PHY115

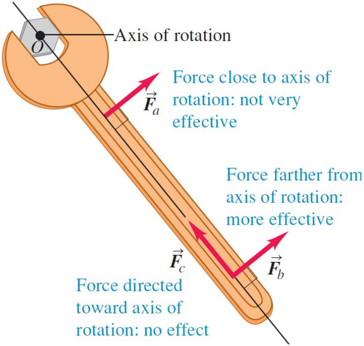
Dynamics of Rotation

Digipen

Spring 2022

Torque

- Direction and magnitud of forces $\rightarrow$ affect Translational Motion.
- The point of appliclation $\rightarrow$ affects Rotational Motion.



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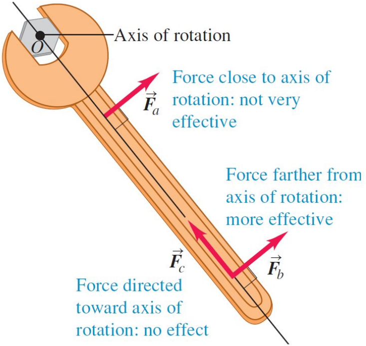
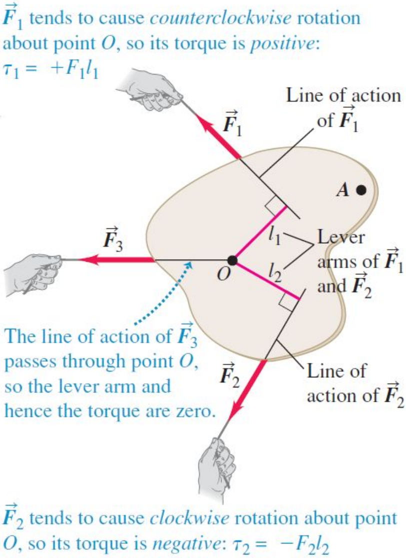
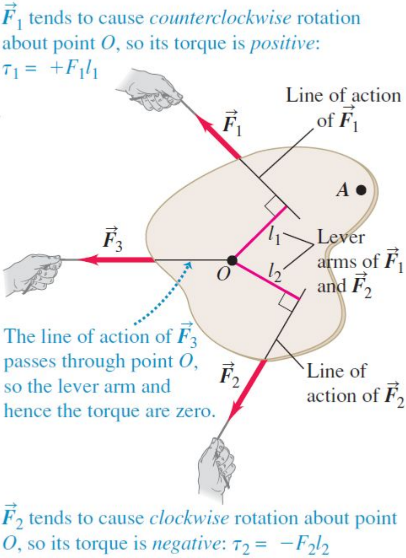


The quantitative measure of the tendency of a force to cause or change a body's rotational motion is called torque;



$$\tau = F\ell$$

ℓ : *lever arm*



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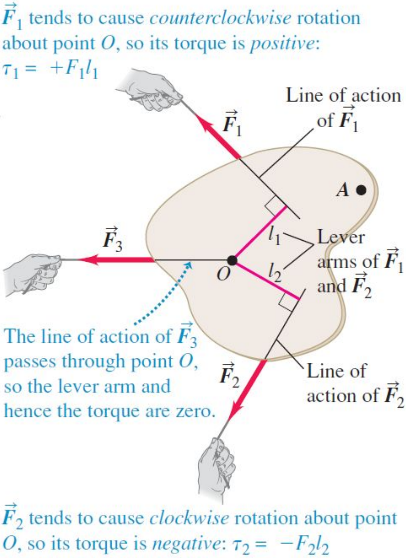
$$\tau_1 = +F_1\ell_1 \ (counterclockwise)$$



$$\tau = F\ell$$

ℓ : *lever arm*

$$\tau_1 = +F_1\ell_1 \ (counterclockwise)$$

$$\tau_2 = -F_2\ell_2 \ (clockwise)$$

EXAMPLE: what is the torque?

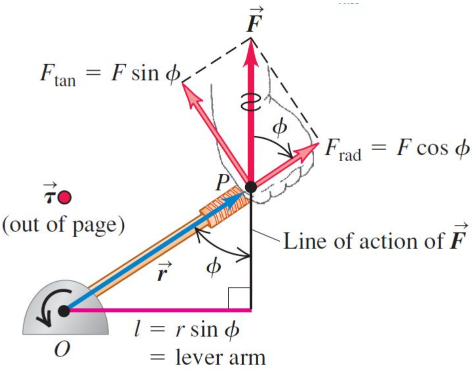


Figure: Image from Sears and Zemansky's University Physics With Modern Physics 13th edition.

