PHY250: FLUIDS IN MOTION

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Digipen

Fall 2021

Fluids Dynamic

- Equation of continuity
- Bernoulli's Equation
- viscosity
- Generalization

Fluids in motion

In general, the equation $\nabla P + \rho \nabla \phi = 0$ has no solution.

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$$\rightarrow \nabla P + \rho \nabla \phi = \rho \vec{a}$$

$\boxed{\textit{Dynamic Fluid}}$

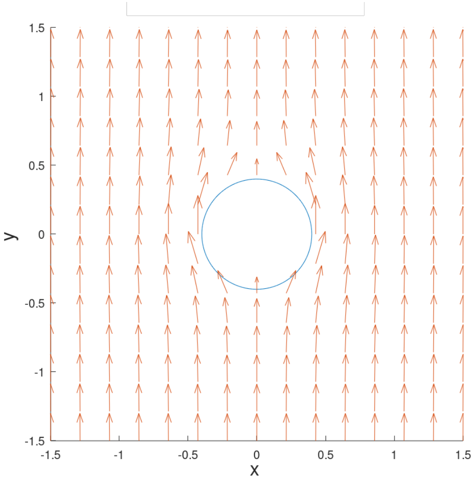
Fluids in motion

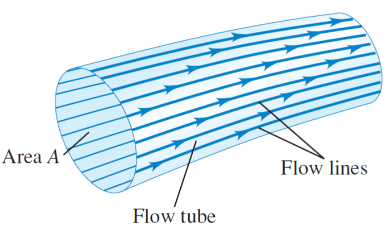
To describe the motion of a fluid, we need:

Fluids in motion

To describe the motion of a fluid, we need:

- An equation of state: $P(\rho)$
- The velocity at every point





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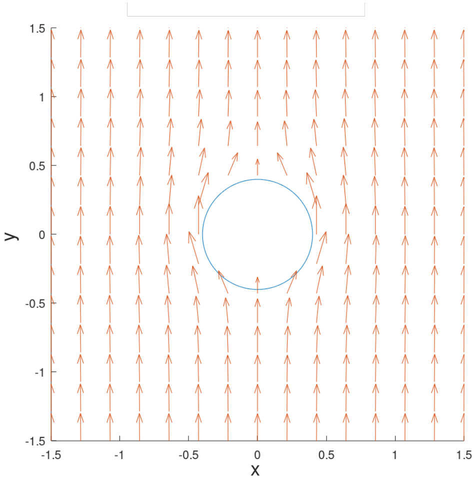
Streamlines: lines which are always tangent to the fluid velocity.



Obs: two streamlines can not cross each other. Why?

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- Steady Flow: at any one place in the fluid the velocity never changes $\rightarrow$ the streamlines are the fluid particles path.

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- Steady Flow: at any one place in the fluid the velocity never changes $\rightarrow$ the streamlines are the fluid particles path.
- Non-Steady Flow: At a given point, the velocity changes with time.

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- The velocity of the flow $<<$speed of sound in the Fluid.
- $\rho = \textit{constant}$ (incompressible fluid)
- steady flow.

Steady flow:

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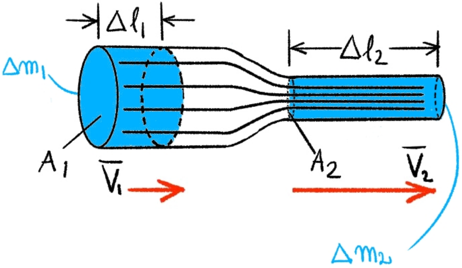
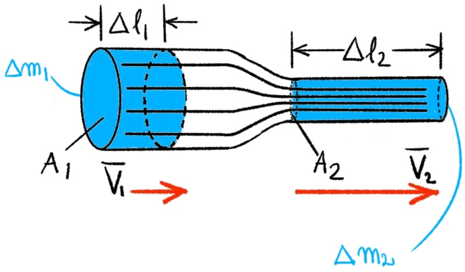
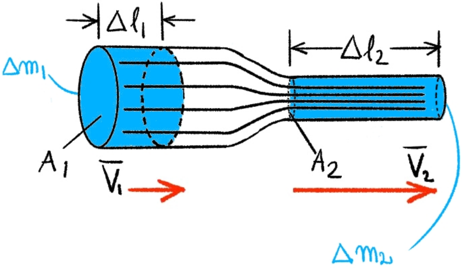
- At any one place in the fluid the velocity never changes.
- Fluid element moves in lines which are always tangent to the fluid velocity (streamlines).


Δl_1
Δl_2
Δm_1
A_1
$\overline{V}_1$
A_2
$\overline{V}_2$
Δm_2



- Mass Flow Rate $\rightarrow \frac{\Delta m}{\Delta t}$



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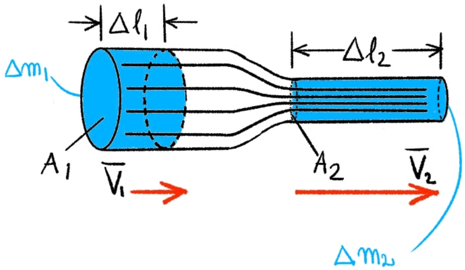

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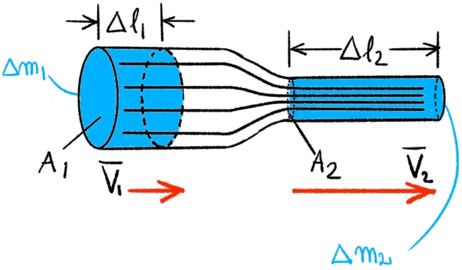
$$\rightarrow \frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \qquad (1)$$


