PHY115

Motion in More Than one Dimension

Digipen

Spring 2021

Free fall:

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2. In 1687 → Newton's Laws
   - He published *"Philosophiæ naturalis principia mathematica"*, the laws that describe this experiment.

Video: `https://www.youtube.com/watch?v=QyeF-_QPSbk`

Free fall: Newton's Law:

The magnitud of the acceletarion that the Earth makes on a body is:

$$g = G\frac{M}{d^2}$$

d is the distance between the body and the center of the Earth.

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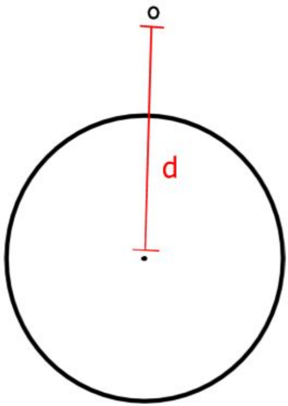
- $M = 5,972 \times 10^{24}$ kg



What happen when the bodyis near the Earth surface?

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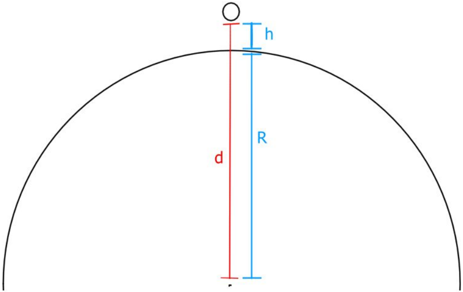
$$g = \frac{GM}{(h+R)^2} = \frac{GM}{R(\frac{h}{R}+1)}$$

$$\rightarrow g \approx \frac{GM}{R} \approx 9.8\ m/s^2$$



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- If the effects of the air can be neglected, all bodies fall with the same downward acceleration, regardless of their size or weight.
- If the fall is small compared with the radius of the earth, and if we ignore small effects due to the earth's rotation, the acceleration is constant.

The constant acceleration of a freely falling body is called the **acceleration due to gravity**, and we denote its magnitude with the letter g.

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Near the surface of the sun:

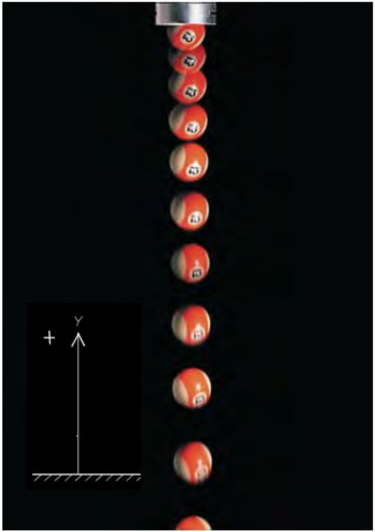
$$g = 270 \ m/s^2 \tag{3}$$

g is always a positive number

Because g is the magnitude of a vector quantity, it is always a positive number.

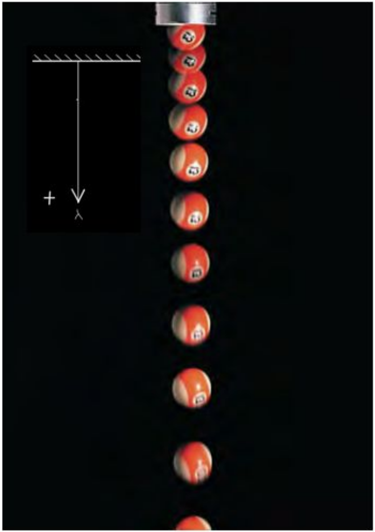
EXAMPLE 3.1

$$x(t) = -\frac{1}{2}gt^2 + v_0 t + x_0 \quad (4)$$



EXAMPLE 3.2

$$x(t) = \frac{1}{2}gt^2 + v_0 t + x_0 \quad (5)$$



EXERCISE 3.1

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A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?

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3. the maximum height reached;
4. the ball's acceleration when it is at its maximum height.

