PHY115

Momentum, Impulse and Collisions

Digipen

Spring 2021

There are many questions involving forces that cannot be answered by directly applying Newton's second law.

There are many questions involving forces that cannot be answered by directly applying Newton's second law.

For example, when a moving van collides head-on with a compact car, what determines which way the wreckage moves after the collision?

There are many questions involving forces that cannot be answered by directly applying Newton's second law.

For example, when a moving van collides head-on with a compact car, what determines which way the wreckage moves after the collision?

We would need to know how are the forces acting between the colliding bodies.

There are many questions involving forces that cannot be answered by directly applying Newton's second law.

For example, when a moving van collides head-on with a compact car, what determines which way the wreckage moves after the collision?

We would need to know how are the forces acting between the colliding bodies.

We will find that we don't have to know anything about these forces to answer questions of this kind!

Our approach uses two new concepts, *momentum and impulse*, and conservation law, *conservation of momentum*.

Our approach uses two new concepts, *momentum and impulse*, and conservation law, *conservation of momentum*.

$$\sum F = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{(\Delta\vec{p})}{\Delta t} \tag{1}$$

Our approach uses two new concepts, *momentum and impulse*, and conservation law, *conservation of momentum*.

$$\sum F = m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta (m\vec{v})}{\Delta t} = \frac{(\Delta \vec{p})}{\Delta t} \tag{1}$$

then...

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \tag{2}$$

Our approach uses two new concepts, *momentum and impulse*, and conservation law, *conservation of momentum*.

$$\sum F = m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{(\Delta \vec{p})}{\Delta t} \tag{1}$$

then...

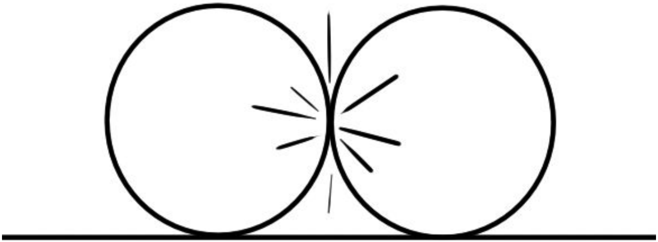
$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \tag{2}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle.

- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$

- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$
- What is $\sum \vec{F}$?

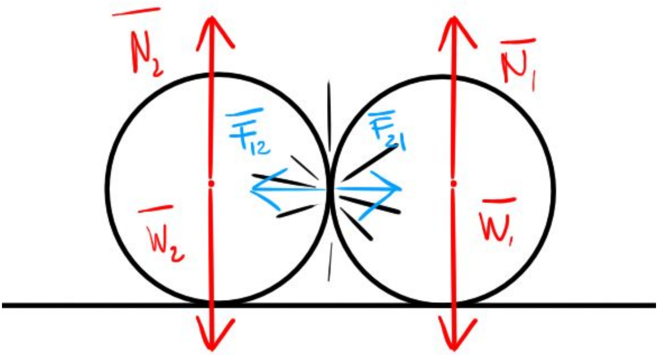
- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$
- What is $\sum \vec{F}$?



- $\sum \vec{F} = 0 \rightarrow \Delta\vec{p} = 0$

- $\sum \vec{F} = 0 \rightarrow \Delta\vec{p} = 0$
- What is $\sum \vec{F}$?

- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$
- What is $\sum \vec{F}$?

