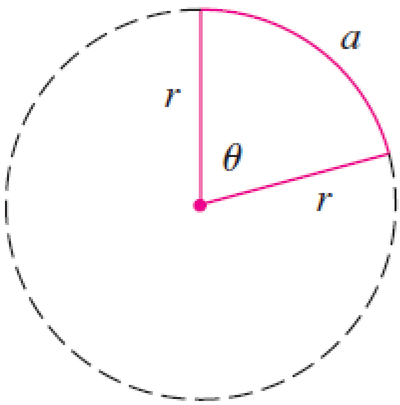
MAT150 - Summer 2023

Phd. Anabela Romina Turlione
Digipen, Bilbao

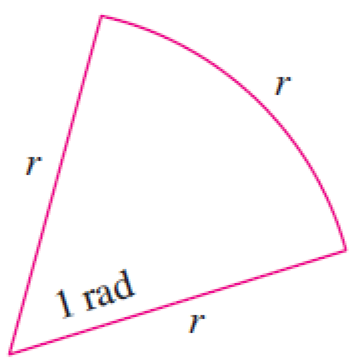
Trigonometry Review

Angles

A radian is the angle contained by an arc of length equal to the radius of the circle.



$$\theta(rad) = \frac{a}{r} \tag{1}$$



- $\pi \; rad = 180^{\circ}$
- $1 \; rad = 180/\pi \sim 57^{\circ}$.

Correspondence between degree and radian measures of some common angles

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$270°$	$360°$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

Standart Position of an angle

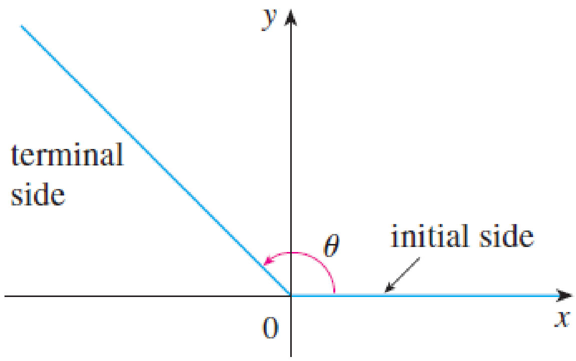


FIGURE 3 $\theta \geqslant 0$

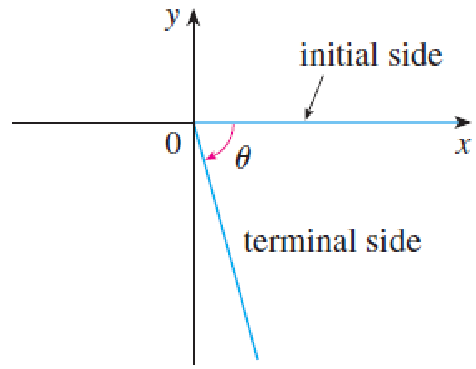
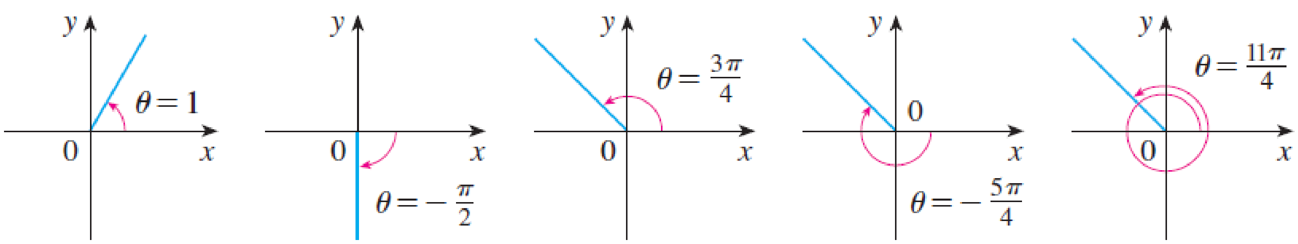


FIGURE 4 $\theta < 0$

Examples:



The Trigonometric Functions

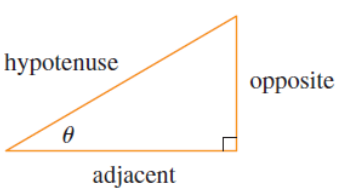


FIGURE 6

4

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

This definition doesn't apply to obtuse or negative angles.

For a general angle in **standard position**...