Δl₁
Δl₂
Δm₁
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A₂
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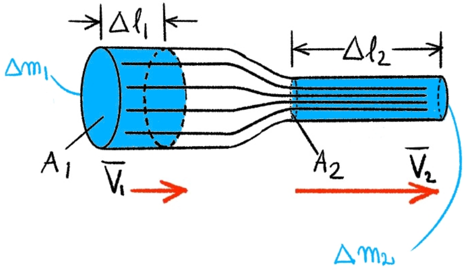
Δm₂

$$Vol_1 = A_1 \Delta \ell_1,$$



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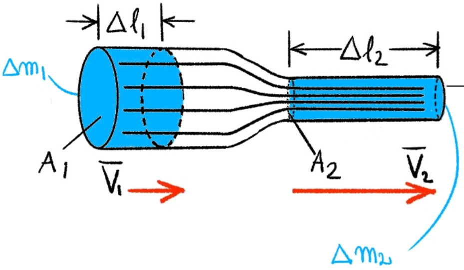

$$\rightarrow \frac{\Delta m_1}{\Delta t} = \frac{\rho_1 Vol_1}{\Delta t} = \frac{\rho_1 A_1 \Delta \ell_1}{\Delta t}$$

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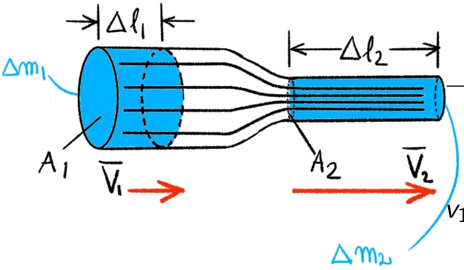

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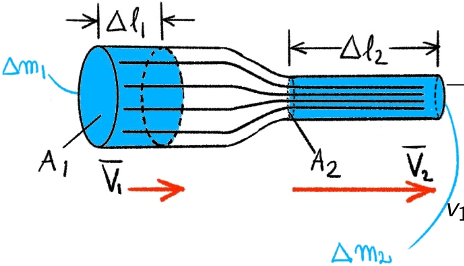

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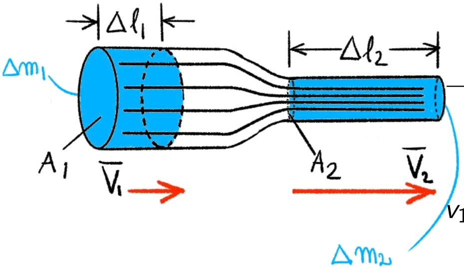

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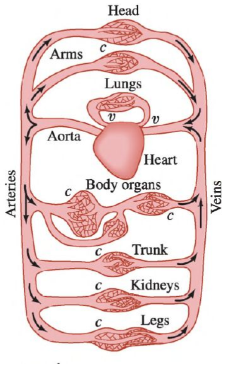
Av is the *volume rate of flow*

$$\boxed{Av = constant}$$

Example 1

Blood flow. In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about $1.2cm$, and the blood passing through it has a speed of about $40cm/s$. A typical capillary has a radius of about $4 \times 10^{-4}cm$, and blood flows through it at a speed of about $5 \times 10^{-4}m/s$.



Estimate the number of capillaries that are in the body.

Bernoulli's Principle

Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

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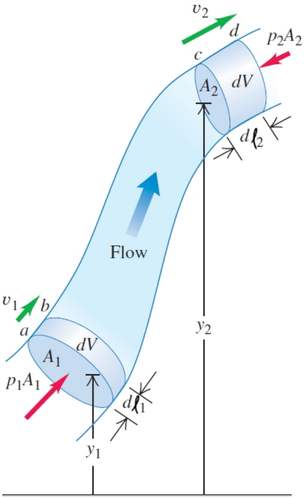
The theorem of Bernoulli is in fact nothing more than a statement of the conservation of energy.



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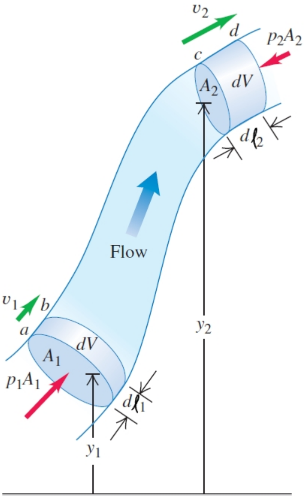
The work done at A_1 is,

$$W_1 = F_1 \Delta \ell_1 = P_1 A_1 \Delta \ell_1 \tag{4}$$



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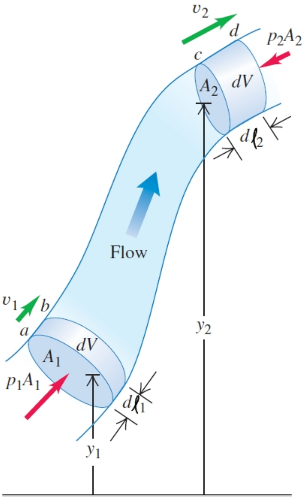

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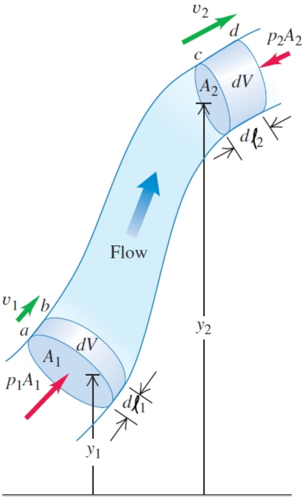

