2025]). We provide proofs below for completeness.

Lemma 4. *Suppose $F(q) = \Theta(q^\alpha)$. Then $F_\ell(q) = \Theta(q^\alpha)$ and $F_h(q) = \Theta(q^{\alpha+1})$.*

Proof. Recall that F is the unconditional cdf, while F_ℓ and F_h are the conditional cdfs given $\theta = \ell$ and $\theta = h$. For any q, these conditional cdfs satisfy

(A.1)
$$F_h(q) = 2\Big(qF(q) - \int_0^q F(x)\,dx\Big),$$
$$F_\ell(q) = 2\Big((1-q)F(q) + \int_0^q F(x)\,dx\Big).$$

Note that $|F_\ell - 2F(q)| \le 3qF(q)$ and that $\lim_{q\to 0} F_\ell(q)/F(q) = 2$. So, if $F(q) = \Theta(q^\alpha)$, then $F_\ell(q) = \Theta(q^\alpha)$.

If $F(q) = \Theta(q^\alpha)$, then there exist constants $0 < c \le C < \infty$ such that for all small q, $cq^\alpha \le F(q) \le Cq^\alpha$. As $F_h(q) \le 2qF(q)$, this gives us the upper bound: $F_h(q) \le 2Cq^{\alpha+1}$.

For the lower bound, fix a small q and some $m \in (0,1)$. We can write the integral as

$$\int_0^q F(x)\,dx = \int_0^{mq} F(x)\,dx + \int_{mq}^q F(x)\,dx.$$

Since F is nondecreasing, we have $F(x) \le F(mq)$ for $x \le mq$ and $F(x) \le F(q)$ for $x \in [mq, q]$. Hence $\int_0^{mq} F(x)\,dx \le mq\,F(mq)$, and $\int_{mq}^q F(x)\,dx \le (1-m)q\,F(q)$. Adding the two bounds and using the fact that $F(mq) \le C(mq)^\alpha$ gives

$$\int_0^q F(x)\,dx \le Cm^{\alpha+1}q^{\alpha+1} + (1-m)q\,F(q).$$

Using $F(q) \ge cq^\alpha$, we have $Cm^{\alpha+1}q^{\alpha+1} \le \frac{Cm^{\alpha+1}}{c}qF(q)$. Choose m small enough so that $\frac{Cm^{\alpha+1}}{c} \le \frac{m}{2}$. Then

$$\int_0^q F(x)\,dx \le \Big(1 - \frac{m}{2}\Big)qF(q).$$

Plugging into (A.1) yields

$$F_h(q) \ge 2\Big(qF(q) - \Big(1 - \frac{m}{2}\Big)qF(q)\Big) = m\,qF(q) \ge mc\,q^{\alpha+1}.$$

This provides the lower bound. Together with the upper bound, it follows that $F_h(q) = \Theta(q^{\alpha+1})$. $\qquad\square$

The next lemma characterizes the asymptotic size of the log-likelihood increments $U_t^h(\nu)$ and $U_t^\ell(\nu)$ in terms of the cutoff $c_t(\nu)$, which will be useful in proving Theorem 1. For notational convenience, we write $\mu_t = c_t(\nu)$ as the private-belief cutoff that agent t uses to choose an action. We will use the Landau notation, $o(\cdot)$ and $O(\cdot)$, i.e., we write $f(x) = o(g(x))$ if $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$, and $f(x) = O(g(x))$ if $\limsup_{x\to 0} \frac{f(x)}{g(x)} < \infty$.

Lemma 5. *For μ_t close to 0, one has $U_t^h(\nu) = \Theta(\mu_t^\alpha)$ and $U_t^\ell(\nu) = \log(\mu_t) + O(1)$, that is*

$$0 < \liminf_{\mu_t \to 0} \frac{U_t^h(\nu)}{\mu_t^\alpha} \leq \limsup_{\mu_t \to 0} \frac{U_t^h(\nu)}{\mu_t^\alpha} < \infty,$$

and there exists a constant $C > 0$ such that for all sufficiently small μ_t,

$$|U_t^\ell(\nu) - \log(\mu_t)| \leq C.$$

Proof. Recall from (5.4) that

$$U_t^h(\nu) = \log \frac{1 - F_h(\mu_t)}{1 - F_\ell(\mu_t)}, \quad \text{and} \quad U_t^\ell(\nu) = \log \frac{F_h(\mu_t)}{F_\ell(\mu_t)}.$$