EXAMPLE: what is the torque?

$$\boxed{\tau = F\ell = rFsin\Phi} \quad (1)$$

TORQUE AS A VECTOR

- Angular velocity $\rightarrow$ vector

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- Magnitude: $\tau = F\ell = rFsin\Phi$
- Direction: Right Hand Rule.
  - $\vec{\tau}$ perpendicular to the plane of $\vec{r}$ and $\vec{F}$
  - $(\cdot) \rightarrow \vec{\tau}$ points out the screen.
  - $(\times) \rightarrow \vec{\tau}$ points into the screen.

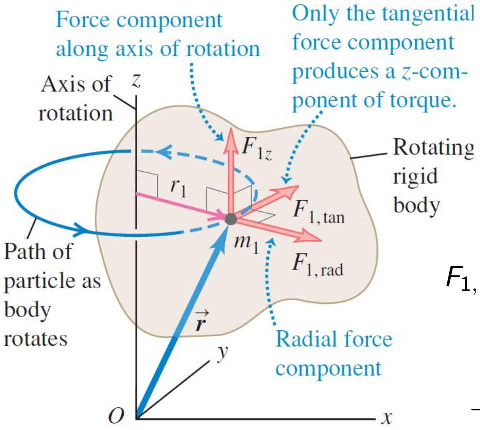
Test Your Understanding of

The figure shows a force P being applied to one end of a lever of length L.

What is the magnitude of the torque of this force about point A?

1. $PLsin(\theta)$;
2. $PLcos(\theta)$;
3. $PLtan(\theta)$.

Torque and Angular Acceleration for a Rigid Body



$$F_{1,tan} = m_1 a_{1,tan}$$

$$F_{1,tan} r_1 = m_1 r_1 a_{1,tan} = m_1 r_1^2 \alpha_z$$

$$\rightarrow \sum \tau_{iz} = \left(\sum m_i r_i^2 \right) \alpha_z$$

Inertia Momentum

$$I = \left(\sum m_i r_i^2\right) \tag{3}$$

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Just as Newton's second law says that the net force on a particle equals the particle's mass times its acceleration, the net torque on a rigid body equals the body's moment of inertia about the rotation axis times its angular acceleration.

Moments of Inertia of Various Bodies

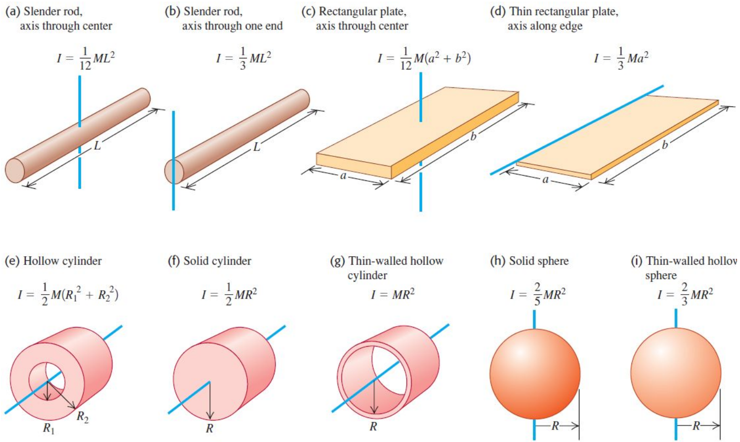
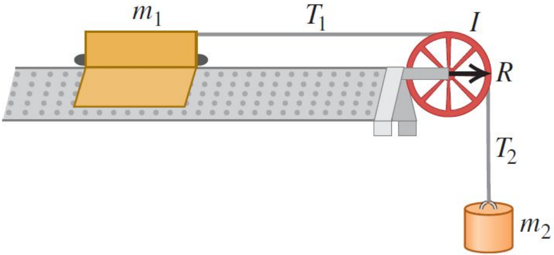


Figure: Image from Sears and Zemansky's University Physics With Modern Physics 13th edition.

Test Your Understanding of the Section

Rank the magnitudes of the following forces that act during the motion, in order from largest to smallest magnitude. (i) the tension force (magnitude T_1) in the horizontal part of the string; (ii) the tension force (magnitude T_2) in the vertical part of the string; (iii) the weight $m_2 g$ of the hanging object.



Rolling Without Slipping: Motion of a wheel

The motion of a rolling wheel is the sum of the translational motion of the center of mass plus the rotational motion of the wheel around the center of mass.

