

Efficiency in Games with Incomplete Information

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Motivation

- Pareto efficiency is a natural benchmark for social welfare: no individual can be made strictly better off without making another worse off.
- Under incomplete information, players possess private information that could improve collective welfare.
- However, individual incentives may lead them to distort what they know.
- **When is a feasible outcome Pareto efficient? When can strategic behavior lead to efficiency?**

Summary of Results

- Excessive randomization over action profiles leads to inefficiency.
 - **Necessary condition for efficiency (generic):**
 $\# \text{ action profiles across states} < \# \text{ players} + \# \text{ states}$.
- Two players: only pure or quasi-pure outcomes can be efficient.

Economics Applications

1. Cheap talk
2. Bayesian persuasion
3. Allocation problem without transfers

Finite game with incomplete information

- **Players:** $i \in \{1, \dots, k\}$.
- **States:** $\omega \in \Omega$ drawn from prior $p \in \text{int}(\Delta\Omega)$.
- **Actions:** $a = (a_1, \dots, a_k) \in A = \prod_{i=1}^k A_i$.
- **Payoffs:** $u_i : \Omega \times A \rightarrow \mathbb{R}$.
- **Types:** $t = (t_1, \dots, t_k) \in T = \prod_{i=1}^k T_i$, with $\pi : \Omega \rightarrow \Delta T$.

Outcomes and feasible payoffs

- **Outcome:** $\mu : \Omega \rightarrow \Delta A$.
- **Induced ex-ante payoff:**

$$u(\mu) = \sum_{\omega \in \Omega} p(\omega) \sum_{a \in A} \mu(a | \omega) u(\omega, a) \in \mathbb{R}^k.$$

- **Feasible payoffs (given p):**

$$F_p = \{u(\mu) \in \mathbb{R}^k : \mu : \Omega \rightarrow \Delta A\}.$$

The set F_p is a convex polytope; extreme points = pure outcomes.

Efficiency

- An outcome μ is **efficient** if there is no outcome η with $u(\eta) \geq u(\mu)$ and at least one strict inequality.
- μ is efficient \iff it maximizes a **strictly positive weighted sum** of players' ex-ante payoff:

$$\exists n \in \mathbb{R}_{++}^k \text{ s.t. } u(\mu) \in \arg \max_{x \in F_p} n^\top x.$$

Necessary Condition for Efficiency

Let $|\mu(\omega)|$ denote number of action profiles taken in state ω .

Generically, an outcome μ is efficient only if $\sum_{\omega} |\mu(\omega)| < k + |\Omega|$.

Two players: generically, efficient outcomes are pure or quasi-pure (binary randomization in exactly one state).

This condition does not depend on the prior, which action profiles are used, or the weight of randomization.

Ex-ante and ex-post efficiency: a geometric link

- **Minkowski sum decomposition:**

$$F_p = \sum_{\omega \in \Omega} p(\omega) F_\omega, \text{ where } F_\omega = \text{Co}\{u(\omega, a) : a \in A\}.$$

- **Ex-ante efficiency:** $u(\mu)$ lies on the Pareto frontier of F_p .
- **Ex-post efficiency:** $u(\mu | \omega)$ lies on the Pareto frontier of F_ω .
- **Key geometric result:**

$$\mu \text{ is efficient} \iff \exists n \in \mathbb{R}_{++}^k \text{ s.t. } u(\mu | \omega) \in \arg \max_{x \in F_\omega} n^\top x \quad \forall \omega \in \Omega$$

Intuition for the result

- Efficiency requires a **common** positive weight vector $n \in \mathbb{R}_{++}^k$ to support **all** states.
- In state ω , ex-post efficiency means that all actions played maximize the same linear objective $n^\top u(\omega, \cdot)$.
- Generically, each additional action profile imposes an additional independent linear constraint on the admissible weight vectors.
- Summing constraints across states, excessive randomization generically rules out any strictly positive weight vector.