By the assumption of tail regularity and Lemma 4, there exist constants $C_h \geq c_h > 0$ and $C_\ell \geq c_\ell > 0$ such that for all sufficiently small μ, $F_h(\mu) \in [c_h \mu^{\alpha+1}, C_h \mu^{\alpha+1}]$ and $F_\ell(\mu) \in [c_\ell \mu^\alpha, C_\ell \mu^\alpha]$. Substituting these into the expression for U_t^ℓ, we obtain

$$\log\Big(\frac{c_h}{C_\ell}\mu_t\Big) \leq U_t^\ell(\nu) \leq \log\Big(\frac{C_h}{c_\ell}\mu_t\Big),$$

which implies

$$U_t^\ell(\nu) = \log(\mu_t) + O(1).$$

From Lemma 4, we have $F_h(\mu_t) = \Theta(\mu_t^{\alpha+1})$ and $F_\ell(\mu_t) = \Theta(\mu_t^\alpha)$, so $F_h(\mu_t) = o(F_\ell(\mu_t))$. We rewrite U_t^h as

$$U_t^h(\nu) = \log\left(\frac{1 - F_h(\mu_t)}{1 - F_\ell(\mu_t)}\right) \leq -\log(1 - F_\ell(\mu_t)).$$

Since $F_\ell(\mu_t) \leq 2F(\mu_t)$ from Lemma 4, we have $U_t^h(\nu) \leq -\log(1 - 2F(\mu_t))$. Using the fact that for small x, $-\log(1-x) \leq x + x^2$, it follows that

$$U_t^h(\nu) \leq 2F(\mu_t)(1 + 2F(\mu_t)),$$

which establishes an upper bound. To obtain a lower bound, notice that since $F_h(\mu_t) \leq 2\mu_t F(\mu_t)$ and $F_\ell(\mu_t) \geq 2(1 - \mu_t)F(\mu_t)$, substituting these bounds into $e^{U_t^h(\nu)}$ yields

$$e^{U_t^h(\nu)} = \frac{1 - F_h(\mu_t)}{1 - F_\ell(\mu_t)} \geq \frac{1 - 2\mu_t F(\mu_t)}{1 - 2(1 - \mu_t)F(\mu_t)}.$$

Using that $1/(1-x) \geq 1 + x + x^2$ for $x \in [0,1)$ for the denominator,

$$\begin{aligned}
e^{U_t^h(\nu)} &\geq \big(1 - 2\mu_t F(\mu_t)\big)\big(1 + 2(1 - \mu_t)F(\mu_t) + (2(1 - \mu_t)F(\mu_t))^2\big) \\
&= 1 + (2 - 4\mu_t)F(\mu_t) + (4 - 12\mu_t + 8\mu_t^2)F(\mu_t)^2 - (8\mu_t - 16\mu_t^2 + 8\mu_t^3)F(\mu_t)^3 \\
&= 1 + 2F(\mu_t) - 4\mu_t F(\mu_t) + 2(2 - O(\mu_t))F(\mu_t)^2
\end{aligned}$$

Applying the logarithm to both sides and using $\log(1+x) \geq x - \frac{x^2}{2}$ for small x, we have

$$U_t^h(\nu) \geq \log\left(1 + 2F(\mu_t) - 4\mu_t F(\mu_t) + (4 - O(\mu_t))F(\mu_t)^2\right)$$

$$\geq 2F(\mu_t) - 4\mu_t F(\mu_t) + (2 - O(\mu_t))F(\mu_t)^2 + o(F(\mu)^2)$$

$$\geq 2F(\mu_t)\left(1 - 2\mu_t + (1 - O(\mu_t))F(\mu_t) + o(F(\mu_t))\right)$$

As $\mu_t \to 0$, $F(\mu_t) \to 0$, the terms $-2\mu_t$ and $(1 - O(\mu_t))F(\mu_t)$ both vanish. Hence, the above upper and lower bounds imply that $\lim_{\mu_t \to 0} \frac{U_t^h(\nu)}{2F(\mu_t)} = 1$. By the tail regularity assumption, we have $F(\mu_t) = \Theta(\mu_t^\alpha)$, and thus it follows that $U_t^h(\nu) = \Theta(\mu_t^\alpha)$. $\qquad\square$

The next lemma establishes the monotonicity of the expected absolute log-likelihood ratio and provides a decomposition of its increments, which will be useful in proving Lemma 3.