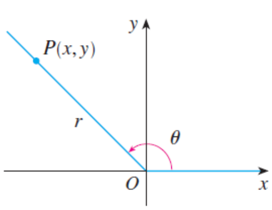


FIGURE 7

5

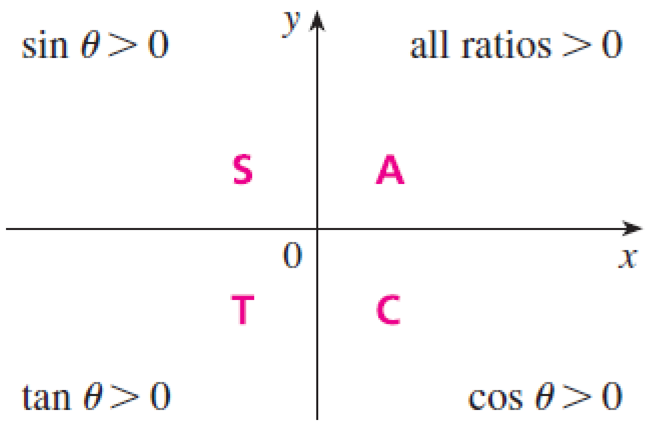
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

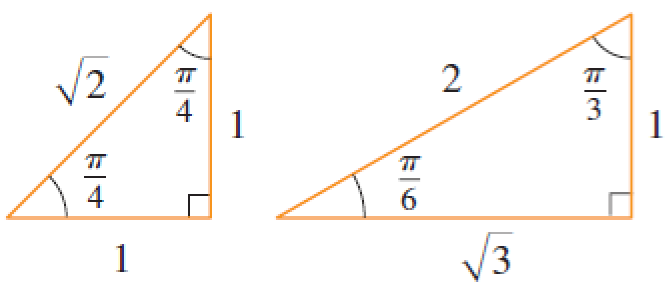
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

- sec and tan are undefined when $x = 0$.
- csc and cot are undefined when $y = 0$.

"**A**ll **S**tudents **T**ake **C**alculus" rule for signs



Exact trigonometric ratios for certain angles



$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan\frac{\pi}{4} = 1 \qquad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

Most common angles:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1

Exercise 1

Compute the values of the following trigonometrical functions according to the quadrant they belong to and using the table of “most common angles”:

1. $\cos(2\pi/3)$
2. $\cos(7\pi/6)$
3. $\sin(-5\pi/4)$
4. $\sin(5\pi/3)$

In [1]:
```
import numpy as np
#Check results
print('1',np.cos(2*np.pi/3),-np.cos(np.pi/3))
print('2',np.cos(7*np.pi/6),-np.cos(np.pi/6))
print('3',np.sin(-5*np.pi/4),np.sin(np.pi/4))
print('4',np.sin(5*np.pi/3),-np.sin(np.pi/3))
```

```
1 -0.4999999999999998 -0.5000000000000001
2 -0.8660254037844388 -0.8660254037844387
3 0.7071067811865475 0.7071067811865476
4 -0.8660254037844386 -0.8660254037844386
```

Trigonometric identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trigonometric identities

1. $\sin^2\theta + cos^2\theta = 1$
2. $\tan^2\theta + 1 = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$

Exercise 2

1. Calculate all the trigonometric functions of the angle α, given that $\cos\alpha = \sqrt{3}/2$, without computing α.
2. Calculate all the trigonometric functions of the angle α, given that $\tan\alpha = 3$, without computing α.

The identities:

- $\sin(-\theta) = -\sin(\theta)$
- $\cos(-\theta) = \cos(\theta)$

show that the sine is an **odd** function and cosine is an **even** function.

The identities:

- $\sin(\theta + 2\pi) = \sin(\theta)$
- $\cos(\theta + 2\pi) = \cos(\theta)$

show that the sine and cosine are periodic functions with period $T = 2\pi$.