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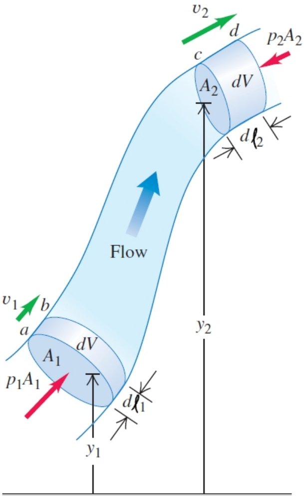

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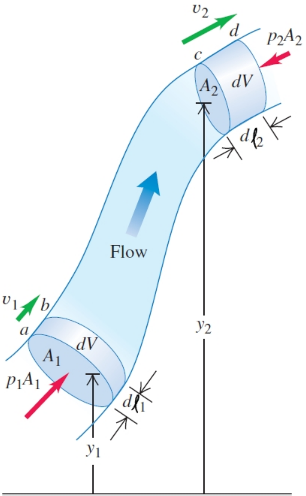

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$$\frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2 = \frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1$$

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$$\boxed{\frac{1}{2}\rho v^2 + P + \rho g y = constant} \tag{7}$$

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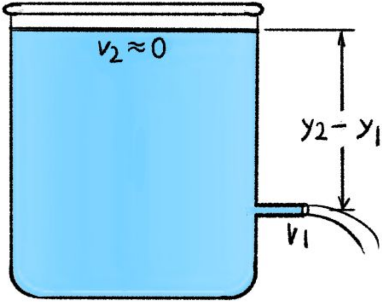
$$P_2 - P_1 = -\rho g (y_2 - y_1) \leftarrow Hydrostatic\ equation \tag{8}$$

Example 2

Calculate the velocity, v_1, of a liquid flowing out of a spigot at the bottom of a reservoir.



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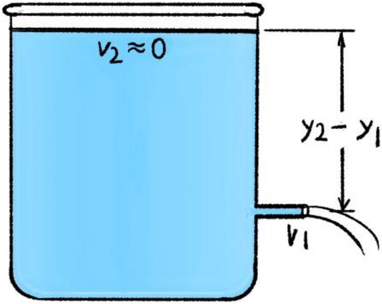
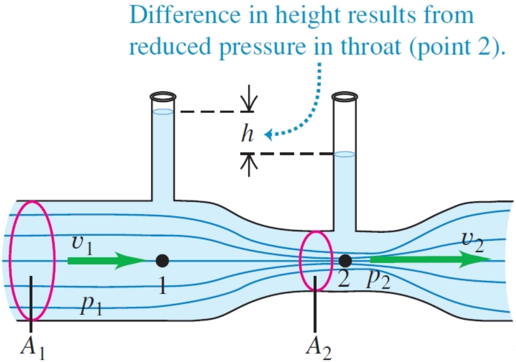


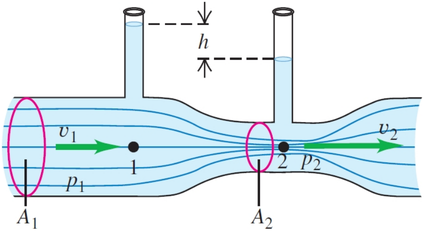
The liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height.

Example 3

Figure 20 shows a Venturi meter, used to measure flow speed in a pipe. Derive an expression for the flow speed in terms of the cross-sectional areas and and the difference in height h of the liquid levels in the two vertical tubes.



Solved in whiteboar



Example 5

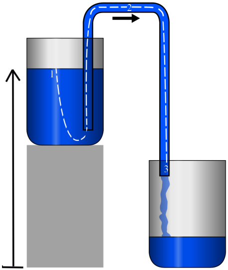
In 1993 the radius of Hurricane Emily was about 350 km. The wind speed near the center ("eye") of a hurricane, whose radius is 30 km, reaches about 200 km/h. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. (a) Estimate the wind speed at the rim of the hurricane. (b) Estimate the pressure difference at the earth's surface between the eye and the rim. Where is the pressure greater?

Example 6

Solved in whiteboar

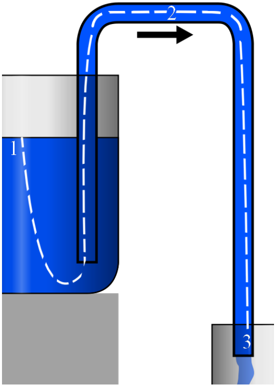
Example 7

A siphon is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density ρ and let the atmospheric pressure be P_{atm}. Assume that the cross-sectional area of the tube is the same at all points along it.



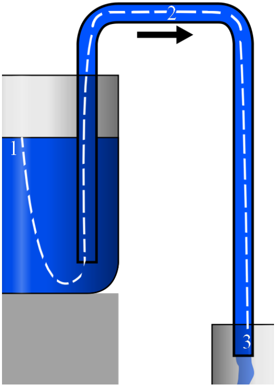
Example 7

1. If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.
2. A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?



(a)

$$P_0 + \rho g h = P_0 + \frac{1}{2}\rho v_3^2$$

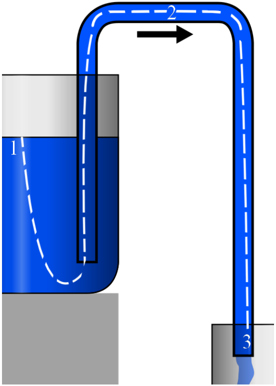


(a)

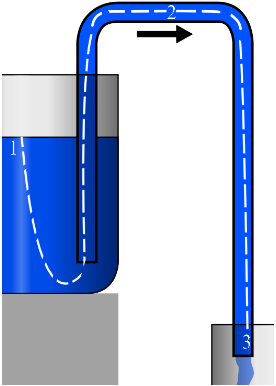
$$P_0 + \rho g h = P_0 + \frac{1}{2}\rho v_3^2$$

$$\rightarrow v_3 = \boxed{\sqrt{2gh}}$$

(b)