Example: Sender–Receiver

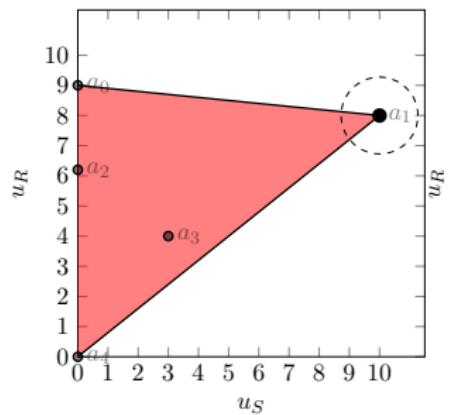
Let $\Omega = \{\omega_0, \omega_1\}$ and $A = \{a_0, a_1, a_2, a_3, a_4\}$. Sender–receiver payoffs (u_S, u_R) :

	a_0	a_1	a_2	a_3	a_4
ω_0	(0, 9)	(10, 8)	(0, 6.2)	(3, 4)	(0, 0)
ω_1	(0, 0)	(10, 4)	(0, 6.2)	(3, 8)	(0, 9)

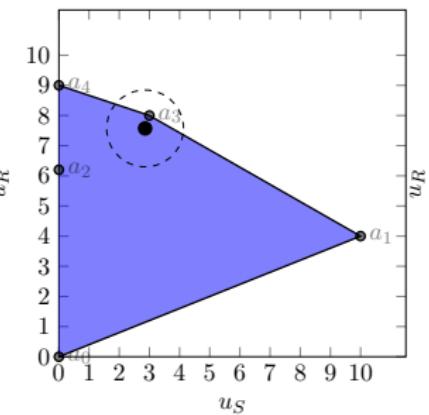
In this environment, we illustrate the efficiency properties of different outcomes.

Example 1: Ex-post inefficient

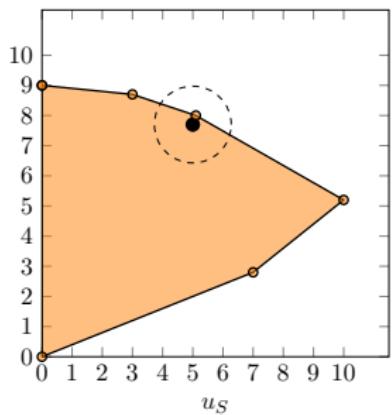
1. Outcome where action a_1 is taken in ω_0 , and actions a_1 and a_4 are taken in ω_1



F_{ω_0}



F_{ω_1}

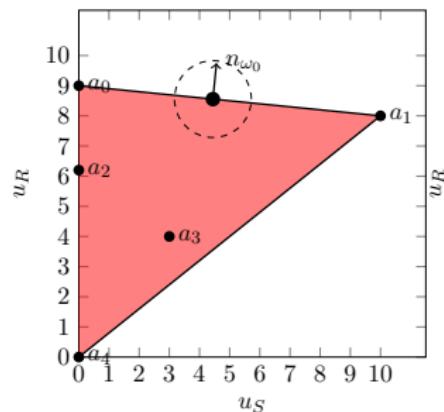


F_p

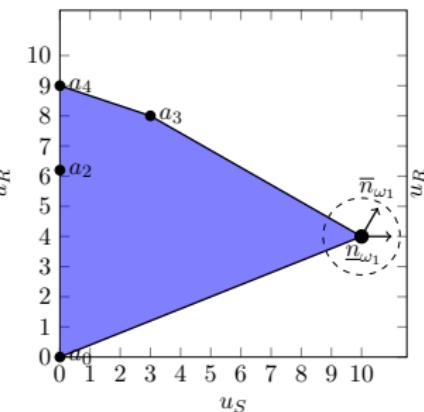
Ex-post inefficient in state $\omega_1 \Rightarrow$ inefficient

Example 2: Ex-post efficient but ex-ante inefficient

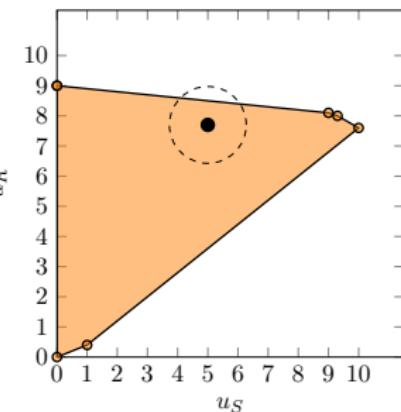
2. Outcome where actions a_0 and a_1 are taken in ω_0 and action a_1 is taken in ω_1



