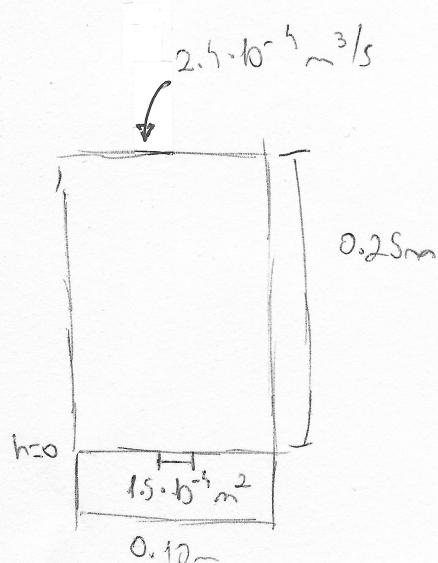


1)



$$A_1 \cdot v_1 = A_2 \cdot v_2 \Rightarrow$$

$$2.5 \cdot 10^{-5} = 1.5 \cdot 10^{-5} \cdot v_2$$

v_2 . We know the Bernoulli's equation, so

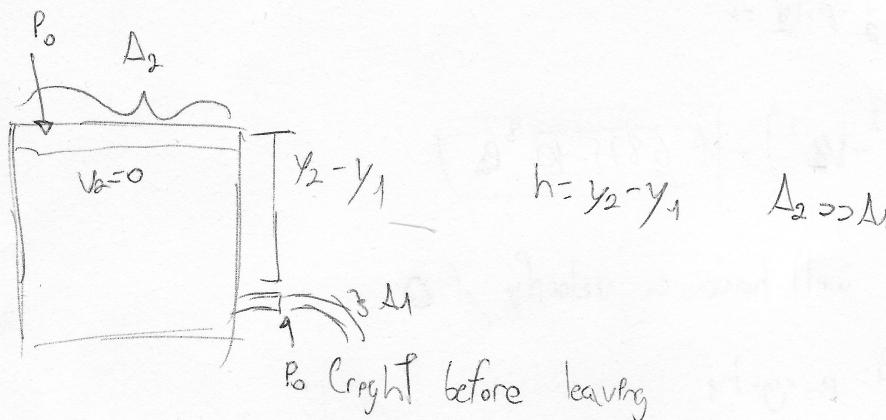
$$\begin{aligned} E_1 = E_2 &\Rightarrow p \cdot g \cdot h + \frac{1}{2} \cdot p \cdot v_1^2 = p \cdot g \cdot h + \frac{1}{2} \cdot p \cdot v_2^2 \\ \text{at nozzle} & \quad \text{after height end} \\ \end{aligned}$$

$$2.5 \cdot 10^{-5} = 1.5 \cdot 10^{-5} \cdot \sqrt{2 \cdot 9.8 \cdot h} \Rightarrow$$

$$\left(\frac{2.5 \cdot 10^{-5}}{1.5 \cdot 10^{-5}} \right)^2 = 2 \cdot 9.8 \cdot h \Rightarrow$$

$$h = \left(\frac{2.5 \cdot 10^{-5}}{1.5 \cdot 10^{-5}} \right)^2 : (2 \cdot 9.8) \Rightarrow h = 0.13 \text{ m}$$

2)

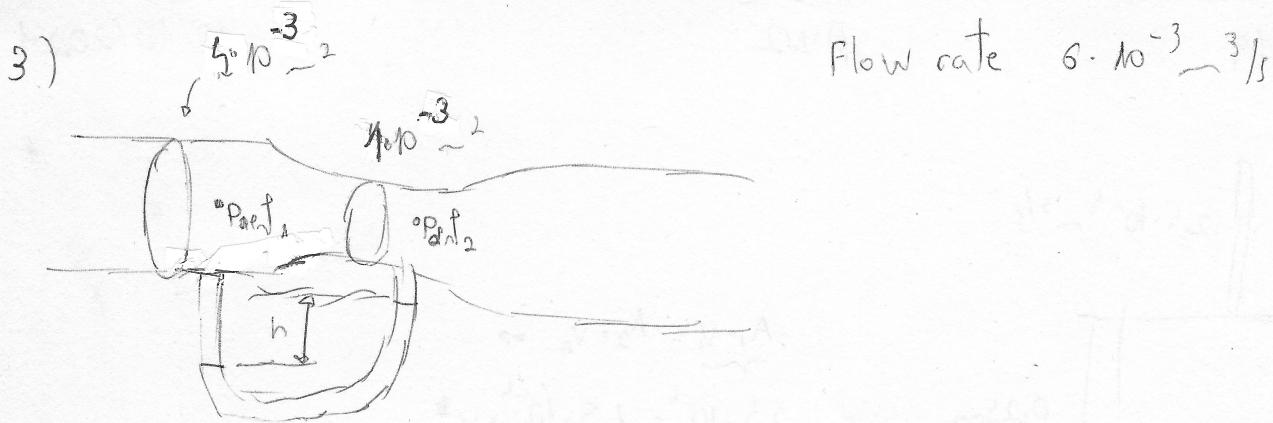


The only force that affects is the gravity, so we know that (Bernoulli's equation)