Exercise 3

Find the set of values of x that satisfies the equations:

1. $\sin(x) = 1/2$
2. $\cos(x) = -\sqrt{3}/2$
3. $\sin(x)\cos(3x) + \cos(3x)\sqrt{2}/2$

Sum and substraction of sin and cos

1. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
2. $\cos(x + y) = \cos x \cos y - \sin x \sin y$

Substituting $-y$ for y in 1 and 2:

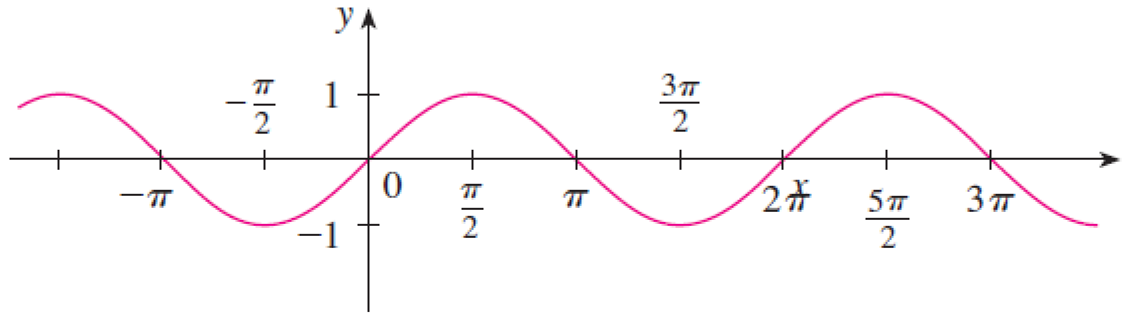
1. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
2. $\cos(x - y) = \cos x \cos y + \sin x \sin y$

Exercise 4

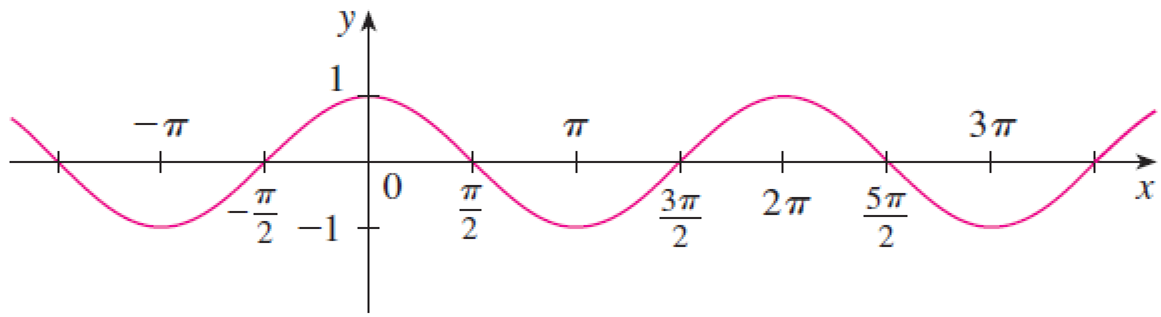
Proof the following expressions:

1. $\sin 2x = 2 \sin x \cos x$
2. $\cos 2x = \cos^2 x - \sin^2 x$
3. $\cos 2x = 1 - 2 \sin^2 x$
4. $\cos 2x = 2 \cos^2 x - 1$

Graph of the Trigonometric Functions



(a) $f(x) = \sin x$



(b) $g(x) = \cos x$

Properties:

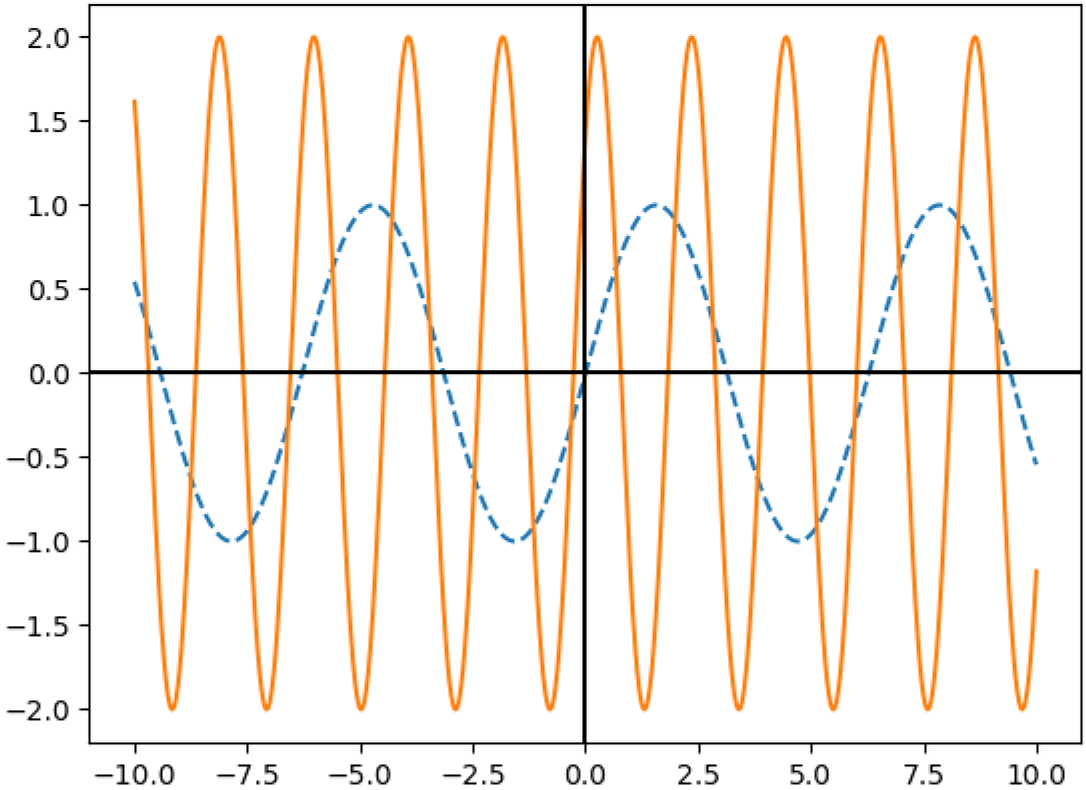
- They are periodic $\rightarrow$ $f(x+T) = f(x)$, where $T = 2\pi$ is the period.
- Range: $[-1, 1]$
- domain: $(-\infty, \infty)$
- zeros of $\cos(x)$: $(2n+1)\pi/2$, $n = \ldots -1, 0, 1, 2, 3 \ldots$
- zeros of $\sin(x)$: $n\pi$, $n = \ldots -1, 0, 1, 2, 3 \ldots$

In general, a sine or a cosine (an oscillation or a wave) is represented as $y = A\sin(\omega x + \theta)$, where

- A is the amplitude and tell us about its range.
- ω is the frequency and tell us about its velocity of oscillation and its period.
- θ is the phase, and tell us about its delay.

In [2]:
```
import matplotlib.pyplot as plt

x=np.linspace(-10,10,1000)
A=2
w=3
theta=np.pi/4

plt.plot(x,np.sin(x),'--')
plt.plot(x,A*np.sin(w*x+theta))
plt.axhline(y = 0, color = 'k', linestyle = '-')
plt.axvline(x = 0, color = 'k', linestyle = '-')
# plt.legend(['$sin(x)$','$Asin(\omega x+\Theta)$'])
```

Out[2]:
```
<matplotlib.lines.Line2D at 0x18651db5d20>
```



Exercise 5

Sketch the functions and find its zeroes, domain and range.

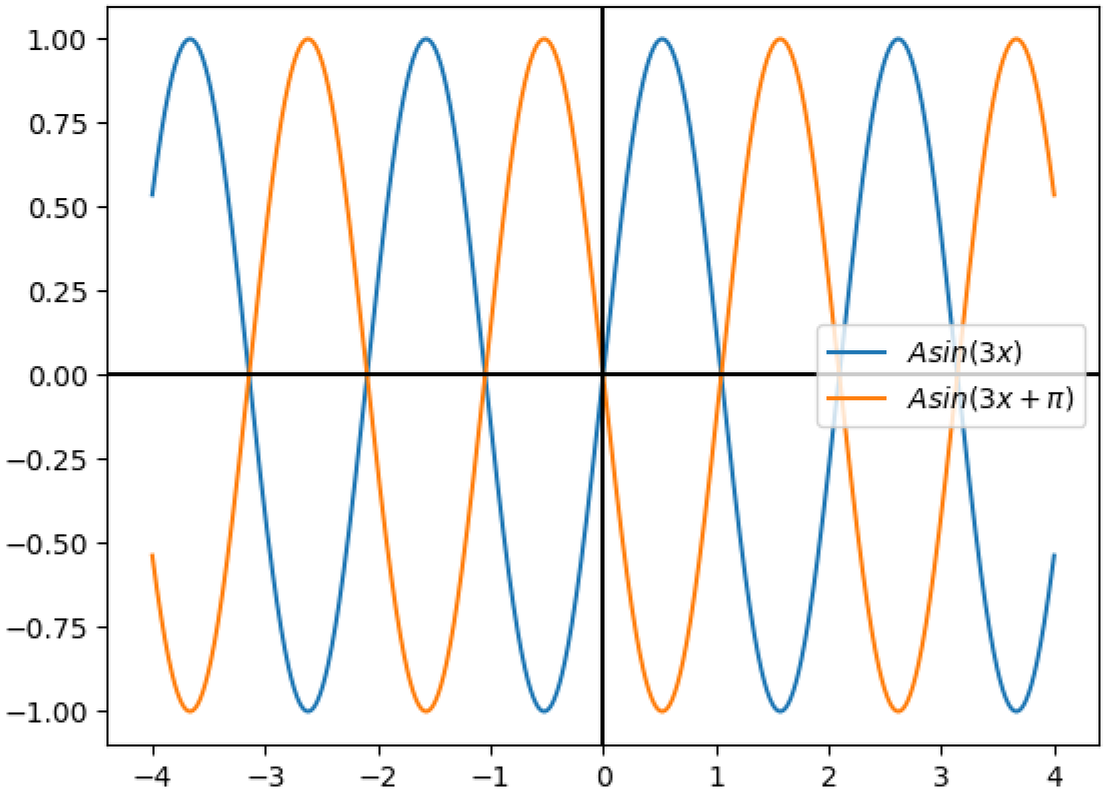
1. $y = \sin(2x)$
2. $y = \sin(x/3)$
3. $y = A\sin(3x - \pi)$ (Final Spring 2018)
4. $y = \tan(x)$
5. $y = \sec(x)$
6. $y = \csc(x)$
7. $y = \csc(x/2 - \pi)$

