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PHY250
HW3

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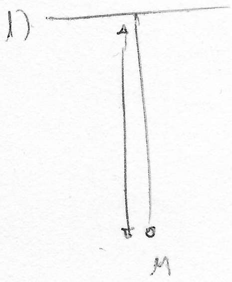
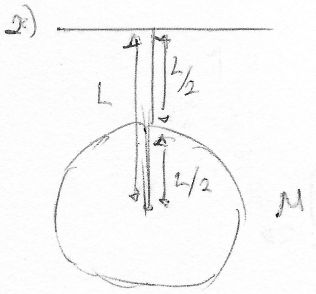
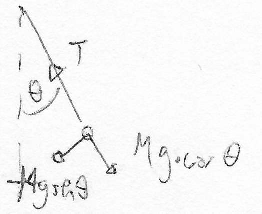
1)





Find T for small displacements, which period is higher

1)



$$T - Mg\cos\theta = M \cdot a_c$$

$$\underbrace{-Mg\sin\theta = M \cdot a_T}$$

$$\Rightarrow -g\sin\theta = a_T \Rightarrow a_T + g\sin\theta = 0$$

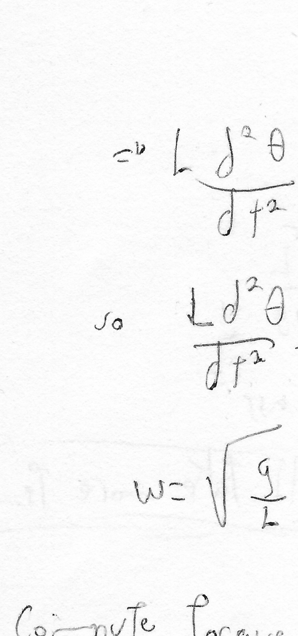
$$a_T = L \cdot \alpha = L \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow L\frac{d^2\theta}{dt^2} + g\sin\theta = 0$$ for small movements $\sin\theta \simeq \theta$

so $$\frac{L d^2\theta}{dt^2} + g\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g\theta}{L} = 0$$ wave equation

$w = \sqrt{\frac{g}{L}}$ and we know $w = \frac{2\pi}{T}$ so $T_1 = 2\pi\sqrt{\frac{L}{g}}$

2) Compute torque



R radius sphere $= L/2$
$r = L$

$$\tau = I_{sp}\alpha \qquad \alpha = \frac{d^2\theta}{dt^2}$$

$$I_{sph} = \frac{2}{5} M \cdot R^2 + r^2 \cdot M$$

$$\Rightarrow I_{sph} = \frac{2}{5} M \cdot \left(\frac{L}{2}\right)^2 + L^2 \cdot M$$

$$\Rightarrow I_{sph} = \frac{2}{5} M \cdot \frac{L^2}{4} + L^2 \cdot M$$

$$\Rightarrow I_{sph} = \frac{1}{10} ML^2 + L^2 M \Rightarrow I_{sph} = \frac{11}{10} ML^2$$

$$\tau = -L \cdot w \cdot \sin\theta$$

$$\Rightarrow -L \cdot M \cdot g \cdot \sin\theta = I\alpha \Rightarrow$$

$$\Rightarrow -L \cdot Mg \cdot \sin\theta = \frac{11}{10} ML^2 \cdot \frac{d^2\theta}{dt^2} \Rightarrow$$

$$0 = \frac{11}{10} ML^2 \frac{d^2\theta}{dt^2} + L \cdot Mg \sin\theta \Rightarrow$$

$$0 = L^2 \frac{d^2\theta}{dt^2} + L \cdot g \cdot \frac{10}{11} \cdot \sin\theta \Rightarrow$$

$$0 = \frac{d^2\theta}{dt^2} + \frac{Lg\,10}{L^2\,11} \cdot \sin\theta$$ small movements so $\sin\theta \simeq \theta$

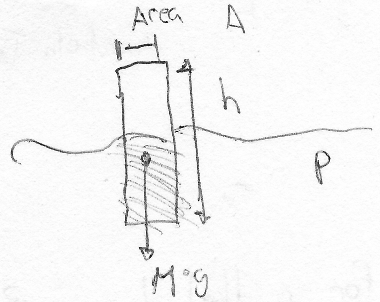
$$\frac{d^2\theta}{dt^2} + \frac{10g}{11L} \cdot \theta = 0 \Rightarrow$$

$w = \sqrt{\frac{10g}{11L}}$ and $T = \frac{2\pi}{w}$ so $T_2 = 2\pi \cdot \sqrt{\frac{11L}{10g}}$

Comparing T_1 and T_2 we can see how T_2 is bigger as

$\sqrt{\frac{11L}{10g}} > \sqrt{\frac{L}{g}}$ so The second pendulum will take more time

