Phy 250

Homework 3

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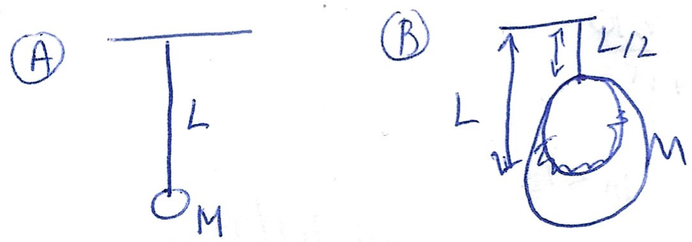


Considering that a is a simple pendulum:

2nd Law of Newton: $T - mg\cos\theta = m \cdot a_c$

$-mg\sin\theta = m \cdot a_T$; $\rightarrow a_T + g\sin\theta = 0$

$a_T = l \cdot \alpha$; α (angular acceleration) $= \frac{d^2\theta}{dt^2}$

$\frac{l\,d^2\theta}{dt^2} + g\sin\theta = 0$; for small angles $\sin\theta \approx \theta$

$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0 \leftarrow \frac{l\,d^2\theta}{dt^2} + g\theta = 0$; Solution is: $\theta(t) = \theta_0 \cos(\omega t)$

where $\omega = \sqrt{\frac{g}{l}}$; $\omega = \frac{2\pi}{T}$; $\underline{T_A = 2\pi \cdot \sqrt{\frac{l}{g}}}$

Considering that B is a physical pendulum

$\vec{\tau} = \vec{r} \times \vec{F}_{mg}$

$\vec{\tau} = I_{sphere} \cdot \alpha = -\frac{L}{2} mg\sin\theta$; for small angles $\sin\theta \approx \theta$

$\alpha = \frac{d^2\theta}{dt^2}$; $I \cdot \frac{d^2\theta}{dt^2} = -\frac{L}{2} mg\theta$

$I \cdot \frac{d^2\theta}{dt^2} + \frac{L}{2} mg\theta = 0$; $\frac{d^2\theta}{dt^2} + \frac{L}{2I} mg\theta = 0$

Solution: $\theta(t) = \theta_0 \cos(\omega t)$; where $\cancel{\omega = \sqrt{\frac{2I}{Lmg}}}$; $\omega = \sqrt{\frac{Lmg}{2I}}$

$I_{sphere} = \frac{2}{5} MR^2$; $R = \frac{L}{2}$; $I_{sphere} = \frac{2}{20} ML^2 = \frac{1}{10} ML^2$; $I_{total} = I_{center\ mass} + ML^2$

$I_{total} = \frac{11}{10} ML^2$;

$T_B = 2\pi \cdot \sqrt{\frac{2I}{Lmg}}$;

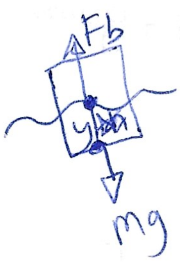
$$T_B = 2\pi\sqrt{\frac{\frac{11}{10}ML^2}{LMg}} = 2\pi\sqrt{\frac{\frac{11L}{10}}{g}} = 2\pi\sqrt{\frac{11L}{10}\cdot\frac{1}{g}} = 2\pi\sqrt{\frac{11L}{10g}}$$

$$\frac{T_A}{T_B} = \frac{2\pi\sqrt{L/g}}{2\pi\sqrt{\frac{11L}{10g}}} = \sqrt{\frac{\frac{L}{g}}{\frac{11L}{10g}}} = \sqrt{\frac{L}{g}\cdot\frac{10g}{11L}} = \sqrt{\frac{10}{11}}$$

$\frac{TA}{TB} = \sqrt{\frac{10}{11}}$; $TA = \sqrt{\frac{10}{11}}\,TB$; $\boxed{T_A = 0.95\,T_B}$ $T_A < T_B$

Ball B takes longer as its period is greater than the period of ball A

(2)

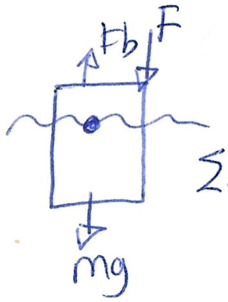


a) $F_b - mg = 0$

$F_b = mg$

$\rho_{Fluid}\cdot g\cdot A\cdot \cancel{\Delta h}\,y = mg$; $\boxed{\cancel{\Delta h}\,y = \frac{m}{\rho_{fluid}\cdot A}}$

b)