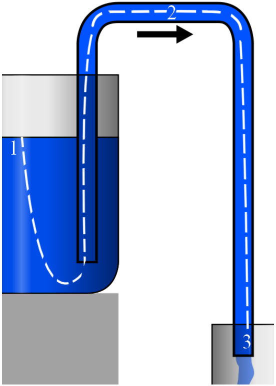


$$Bernoulli\ (1) \quad = \quad Bernoulli\ (3)$$



(b)

$$
\begin{aligned}
Bernoulli\ (1) &= Bernoulli\ (3) \\
P_2 + \frac{1}{2}\rho v_2^2 + \rho g(H+h) &= P_0 + \frac{1}{2}\rho v_1^2
\end{aligned}
$$



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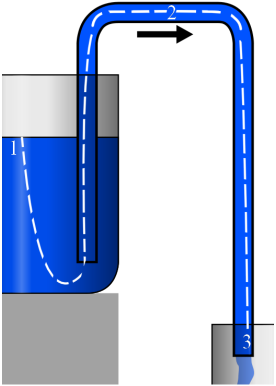

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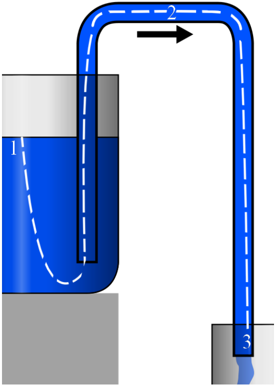

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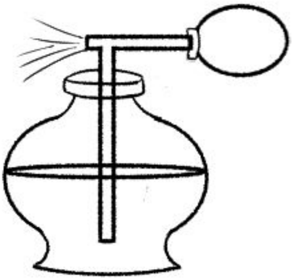
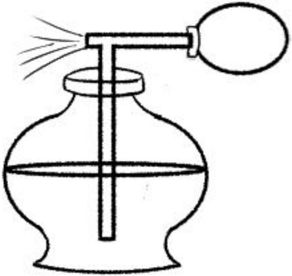
$$

$$
min \quad P_2 = 0 \ \rightarrow H_{max} = \boxed{\frac{P_0}{\rho g} - h}
$$

Conceptual example: How does a Perfume Atomizer work?



Conceptual example: How does a Perfume Atomizer work?



The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top.

Viscosity

Real Fluids have certain amount of internal friction called **viscosity**. It is a frictional force between adjacent layers of fluid.

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The viscosity can be expressed quantitatively by a *coefficient of viscosity* η

Viscosity

How do we calculate it?

Viscosity

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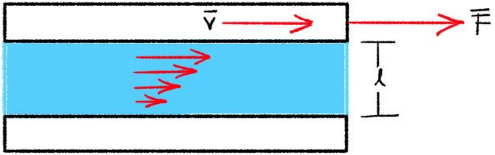

Viscosity

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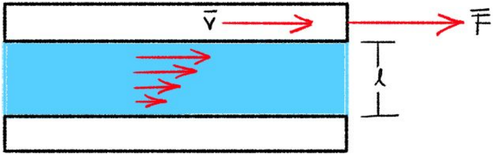

$$F = \propto A\frac{v}{\ell} \tag{9}$$

Viscosity

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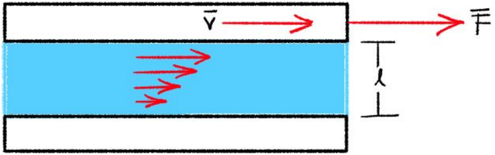

$$F = \propto A\frac{v}{\ell} \tag{9}$$

$$F = \eta A\frac{v}{\ell} \rightarrow \eta = \boxed{\frac{F\ell}{Av}} \tag{10}$$

Viscosity

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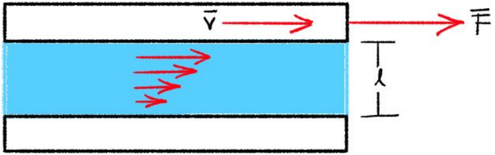


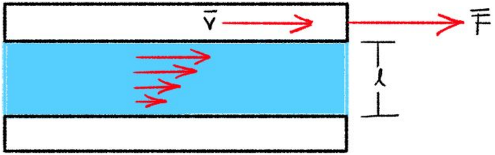
$$F = \propto A\frac{v}{\ell} \tag{9}$$

$$F = \eta A\frac{v}{\ell} \rightarrow \eta = \boxed{\frac{F\ell}{Av}} \tag{10}$$

The unit of η is $N \cdot s/m^2 = Pa \cdot s$

Viscosity

How do we calculate it?



There is always a thin boundary layer of fluid near the surface, in which the fluid is nearly at rest with respect to the surface. That's why dust particles can cling to a fan blade even when it is rotating rapidly

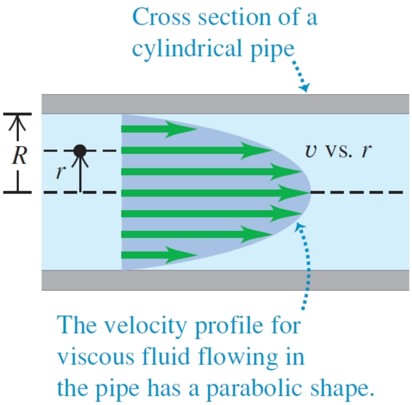
TABLE 13–3
Coefficients of Viscosity

Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s)[†]
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

[†] 1 Pa·s = 10 P = 1000 cP.

