PHY 250

1.

a) Find period of each pendulum.

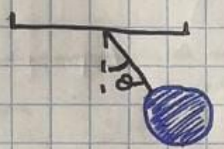
b) Which ball takes longer to complete a swing?

a)

Pendulum A → $\omega = \sqrt{\frac{g}{l}} = \omega_1 = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega}$

$T_1 = \frac{2\pi}{\sqrt{\frac{g}{l}}}$

Pendulum B →



e generated by w

↓

$-l \cdot w \cdot \sin(\theta) \rightarrow -lw\theta = e_w$

$\theta'' + \omega^2\theta = 0$

when the displacement is small $\sin(\theta) \simeq \theta$

$I = \frac{2}{5} l^2 M + (l^2 M) = \frac{2}{5} l^2 2M$

$-l w \sin(\theta) = \frac{2}{5} l^2 2M \, \theta''$

$\frac{2}{5} l^2 2M \theta'' + l w \sin(\theta) = 0$ $w = m g$

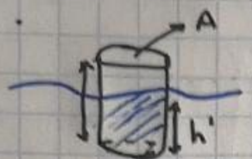
$\theta'' + \frac{l \cdot w}{\frac{2}{5} l^2 2 \cdot m} \cdot \theta = 0$ $T_2 = \frac{2\pi}{\sqrt{\frac{g}{\frac{4}{5} l}}}$

↓

$\frac{\cancel{l} \cdot \cancel{m} g}{\frac{2}{5} l^{\cancel{2}} 2 \cancel{m}} = \frac{g}{\frac{2}{5} l \cdot 2} = \omega^2 \rightarrow$ $\boxed{\omega_2 = \sqrt{\frac{g}{\frac{4}{5} l}}}$

$\boxed{\text{As } \omega_1 > \omega_2 \rightarrow T_1 < T_2}$

2.



Height = h
Mass = M
Area = A
Density = ρ

a) Calculate: h'

We know that $\rho = M / Vol = M / \underbrace{A \cdot h}_{\text{data}}$

$F_B = \rho \cdot g \cdot hA$

Acording to Archimede's principle

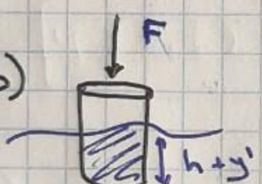
$F_B = W$

$\downarrow$

$\rho g h \cdot A = M \cdot g$

$h = \frac{m \cancel{g}}{\rho \cdot \cancel{g} \cdot A} \rightarrow \boxed{h = \frac{m}{\rho \cdot A}}$

b)



Now $V \neq h \cdot A$ as h is chaged

$V' = (h + y') \cdot A$

Equlibrium $\Sigma F = 0$

$F_B = \rho g (h + y') \cdot A$

$F_B - F - W = 0$

$\rho g (h + y') \cdot A = Mg + F$

$y' = \frac{Mg + F}{\rho g A} - h$ → we know h from "a"

$y' = \frac{Mg + F}{\rho g A} - \frac{m}{\rho A} = \frac{\cancel{mg} + F - \cancel{mg}}{\rho g A}$

$\boxed{y' = \frac{F}{\rho g A}}$

c) F is now suddenly removed → $\underline{\text{oscilation?}}$

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$F_{net} = -kx$

↓

$F_B + \omega$

$$Mg - \rho g\,(h + x)\,A = -Kx$$

Knowing that
$mg = \rho g h A$

$$\to Mg - [\rho g h A + \rho g h x A]$$

$$mg - [mg + \rho g h x A] = -kx$$

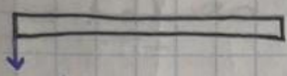
$$-A\rho g h x = -kx = A\rho g h \cancel{x} = k\cancel{x}$$

$$A\rho g h = k$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{\rho g h A}}}$$

→ the movement will be an oscilation with a period T

3.



oscilator point

$f = 40 Hz$

$A = 3.00 cm$

$\mu = 50 g/m$

$T = 5N$

a) Speed?
b) λ
c) $y(x,t)$?
d) MAX a
e) Is aprox right?

a) Speed of the wave

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{5}{5 \cdot 10^{3}}} = \underline{10 m/s}$$

$$50 g/m = 5 \cdot 10^{-3} Kg/m$$

b) λ?

$$v = \frac{\lambda}{T} \Rightarrow \lambda = v \cdot T = \frac{v}{f} = \frac{10}{40} = 1/4 = \boxed{0,25m}$$

c) Ecuation?

$$\boxed{y(x,t) = A \cdot \cos(Kx - \omega t)}$$

$\begin{cases} \text{at } t = 0 \\ \quad\; x = 5 \\ y = A \end{cases}$

d) Max a?

$$y' = v = +A \sin(kx - \omega t) \cdot \omega$$
$$v' = a = +A\omega^2 \cos(kx - \omega t)$$

$$a_{MAX} \Rightarrow \cos(kx - \omega t) = 1 \rightarrow a_{max} = A\omega^2$$

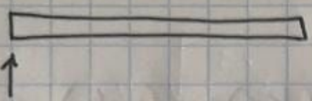
Knowing that

$\omega = 2\pi \cdot f$

$$a_{MAX} = 3 \cdot 10^{-2} \cdot (2\pi \cdot 40)^2$$

$$\boxed{1894,96 m/s^2}$$

4.



free end

a)

Wave function = $y(x,t) = A \cdot \cos(kx + \omega t)$
(towards the right)

Wave function = $y(x,t) = A \cdot \cos(kx - \omega t)$
(towards the left)

if both waves interfere:

$\cancel{y}\ \omega_1 + \omega_2 = \omega_T$

$\omega_T = y(x,t) = 2A\left[\cos(kx+\omega t) + \cos(kx-\omega t)\right]$

↓ Expanding this

$y(x,t) = A\left[\cos(kx)\cos(\omega t) - \cancel{\sin(kx)\sin(\omega t)} + \cos(kx)\cos(\omega t) + \cancel{\sin(kx)\sin(\omega t)}\right]$

$$\boxed{y(x,t) = 2A\cos(kx)\cos(\omega t)}$$

b) Show that the wave has an antinode at $x=0$

if $x=0$ $\quad y(0,t) = 2A\cos(\omega t)$

↓

Amplitude $= 2A$

this is an equation for a simple harmonic motion,

so at $t=0$ $\boxed{y = 2A}$ → which means that $x=0$ we have an antinode

c) Max displacement, a and v

$\boxed{y_{MAX} = 2A}$ from ~~[illegible]~~ $y(0,t)$

$v = \frac{dy}{dt} \rightarrow$ ~~[illegible]~~ $v = -2A \cos(kx)\sin(\omega t)\cdot\omega$

$v(0,t) = -2A \sin(\omega t)\cdot \omega$

$\boxed{V_{MAX} = 2\omega A}$ when $\boxed{\sin(\omega t) = 1/-1}$

$a = \frac{dv}{dt} = -2\omega^2 A \cos(kx)\cos(\omega t)$

at $x = 0$ $\quad a = -2\omega^2 A \cos(\omega t)$

$\boxed{a_{MAX} = +2\omega^2 A}$ for $\cos(\omega t) = 1/-1$