PHY115

Week2: Motion Along a Straight Line

Digipen

Spring 2021

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- We introduce the physical quantities velocity and acceleration.

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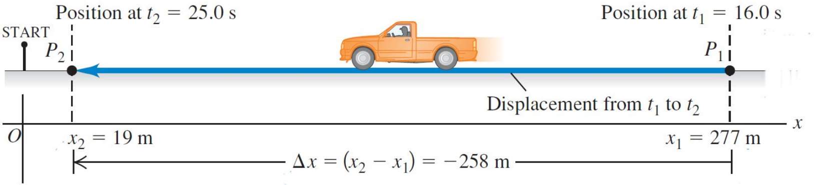
- We introduce the physical quantities velocity and acceleration.
- Both are vectors, they have magnitud and direction.

Average Velocity

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \tag{1}$$



Position as a function of time

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- We can represent the variation of the position respect to the time as a function: $x = x(t)$.
- $x(t)$ *Does not* represent the object's path in space.
- Even for a straight-line motion, $x(t)$ may not be a strigh line.

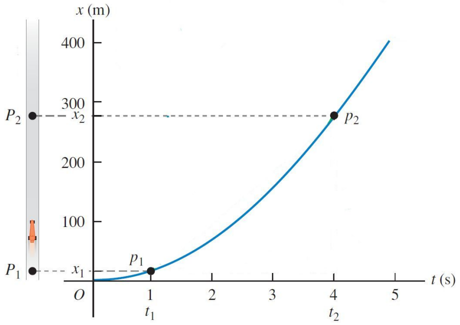


Figure: Position vs. time. Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Plot of the average velocity

- The average velocity is the *slope* of the line $p_1 p_2$.
- The average velocity depends only on the total displacement, not on the details of what happens inbetween.
- If distance is given in m and time in s, average velocity is measured in (m/s)

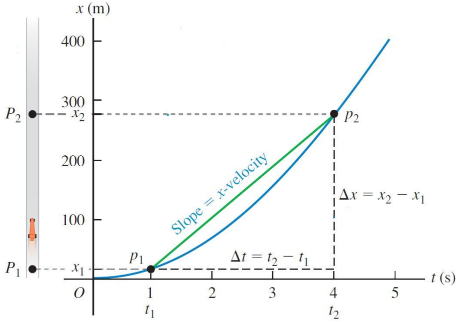


Figure: Position vs. time. Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Test your understanding.

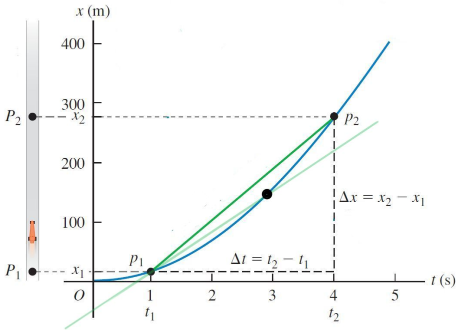
Each of the following automobile trips takes one hour. The positive x-direction is to the east. Rank the five trips in order of average velocity from most positive to most negative. Which trips, if any, have the same average velocity? For which trip, if any, is the average xvelocity equal to zero?

1. Automobile A travels 50 km due east.
2. Automobile B travels 50 km due west.
3. Automobile C travels 60 km due east, then turns around and travels 10 km due west.
4. Automobile D travels 70 km due east.
5. Automobile E travels 20 km due west, then turns around and travels 20 km due east.

Instantaneous Velocity

- if we move the second point closer and closer to the first point
- the average velocity becomes the **instantaneous velocity** at point p_1.

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$



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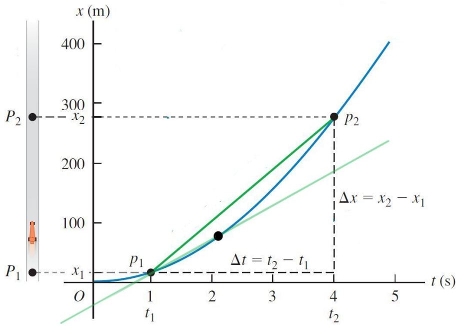

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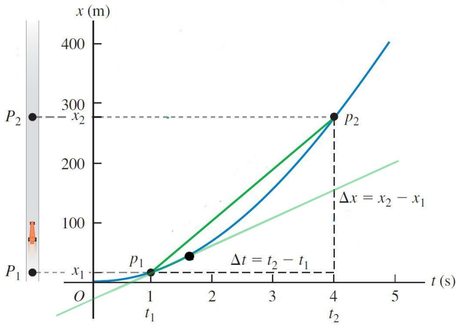