Viscous fluid in a cylindrical pipe:

Viscous fluid in a cylindrical pipe: *"The motion is like a lot of concentric tubes sliding relative to one another".*



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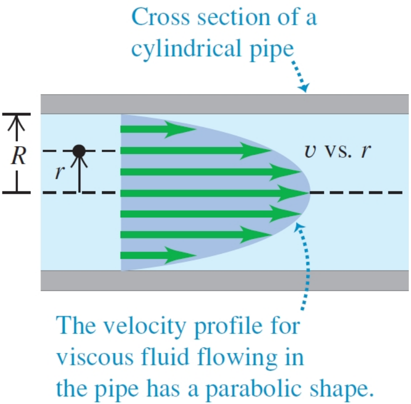
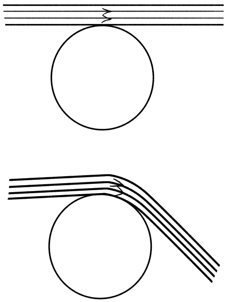


A pressure difference $\propto L/R^4$ is required to sustain the motion.

- This simple relationship, $\propto L/R^4$, explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure.

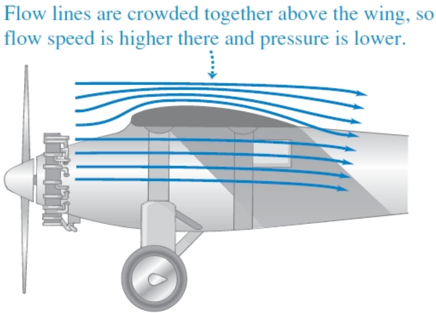
- This simple relationship, $\propto L/R^4$, explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure.
- That's why you have to keep squeezing a tube of toothpaste or a packet of ketchup to keep the fluid coming out of its container.

Coanda effect



https://www.youtube.com/watch?v=NvzXKZNJ7ZU&t=4s

Airplane Wings and Dynamic Lift



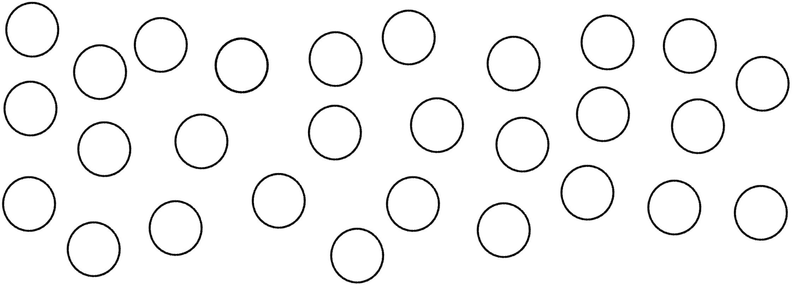
Surface Tension and Capillarity

Surface Tension and Capillarity

Why a body with a density several times the density of water cans rest *atop* the water surface?

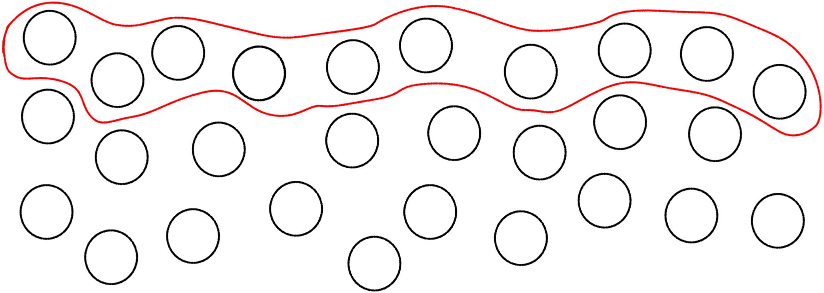
Surface Tension and Capillarity

The particles that make up the liquid are in constant random motion.



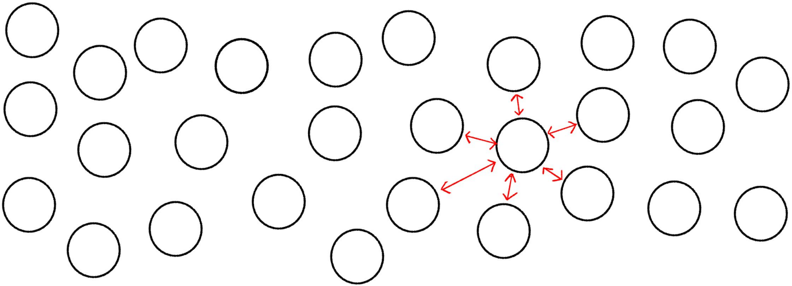
Surface Tension and Capillarity

Do the particles at the surface form a random irregular surface?



Surface Tension and Capillarity

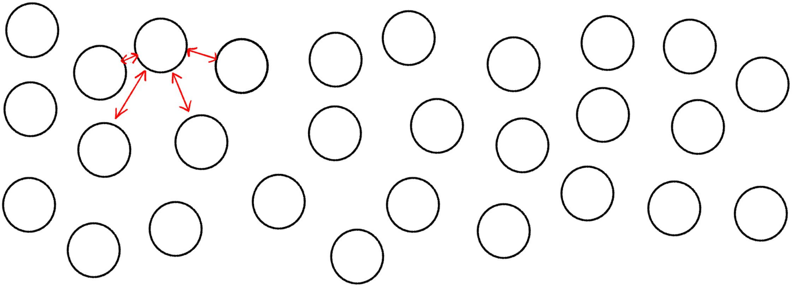
Molecules in a liquid are attracted by neighboring molecules.