Lemma 6. *The sequence $(\mathbb{E}[|\ell_t|])_{t \geq 0}$ is nondecreasing. Furthermore, the increments satisfy*

$$\mathbb{E}[|\ell_{t+1}|] - \mathbb{E}[|\ell_t|] = \mathbb{E}[\Delta(\pi_t, \nu_t)],$$

where

$$\Delta(\pi_t, \nu_t) = f(\pi_t, \nu_t)\left(|\ell_t + U_t^\ell(\nu_t)| - |\ell_t|\right) + \left(1 - f(\pi_t, \nu_t)\right)\left(|\ell_t + U_t^h(\nu_t)| - |\ell_t|\right),$$

and $f(\pi_t, \nu_t) = \pi_t F_h(c_t(\nu_t)) + (1 - \pi_t)F_\ell(c_t(\nu_t))$.

Proof. Consider the function $g(x) = \left|\log \frac{x}{1-x}\right|$. Since g is convex on $(0,1)$, Jensen's inequality implies

$$\mathbb{E}[g(\pi_{t+1}) \mid H_t] \geq g(\mathbb{E}[\pi_{t+1} \mid H_t]),$$

where $H_t = (a_1, \ldots, a_{t-1})$ is the history of actions available to the planner at time t. Since (π_t) is a martingale, we have $\mathbb{E}[\pi_{t+1} \mid H_t] = \pi_t$. Thus,

$$\mathbb{E}[|\ell_{t+1}| \mid H_t] = \mathbb{E}[g(\pi_{t+1}) \mid H_t] \geq g(\pi_t) = |\ell_t|.$$

Taking the total expectations yields $\mathbb{E}[|\ell_{t+1}|] \geq \mathbb{E}[|\ell_t|]$.

To derive the decomposition, we condition on the planner's belief π_t and the induced social belief ν_t. Note that the probability that agent t chooses the low action is

$$\mathbb{P}[a_t = \ell \mid \pi_t, \nu_t] = \pi_t F_h(c_t(\nu_t)) + (1 - \pi_t)F_\ell(c_t(\nu_t)) =: f(\pi_t, \nu_t).$$

The probability of agent t choosing the high action is thus $1 - f(\pi_t, \nu_t)$. By the law of iterative expectations and the update rule for ℓ_t in (5.4), we have

$$\mathbb{E}[|\ell_{t+1}|] = \mathbb{E}[\mathbb{E}[|\ell_{t+1}| \mid \pi_t, \nu_t]]$$

$$= \mathbb{E}[f(\pi_t, \nu_t) \cdot |\ell_t + U_t^\ell(\nu_t)|] + \mathbb{E}[(1 - f(\pi_t, \nu_t)) \cdot |\ell_t + U_t^h(\nu_t)|].$$

Using the identity $|\ell_t| = f(\pi_t, \nu_t)|\ell_t| + (1 - f(\pi_t, \nu_t))|\ell_t|$, we obtain

$$\mathbb{E}[|\ell_{t+1}|] - \mathbb{E}[|\ell_t|] = \mathbb{E}[\Delta(\pi_t, \nu_t)],$$

where $\Delta(\pi_t, \nu_t)$ is defined as the conditional expected change in the absolute LLR:

$$\Delta(\pi_t, \nu_t) = f(\pi_t, \nu_t) \cdot (|\ell_t + U_t^\ell(\nu_t)| - |\ell_t|) + (1 - f(\pi_t, \nu_t)) \cdot (|\ell_t + U_t^h(\nu_t)| - |\ell_t|).$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\square$