EXERCISE 3.2

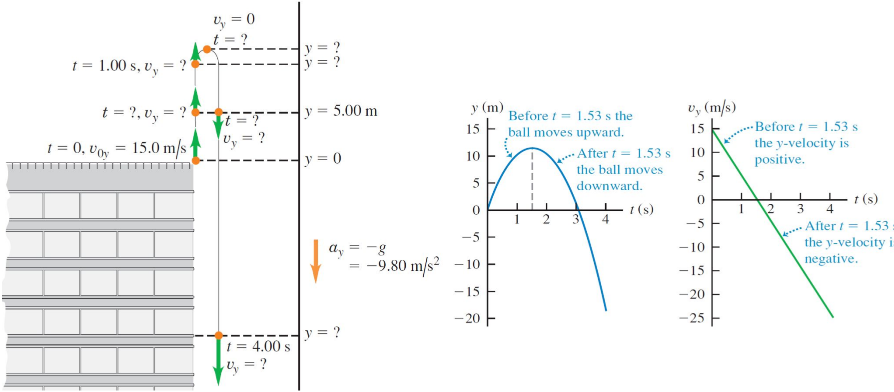

Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Test Your Understanding

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  1. $h\sqrt{2}$
  2. $2h$
  3. $4h$
  4. $8h$
  5. $16h$

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- If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height?
  1. $t/2$
  2. $t/\sqrt{2}$
  3. t
  4. $t\sqrt{2}$
  5. $2t$

Test Your Understanding

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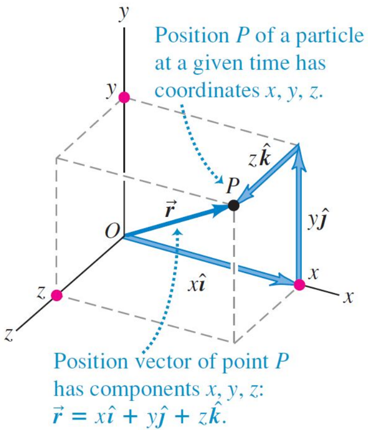
1. a straight line,
2. concave up (i.e., with an upward curvature), or
3. concave down (i.e., with a downward curvature)?

Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (6)$$



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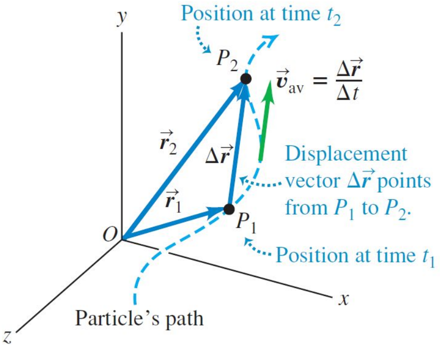
Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (7)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \qquad (8)$$



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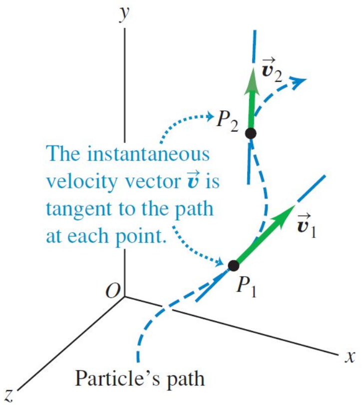
Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (9)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t} \qquad (10)$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t} \qquad (11)$$



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The Instantaneous velocity is,

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{12}$$

where,

$$v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt}\ and\ v_z = \frac{dz}{dt} \tag{13}$$

Its magnitude is,

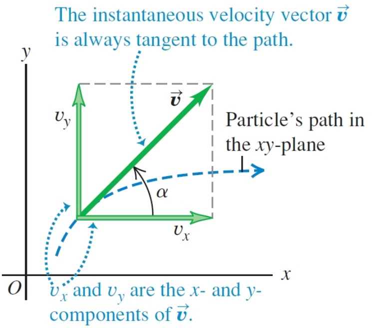
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{14}$$

When the motion is in the $x - y$-plane

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \qquad (15)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} \qquad (16)$$

$$tan\alpha = \frac{v_y}{v_x} \qquad (17)$$



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EXERCISE 3.3

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = (2.00 - 0.25t^2)\ m \tag{18}$$

$$y = (t + 0.025t^3)\ m \tag{19}$$

(a) Find the rover's coordinates and distance from the lander at $t = 2\ s$. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0\ s$ to $t = 2\ s$.