Molecules in the interior are equally attracted in all directions.

Surface Tension and Capillarity

Molecules in a liquid are attracted by neighboring molecules.



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Surface Tension and Capillarity

Surface molecules form a much smoother surface.

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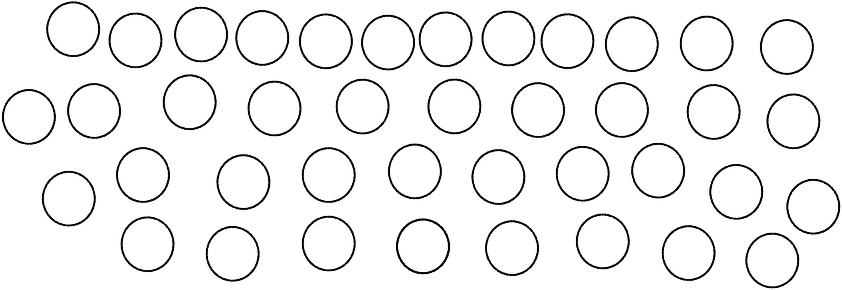
Molecules are compressed and minimizes the surface $\rightarrow$

Surface Tension and Capillarity

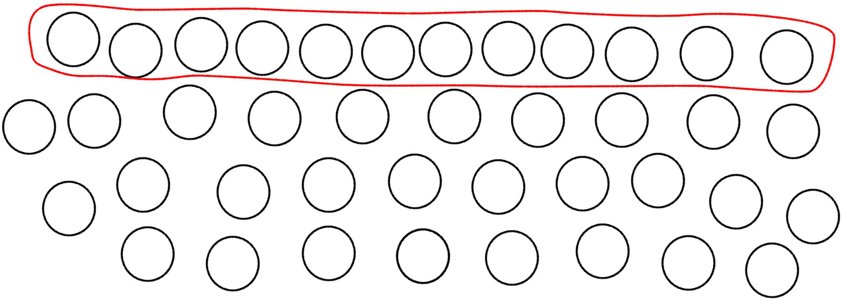
Surface molecules form a much smoother surface. Surface

Molecules are compressed and minimizes the surface $\rightarrow$ Higher energy at the surface.



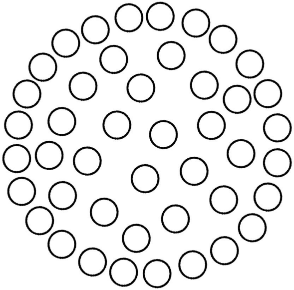
Surface Tension and Capillarity

Liquid surface $\leftrightarrow$ Stretched membrane under tension



Surface Tension and Capillarity

Liquid not confined in a container.



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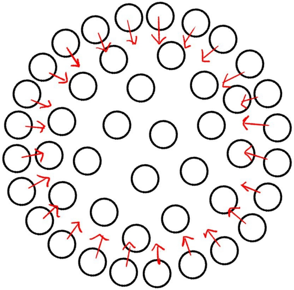

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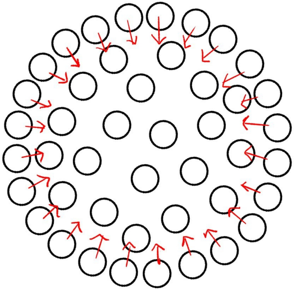
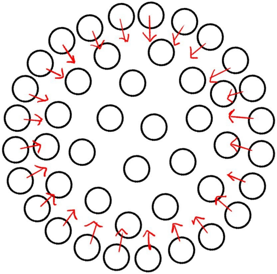


The ratio of surface area to volume is $4\pi r^2/(4\pi r^3/3) = 3/r$

Surface Tension and Capillarity



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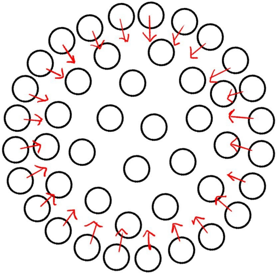
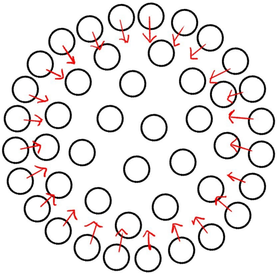
Surface Tension and Capillarity



- The ratio of surface area to volume is $4\pi r^2/(4\pi r^3/3) = 3/r$
- Large quantities of liquid $\rightarrow$ surface tension $<<$ pressure forces.

Surface Tension and Capillarity



- The ratio of surface area to volume is $4\pi r^2/(4\pi r^3/3) = 3/r$
- Large quantities of liquid $\rightarrow$ surface tension $<<$ pressure forces.
- Small quantities of liquid $\rightarrow$ surface tension $>>$ pressure forces.

Surface Tension and Capillarity

How do we calculate the surface tension?

Surface Tension and Capillarity

How do we calculate the surface tension?