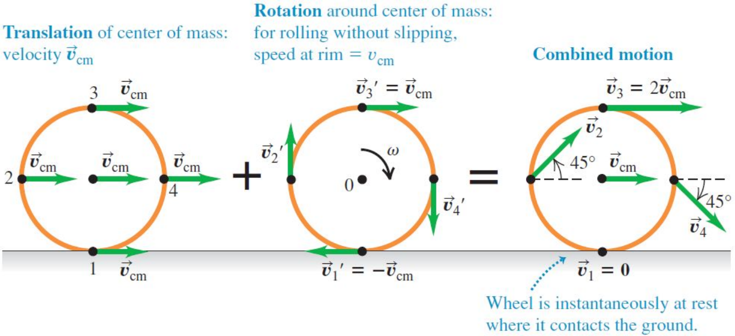


Figure: Image from Sears and Zemansky.

Acceleration of a rolling sphere

A bowling ball rolls without slipping down a ramp, which is inclined at an angle β to the horizontal. What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.

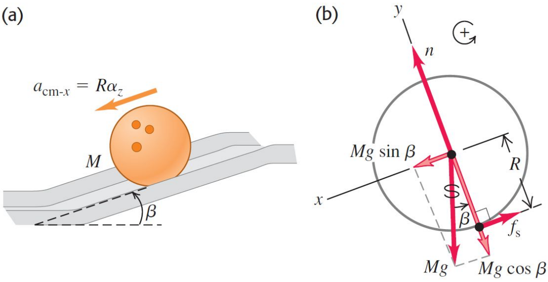
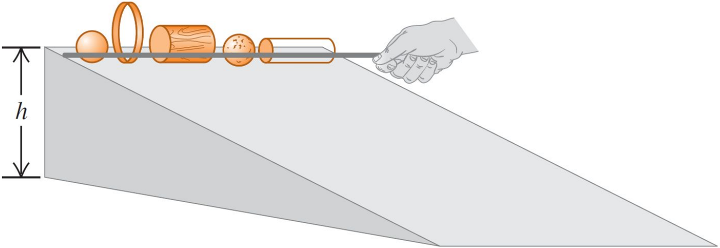


Figure: Image from Sears and Zemansky.

Race of the rolling bodies

What shape should a body have to reach the bottom of the incline first?



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Angular Momentum

Every rotational quantity that we have encountered last sections is the analog of some quantity in the translational motion of a particle.

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$\vec{\tau} \rightarrow \vec{F}$

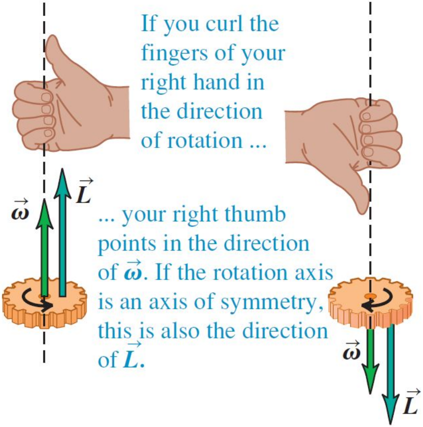
The analog of $\vec{P}$ of a particle is angular momentum $\vec{L}$.

$$\vec{L} = \vec{r} \times \vec{p} \tag{5}$$

Angular Momentum

For a rigid body rotating around a symmetry axis:

$$\vec{L} = I\vec{\omega} \qquad (6)$$



Angular Momentum and Torque

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No external torque $\rightarrow \vec{L} = constant \rightarrow \omega I = constant$

Test Your Understanding of Section

If the polar ice caps were to completely melt due to global warming, the melted ice would redistribute itself over the earth. This change would cause the length of the day (the time needed for the earth to rotate once on its axis) to

1. increase;
2. decrease;
3. remain the same.

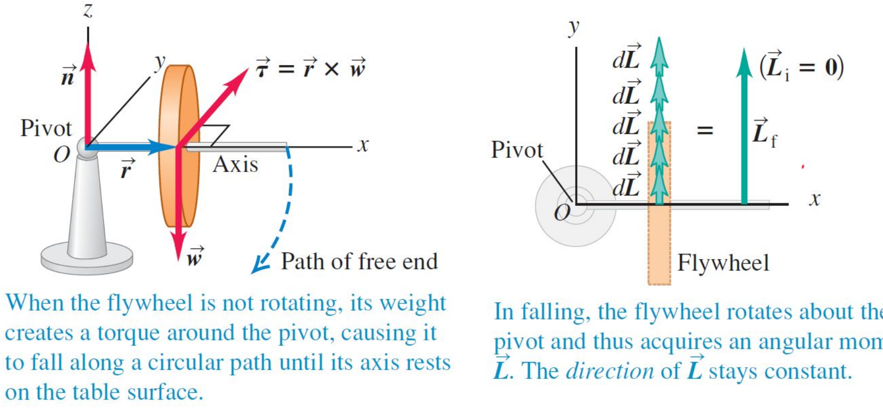
(Hint: Use angular momentum ideas. Assume that the sun, moon, and planets exert negligibly small torques on the earth.)