Proof of Lemma 3. Let b_t be the action chosen by an agent with social belief ν_t receiving transfer τ_t. To simplify notation, throughout the proof, C denotes a constant that may change from line to line. To establish the desired upper bound on $\mathbb{E}[|\ell_{t+1}|] - \mathbb{E}[|\ell_t|]$, by Lemma 6, it suffices to show that for some $M < \infty$,

$$\mathbb{E}\big[\Delta(\pi_t, \nu_t)\big] \leq M \cdot \big(\mathbb{P}[b_t \neq \theta] + \mathbb{E}[|\tau_t|]\big).$$

To this end, observe first that by the triangle inequality, one has

(A.2) $$\Delta(\pi_t, \nu_t) \leq f(\pi_t, \nu_t) \cdot |U_t^\ell(\nu_t)| + (1 - f(\pi_t, \nu_t)) \cdot |U_t^h(\nu_t)|.$$

Fix some small $\varepsilon \in (0, 1/4]$ and let $\mu_t = c_t(\nu_t)$. We consider two cases: (i) the tail region where either $\mu_t \leq \varepsilon$ or $\mu_t \geq 1 - \varepsilon$ and (ii) the middle region where $\mu_t \in (\varepsilon, 1 - \varepsilon)$.

Case (i). Suppose $\mu_t \leq \varepsilon$. Conditioning on the social belief ν_t, since ν_t is a mean-preserving contraction of π_t, we have $\mathbb{E}[\pi_t \mid \nu_t] = \nu_t$. Thus, the conditional expectation of $f(\pi_t, \nu_t)$ is

$$\mathbb{E}[f(\pi_t, \nu_t) \mid \nu_t] = \mathbb{E}[\pi_t \mid \nu_t] \cdot F_h(\mu_t) + (1 - \mathbb{E}[\pi_t \mid \nu_t]) \cdot F_\ell(\mu_t) = \nu_t F_h(\mu_t) + (1 - \nu_t) F_\ell(\mu_t).$$

By Lemma 5 and Lemma 4, when $\mu_t \leq \varepsilon$, we have $|U_t^\ell(\nu_t)| \leq C \cdot |\log \mu_t|$ and $F_h(\mu_t) = \Theta(\mu_t^{\alpha+1})$ and $F_\ell(\mu_t) = \Theta(\mu_t^\alpha)$. Using the inequality $(1 - \nu_t) \leq \mu_t + |\tau_t|$ from Lemma 1, we obtain

$$\begin{aligned}
\mathbb{E}[f(\pi_t, \nu_t) \mid \nu_t] \cdot |U_t^\ell(\nu_t)| &\leq C\big(\nu_t \mu_t^{\alpha+1} + (\mu_t + |\tau_t|)\mu_t^\alpha\big)|\log \mu_t| \\
&\leq C\big(\mu_t^{\alpha+1}|\log \mu_t| + |\tau_t|\mu_t^\alpha|\log \mu_t|\big) \\
&\leq C(\mu_t + |\tau_t|),
\end{aligned}$$

where the last inequality follows since $x^\alpha |\log x| \to 0$ as $x \to 0$, for any $\alpha > 0$. By Lemma 5, $U_t^h(\nu_t) \leq C\mu_t^\alpha$, and since $\alpha \geq 1$, $U_t^h(\nu_t) \leq C\mu_t$ and thus it follows from (A.2) that

(A.3) $$\mathbb{E}[\Delta(\pi_t, \nu_t)\mathbb{1}(\mu_t \leq \varepsilon) \mid \nu_t] \leq C(\mu_t + |\tau_t|).$$