In [3]:

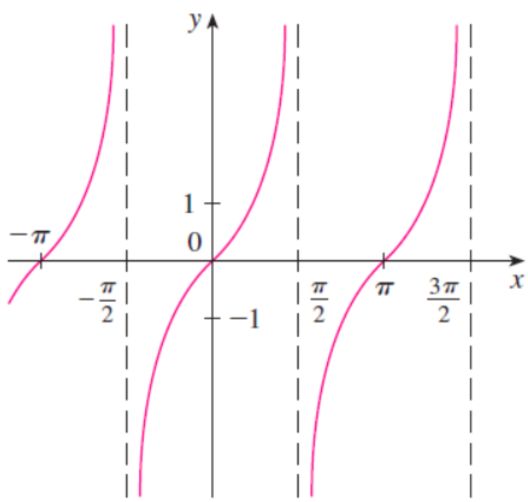
```
#Check 3.
x=np.linspace(-4,4,1000)

A=1

plt.plot(x,A*np.sin(3*x))
plt.plot(x,A*np.sin(3*x-np.pi))
plt.axhline(y = 0, color = 'k', linestyle = '-')
plt.axvline(x = 0, color = 'k', linestyle = '-')
plt.legend(['$Asin(3x)$','$Asin(3x+\pi)$'])
```

Out[3]:

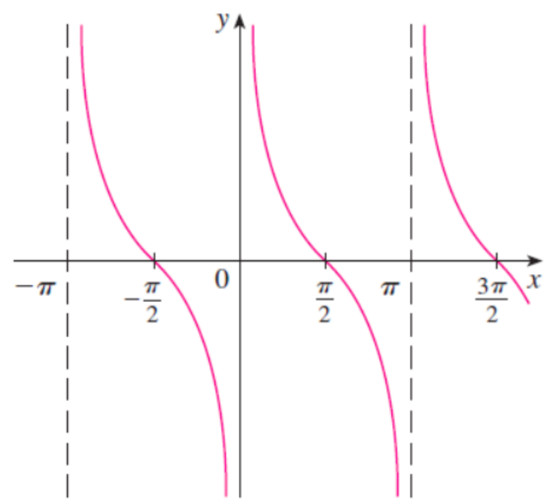
```
<matplotlib.legend.Legend at 0x18654c6e3e0>
```



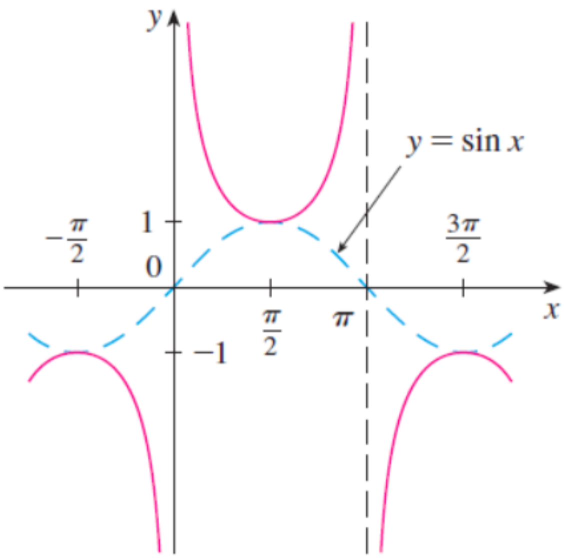
More Plots...



