Phy 250
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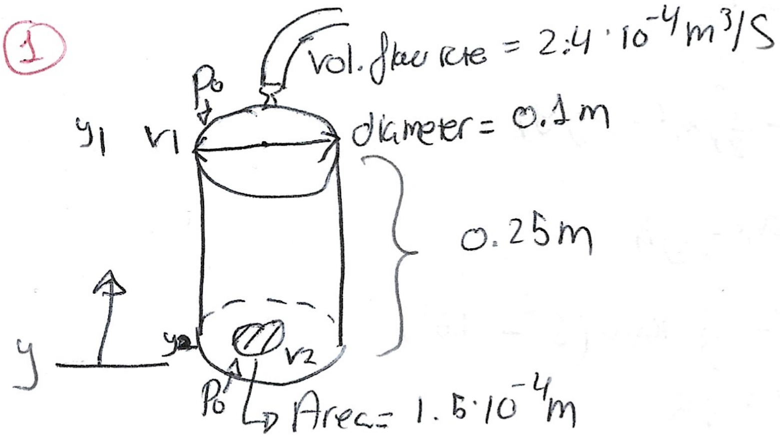
Homework 2

(1)



Bernoulli's equation: $\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$

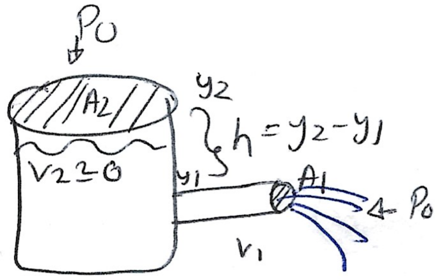
Vol. rate of flow $= A \cdot V$

$V_2 = \frac{\text{vol. rate of flow}}{A} = \frac{2.4 \cdot 10^{-4}}{1.5 \cdot 10^{-4}} = 1.6\,m/s$

$\cancel{\frac{1}{2}\rho v_1^2}^{\to 0} + \cancel{P_0} + \rho g y_1 = \frac{1}{2}\rho v_2^2 + \cancel{P_0} + \cancel{\rho g y_2}^{\to 0}$

$\cancel{\rho} g y_1 = \frac{1}{2}\cancel{\rho} v_2^2$; $\quad y_1 = \frac{v_2^2}{2g}$; $\quad y_1 = \frac{(1.6)^2}{2 \cdot 9{,}8} = 0.13\,m$

(2)



Find V1.

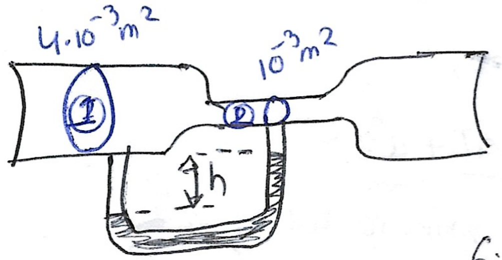
Bernoulli's equation: $\frac{1}{2}\rho v_2^2 + P_0 + \rho g y_2 = \frac{1}{2}\rho v_1^2 + P_0 + \rho g y_1$

$\cancel{\frac{1}{2}\rho v_2^2}^{\to 0} + \cancel{P_0} + \rho g y_2 = \frac{1}{2}\rho v_1^2 + \cancel{P_0} + \rho g y_1$

$\frac{1}{2}\rho v_1^2 = \rho g y_2 - \rho g y_1$; $\quad \frac{1}{2}\cancel{\rho} v_1^2 = \cancel{\rho} g (y_2 - y_1)$; $\quad v_1^2 = 2gh$;

$v_1 = \sqrt{2gh} = \sqrt{2g(y_2 - y_1)}$

(3)



Rate of flow $= 6 \cdot 10^{-3}\,m^3/s$

a) volume flow rate $= A \cdot v$; $\quad v_1 = \frac{6 \cdot 10^{-3}}{4 \cdot 10^{-3}} = 1{,}5\,m/s$

$v_2 = \frac{6 \cdot 10^{-3}}{10^{-3}} = 6\,m/s$

b) Bernoulli's equation: $\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$

$y_1 = y_2 = h$

$P_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2 - \frac{1}{2}\rho v_1^2 - \rho g y_1$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2 + \cancel{\rho g h} - \frac{1}{2}\rho v_1^2 - \cancel{\rho g h}$