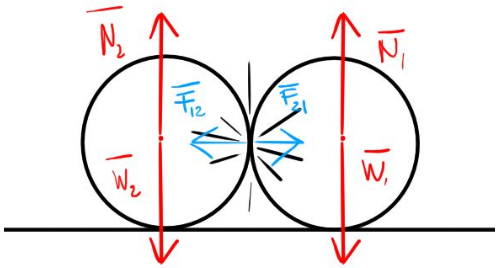


- $\sum \vec{F} = \vec{N}_1 + \vec{W}_1 + \vec{N}_2 + \vec{W}_2 + \vec{F}_{12} + \vec{F}_{21} = 0 \rightarrow \Delta p = 0$

- $\sum \vec{F} = 0 \rightarrow \Delta\vec{p} = 0$

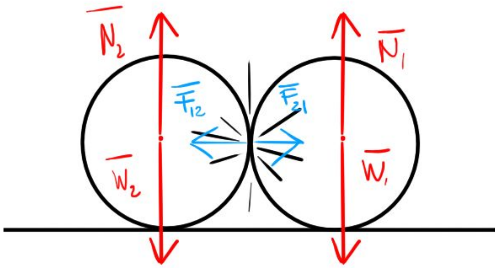
- $\sum \vec{F} = 0 \rightarrow \Delta\vec{p} = 0$
- What is $\sum \vec{F}$?

- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$
- What is $\sum \vec{F}$?

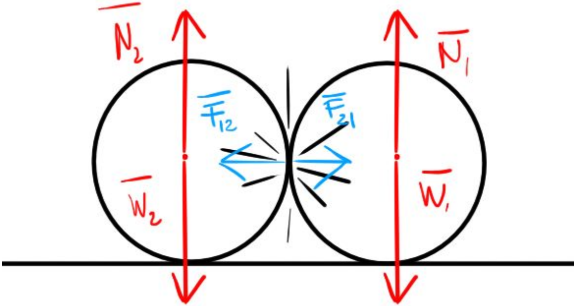


- $\sum \vec{F}_1 = \vec{N}_1 + \vec{W}_1 + \vec{F}_{21} = \frac{\Delta \vec{p}_1}{\Delta t}$
- $\sum \vec{F}_2 = \vec{N}_2 + \vec{W}_2 + \vec{F}_{21} = \frac{\Delta \vec{p}_2}{\Delta t}$

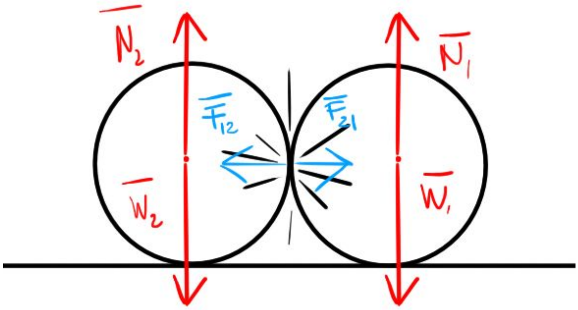
- $\sum \vec{F} = 0 \rightarrow \Delta\vec{p} = 0$
- What is $\sum \vec{F}$?



- $\sum \vec{F}_1 = \vec{N}_1 + \vec{W}_1 + \vec{F}_{21} = \frac{\Delta\vec{p}_1}{\Delta t}$
- $\sum \vec{F}_2 = \vec{N}_2 + \vec{W}_2 + \vec{F}_{21} = \frac{\Delta\vec{p}_2}{\Delta t}$
- $\rightarrow \vec{p}_1 = -\vec{p}_2$

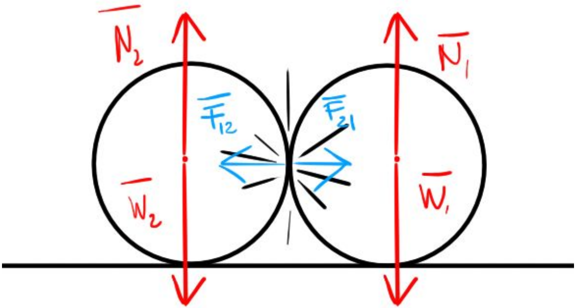


If we consider the twoo balls as a system...