To relate μ_t to the mistake probability, recall from Lemma 1 that $|\mu_t - (1 - \nu_t)| \leq |\tau_t|$. If $\nu_t \leq 1/2$, the condition $\mu_t \leq \varepsilon$ implies $|\tau_t| \geq 1/2 - \varepsilon \geq 1/4$, so $\mathbb{1}(\nu_t \leq 1/2) \leq 4|\tau_t|$. Note that we can write

$$1 - \nu_t \leq \min\{\nu_t, 1 - \nu_t\} + \mathbb{1}(\nu_t \leq 1/2),$$

and by the triangle inequality,

$$\mu_t \leq (1 - \nu_t) + |\mu_t - (1 - \nu_t)| \leq (1 - \nu_t) + |\tau_t|.$$

Thus,
$$\mu_t \leq \min\{\nu_t, 1 - \nu_t\} + C|\tau_t|.$$

It then follows from (A.3) that
$$\mathbb{E}[\Delta(\pi_t, \nu_t) \cdot \mathbb{1}(\mu_t \leq \varepsilon) \mid \nu_t] \leq C \cdot (\min\{\nu_t, 1 - \nu_t\} + |\tau_t|).$$

Notice that $\min\{\nu_t, 1 - \nu_t\}$ equals the mistake probability for an agent with induced social belief ν_t and zero transfer. Let b_t^0 denote the action chosen by such an agent under zero transfer. Then,
$$\min\{\nu_t, 1 - \nu_t\} = \mathbb{P}[b_t^0 \neq \theta \mid \nu_t].$$

Since b_t^0 is the Bayes-optimal action for an agent with belief ν_t, any non-zero transfer τ_t that induces a different action $b_t \neq b_t^0$ can only increase the mistake probability: $\mathbb{P}[b_t^0 \neq \theta \mid \nu_t] \leq \mathbb{P}[b_t \neq \theta \mid \nu_t]$. Hence,

(A.4) $$\mathbb{E}[\min\{\nu_t, 1 - \nu_t\}] = \mathbb{P}[b_t^0 \neq \theta] \leq \mathbb{P}[b_t \neq \theta].$$

Finally, combining the above observations and taking expectations yields
$$\mathbb{E}[\Delta(\pi_t, \nu_t)\mathbb{1}(\mu_t \leq \varepsilon)] \leq C\big(\mathbb{P}[b_t \neq \theta] + \mathbb{E}[|\tau_t|]\big).$$

The same bound holds for the right tail $\mu_t \geq 1 - \varepsilon$ by symmetry.

Case (ii). Consider the case $\mu_t \in (\varepsilon, 1 - \varepsilon)$. In this region, μ_t is bounded away from 0 and 1, ensuring the updates $|U_t^h|$ and $|U_t^\ell|$ are bounded by a constant C. It follows from (A.2) that $\Delta(\pi_t, \nu_t) \leq C$ and thus
$$\mathbb{E}[\Delta(\pi_t, \nu_t) \cdot \mathbb{1}(\mu_t \in (\varepsilon, 1 - \varepsilon))] \leq C \cdot \mathbb{E}[\mathbb{1}(\mu_t \in (\varepsilon, 1 - \varepsilon))]$$

If $\mu_t \in (\varepsilon, 1-\varepsilon)$ and $\nu_t \leq \varepsilon/2$ (or $\nu_t \geq 1-\varepsilon/2$) then by Lemma 1, $|\tau_t| \geq |\mu_t - (1-\nu_t)| \geq \varepsilon/2$. Consequently, we can bound the indicator by
$$\mathbb{1}(\mu_t \in (\varepsilon, 1 - \varepsilon)) \leq \mathbb{1}(\nu_t \in (\varepsilon/2, 1 - \varepsilon/2)) + \mathbb{1}(|\tau_t| \geq \varepsilon/2)$$
$$\leq \frac{2}{\varepsilon}\min\{\nu_t, 1 - \nu_t\} + \frac{2}{\varepsilon}|\tau_t|.$$

Substituting these back into the expectation, we thus obtain
$$\mathbb{E}[\Delta(\pi_t, \nu_t) \cdot \mathbb{1}(\mu_t \in (\varepsilon, 1 - \varepsilon))] \leq C \cdot \mathbb{E}\left[\frac{2}{\varepsilon}\min\{\nu_t, 1 - \nu_t\} + \frac{2}{\varepsilon}|\tau_t|\right]$$
$$\leq C \cdot (\mathbb{P}[b_t \neq \theta] + \mathbb{E}[|\tau_t|]),$$

where the second inequality follows from (A.4). This concludes the proof of Lemma 3. $\square$

Proof of Theorem 1. Let a_t^x be the action chosen by the planner based on his belief π_t at time t. Let a_t^0 be the action chosen by the agent with private belief q_t and induced social belief ν_t under zero transfer. As before, b_t is the action chosen by an agent with induced social belief ν_t under transfer τ_t, and b_t^0 is the action chosen by the same agent

under zero transfer. To simplify notation, throughout the proof, C denotes a constant that may change from line to line.