F_{ω_0}



F_{ω_1}

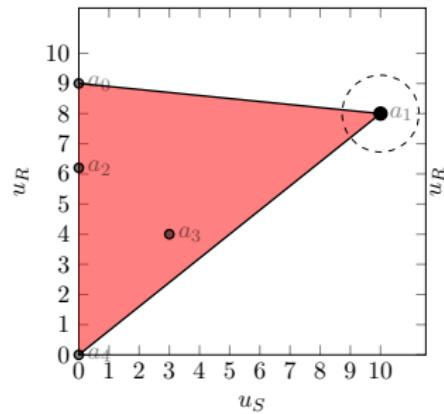


F_P

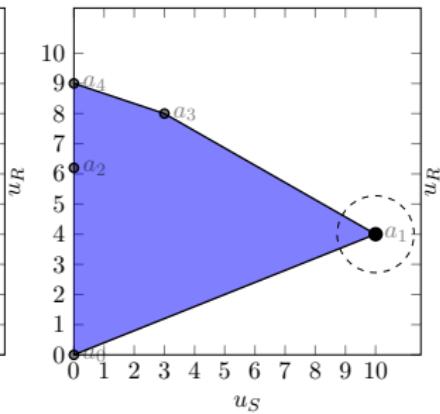
Ex-post efficient in both states but no common positive weight vector ($n_{\omega_0} \notin \text{cone}\{\underline{n}_{\omega_1}, \bar{n}_{\omega_1}\}$) \Rightarrow inefficient

Example 3: Ex-ante efficient

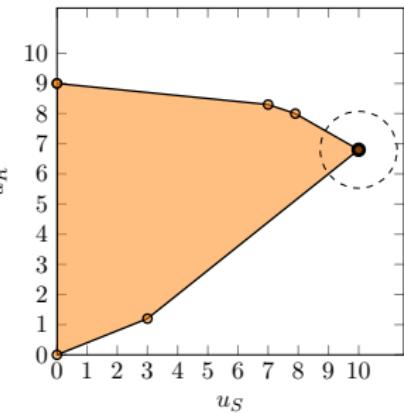
3. Outcome where only action a_1 is taken in both states



F_{ω_0}



F_{ω_1}



F_p

Outcome is ex-post efficient in both states and a common positive weight vector exists \Rightarrow efficient

Cheap Talk

Cheap Talk: Model

- State $\omega \in \Omega$ drawn according to prior $p \in \text{int}(\Delta\Omega)$.
- Sender observes ω and chooses a message $m \in M$ to send.
- Upon seeing m , receiver chooses an action $a \in A$.
- Results in payoffs $u_S(\omega, a)$ and $u_R(\omega, a)$.

When is a cheap talk outcome efficient?

Generically, a cheap talk outcome is efficient only if it is pure.

- Suppose a cheap talk equilibrium induces a **stochastic outcome**.
- Generically, efficiency implies that in some state ω^* no more than two actions a_1, a_2 are played.

Sender's equilibrium: $u_S(\omega^*, a_1) = u_S(\omega^*, a_2)$

Ex-post efficiency: $u_R(\omega^*, a_1) = u_R(\omega^*, a_2)$

- Non-generic condition! Any small perturbation breaks this knife-edge indifferences.

Cheap talk with state-independent sender payoff

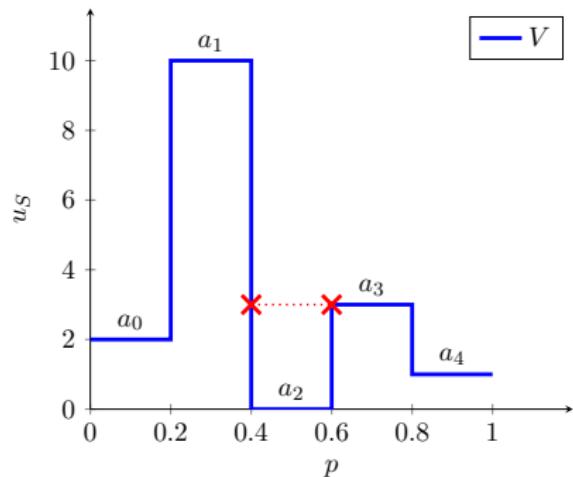
- Let a^* denote the sender's most preferred action among receiver's best responses.

Suppose $u_S(a)$. A cheap talk outcome is efficient $\iff a^*$ is induced with certainty.

- Any non-trivial cheap talk outcome, where communication affects the receiver's action, is inefficient.