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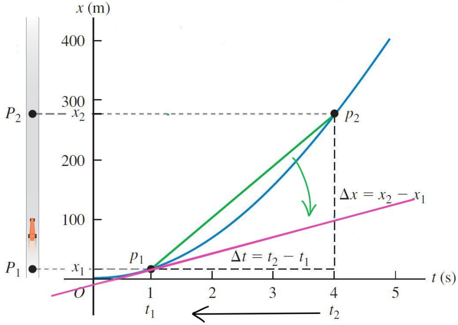

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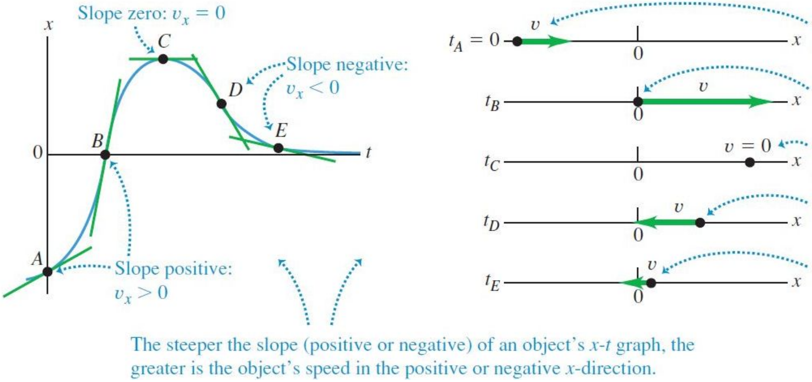
- It has the same sign than Δx.
- The instantanous speed is the magnitude of the instantaneous velocity.

CAUTION!

The average speed is not the magnitude of the average velocity.

Finding Velocity on an x-t Graph

- The isntantaneous velocity is the slope of the curve.



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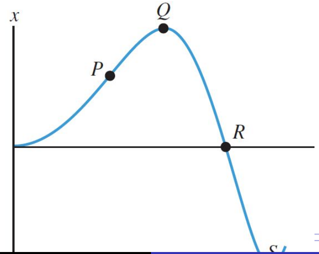
Test your undertanding

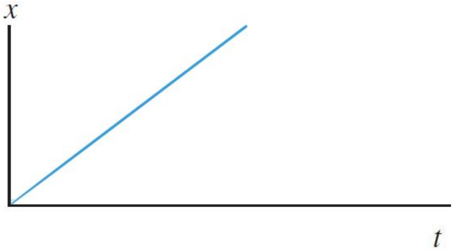
The figure is an x-t graph of the motion of a particle.

1. Rank the values of the particle's velocity at the points P, Q, R, and S from most positive to most negative.
2. At which points v is positive?
3. At which points v is negative?
4. At which points v is zero?
5. Rank the values of the particle's speed at the points P, Q, R, and S from fastest to slowest.



When the velocity is constant, the average velocity is equal to the isntantaneous velocity.

$$v = \frac{\Delta x}{\Delta t} \rightarrow x(x) = vt + x_0 \quad (2)$$



How is the velocity vs. time plot?

Average and Instantaneous Acceleration

- The average acceleration is the change of the velocity in the time interval Δt

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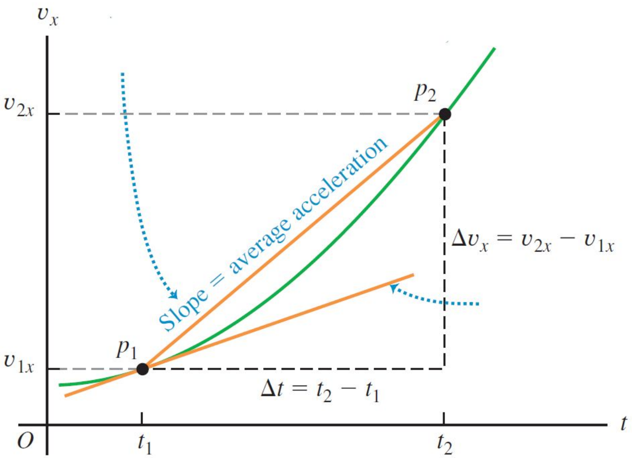
$$a = \frac{dv}{dt} \tag{4}$$