If we consider the twoo balls as a system... We define the linear momentum of the system to be,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \tag{3}$$



If we consider the twoo balls as a system... We define the linear momentum of the system to be,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \tag{3}$$

- $\sum \vec{F} = \vec{N}_1 + \vec{W}_1 + \vec{N}_2 + \vec{W}_2 + \vec{F}_{12} + \vec{F}_{21} = 0 \Rightarrow \Delta \vec{p} = 0$

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

Example: `https://www.youtube.com/watch?v=4IYDb6K5UF8`

More than one particle?

More than one particle?

We define the total momentum as,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5$$

More than one particle?

We define the total momentum as,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5$$

No external forces $\rightarrow \Delta\vec{p} = 0$

More than one particle?

We define the total momentum as,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5$$

No external forces $\rightarrow \Delta\vec{p} = 0$

$$\rightarrow \Delta\vec{p}_x = 0$$

$$\rightarrow \Delta\vec{p}_y = 0$$

External force...

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\rightarrow \Delta \vec{p}_x = F_x$$

$$\rightarrow \Delta \vec{p}_y = F_y$$

External force...

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\rightarrow \Delta \vec{p}_x = 0$$

$$\rightarrow \Delta \vec{p}_y = \sum_i m_i \vec{g}_i$$

Example

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum of the bullet and rifle?

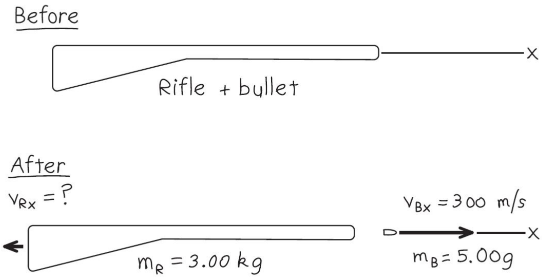
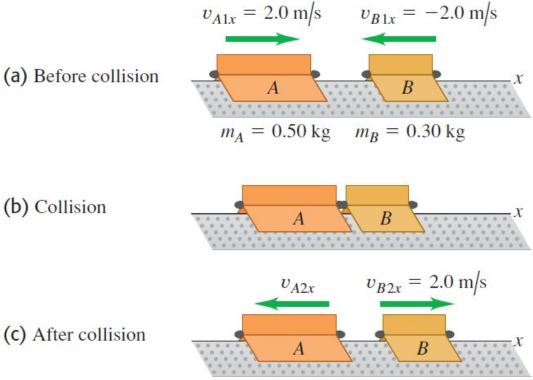


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Example

Two gliders with different masses move toward each other on a frictionless air track. After they collide, glider B has a final velocity of $+2.0$ m/s. What is the final velocity of glider A? How do the changes in momentum and in velocity compare?



Example

Test Your Understanding

A spring-loaded toy sits at rest on a horizontal, frictionless surface. When the spring releases, the toy breaks into three equal-mass pieces, A, B, and C, which slide along the surface. Piece A moves off in the negative x-direction, while piece B moves off in the negative y-direction.

1. What are the signs of the velocity components of piece C?
2. Which of the three pieces is moving the fastest?

Example

Two kind of collisions

- Elastic Collisions: the total kinetic energy of the system is the same after the collision as before (billiard balls).
- Inelastic Collisions: the total kinetic energy of the system is not conserved (collision between 2 pieces of clay).

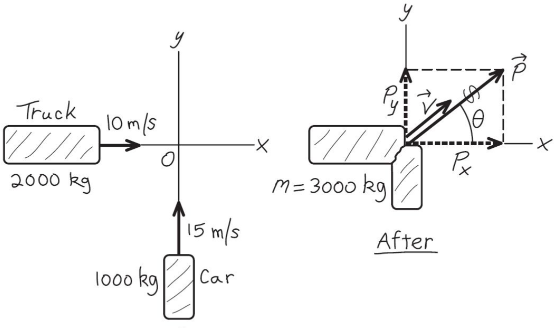
Example

Example: Collision with a wall

Example

Example of inelastic collision in 2D: An automobile collision

A 1000 *kg* car traveling north at collides with a 2000 *kg* truck traveling east at The occupants, wearing seat belts, are uninjured, but the two vehicles move away from the impact point as one. The insurance adjustor asks you to find the velocity of the wreckage just after impact. What is your answer?