The Instantaneous acceleration is,

$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} \tag{20}$$

where,

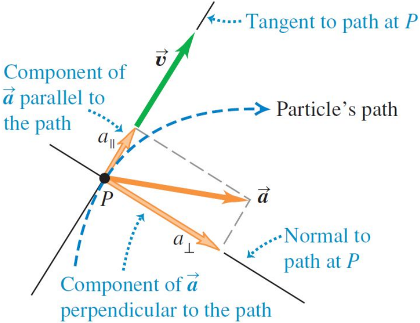
$$a_x = \frac{dv_y x}{dt}, \; a_y = \frac{dv_y}{dt} \; and \; a_z = \frac{dv_z}{dt} \tag{21}$$

Its magnitude is,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{22}$$

Parallel and Perpendicular Components of Acceleration

A useful way to think about the acceleration is in terms of its component parallel to the path and its component perpendicular to the path.



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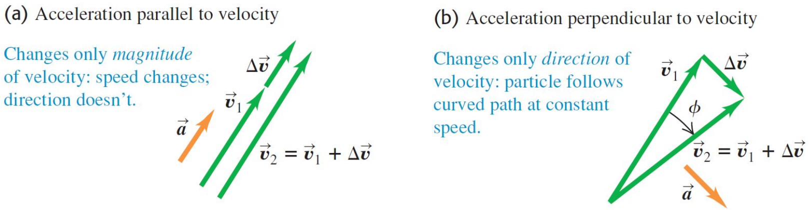

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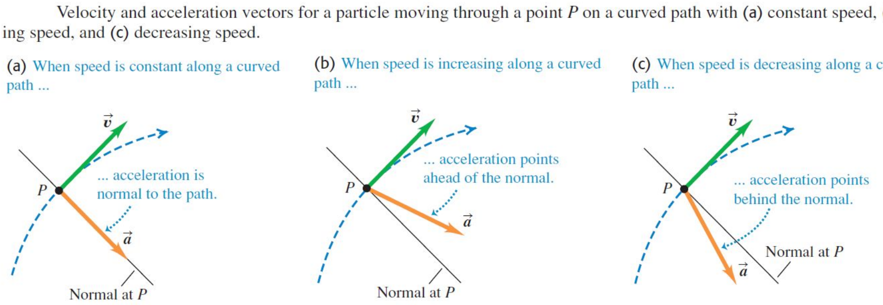

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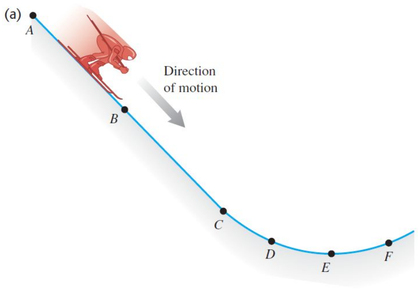
Conceptual Example:

A skier moves along a ski-jump ramp. The ramp is straight from point A to point C and curved from point C onward. The skier speeds up as she moves downhill from point A to point E, where her speed is maximum. She slows down after passing point E. Draw the direction of the acceleration vector at each of the points B, D, E, and F.