Finding Acceleration on a v-t Graph



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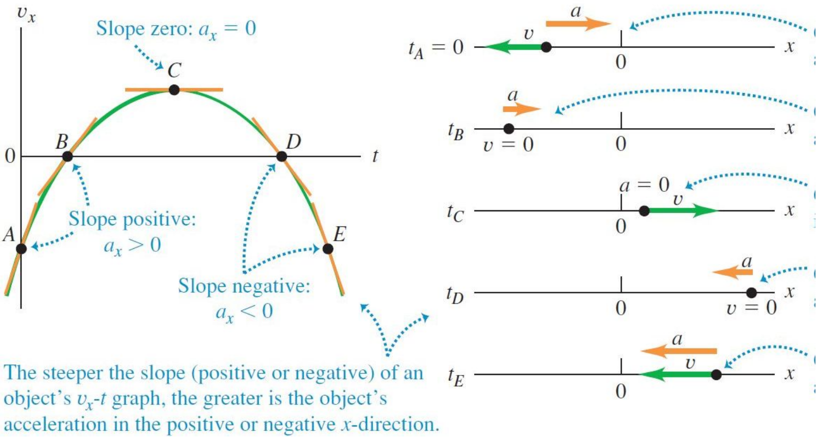
The acceleration is the slope of $v(t)$



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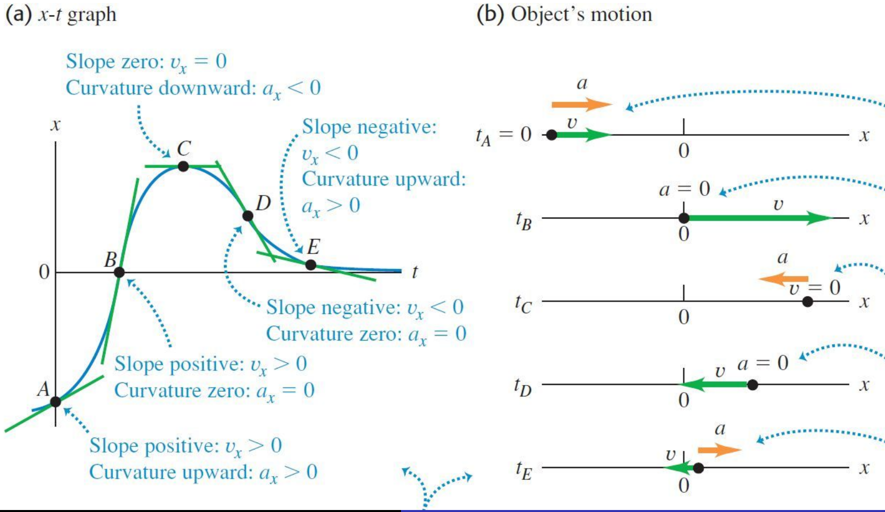
Finding Acceleration on a x-t Graph



- If the curve is concave up, the acceleration is positive.

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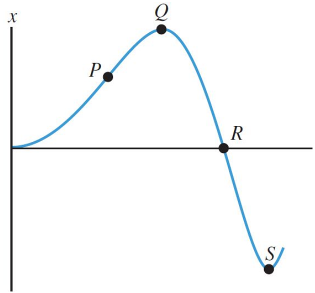
- If the curve is concave dow, the aceleration is negative.
- if the velocity has the same sign than the acceleration, the motion accelerated.
- If the velocity has a opposite sign to the acceleration, the motion is decelerated.

Test your undertanding

1. At which of the points P, Q, R, and S is the x-acceleration positive?
2. At which points is the x-acceleration negative?
3. At which points does the x-acceleration appear to be zero?
4. At each point state whether the velocity is increasing, decreasing, or not changing.



- straight-line motion with constant acceleration

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- the velocity changes at the same rate throughout the motion.

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- the velocity changes at the same rate throughout the motion.
  - falling body when the effects of the air are not important.

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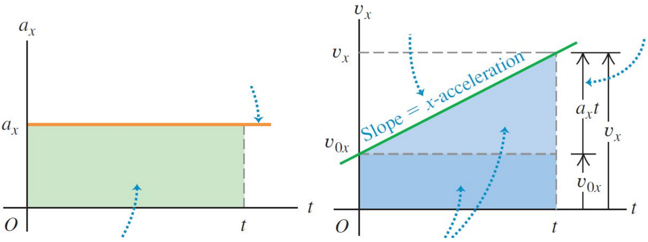
if $t_1 = 0$, $v_1 = v_0$ and $v_2 = v \rightarrow v = at + v_0$



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What is x(t)?

