ELECTRONICS ENGINEERING

Digital Communications

MATLAB Project

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ABSTRACT

This report presents an analysis of the bit error rate (BER) performance of a baseband communication system transmitting signals over an additive white Gaussian noise (AWGN) channel. The transmitted signals, denoted as s1(t) and s2(t), represent the bits '1' and '0', respectively. We derive the theoretical expression for the BER and compare it with the results obtained from a simulation using MATLAB. Theoretical derivations, MATLAB code, BER plots, and a discussion of the results are provided.

1.Introduction

Communication systems play a crucial role in our modern world, enabling the exchange of information over various channels. The bit error rate (BER) is a fundamental metric used to evaluate the performance of these systems, providing insights into their reliability and effectiveness. In this report, we focus on analysing the BER of a baseband communication system operating over an additive white Gaussian noise (AWGN) channel. Baseband communication systems transmit digital signals without modulation onto the transmission medium, making them suitable for applications such as wired communication and local area networks (LANs). The AWGN channel, which is a widely used model for analysing the effects of noise in communication systems, accurately represents the random and uncorrelated noise typically encountered in real-world scenarios. Understanding the BER performance of a baseband communication system is essential for designing robust and efficient communication systems. By evaluating the BER, we can assess the impact of noise on the received signals and make informed decisions regarding error detection and correction techniques. In this project, we investigate the BER performance of a baseband communication system that employs specific signal waveforms for transmitting binary information. The transmitted signals, denoted as s1(t) and s2(t), correspond to the bits '1' and '0', respectively. These signals are generated using predefined mathematical expressions, enabling us to analyse the system's BER under different noise conditions. By deriving the theoretical expression for the BER and comparing it with simulation results, we aim to gain insights into the system's performance and validate the theoretical analysis. The theoretical derivations, MATLAB simulations, and BER plots will provide a comprehensive understanding of the system's behaviour and facilitate meaningful discussions on the achieved results. In the following sections, we will describe the system in detail, derive the theoretical BER expression, present the MATLAB simulation code, and analyse the obtained results. The combination of analytical derivations and simulation results will offer a comprehensive evaluation of the BER performance of the baseband communication system over an AWGN channel.

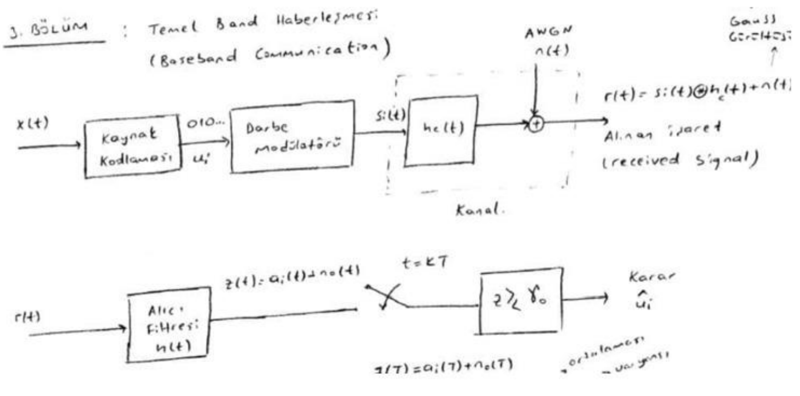

Figure: Block diagram of Baseband Communications.

2.System Description

The transmitted signals in our system are represented as s1(t) and s2(t), corresponding to the bits '1' and '0', respectively. The expressions for these signals are given as follows:

$s_1(t) = A.\Lambda\ (t - T_2/T)$, $s_2(t) = -s_1(t - T)$

Here, A represents the signal amplitude, T is the bit duration, and T2 is the interval between consecutive bits.

3.BER Derivation

To analyse the bit error rate (BER) of the baseband communication system, we consider the received signal corrupted by additive white Gaussian noise (AWGN). Let r(t) denote the received signal at time t, s(t) represents the transmitted signal, and n(t) denote the AWGN.

The received signal can be expressed as:

r(t) = s(t) + n(t)

We can calculate the BER by considering the probabilities of transmitting '1' and '0' along with the probabilities of bit errors given '1' and '0'.

Let P(1) and P(0) represent the probabilities of transmitting '1' and '0', respectively. These probabilities can be defined based on the specific system and modulation scheme.