2)

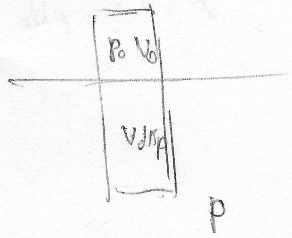


a) Vertical distance from surface to bottom

b) Downwards force F applied to top, how much farther below the surface is the bottom (some of the object doesn't sink)

c) If F is removed. Object will oscilate up and down as SHM. Compute T in terms of ρ, M and A. Ignore damping

a)



In this case $\frac{\rho_0}{\rho} < 1 \Rightarrow V_0 > V_{eldisp}$

$m_L \cdot g = \cancel{m_0} \overset{M}{} \cdot g \Rightarrow$

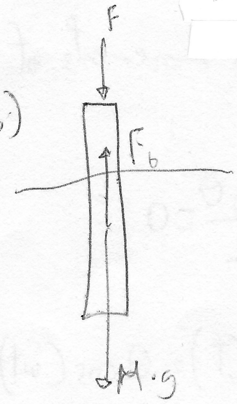
$\rho = \frac{M}{V}$

$\rho \cdot V_{dis} = \rho_0 \cdot V_0 \Rightarrow \frac{V_{dis}}{V_0} = \frac{\rho_0}{\rho} \Rightarrow V_{dis} = \frac{\rho_0 \cdot V_0}{\rho} \Rightarrow V_{dis} = \frac{M}{\rho}$

$V_{dis} = A \cdot L_{dis} \Rightarrow A \cdot L_{dis} = \frac{M}{\rho} \Rightarrow \boxed{L_{dis} = \frac{M}{A \cdot \rho}} \Rightarrow L_{dis} = \frac{\rho_0 \cdot V_0}{A \cdot \rho} \Rightarrow$

$L_{dis} = \frac{\rho_0 \cdot A \cdot L}{A \cdot \rho} \Rightarrow \boxed{L_{dis} = \frac{\rho_0}{\rho} \cdot L}$

b)



$F_b = M \cdot g + F \Rightarrow m_L \cdot g = M \cdot g + F \Rightarrow \rho \cdot V_{dis}' \cdot g = M \cdot g + F \Rightarrow$

$\rho \cdot V_{dis}' \cdot g = \rho_0 \cdot V_0 \cdot g + F \Rightarrow$

$V_{dis}' = \frac{\rho_0 \cdot V_0 \cdot g + F}{\rho \cdot g} \Rightarrow L_{dis}' = \frac{\rho_0 \cdot V_0 \cdot g + F}{\rho \cdot g \cdot A} \Rightarrow$

$L_{dis}' = \frac{\rho_0 \cdot A \cdot L \cdot g + F}{\rho \cdot g \cdot A} \Rightarrow L_{dis}' = \frac{\rho_0 \cdot A \cdot L \cdot g}{\rho \cdot g \cdot A} + \frac{F}{\rho \cdot g \cdot A} \Rightarrow$

$\boxed{L_{dis}' = L_{dis} + \frac{F}{\rho \cdot g \cdot A}}$

So the extra height is $\frac{F}{\rho g A}$

*2) c) Oscillation

In harmonic oscillation we now

$Kx = F_{net}$

And in this scenario

$F_b - M \cdot g = F_{net}$ so $\Rightarrow$

$m_f \cdot g - Mg = F_{net} \Rightarrow$

$\rho \cdot V'_{dis} g - Mg = F_{net} \Rightarrow$

$\rho \cdot A \cdot (\underbrace{L_{dis} + x}_{\text{extra height}}) g - Mg = F \Rightarrow \rho \cdot A \cdot L_{dis} g + \rho \cdot A \cdot x \cdot g - Mg = F \Rightarrow$

Knowing by Δ $\rho \cdot A \cdot L_{dis} g - Mg = 0$ so $\Rightarrow \rho A x g = F \Rightarrow$

$\rho \cdot A x g = Kx \Rightarrow K = \rho A g$ We also know $w = \sqrt{\frac{K}{m}}$ so $w = \sqrt{\frac{\rho A g}{M}}$

and that $T = \frac{2\pi}{w} \Rightarrow \boxed{T = \frac{2\pi}{\sqrt{\frac{\rho A g}{M}}}} = \boxed{2\pi \cdot \sqrt{\frac{M}{\rho \cdot A g}}}$