$$\frac{1}{2} \cdot p \cdot v_2^2 + P_0 + p \cdot g \cdot y_2 = \frac{1}{2} \cdot p \cdot v_1^2 + P_0 + p \cdot g \cdot y_1 \Rightarrow$$

$$p \cdot g \cdot y_2 = \frac{1}{2} \cdot p \cdot v_1^2 + p \cdot g \cdot y_1 \Rightarrow g \cdot y_2 = \frac{1}{2} \cdot v_1^2 + g \cdot y_1 \Rightarrow$$

$$\sqrt{2 \cdot g \cdot (y_2 - y_1)} = v_1 \Rightarrow \boxed{2 \cdot 9.8 \cdot (y_2 - y_1) = v_1^2}$$



Flow rate $6 \cdot 10^{-3} \text{ m}^3/\text{s}$

a) v_1 v_2

$$A_1 \cdot v_1 = 6 \cdot 10^{-3} \Rightarrow v_1 = 15 \text{ m/s}$$

$$A_2 \cdot v_2 = 6 \cdot 10^{-3} \Rightarrow v_2 = 6 \text{ m/s}$$

b) Bernoulli's equation

$$\underbrace{\frac{1}{2} \cdot p \cdot v_1^2 + p_1 + \rho \cdot g \cdot h}_{\text{Point 1}} = \underbrace{\frac{1}{2} \cdot p \cdot v_2^2 + p_2 + \rho \cdot g \cdot h}_{\text{Point 2}}$$

Same height
so

$$\Rightarrow \frac{1}{2} p \cdot v_1^2 + p_1 = \frac{1}{2} p \cdot v_2^2 + p_2 \Rightarrow$$

$$p_1 - p_2 = \frac{1}{2} p \cdot v_2^2 - \frac{1}{2} p \cdot v_1^2 \Rightarrow$$

$$p_1 - p_2 = \frac{1}{2} \cdot p \cdot (v_2^2 - v_1^2) = 16875 \text{ Pa} \dots$$

c) The mercury well have a velocity of 0

$$\text{Column 1: } p_1 = p_m \cdot g \cdot h_1$$

$$\text{Column 2: } p_2 = p_m \cdot g \cdot h_2$$

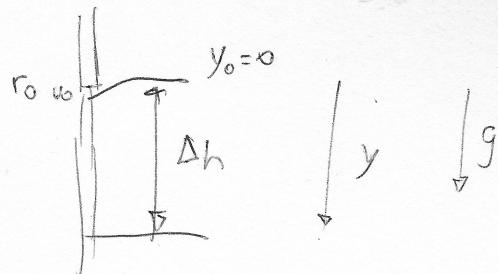
Bernoulli eq.

$$p_1 - p_2 = p \cdot g \cdot \Delta h = \frac{16875}{13,6 \cdot 10^3 \cdot 9,8} = 0,127 \text{ m}$$

5) Equations of motion

a) $y = y_0 + v_0 t + \frac{1}{2} \cdot g \cdot t^2$

$$v = v_0 + g \cdot t$$



$$\Rightarrow \frac{v - v_0}{g} = t$$

$$y = v_0 \cdot \left(\frac{v - v_0}{g} \right) + \frac{1}{2} \cdot g \cdot \left(\frac{v - v_0}{g} \right)^2 \Rightarrow$$

$$y = \frac{v v_0 - v_0^2}{g} + \frac{1}{2} \cdot \frac{v^2 + v_0^2 - 2vv_0}{g} \Rightarrow$$

$$g \cdot y = \frac{2vv_0 - 2v_0^2 + v^2 + v_0^2 - 2vv_0}{2} \Rightarrow$$

$$2g \cdot y = v^2 - v_0^2 \Rightarrow \boxed{v = \sqrt{2gy + v_0^2}}$$

$$A_1 \cdot v_1 = A_2 \cdot v_2 \Rightarrow \quad v_1 = v_0 \quad v_1 = v_0 \quad v_2 \quad A_2 = r^2 \pi \quad A_1 = r_0^2 \pi$$

$$\pi \cdot r_0^2 \cdot v_0 = \pi \cdot r^2 \cdot \sqrt{2g \cdot y + v_0^2} \Rightarrow$$

$$r^2 = \frac{\pi \cdot r_0^2 \cdot v_0}{\pi \cdot \sqrt{2g \cdot y + v_0^2}} \Rightarrow \boxed{r = \frac{r_0 \sqrt{v_0}}{\sqrt{2g \cdot y + v_0^2}}}$$

6) $v_0 = 12 \text{ m/s}$ $r = \frac{1}{2} r_0$

$$\frac{1}{2} r_0 = \frac{r_0 \sqrt{1.2}}{\sqrt{2 \cdot 9.8 \cdot y + 1.44}} \Rightarrow \sqrt{2 \cdot 9.8 \cdot y + 1.44} = 2 \cdot \sqrt{1.2} \Rightarrow (\text{to the power of } 4)$$

$$2 \cdot 9.8 \cdot y + 1.44 = 23.04 \Rightarrow y = \boxed{1.1}$$