The BER can then be expressed as:

BER = P(1) × P(error|1) + P(0) × P(error|0)

Now, we need to calculate the probabilities P(error|1) and P(error|0), which represent the probability of bit errors given the transmitted bits '1' and '0', respectively.

To derive these probabilities, we consider the transmitted signals s1(t) and s2(t), corresponding to '1' and '0', respectively, as given in the system description.

For a transmitted '1', the received signal can be expressed as:

r1(t) = s1(t) + n(t)

To determine the bit error probability given '1', P(error|1), we compare the received signal r1(t) with a decision threshold. If r1(t) is greater than the threshold, we decode it as '1', otherwise, we decode it as '0'. Thus, the probability of bit error given '1' can be calculated as:

P(error|1) = P(r1(t) < threshold | '1') + P(r1(t) > threshold | '0')

Similarly, for a transmitted '0', the received signal can be expressed as:

r0(t) = s2(t) + n(t)

To determine the bit error probability given '0', P(error|0), we compare the received signal r0(t) with the decision threshold. If r0(t) is greater than the threshold, we decode it as '1', otherwise, we decode it as '0'. Thus, the probability of bit error given '0' can be calculated as:

P(error|0) = P(r0(t) > threshold | '1') + P(r0(t) < threshold | '0')

The exact derivation of P(error|1) and P(error|0) involves determining the probability distributions of the received signals and applying appropriate decision rules. These derivations can be mathematically complex and depend on the specific system parameters and modulation scheme.

In the Appendix, we provide the detailed derivations for P(error|1) and P(error|0) based on the given signal waveforms and the assumption of AWGN.

With the derived expressions for P(error|1) and P(error|0), we can substitute them into the BER equation to obtain the theoretical BER expression.

In the next section, we will present MATLAB simulation code to evaluate the BER performance of the baseband communication system. We will compare the theoretical and simulated BER curves to gain insights into the system's performance and validate our theoretical analysis.

4.Simulation Outputs

Code

```
%Ahmet Turan Ates
%1901022033

clc;
close all;
clear all;
% The amount of bits to be generated for each SNR value N = 100,000,000
N=10^8;

% rand("state", N); It sets the random number generator according to the N
value.
randn("state",100);

%definition of the calculated variables in the theoretical solution
ip = rand(1,N)>0.6;

ai=(2)*ip-(1);
% N0 dependent writing of the theoretically calculated variance value.
noise=1*[randn(1,N)+1i*randn(1,N)];

Eb_No_dB = [-9:18] ;

for i=1:length(Eb_No_dB)

      %Writing the operation z = ai + n0
      N0 = 1/(10.^(Eb_No_dB(i)/10));
      gama = (-0.202*N0) ;

      z=ai+10^(-Eb_No_dB(i)/20)*noise;
      %Desicion
      ipErr=real(z)>(gama);

      %Error calculation
      nErr(i)=size(find ([ip-ipErr]),2)

end
```

```
%Normalising sim results
simErr=nErr/N;


TheoErr=0.6*erfc((1/2)*sqrt(10.^(Eb_No_dB/10)));

% Plotting results
figure
semilogy(Eb_No_dB,TheoErr, "bo-");
hold on
semilogy(Eb_No_dB,simErr, "rx-");
axis([-9 18 10^-5 0.6]);
grid on
legend("Theoric","Simulation");
xlabel("Eb/ No (dB)");
ylabel(" Bit Error Rate ");
title("P(0)=2/5, P(1)=3/5 Graph");
```