Surface Tension (definition)

Force per unit length that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

Surface Tension and Capillarity

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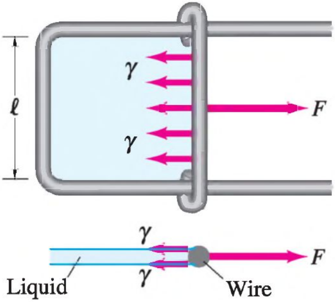
Force per unit length that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\boxed{\gamma = \frac{F}{\ell}} \tag{11}$$

Surface Tension and Capillarity



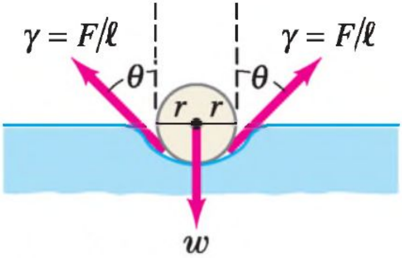
$$2\gamma\ell = F \rightarrow \gamma = \boxed{\frac{F}{2\ell}}$$

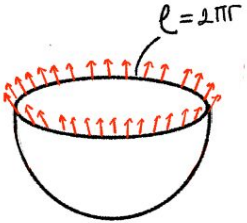
TABLE 13–4 Surface Tension of Some Substances

Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059

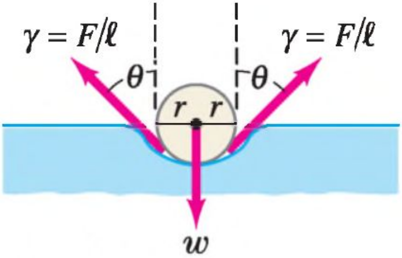
Example 8

Insect walks on water. The base of an insect's leg is approximately spherical in shape, with a radius of about $2.0 \times 10^{-5} m$. The 0.0030 g mass of the insect is supported equally by its six legs. Estimate the angle θ for an insect on the surface of water. Assume the water temperature is 20°C.



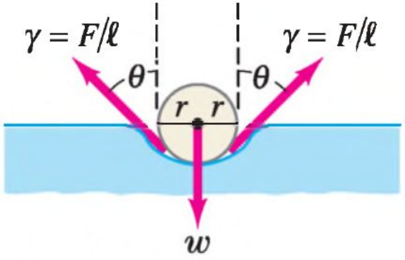


Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

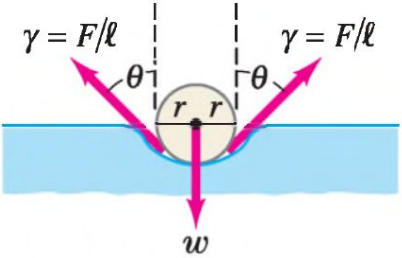
Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

Surface Tension and Capillarity



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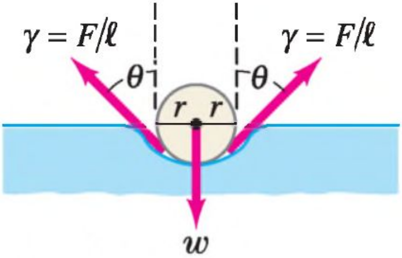
$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

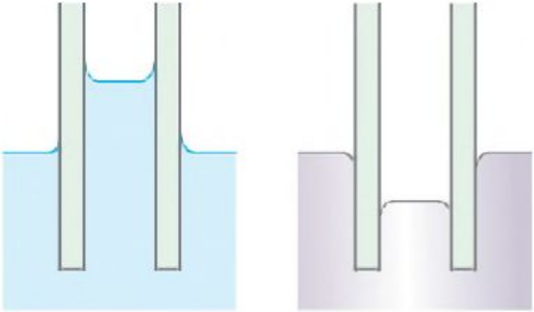
$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

$$\rightarrow cos\theta \simeq 0.54 \rightarrow \theta \simeq 57$$

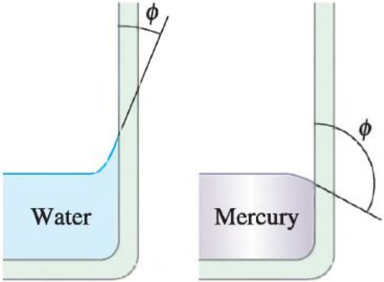
Surface Tension and Capillarity

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called capillarity



Surface Tension and Capillarity

Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. Cohesion refers to the force between molecules of the same type, whereas adhesion refers to the force between molecules of different types.



Fluids in motion

We can distinguish two main types of fluid flow:

- Laminar Flow: Smooth flow $\rightarrow$ the neighbouring layers of the fluid slide by each other smoothly

Fluids in motion

We can distinguish two main types of fluid flow:

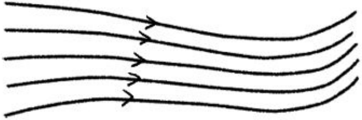
- Laminar Flow: Smooth flow $\rightarrow$ the neighbouring layers of the fluid slide by each other smoothly
- Turbulent Flow: Above a certain speed, the flow is characterized by erratic, small, whirlpool-like circles called eddy currents or eddies

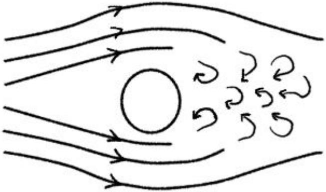
https://www.youtube.com/watch?v=WG-YCpAGgQQ

Fluids in motion

In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another.

In a turbulent Flow, eddies absorb a great deal of energy, and although a certain amount of internal friction called viscosity is present even during streamline flow, it is much greater when the flow is turbulent.





How do we know if the flow is laminar or turbulent?