Testing you understanding about Linear Momentum.

(a) When a large car collides with a small car, which one undergoes the greater change in momentum: the large one or the small one? Or is it the same for both? (b) In light of your answer to part (a), why are the occupants of the small car more likely to be hurt than those of the large car, assuming that both cars are equally sturdy?

Testing you understanding about Linear Momentum.

A woman holding a large rock stands on a frictionless, horizontal sheet of ice. She throws the rock with speed v_0 at an angle above the horizontal. Consider the system consisting of the woman plus the rock. Is the momentum of the system conserved? Why or why not? Is any component of the momentum of the system conserved? Again, why or why not?

EXANPLE: `https://www.youtube.com/watch?v=Kf0bBxmNeec`

Testing you understanding about Linear Momentum.

A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why?

CENTER OF MASS

The total mass of the system is: $M = \sum_i m_i$

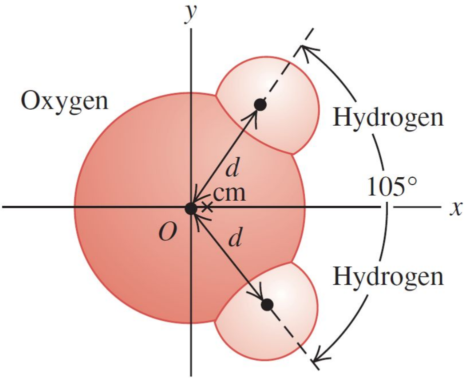
We define the **Center of Mass** of the system as,

$$\vec{R} = \frac{1}{M}\sum_i m_i \vec{r}_i \tag{4}$$

CENTER OF MASS

EXAMPLE: Center of mass of a water molecule

$d = 9.57 \times 10^{-11}$ m, $m_o = 16\ u$, $m_H = 1\ u$



CENTER OF MASS

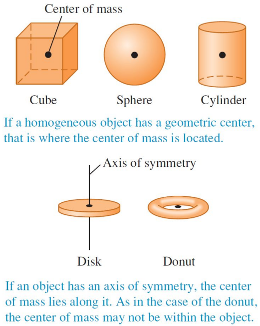


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

For solid bodies, in which we have (at least on a macroscopic level) a continuous distribution of matter, the sums in eq. 4 have to be replaced by integrals.

MOTION OF THE CENTER OF MASS

We can show that,

$$\vec{P} = M\vec{V}_{CM} \tag{5}$$

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} \tag{6}$$

MOTION OF THE CENTER OF MASS

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

This is a very impressive result!!

EXAMPLE

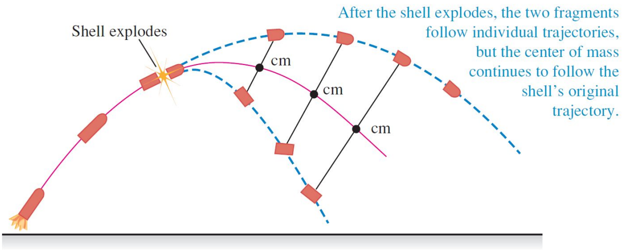


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

EXAMPLE

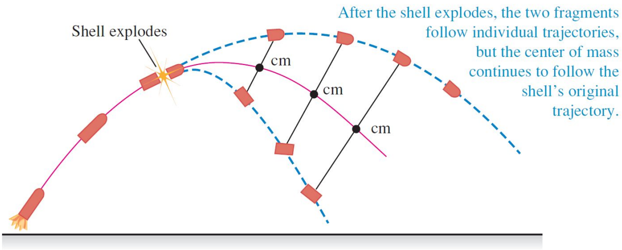


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Will the center of mass in the figure continue on the same parabolic trajectory even after one of the fragments hits the ground? Why or why not?