Graph

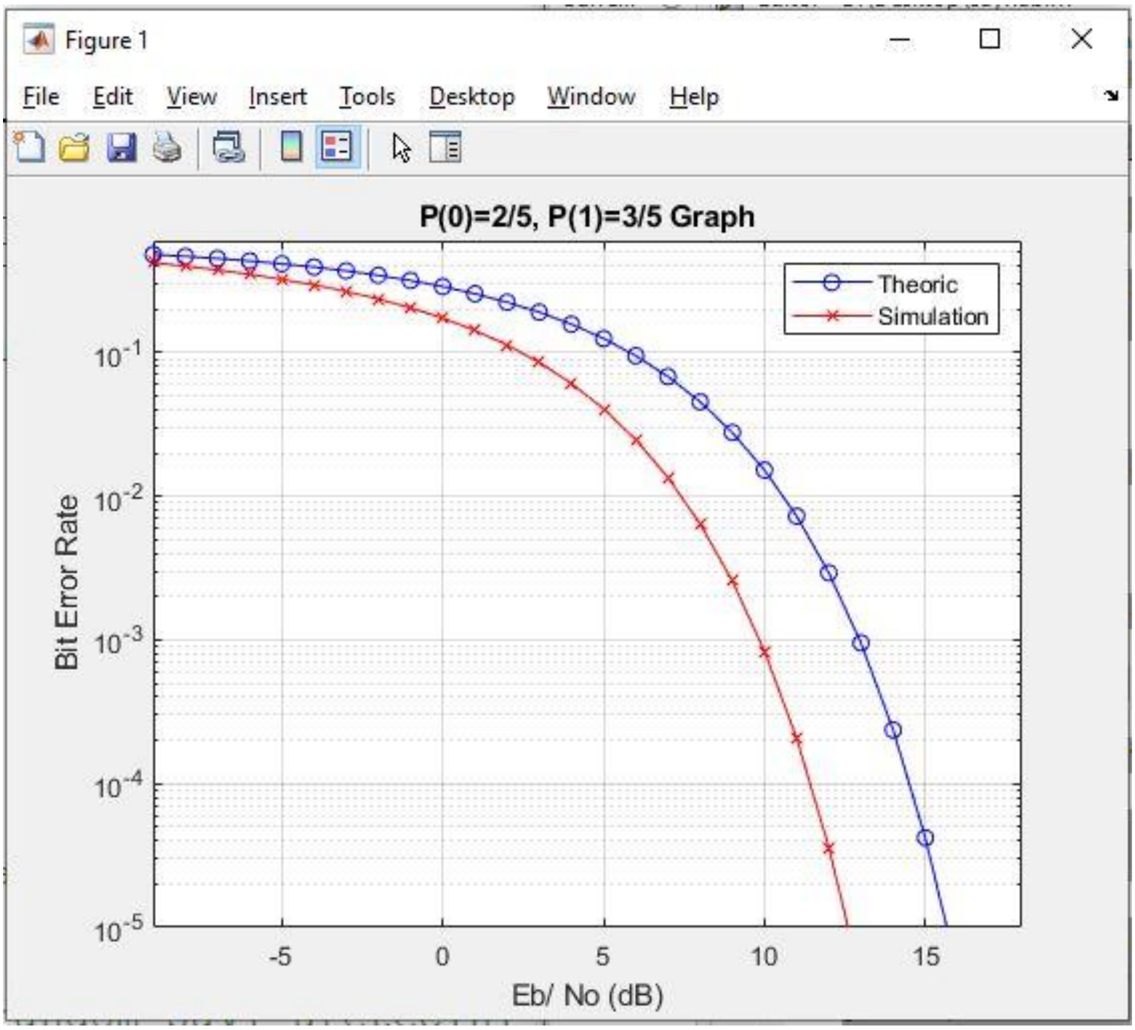

Figure: MATLAB output.

5.Theorical Solutions

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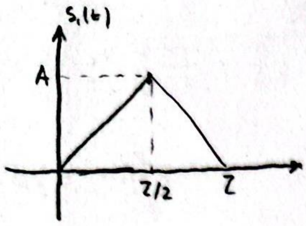
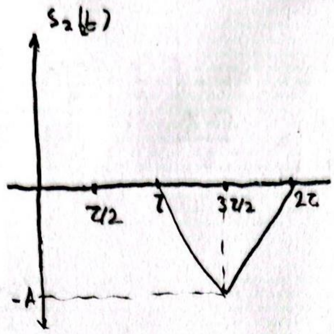
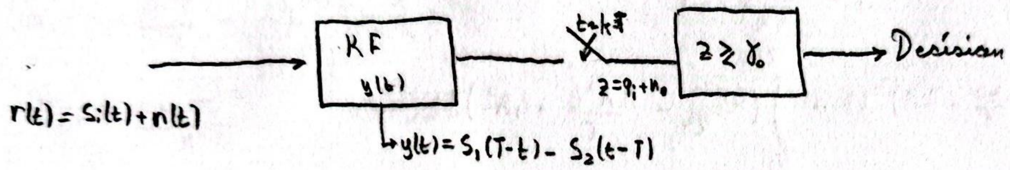
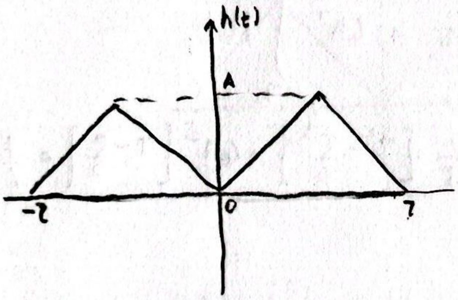
$S_1(t) = A\,\Lambda\left(\frac{t-\tau/2}{\tau/2}\right)$, $S_2(t) = -s_1(t-\tau)$