How do we know if the flow is laminar or turbulent?$\rightarrow$ *Reynold's number*

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The equations that describe the motion of the elements of fluids are:

$$\boxed{\rho\vec{a} = -\nabla P - \rho\nabla\phi + \vec{f}_{vis}} \quad \textit{Navier Stokes eq.}$$

We can write an equivalent dimensionless equation, so we can get rid of the units:

$$\frac{d\vec{v'}}{dt'} = -\nabla' P' - \nabla'\phi' + \frac{1}{\mathrm{R}_e}\nabla'^2\vec{v'}$$

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$$\boxed{\mathrm{R}_e = \frac{\rho v D}{\eta}}$$

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v : *caracteristic velocity*
D : *caracteristic dimension*
ρ : *caracteristic density*

We can write an equivalent dimensionless equation, so we can get rid of the units:

$$\underbrace{\frac{d\vec{v'}}{dt'}}_{acceleration} = \underbrace{-\nabla' P'}_{pressure} - \underbrace{-\nabla' \phi'}_{gravity} + \overbrace{\frac{1}{\mathrm{R}_e}\nabla'^2 \vec{v'}}^{viscosity}$$

We can write an equivalent dimensionless equation, so we can get rid of the units:

$$\underbrace{\frac{d\vec{v}'}{dt'}}_{acceleration} = \underbrace{-\nabla' P'}_{pressure} - \underbrace{-\nabla'\phi'}_{gravity} + \overbrace{\frac{1}{R_e}\nabla'^2\vec{v}'}^{viscosity}$$

Now we can compare acceleration with pressure with gravity.

$R_e >> 1$

$$\frac{d\vec{v'}}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v'}$$

$R_e >> 1$

$$\frac{d\vec{v}'}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v}'$$

Viscosity is not important

$\mathrm{R}_e << 1$

$$\mathrm{R}_e \frac{d\vec{v'}}{dt'} = -\frac{1}{\eta}\nabla P - \frac{\rho}{\eta}\nabla\phi + \nabla'^2\vec{v'}$$

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$$\mathrm{R}_e \frac{d\vec{v'}}{dt'} = -\frac{1}{\eta}\nabla P - \frac{\rho}{\eta}\nabla\phi + \nabla'^2\vec{v}'$$

We no longer have time in our equation!!!

$\mathrm{R}_e << 1$

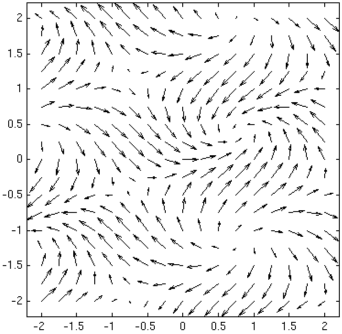
$$\mathrm{R}_e \frac{d\vec{v'}}{dt'} = -\frac{1}{\eta}\nabla P - \frac{\rho}{\eta}\nabla\phi + \nabla'^2\vec{v'}$$

We no longer have time in our equation!!!

You can go forward in time and then do the same thing backward in time and "undo" the process.

Generalization

To fully describe a fluid, we need the velocity vector field, that is $\vec{v}(x, y, z, t)$.



→ https://www.youtube.com/watch?v=VJ2ZDLQk3IQ

Generalization

Which are the equations that describe the velocity field?

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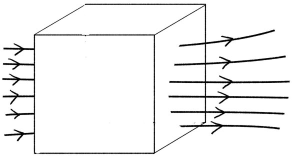

Generalization

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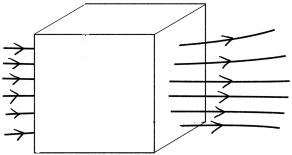
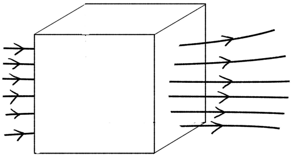


The mass flow rate passing A_{X1} is

$$\rho v_x dydz$$

Generalization

Which are the equations that describe the velocity field?



The mass flow rate passing A_{X1} is

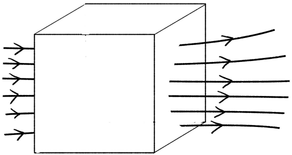
$$\rho v_x dydz$$

The mass flow rate passing A_{x2} is

$$\rho(v_x + \frac{\partial v_x}{\partial x}dx)dydz$$

Generalization

Which are the equations that describe the velocity field?



The mass flow rate passing A_{X1} is

$$\rho v_x dydz$$

The mass flow rate passing A_{x2} is

$$\rho(v_x + \frac{\partial v_x}{\partial x}dx)dydz$$

The difference between the mass flow rate entering through A_{X2} and A_{X1} is,

$$\rho\frac{\partial v_x}{\partial x}dydzdx$$

Generalization

If we do the same for y and z,

$$\rho\frac{\partial v_y}{\partial y}dydzdx$$

$$\rho\frac{\partial v_z}{\partial z}dydzdx$$

Then, the difference between the total flow entering and exiting the volume is

$$\rho(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z})dydzdx$$

Generalization

If the entering flux is equatl to the exiting flux

$$\rho(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z})dydzdx = 0$$

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or using the nabla operator,

$$\nabla \cdot \vec{v} = 0$$

Generalization

If the density is not constant,

$$\nabla \cdot (\rho \vec{v}) = 0$$

If the entering flux is not equal to exiting flux,

$$\nabla \cdot (\rho \vec{v}) = -\frac{\partial \rho}{\partial t}$$

This is the Hydrodynamic Equation of Continuity.

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We suppose that the liquid is "thin" the viscosity is unimportant, so we will omit $f_{vis} \rightarrow$ *Dry Water Flow*

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To obtain the acceleration we must calculate,

$$\frac{\Delta \vec{v}}{\Delta t}$$

for an actual displacement of an element of fluid. That is,

$$\frac{\Delta \vec{v}}{\Delta t} = v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}$$

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$$\vec{a} = (\vec{v} \cdot \nabla)\vec{v} + \frac{\partial \vec{v}}{\partial t}$$

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$$\Omega \times \vec{v} + \frac{1}{2}\nabla v^2 + \frac{\partial \vec{v}}{\partial t} = -\frac{\nabla P}{\rho} - \nabla\phi$$

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Is the curl of $\vec{v}$ and is called vorticity.

Physical interpretation of the curl

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$\rightarrow$ `https://mathinsight.org/curl_idea`