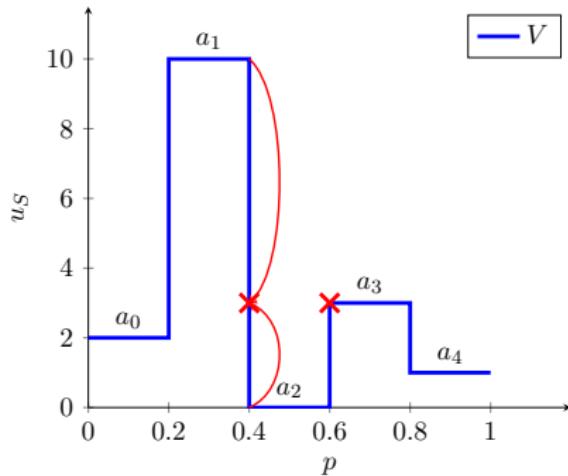
$u_S(a)$: Influential equilibrium

- (i) Suppose $p = 0.5$ and posteriors $q_1 = 0.4$ and $q_2 = 0.6$ are induced.



$u_S(a)$: Influential equilibrium

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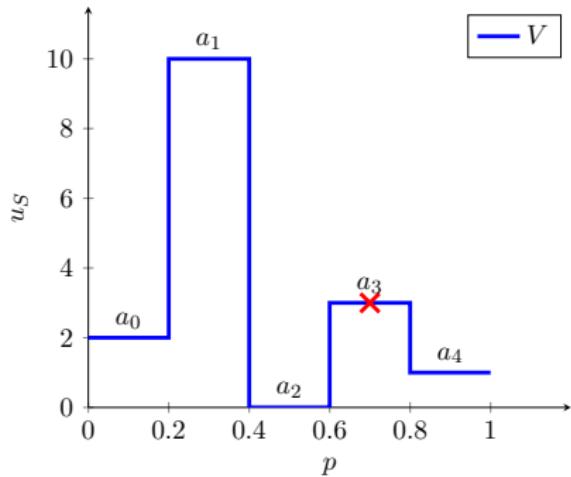


Given posterior $q_1 = 0.4$, the receiver must randomize between a_1 and a_2 to satisfy sender's indifference condition.

Both players strictly prefer a_1 over a_2 in state $\omega_0 \Rightarrow$ inefficient.

$u_S(a)$: Babbling equilibrium

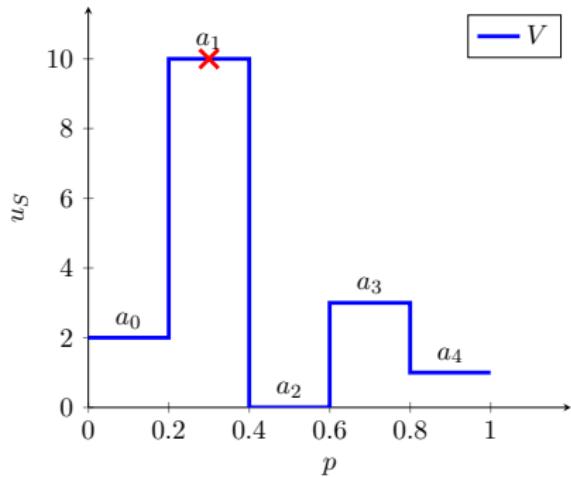
- (ii) Suppose $p = 0.7$ where the action $a_3 \neq a^*$ is played with certainty.



Again, both players strictly prefer a_1 over a_3 in state $\omega_0 \Rightarrow$ inefficient

$u_S(a)$: Babbling equilibrium

(iii) Suppose $p = 0.3$ where the action $a_1 = a^*$ is played with certainty.



This babbling equilibrium is the sender's most preferred outcome
⇒ efficient

Relation to previous literature

- **Pareto efficiency:** Rudov, Sandomirskiy, and Yariv, 2025, Arieli and Babichenko, 2012, Pradelski and Young, 2012, Marden, Young, and Pao, 2014, Jindani, 2022, ...
- **Strategic communication:** Ichihashi, 2019, Doval and Smolin, 2024, Crawford and Sobel, 1982, Kamenica and Gentzkow, 2011, ...

Summary

- We analyzed Pareto efficiency in finite games with incomplete information.
- Generically, excessive randomization across action profiles leads to inefficiency.
- Necessary condition (generic):

$$\# \text{ action profiles across states} < \# \text{ players} + \# \text{ states}.$$

- Incentive constraints often prevent efficiency in natural strategic environments.

Thank you! Any questions?