Test Your Understanding of Section

A ball is attached to one end of a piece of string. You hold the other end of the string and whirl the ball in a circle around your hand.

1. If the ball moves at a constant speed, is its linear momentum constant? Why or why not?
2. Is its angular momentum constant? Why or why not?

Gyroscopes



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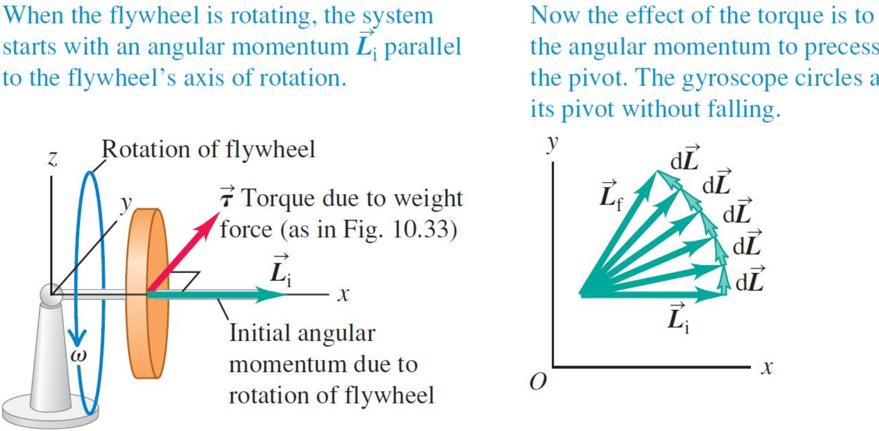
Gyroscopes


When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.
Rotation of flywheel
z
y
x
ω
$\vec{\tau}$ Torque due to weight force (as in Fig. 10.33)
$\vec{L}_i$
Initial angular momentum due to rotation of flywheel
Now the effect of the torque is to the angular momentum to precess the pivot. The gyroscope circles a its pivot without falling.
y
x
O
$\vec{L}_f$
$d\vec{L}$
$d\vec{L}$
$d\vec{L}$
$d\vec{L}$
$d\vec{L}$
$\vec{L}_i$

1. Can a single force applied to a body change both its translational and rotational motion? Explain.

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2. When an acrobat walks on a tightrope, she extends her arms straight out from her sides. She does this to make it easier for her to catch herself if she should tip to one side or the other. Explain how this works.
3. Experienced cooks can tell whether an egg is raw or hardboiled by rolling it down a slope (taking care to catch it at the bottom). How is this possible? What are they looking for?

Example 1

A square metal plate $0.180m$ on each side is pivoted about an axis through point O at its center and perpendicular to the plate. Calculate the net torque about this axis due to the three forces shown in the figure if the magnitudes of the forces are $F_1 = 18N$, $F_2 = 26N$ and $F_3 = 14N$ The plate and all forces are in the plane of the page. The plate and all forces are in the plane of the screen.

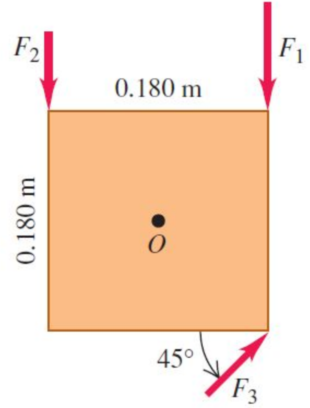
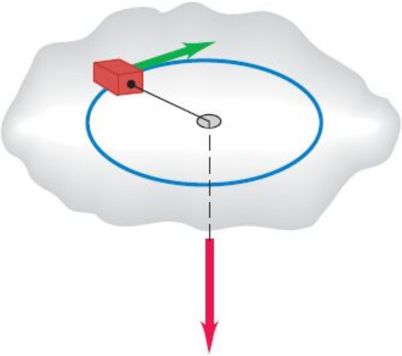


Figure: Image from Sears and Zemansky.

Example 2

A small block on a horizontal surface ($\mu = 0$) withmass $0.025kg$ is attached to a massless cord passing through a hole. The block is originally revolving at a distance of $0.3m$ from the hole with $\omega = 1.75\ rad/s$. The cord is then pulled from below, shortening the radius to $0.15m$. (a) Is the angular momentum of the block conserved? (b) What is the new angular speed?



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Example 3

A cue ball (mass m, radius R) is at rest on a level pool table. You give the ball a horizontal hit of magnitude F at a height h above the center of the ball. The force of the hit is much greater than the friction force . The hit lasts for a short time Δt . (a) For what value of h will the ball roll without slipping? (b) If you hit the ball center ($h = 0$), the ball will slide across the table for a while, but eventually it will roll without slipping. What will the speed of its center of mass be then?