EXAMPLE

Explain the motion of a Newton Cradle

`https://www.youtube.com/watch?v=8dgyPRA86K0`

Rotation of Rigid Bodies

- We are going to start studing motion of Rigid Bodies.

Rotation of Rigid Bodies

- We are going to start studing motion of Rigid Bodies.
- They are systems of particles with constant distance between them.

Rotation of Rigid Bodies

- We are going to start studing motion of Rigid Bodies.
- They are systems of particles with constant distance between them.
- Macroscopically we see a continuous body.

Rotation of Rigid Bodies

- We are going to start studing motion of Rigid Bodies.
- They are systems of particles with constant distance between them.
- Macroscopically we see a continuous body.
- We can not represented adequately their motion as a moving point.

We are going to represent the motion of rigid bodies as the superposition of two motions:

We are going to represent the motion of rigid bodies as the superposition of two motions:

- Translation of center of Mass + rotation around the Center of Mass.

We are going to represent the motion of rigid bodies as the superposition of two motions:

- Translation of center of Mass + rotation around the Center of Mass.
- We are going to consider only planar rotations, that is rotations in 2D (example: rotation of a wheel).

We are going to represent the motion of rigid bodies as the superposition of two motions:

- Translation of center of Mass + rotation around the Center of Mass.
- We are going to consider only planar rotations, that is rotations in 2D (example: rotation of a wheel).
- Rotations in 3D have a much more complex mechanics.

- To describe rotations, we need angles

- To describe rotations, we need angles
- We are going to use θ, ω and α

- To describe rotations, we need angles
- We are going to use θ, ω and α
- Till now, we have been using just numbers to represent them,

$$\omega = \frac{\Delta\theta}{\Delta t},\ \alpha = \frac{\Delta\omega}{\Delta t}\ (\alpha\ constant)$$

- To describe rotations, we need angles
- We are going to use θ, ω and α
- Till now, we have been using just numbers to represent them,

$$\omega = \frac{\Delta\theta}{\Delta t}, \ \alpha = \frac{\Delta\omega}{\Delta t} \ (\alpha \ \mathit{constant})$$

- But rotations can have two different sense, so to specify a rotation, we need a numbersand a sense.
- Angular velocity and Acceleration are represented by vectors!

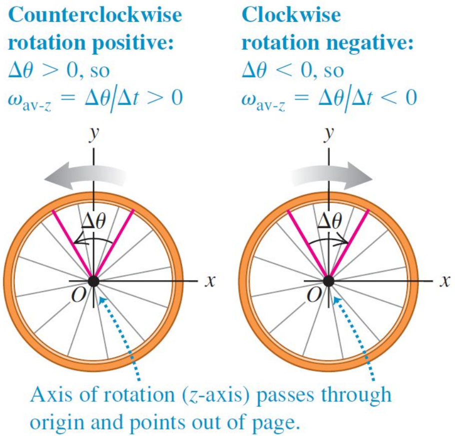
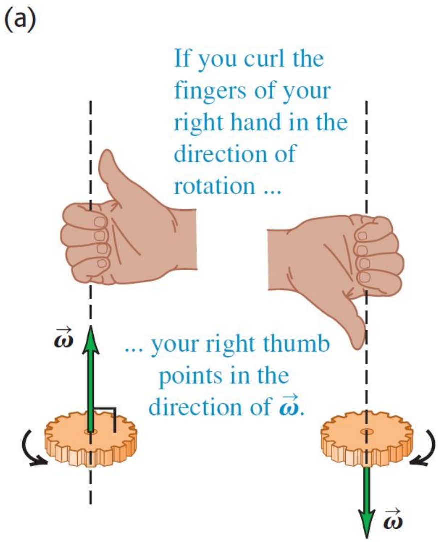