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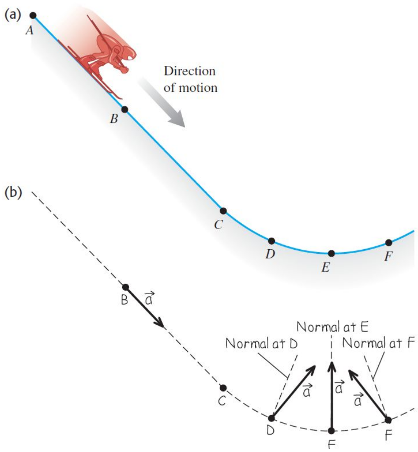

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Test Your Undertanding

A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)

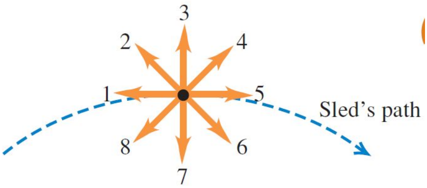


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

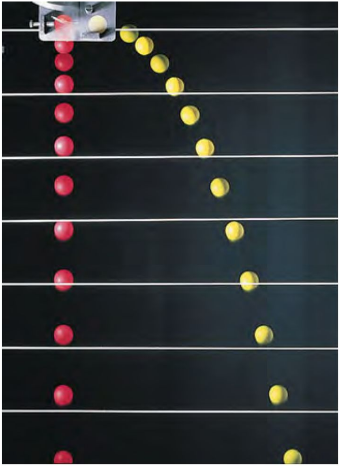
Projectile Motion

A projectile is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

The path followed by a projectile is called its trajectory.

Projectile Motion

- we can treat the x- and y-coordinates separately.
- The x-component of acceleration is zero, and the y-component is constant and equal to $-g$
- So, projectile motion is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



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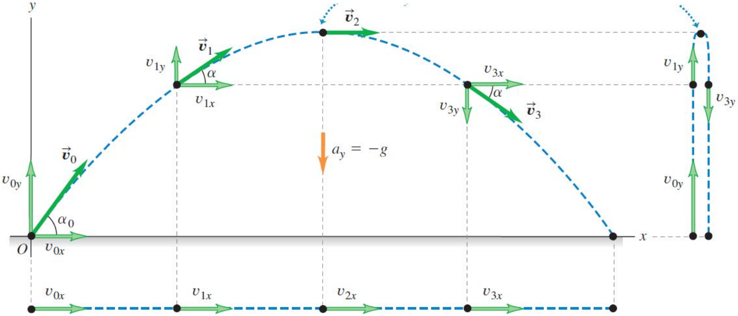
$$x - motion: \quad x = v_{x0}t + x_0 \tag{23}$$

$$y - motion: \quad y = -\frac{1}{2}gt^2 + v_{y0}t + y_0 \tag{24}$$



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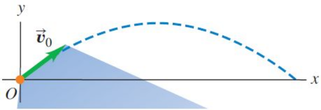

$v_{x0} = ? \quad v_{y0} = ?$

$$v_{x0} = v_0 cos\alpha_0, \quad v_{y0} = v_0 sin\alpha_0$$

$$tan\alpha_0 = \frac{v_{x0}}{v_{y0}}$$

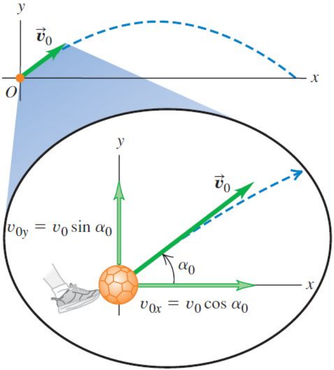

Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

$$x = (v_0 \cos \alpha_0)t \qquad \text{(projectile motion)}$$

$$y = (v_0 \sin \alpha_0)t - \frac{1}{2}gt^2 \qquad \text{(projectile motion)}$$

$$v_x = v_0 \cos \alpha_0 \qquad \text{(projectile motion)}$$

$$v_y = v_0 \sin \alpha_0 - gt \qquad \text{(projectile motion)}$$

Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Motion Path?

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$$y(x) = (tan\alpha_0)x - \frac{g}{2v_0^2 cos^2\alpha_0}x^2$$

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PARABOLA

$$\rightarrow y(x) = bx - cx^2 \quad parabola$$

What if we include air resistance?