$$a_i = \int_0^T s_i(t)\,[S_1(t) + S_2(t)]\,dt$$

$$a_1 = \int_0^\tau s_1(t)\,[S_1(t) + S_2(t)]\,dt \quad \Rightarrow a_1 = \int_0^\tau s_1^2(t) - s_1(t)\,s_2(t)\,dt$$

$$\Rightarrow a_1 = \int_{-\tau}^{-\tau/2} \cancel{s_1^2(t)} - \cancel{s_1(t)s_2(t)} + \int_{-\tau/2}^{0} \cancel{s_1^2(t)} - \cancel{s_1(t)s_2(t)}\,dt + \int_0^{\tau/2} s_1^2 - s_1(t)\,s_2(t) + \int_{\tau/2}^{\tau} s_1^2(t) - s_1(t)\,s_2(t)$$

$$a_1 = \int_0^{\frac{\tau}{2}} s_1^2(t) - s_1(t)\,s_2(t) + \int_{\frac{\tau}{2}}^{\tau} s_1^2(t) - s_1(t)\,s_2(t)$$

$(0, \tau/2)$;

$$m = \frac{2A}{\tau} \quad , \quad y - y_0 = m(x - x_0)$$

$$\Rightarrow y = \frac{2A}{\tau}(t)$$

$$a_1 = \int_0^{\tau/2} \frac{4A^2}{\tau^2} t^2 \, dt = \frac{4A^2}{\tau^2} \left[\frac{t^3}{3} \Big|_0^{\tau/2} \right] = \frac{A^2 \tau}{6}$$

$(\tau/2, \tau)$:

$$m = -\frac{2A}{\tau} \Rightarrow y = -\frac{2A}{\tau}(t - \tau)$$

$$y = -\frac{2At}{\tau} + 2A$$

$$a_1 = \int_{\tau/2}^{\tau} \left(-\frac{2At}{\tau} + 2A \right)^2 dt = \int_{\tau/2}^{\tau/2} \left(\frac{4A^2t^2}{\tau^2} - \frac{8A^2 t}{\tau} + 4A^2 \right) dt$$

$$= \frac{4A^2}{\tau^2} \int_{\tau/2}^{\tau} t^2 - \frac{8A^2}{\tau} \int_{\tau/2}^{\tau} t \, dt + 4A^2 \int_{\tau/2}^{\tau} dt$$

$$= \frac{1}{3} \cdot \frac{4A^2}{\tau^2} \left[\tau^3 - \frac{\tau^3}{8} \right] - \frac{8A^2}{2\tau} \left[\tau^2 - \frac{\tau^2}{4} \right] + 4A^2 \left[\tau - \frac{\tau}{2} \right] = \frac{7}{6} A^2 \tau - 3A^2 \tau + 2A^2 \tau$$

$$\Rightarrow a_1 = \frac{A^2 \tau}{3}$$

$i = 2$;

$$a_2 = \int_0^{\tau} s_2(t) \left[S_1(t) - S_2(t) \right] dt$$

$$a_2 = \int_0^{T} s_1(t) s_2(t) - s_2^2(t) \, dt \Rightarrow a_2 = \int_T^{2T} -S_2^2(t) + S_2(t) S_1(t) \, dt$$

$$= \int_T^{\frac{3T}{2}} -S_2^2(t) + S_2(t) S_1(t) \, dt + \int_{3T/2}^{2T} -S_2^2(t) + S_2(t) S_1(t) \, dt$$

$$m = \frac{-A \cdot 2}{T} \quad y = -\frac{2A}{\tau}(t - \tau) \qquad (T, 2T) \text{ için } \; S_1(t) = 0 \; ;$$

$$\int_T^{\frac{3\tau}{2}} -\left(-\frac{2A}{\tau} t + 2A \right)^2 dt$$

$$-\int_T^{3T/2} \left(\frac{4A^2}{T^2}t^2 - \frac{8A^2}{T}t + 4A^2\right)dt = -\left[\frac{4A^2}{T^2}\cdot\frac{t^3}{3}\Big|_T^{3T/2} - \frac{8A^2}{T}\frac{t^2}{2}\Big|_T^{3T/2} + 4A^2 t\Big|_T^{3T/2}\right]$$