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Angular Velocity As a Vector



Angular Acceleration As a Vector

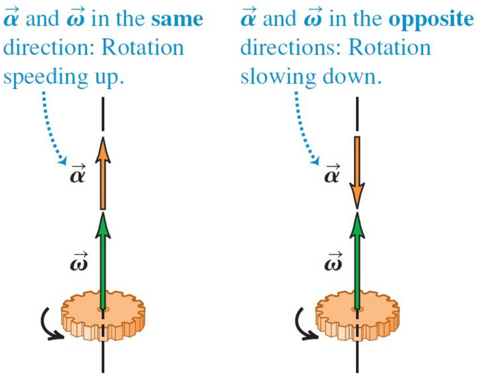


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Test Your Understanding The figure shows a graph of ω_z and α_z versus time for a particular rotating body. (a) During which time intervals is the rotation speeding up?

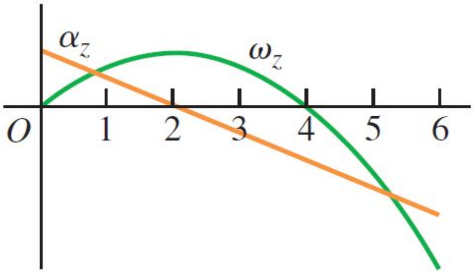


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Comparison of Linear and Angular Motion with Constant Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$

Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Rotation with constant angular acceleration

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is $27.5\ rad/s$, and its angular acceleration is a constant $-10\ rad/s^2$. A line PQ on the disc's surface lies along the at $t = 0$. (a) What is the disc's angular velocity at $t = 0.3\ s$ (b) What angle does the line PQ make with the $+x - axis$ at this time?

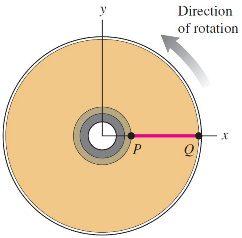


Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Relating Linear and Angular Kinematics

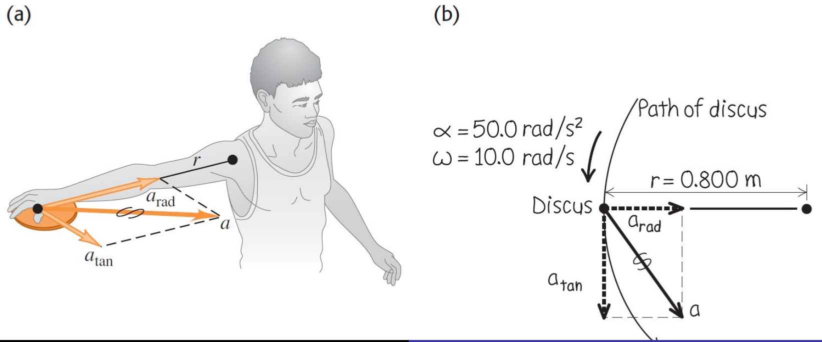
$$\boxed{v = r\omega, \;\; a_{rad} = \omega^2 r}$$

Relating Linear and Angular Kinematics

$$\boxed{v = r\omega,\ \ a_{rad} = \omega^2 r,\ \ a_{tan} = r\alpha}$$

Throwing a discus

An athlete whirls a discus in a circle of radius 80.0cm. At a certain instant, the athlete is rotating at 10 rad/s and the angular speed is increasing at 50 rad/s^2. At this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.



Throwing a discus

Information is stored on a disc (see Fig. 13) in a coded pattern of tiny pits. The pits are arranged in a track that spirals outward toward the rim of the disc. As the disc spins inside a player, the track is scanned at a constant linear speed. How must the rotation speed of the disc change as the player's scanning head moves over the track? (i) The rotation speed must increase. (ii) The rotation speed must decrease. (iii) The rotation speed must stay the same.