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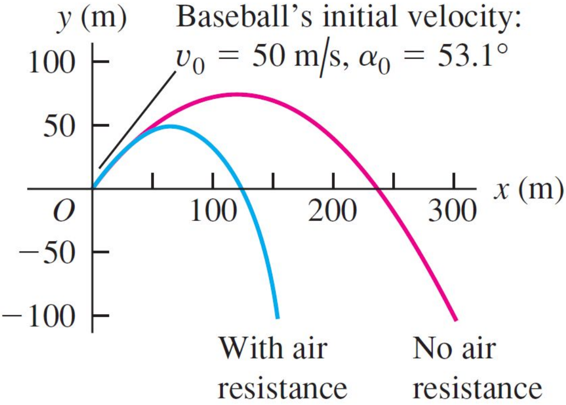

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HEIGHT AND RANGE OF A PROJECTILE

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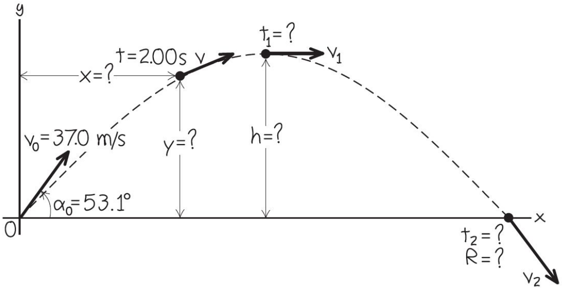
A batter hits a baseball so that it leaves the bat at speed $v_0 = 37\ m/s$ at an angle $\alpha_0 = 53.1°$. Find:

1. the time for the highest point, and its height h at this times
2. the horizontal range R



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HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 sin\ \alpha_0}{g}$$

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$$\rightarrow h = \frac{v_0^2 sin\ \alpha_0^2}{2g}$$

HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 sin\ \alpha_0}{g}$$

$$h = v_{0y} t - \frac{1}{2} g t^2$$

$$\rightarrow h = \frac{v_0^2 sin\ \alpha_0^2}{2g}$$

EXTRA-CREDIT: PROOF IT

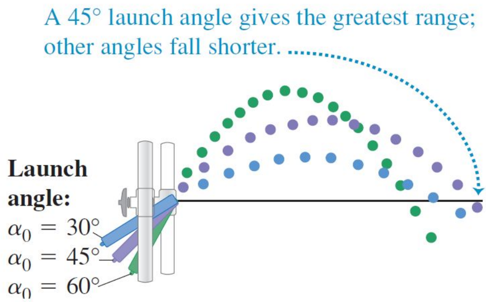
RANGE

$$t_l = 2\frac{v_0 sin\ \alpha_0}{g}$$

$$R = v_{0x}t = 2v_{0x}\frac{v_0 sin\ \alpha_0}{g} = 2v_0 cos\ \alpha\frac{v_0 sin\ \alpha_0}{g}$$

$$\rightarrow R = \frac{v_0^2 sin\ 2\alpha_0}{g}$$

VARIATION OF RANGE WITH INITIAL INCLINATION



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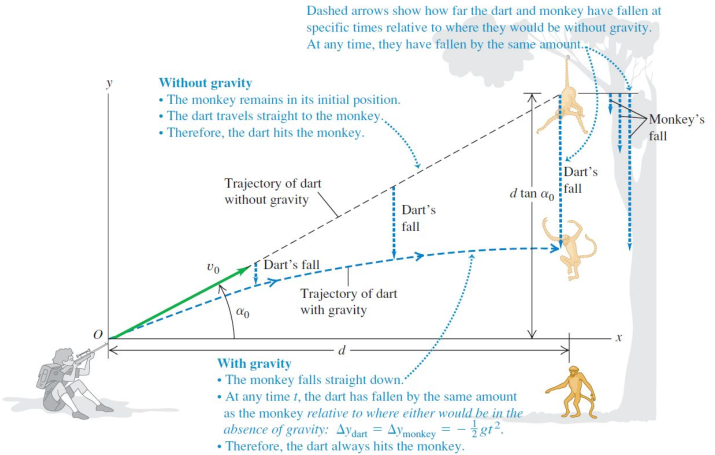
A classic Physics problem: "The zookeeper and the monkey"