C

3) $x=0$ wave on a rope $f=40hz$ $A=0.03m$

$\mu=50g/m$ $T=5N$ a) Speed of the wave

$\mu=0.05\frac{kg}{m}$

b) Wavelength

c) Wave function $y(x,t)$, time 0 maximum displaced

d) Find maximum transverse acceleration of points on the rope

e) Is reasonable to ignore gravity here

a) $v=\sqrt{\frac{F_T}{\mu}}=\sqrt{\frac{5}{0.05}}=\boxed{10\,m/s}$

b) $v=\lambda f \Rightarrow \frac{v}{f}=\lambda=\boxed{0.25m}$

c) $y(x,t)=A\cdot\sin(Kx-\omega t+\phi)$

$K=\frac{2\pi}{\lambda}=\cancel{25.13}$ $\omega=\frac{2\pi}{T}=2\pi\cdot f=80\pi$

25,13

$y(x,t)=A\cdot\sin(2{,}5\cdot10^1\cdot x-80\pi\cdot t+\phi)\Rightarrow \boxed{y(x,t)=0.03\cdot\sin(25{,}13x-80\pi t)}$ $+\phi$

When $t=0$ y is maximum

$y(0,0)=0.03\cdot\sin(2.5\cdot10^1x-80\pi\cdot t+\phi)$ $f=\frac{\omega}{2\pi}$

So in order to be maximum $\phi=\frac{\pi}{2}\cdot(2K+1)$ $K\in\mathbb{Z}$

sin result has to be one $\sin(\phi)=1$ so

for example $K=0$ $\phi=\frac{\pi}{2}$

$\boxed{y(x,t)=0.03\cdot\sin\left(25{,}13x-80\pi t+\frac{\pi}{2}\right)}$

d) acceleration

Second derivative

$a=\omega^2\cdot A\cdot\cos(Kx-\omega t+\phi)\Rightarrow(80\pi)^2\cdot0.03\cdot\sin\left(25{,}13\cdot0-80\pi\cdot0+\frac{\pi}{2}\right)\Rightarrow$

$a=\boxed{1894.96\,m/s^2}$

*3) Can we ignore gravity? Yes, because the maximum acceleration is much bigger than g. So the force that causes the wave is much bigger than the weight of it

5) String along +x axis, free end at $x=0$

a) Incident travelling wave $y_1(x,t) = A\cos(Kx+\omega)$ gives rise to a standing wave to $y(x,t) = 2A\cos(\omega t)\cos(Kx)$

b) Show that the standing wave has an antinode at $x=0$

c) Find maximum displacement, maximum speed and maximum acceleration of the free end of the string

d) Find the resonance frequencies for the system

a) $y_1(x,t) = A\cos(Kx+\omega t)$ the other wave will be the same but travelling to the right so $y_2(x,t) = A\cos(Kx-\omega t)$

$y_1(x,t)+y_2(x,t) = A\cos(Kx+\omega t) + A\cos(Kx-\omega t) = A\cdot(\cos(Kx+\omega t) + \cos(Kx-\omega t)) =$
$A\cdot(2\cos(Kx)\cdot\cos(\omega t)) = \boxed{2A\cos(\omega t)\cos(Kx)}$

b) $y(x,t) = 2A\cos(\omega t)\cos(Kx)$

So $x=0$

$y(0,t) = 2A\cos(\omega t)\underbrace{\cos(0)}_{\text{maximum}}$

~~ASTh~~ As the cosine is maximum, the amplitude will be the maximum too, so (in the correct time) $x=0$ is an antinode

c) Maximum displacement $\Rightarrow \boxed{2A}$ m at time $t=0$ for example

Maximum speed: First derivative $v(x,t) = -2\cdot A\cdot\omega\cdot\sin(\omega t)\cdot\cos(K\cancel{x})$

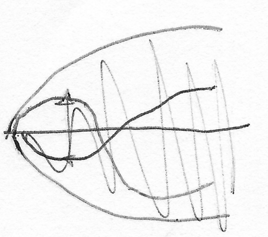
$v(0,0) = \boxed{-2\cdot A\cdot\omega}$ m/s

Acceleration: second derivative $a(x,t)$ ~~ax~~ $= -2\cdot A\cdot\omega^2\cdot\cos(\omega t)\cdot\cos(Kx)$

$a(0,0) = -2\cdot A\cdot\omega^2\cdot\cos(0)\cdot\cos(0) \Rightarrow \boxed{-2A\cdot\omega^2}$

¿

d)



$\frac{\lambda}{4} = L$

$\boxed{\lambda = \frac{4L}{2n+1}}$ $n \in \mathbb{Z}$

obtain frequency with this

$$f = \frac{v}{\lambda} \qquad v = \frac{\omega}{K}$$

$$\text{so } f = \frac{\omega}{K\lambda} = \boxed{\frac{\omega \cdot (2n+1)}{4L \cdot K}}$$