Recall that p_t is the agent t's posterior belief after combining her private belief q_t with her induced social belief ν_t. From the perspective of the agent, note that $\min\{p_t, 1 - p_t\}$ equals her mistake probability under zero transfer, that is, $\min\{p_t, 1-p_t\} = \mathbb{P}[a_t^0 \neq \theta \mid p_t]$. Since transfers can only increase the mistake probability relative to the Bayes-optimal choice, we have

$$\mathbb{E}[\min\{p_t, 1 - p_t\}] = \mathbb{P}[a_t^0 \neq \theta] \leq \mathbb{P}[a_t \neq \theta].$$

By Lemma 2 (with zero transfers), there exists ε_F such that for all t,

$$\mathbb{P}[a_t^0 \neq \theta] \geq \varepsilon_F \cdot \mathbb{P}[b_t^0 \neq \theta].$$

Furthermore, since the planner's belief π_t is more informative than agent t's induced social belief ν_t, one has $\mathbb{P}[b_t^0 \neq \theta] \geq \mathbb{P}[a_t^x \neq \theta]$. Combining these yields

(A.5) $$\mathbb{P}[a_t \neq \theta] \geq \varepsilon_F \cdot \mathbb{P}[a_t^x \neq \theta].$$

Suppose $\alpha \geq 1$ and by assumption we have $F = \Theta(q^\alpha)$. Assume, towards a contradiction, that there exists a disclosure and transfer scheme such that learning is efficient and the expected sum of absolute transfers is finite:

$$\sum_{t \geq 1}^{\infty} \mathbb{P}[a_t \neq \theta] < \infty \quad \text{and} \quad \sum_{t \geq 1}^{\infty} \mathbb{E}[|\tau_t(\nu_t)|] < \infty.$$

By Lemma 3, for $\alpha \geq 1$, the incremental growth of the expected absolute log-likelihood ratio is bounded above by

$$\mathbb{E}[|\ell_{t+1}|] - \mathbb{E}[|\ell_t|] \leq C \cdot (\mathbb{P}[b_t \neq \theta] + \mathbb{E}[|\tau_t|])$$

(A.6) $$\leq C \cdot (\mathbb{P}[a_t \neq \theta] + \mathbb{E}[|\tau_t|]),$$

where the second inequality follows from Lemma 2. Summing (A.6) over all t, the hypothesis implies that the sequence $(\mathbb{E}[|\ell_t|])_{t \geq 1}$ is uniformly bounded:

$$\lim_{t \to \infty} \mathbb{E}[|\ell_t|] = \mathbb{E}[|\ell_1|] + \sum_{t \geq 1}(\mathbb{E}[|\ell_{t+1}|] - \mathbb{E}[|\ell_t|]) < \infty.$$

Notice that $\pi_t = \frac{e^{\ell_t}}{e^{\ell_t}+1}$ and thus the planner's mistake probability is

$$\mathbb{P}[a_t^x \neq \theta] = \mathbb{E}[\min\{\pi_t, 1 - \pi_t\}] = \mathbb{E}\left[\frac{1}{e^{|\ell_t|} + 1}\right] \geq \frac{1}{e^{\mathbb{E}[|\ell_t|]} + 1},$$

where the inequality follows from Jensen's inequality. As $\mathbb{E}[|\ell_t|]$ is bounded by some constant $L < \infty$ for all t, it follows that

$$\mathbb{P}[a_t^x \neq \theta] \geq \frac{1}{e^L + 1} =: \delta > 0.$$

By (A.5), this implies $\mathbb{P}[a_t \neq \theta] \geq \delta \cdot \varepsilon_F > 0$ for all t, which contradicts the assumption that $\sum_t \mathbb{P}[a_t \neq \theta] < \infty$. $\qquad\square$

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