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DART:

$$x_d = v_{0x} t$$

$$y_d = v_{0y} t - (\frac{1}{2}) g t^2$$

MONKEY:

$$x_m = D$$

$$y_m = -(\frac{1}{2}) g t^2 + H$$

$$\rightarrow tan\ \alpha = \frac{H}{D}$$

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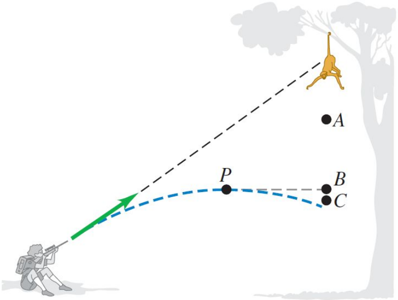
The dart is going to hit the monkey as long the zookeeper points right to the monkey at the begining.

$$\rightarrow tan\ \alpha = \frac{H}{D}$$

The dart is going to hit the monkey as long the zookeeper points right to the monkey at the begining.

EXTRA-CREDIT: PROOF IT

Suppose the tranquilizer dart has a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.



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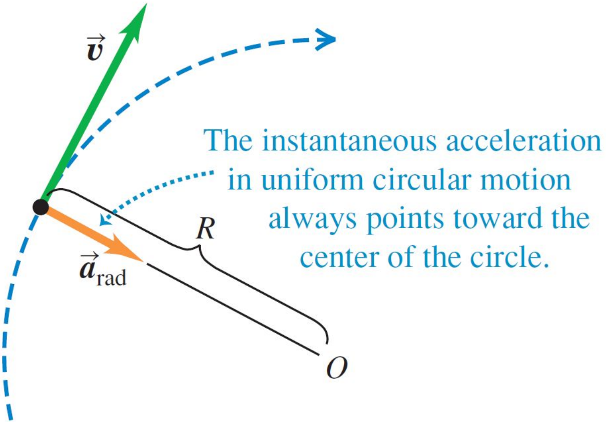
Uniform Circular Motion

- Motion in a circle with constant speed.
- There is no component of acceleration parallel (tangent) to the path.
- The acceleration vector is perpendicular to the path and hence directed inward.
- This causes the direction of the velocity to change without changing the speed



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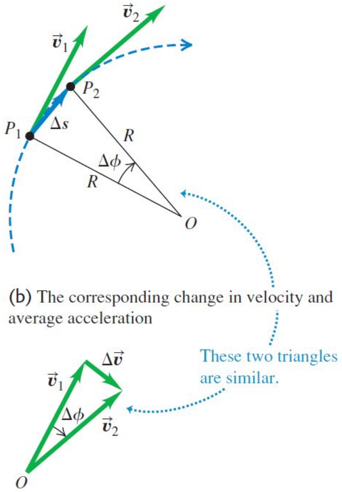
Magnitude of $\vec{a}$?

$$\Delta\Phi R = \Delta s$$



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Magnitude of $\vec{a}$?

$$\Delta\Phi R = \Delta s$$

$$\Delta\Phi v_1 = |\Delta\vec{v}| \rightarrow \frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R}$$