Let's use the expression for the average velocity,

$$v_{av} = \frac{x - x_0}{t}$$

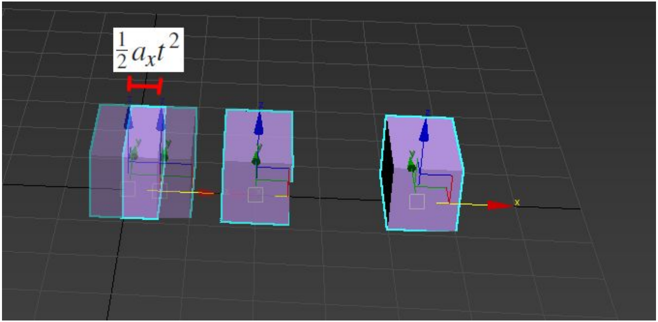
When the acceleration is constant, the average velocity is also

$$v_{av} = \frac{v + v_0}{2}$$

And..we already know that $v = at + v_0$
Then...

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \tag{6}$$

What is x(t)? Example: object motion, $v_0 = 0$ and $x_0 = 0$



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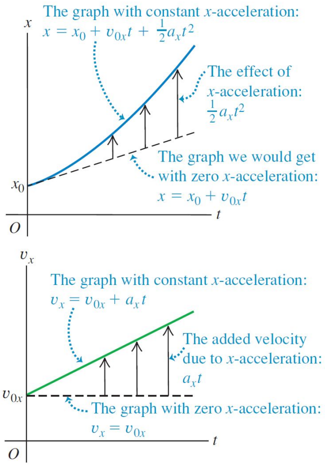
x(t) is a parababola



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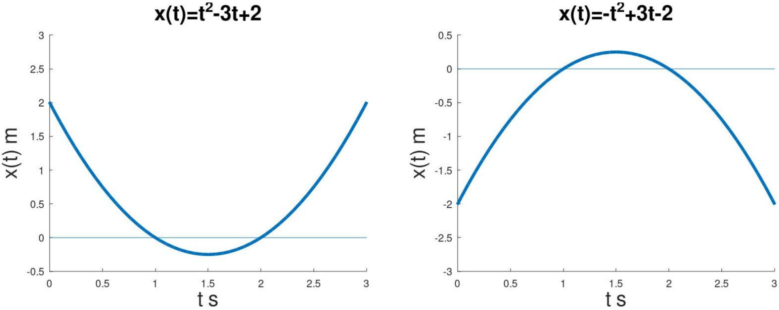
Example: Let's analyze the motion for the following graphs



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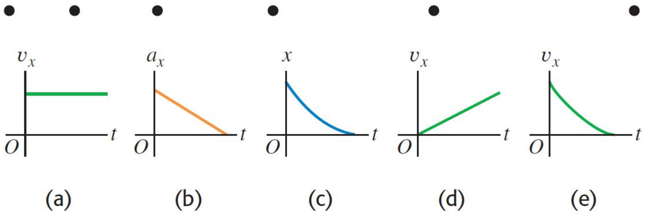
DISCUSSION QUESTIONS

The top diagram in the figure represents a series of highspeed photographs of an insect flying in a straight line from left to right (in the positive x-direction). Which of the graphs of the bottom graphs most plausibly depicts this insect's motion?



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Another usefull expression valid for constant acceleration motion:

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$$\rightarrow x = \frac{1}{2}a(\frac{v - v_0}{a})^2 + v_0(\frac{v - v_0}{a}) + x_0$$

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$$v = at + v_0 \rightarrow t = \frac{v - v_0}{a}$$

$$\rightarrow x = \frac{1}{2}a(\frac{v - v_0}{a})^2 + v_0(\frac{v - v_0}{a}) + x_0$$

$$\rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a}$$

By using these equations, we can solve any problem involving straight-line motion of a particle with constant acceleration.

Equation	Includes Quantities			
$v_x = v_{0x} + a_x t$	t		v_x	a_x
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$ (2.12)	t	x		a_x
$v_x{}^2 = v_{0x}{}^2 + 2a_x (x - x_0)$		x	v_x	a_x
$x - x_0 = \left(\dfrac{v_{0x} + v_x}{2} \right) t$	t	x	v_x	

Figure: © University Physics with Modern Physics, 13th Edition.

Exercise:

A motorist traveling with a constant speed of 15 m/s passes a school-crossing corner, where the speed limit is 10 m/s. Just as the motorist passes the schoolcrossing sign, a police officer on a motorcycle stopped there starts in pursuit with a constant acceleration of 3 m/s^2

1. How much time elapses before the officer passes the motorist?
2. What is the officer's speed at that time?
3. At that time, what distance has each vehicle traveled?

Testing your understanding

- Can an object with constant acceleration reverse its direction of travel? Can it reverse its direction twice?
- Under what conditions is average velocity equal to instantaneous velocity?
- Is it possible for an object to be slowing down while its acceleration is increasing in magnitude?
- To be speeding up while its acceleration is decreasing? $+98$