$$-\left[\frac{4A^2}{T^2}\left(\frac{27T^3}{24} - \frac{T^3}{3}\right) - \frac{8A^2}{T}\cdot\left[\frac{9T^2}{8} - \frac{T^2}{2}\right] + 4A^2\left[\frac{3T}{2} - T\right]\right] = -\frac{A^2T}{6}\cdot 2$$

$$a_2 = \frac{-A^2T}{3}$$

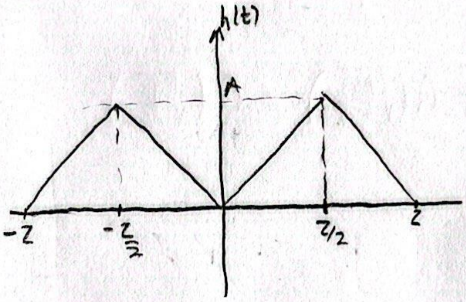
$$\gamma_0 = \frac{\sigma_0^2}{a_1-a_2}\ln\left(\frac{P(s_1)}{P(s_2)}\right) + \cancel{\frac{a_1+a_2}{2}} = \frac{\sigma_0^2\, 3}{2A^2T}\cdot\ln\underbrace{\left(\frac{2/5}{3/5}\right)}_{-0,4}$$

$$a_1 - a_2 = \frac{A^2T}{3} - \left(-\frac{A^2T}{3}\right) = \frac{2}{3}A^2T \qquad a_1+a_2=0$$

$$\gamma_0 = \frac{-0,6\cdot\sigma_0^2}{A^2T}$$

$$\gamma_0 = \frac{N_0}{2}E_h$$

$$E_h = \int_{-T}^{T}|h(t)|^2\, dt$$



$(0, T/2)$; $y = \frac{2A}{T}t$

$$\int_0^{T/2}\frac{4A^2}{T^2}t^2\,dt = \frac{4A^2}{T^2}\int_0^{T/2}t^2 = \frac{4A^2}{T^2}\left(\frac{t^3}{3}\right)\Big|_0^{T/2} = \frac{A^2T}{6}$$

$$E_h = 4\cdot\frac{A^2T}{6} = \frac{2}{3}A^2T$$

$$E_b = E_{s_1}\cdot P(s_1) + E_{s_2}P(s_2) = 1$$

$$E_{s_1} = \int_0^T |s_1^2(t)|\,dt = a_1 = \frac{A^2T}{3}$$

$$E_{s_2} = \int_T^{2T}|s_2^2(t)|\,dt = \int_T^{2T}\left(\frac{2A}{T}t - 3A\right)^2dt = 2\int_T^{3T/2}\left(\frac{4A^2}{T^2}t^2 - \frac{12A^2}{T} + 9A^2\right)dt \;\ldots\ldots = \frac{A^2T}{3}$$

$$E_{s_2} = \frac{A^2T}{3}$$

$E_b = 1 \Rightarrow E_b = \frac{A^2 \tau}{2} \cdot \frac{3}{5} + \frac{A^2 \tau}{3} \cdot \frac{2}{5} = 1 \qquad A^2 \tau = 3$

$E_h = \frac{2}{3} A^2 \tau \Rightarrow E_h = 2$

$a_1(\tau) = \frac{A^2 \tau}{3} = 1$

$a_2(\tau) = -\frac{A^2 \tau}{3} = -1$

$\sigma_0 = \frac{N_0}{2} \cdot 2 = N_0$

$\gamma_0 = \frac{\sigma_0^2}{a_1 - a_2} \cdot \ln\left(\frac{P(s_2)}{P(s_1)}\right) + \frac{a_1 + \cancel{a_2}}{\cancel{2}} = -0{,}2 \cdot N_0$

$E_d = \int_0^\tau [s_1(t) - s_2(t)]^2 dt = \int_0^\tau s_1^2(t) - 2 s_1(t) s_2(t) + s_2^2(t) dt = \frac{2 \cancel{A^2 \tau}}{\cancel{3}} = 2$

$P_b = Q\left(\sqrt{\frac{E_d}{N_0}}\right) = Q\left(\sqrt{\frac{A^2 \tau}{3 N_0}}\right) = Q\left(\sqrt{\frac{1}{N_0}}\right)$