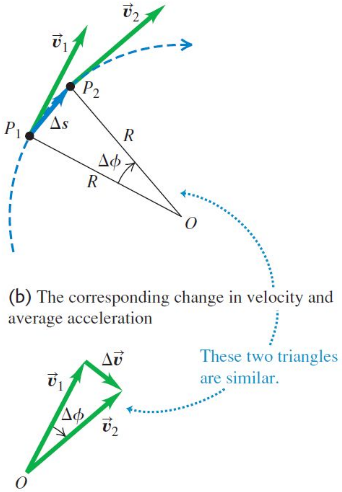

Figure: Figure from Sears and Zemansky's University Physics

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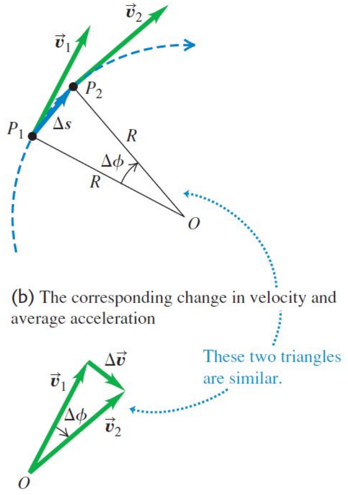

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Taking the limit for $\Delta t \rightarrow 0$

$$\boxed{a = \frac{v^2}{R}}$$

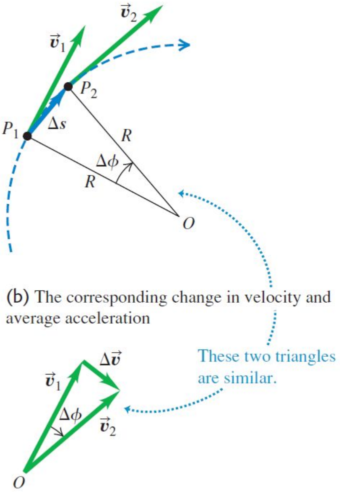


Figure: Figure from Sears and Zemansky's University Physics

Uniform Motion vs. Projectile Motion

1. The magnitude of acceleration is the same at all times.
2. In uniform circular motion the direction of a always points toward the center of the circle.

Uniform Motion vs. Projectile Motion

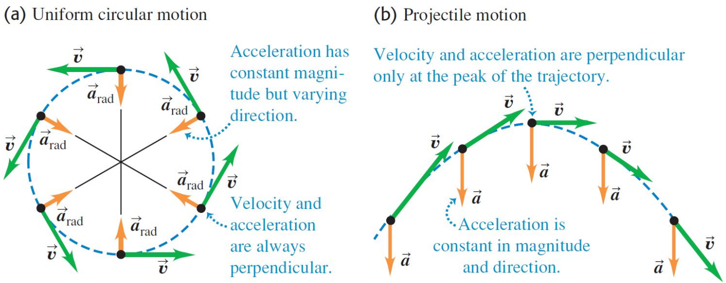
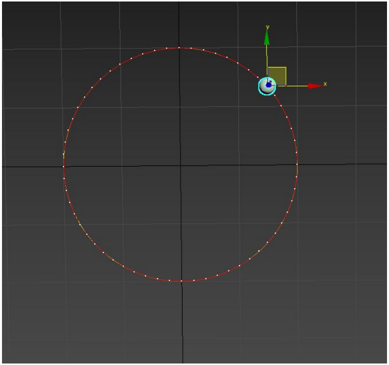


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Coordinates in circular motion?



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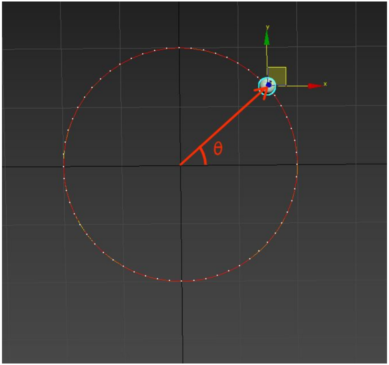

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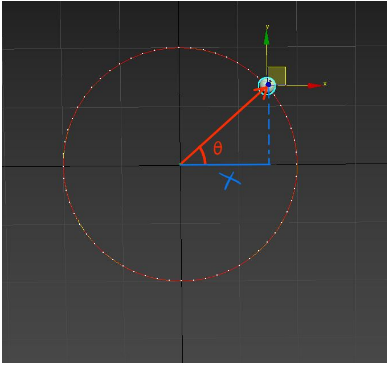