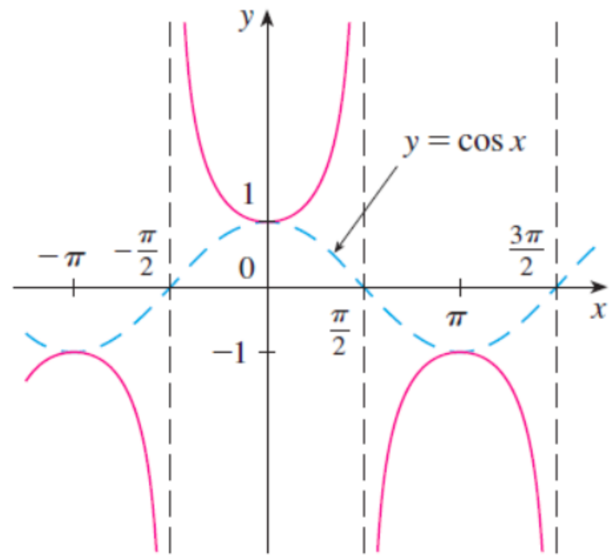
(a) $y = \tan x$



(b) $y = \cot x$



(c) $y = \csc x$



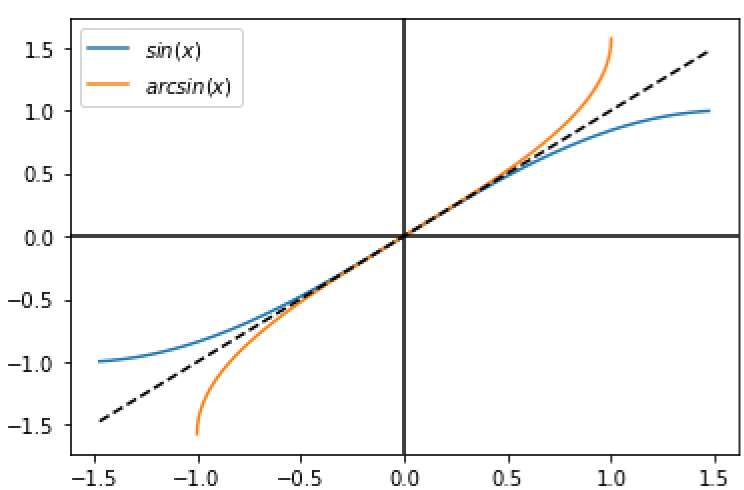
(d) $y = \sec x$

Inverse trigonometric functions

- They are not one-to-one.
- We have to restrict the domain.

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt

x=np.linspace(-np.pi/2+0.1,np.pi/2-0.1,1000)
x1=np.linspace(-1,1,1000)

plt.plot(x,np.sin(x))
plt.plot(x1,np.arcsin(x1))
plt.plot(x,x,'--',color='k')

plt.axhline(y = 0, color = 'k', linestyle = '-')
plt.axvline(x = 0, color = 'k', linestyle = '-')