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$; $\Delta P = \frac{1}{2}\cdot 1000\,(6^2 - 1.5^2)$

$\Delta P = 500\cdot(33.75) = 16875$ Pa

c)

$\cancel{P_1 - P_2 = \rho_{water}}$ $\cancel{P_1 = P_0 + \rho_{Hg} g h_1}$

$\cancel{(P_1-P_2) = \rho_{Hg} - \rho_{water}}$ $\cancel{P_2 = P_0 + \rho_{water} g h_2}$ $\rho_{Hg} = 13.6\cdot 10^3\ kg/m^3$

$\cancel{g\Delta h}$

$\cancel{g\Delta h = P}$

$\rho_{water}\, g h + P_1 = \rho_{Hg}\, g h + P_2$;

$P_1 - P_2 = \rho_{Hg}\, g h - \rho_{water}\, g h$; $P_1 - P_2 = gh(\rho_{Hg} - \rho_{water})$

$gh = \frac{P_1 - P_2}{\rho_{Hg} - \rho_{water}}$; $h = \frac{P_1 - P_2}{(\rho_{Hg} - \rho_{water})g} = \frac{16875}{(13.6\cdot 10^3 - 1000)\cdot 9.8} = 0.13\ m$

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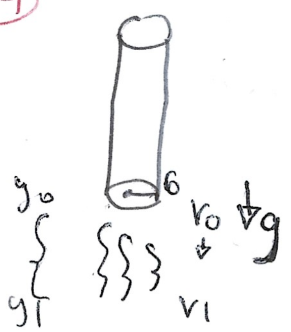

a) Equations of motion with constant acceleration

$v_1 = v_0 + at$

$y_1 = y_0 + v_0 t + \frac{1}{2}at^2$;

$v_1^2 = (v_0 + at)^2$; $v_1^2 = v_0^2 + \underbrace{2v_0 a t + a^2t^2}$

This is similar to $v_0 t + \frac{1}{2}at^2$.
We need to get rid of the $\frac{1}{2}$ and multiply by an extra a.

$y_1 - y_0 = v_0 t + \frac{1}{2}at^2$; $2a(y_1 - y_0) = 2av_0 t + a^2t^2$

$v_1^2 = v_0^2 + 2a(y_1 - y_0)$; $\boxed{v_1 = \sqrt{v_0^2 + 2a(y_1 - y_0)}}$

Equation of continuity: $V_0 A_0 = V_1 A_1$; $A = \pi r^2$;

$$V_0 \cdot \pi r_0^2 = V_1 \pi r_1^2;$$

$$V_1 = V_0 \cdot \frac{\cancel{\pi} r_0^2}{\cancel{\pi} r_1^2}; \quad V_1 = V_0 \frac{r_0^2}{r_1^2}$$

$$\frac{V_0 \cdot r_0^2}{r_1^2} = \sqrt{V_0^2 + 2a(y_1 - y_0)}; \quad V_0 \cdot r_0^2 = r_1^2 \sqrt{V_0^2 + 2a(y_1 - y_0)};$$

$$\frac{V_0 \cdot r_0^2}{\sqrt{V_0^2 + 2a(y_1 - y_0)}} = r_1^2; \quad r_1 = \sqrt{\frac{V_0 \cdot r_0^2}{\sqrt{V_0^2 + 2a(y_1 - y_0)}}}$$

b) $V_0 = 1.2\,m/s$

$r_1 = \frac{1}{2} r_0$

y? $y = y_1 - y_0$;

$$\frac{1}{2} r_0 = \sqrt{\frac{V_0 r_0^2}{\sqrt{V_0^2 + 2ay}}};$$

$$\frac{1}{4} r_0^2 = \frac{V_0 r_0^2}{\sqrt{V_0^2 + 2ay}}; \quad \frac{1}{4} \cancel{r_0^2} \cdot \sqrt{V_0^2 + 2ay} = V_0 \cancel{r_0^2};$$

$$\sqrt{V_0^2 + 2ay} = 4V_0; \quad V_0^2 + 2ay = 16 V_0^2; \quad y = \frac{16V_0^2 - V_0^2}{2a}; \quad a = g = 9{,}8\,m/s^2$$

$$y = \frac{16 \cdot (1.2)^2 - (1.2)^2}{2 \cdot 9{,}8} = 1{,}1\,m$$