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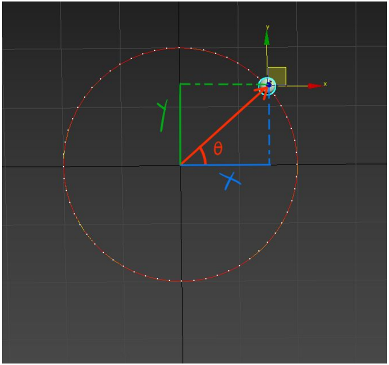

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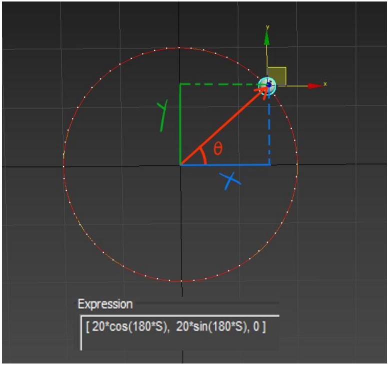
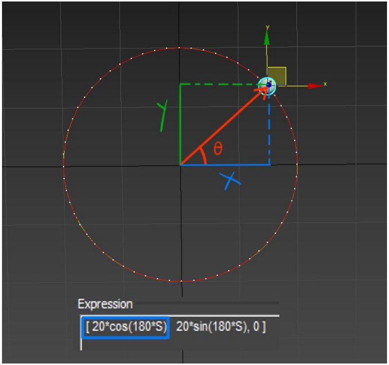


Coordinates in circular motion?

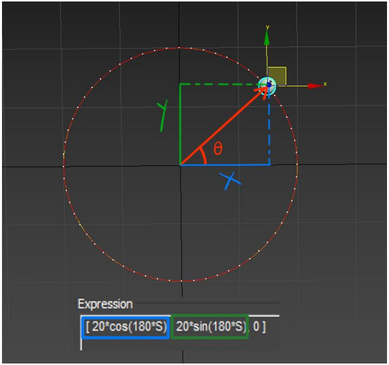
$$x = R\ cos\ \theta(t)$$



Coordinates in circular motion?

$$x = R \cos \theta(t)$$

$$y = R \sin \theta(t)$$



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Let's define the angular velocity:

$$\boxed{\omega = \frac{d\theta}{dt}} \tag{25}$$

ω counts how many turns per unit time.

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ω counts how many turns per unit time.

Let's define the period of rotation:

$$\boxed{T = \frac{2\pi}{\omega}} \tag{26}$$

T counts how long a single turn takes.

Example, what is the angular velocity if the Space station in 2001: A Space Odyssey?

https://www.youtube.com/watch?v=0ZoSYsNADtY

When ω is constant,

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$$\rightarrow v = R\omega$$

Nonuniform Circular Motion

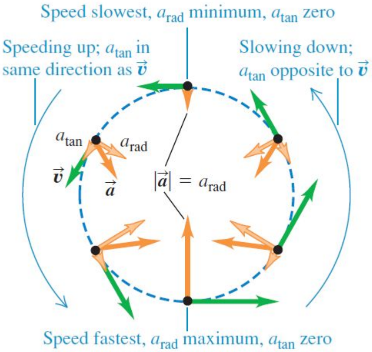

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Nonuniform Circular Motion

1. The speed varies.
2. the radial component of acceleration is $a_{rad} = \frac{v^2}{R}$.
3. v changes $\rightarrow$ a changes.
4. ω also changes.
5. There is also a tangential component, $a_{tan} = \frac{d|\vec{v}|}{dt}$

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Let's define the angular acceleration:

$$\alpha = \frac{d\omega}{dt} \tag{28}$$

quantity	spacial	angular
coordinates	x	θ
velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Motion with constant angular acceleration:

$$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \tag{29}$$

Test your undertanding of the section:

What is the difference between $\frac{d|\vec{v}|}{dt}$ and $|\frac{d\vec{v}}{dt}|$

Test your undertanding of the section:

Suppose that the particle in the previous figure experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i)$\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great?