$\Sigma F = 0$; $F_b - mg - F = 0$;

$F_b = mg + F$

$A\cdot\cancel{\Delta h}\,y =$ Volume submerged; The object will be submerged more; Volume submerged $= A\cdot$ amount submerged;

$= A\cdot(y + y_2)$

$\rho_{fluid}\cdot g\cdot A(y+y_2) = mg + F$; as $y = \frac{m}{\rho_{fluid}\cdot A}$;

$\rho_{fluid}\cdot g\cdot A\left(\frac{m}{\rho_{fluid}\cdot A} + y_2\right) = mg + F$;

$\frac{\cancel{\rho_{fluid}}\cdot g\cdot \cancel{A}m}{\cancel{\rho_{fluid}\cdot A}} + \rho_{fluid}\cdot g\cdot A\cdot y_2 = mg + F$;

$A\cdot\rho_{fluid}\cdot g\cdot y_2 = \cancel{mg} + F - \cancel{mg}$; $\boxed{y_2 = \frac{F}{\rho_{fluid}\cdot g\cdot A}}$

c)

3 a) $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{5}{0{,}05}} = 10 m/s$

$f = 40 Hz$

$A = 3cm = 0{,}03m$

$\mu = 50 g/m = 0{,}05 kg/m$

$F_T = 5N$

b) $v = \lambda f$; $\lambda = \frac{v}{f}$; $\lambda = \frac{10}{40} = 0{,}25m$

c) $D(x,t) = A\cos(kx - \omega t)$ → moving to the right

$k = \frac{2\pi}{\lambda} = 25{,}12 \ rad/m$

$\omega = \frac{2\pi}{T} = 2\pi f = 251{,}2 \ rad/s$

$D(x,t) = 0{,}03 \cdot \cos(25{,}12x - 251{,}2t)$

d) $v_y = \frac{dy}{dt} = A\omega \sin(kx - \omega t)$;

$a_y(x,t) = \frac{dv_y}{dt} = -A\omega^2\cos(kx - \omega t)$

acceleration is maximum at $t=0, x=0$; $a_y(0,0) = -A\omega^2\cos(0) = -A\omega^2$

$= -0{,}03 \cdot (251{,}2)^2 = 1893{,}04 m/s^2$

e) $g = 9{,}8 m/s^2$

$a = 1893{,}04 m/s^2$

Gravity is so small compared to the acceleration of a point in the wave that it can be ignored, so the results are reasonable approximations

4 a) $y_1(x,t) = A\cos(kx + \omega t)$; 2 waves generated

$D_1(x,t) = A\cos(kx + \omega t)$

$D_2(x,t) = A\cos(kx - \omega t)$

$A\cos(kx + \omega t) + A\cos(kx - \omega t) = A \cdot (\cos(kx + \omega t) + \cos(kx - \omega t))$

$= A \cdot \left(2\cos\left(\frac{(kx + \omega t) + (kx - \omega t)}{2}\right) \cdot \cos\left(\frac{(kx + \omega t) - (kx - \omega t)}{2}\right)\right)$

$= 2A\cos(kx) \cdot \cos(\omega t)$

b) At $x=0$; $y(x,t) = 2A\cos(\omega t)$; Maximum amplitude $= 2A$

Amplitude at $x=0$ is $2A$

Hence $x=0$ there is an antinode

c) Max displacement, max speed and max acceleration?

We know already that the maximum displacement is $2A$;

$y_{max} = 2A$;

$$v_{y} = \frac{dy}{dt} = 2A\cos(kx) \cdot -\omega\sin(\omega t) = -2A\omega\cos(kx)\sin(\omega t)$$

v_y at $x=0$; $\quad -2A\omega\cos(0)\sin(\omega t) = -2A\omega\sin(\omega t)$

The maximum velocity is $|-2A\omega| = 2A\omega$

$$a_y = \frac{dv}{dt} = -2A\omega\cos(kx) \cdot \omega\cos(\omega t) = -2A\omega^2\cos(kx) \cdot \cos(\omega t)$$

At $x=0$; $-2A\omega^2 \cdot \cos(\omega t)$;

Maximum acceleration $= 2A\omega^2$

d)

$$f_n = \frac{nv}{2L} = \frac{n}{2l}\sqrt{\frac{F_T}{\mu}}$$

Resonant frequency : frequency where it tends to vibrate at a higher amplitude

$$f_n = \frac{n \cdot v_{max}}{2L}; \quad \boxed{f_n = \frac{n \cdot 2A\omega}{2L}}$$