plt.legend(['$sin(x)$','$arcsin(x)$'])
```

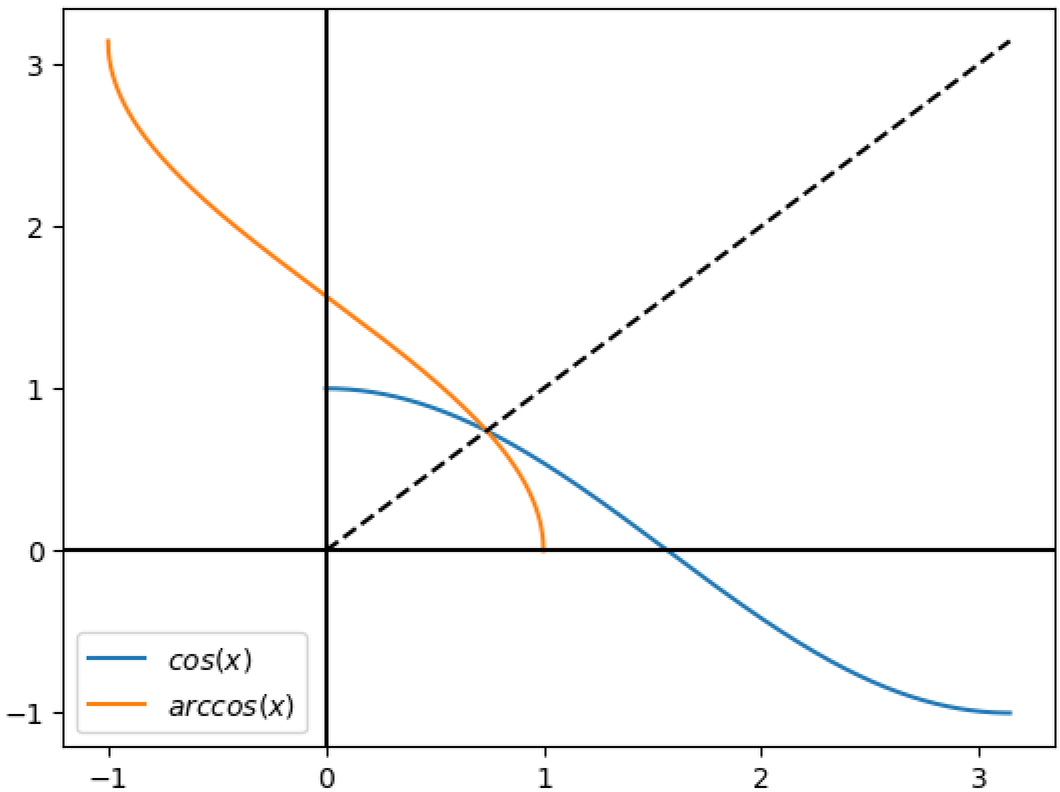
Out[2]:
```
<matplotlib.legend.Legend at 0x1efa1e0ccd0>
```



In [5]:
```
x=np.linspace(0,np.pi,1000)
x1=np.linspace(-1,1,1000)
```

```
plt.plot(x,np.cos(x))
plt.plot(x1,np.arccos(x1))
plt.plot(x,x,'--',color='k')

plt.axhline(y = 0, color = 'k', linestyle = '-')
plt.axvline(x = 0, color = 'k', linestyle = '-')
plt.legend(['$cos(x)$','$arccos(x)$'])
```

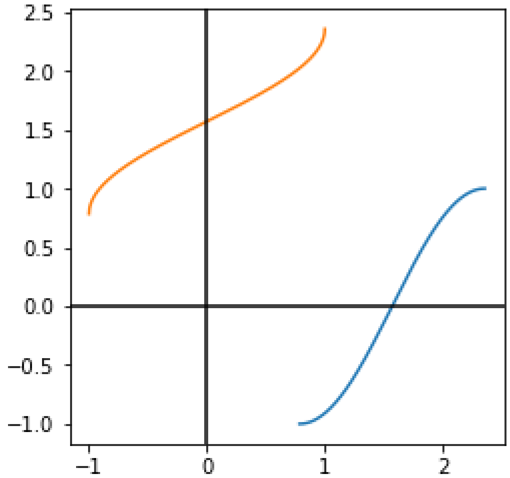
Out[5]: <matplotlib.legend.Legend at 0x1ccdf455ed0>



Exercise 6

1. Sketch $y = \sin(2x - \pi)$ and $y = \arcsin(2x - \pi)$. Find their domain and range.
2. Sketch $y = \arctan(x/2)$. Indicate its domain and range.
3. Find the inverse of $y = 2\sec(3x - \pi)$ and sketch it. Indicate its domain. (Final Spring 2021)

In [3]:
```
#Check 1.
x=np.linspace(np.pi/4,3*np.pi/4,1000)
x1=np.linspace(-1,1,1000)

plt.plot(x,np.sin(2*x-np.pi))
plt.plot(x1,(np.arcsin(x1)+np.pi)/2)
plt.axhline(y = 0, color = 'k', linestyle = '-')
plt.axvline(x = 0, color = 'k', linestyle = '-')
# plt.legend(['$Asin(3x)$','$Asin(3x+\pi)$'])
ax = plt.gca()
ax.set_aspect('equal')
```



In [ ]: