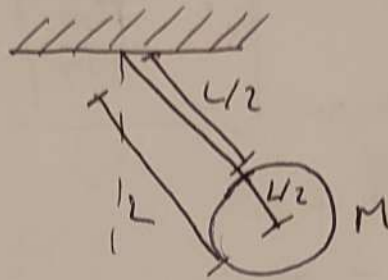
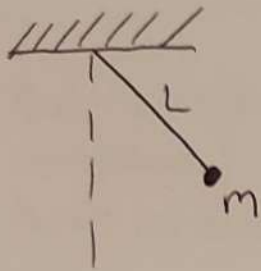


PHY250

HW-3

Asier Azpiri

①



$$T = 2\pi \sqrt{\frac{I}{mgL}} \cdot \sqrt{\frac{I}{mL^2}}$$

I : Moment of inertia about axis of rotation
 l : Distance of center of mass and axis.

$$T_1 = 2\pi \sqrt{\frac{L}{g}} \cdot \sqrt{\frac{mL^2}{mL^2}}$$

$I = mL^2$ for point with mass m .

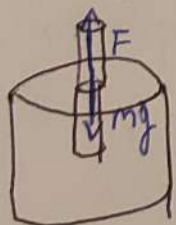
$$T_2 = 2\pi \sqrt{\frac{L}{g}} \cdot \sqrt{\frac{I}{mL^2}}$$

$$l = L \quad I = \frac{2}{5} MA^2 + MR_1^2$$

$$= \frac{2}{5} M \left(\frac{L}{2}\right)^2 + ML^2 = \frac{11}{10} ML^2$$

$$T_2 = 2\pi \sqrt{\frac{L}{g}} \cdot \sqrt{\frac{\frac{11}{10} ML^2}{ML^2}} \Rightarrow \boxed{2\pi \sqrt{\frac{11L}{10g}} = T_2}$$

T_2 is greater than T_1 . Hence, The large sphere Takes longer to complete a swing.



②

a) Here buoyancy force is given by

$$F = \rho V g$$

$$F = mg \text{ (force upwards)}$$

$$mg = \rho V g$$

$$m = \rho V$$

$$V = A \cdot l$$

$$m = \rho A l$$

$$l = \frac{m}{\rho A} \Rightarrow l = \frac{\rho_0 A h}{\rho A} \Rightarrow \boxed{l = \frac{\rho_0 h}{\rho}}$$

b) consider x' to be distance which is further displacement downwards force F , we can write:

$$F = Ax' \rho g$$

$$x' = \frac{F_m}{A \rho g}$$

c) Time period T ?

Force constant is given by:

$$k = \frac{F}{x} = \frac{Ax \rho g}{x}$$

Now angular frequency is: $\omega = \sqrt{\frac{k}{m}}$

$$\omega = \sqrt{\frac{A \rho g}{M}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{A \rho g}{M}}} = 2\pi \sqrt{\frac{M}{A \rho g}}$$

3

a) $v = \sqrt{\frac{F}{\mu}}$

$$v = \sqrt{\frac{5 \text{ N}}{50 \cdot 10^{-3} \text{ kg/m}}} = 10 \text{ m/s}$$

v : velocity
 F : force
 μ : mass density

b) $\lambda = \frac{v}{f}$

$$\lambda = \frac{10 \text{ m/s}}{40 \text{ Hz}} = 0.25 \text{ m}$$

λ : wavelength
 v : velocity
 f : frequency

c) $T = \frac{1}{f}$ $T = \frac{1}{40 \text{ Hz}} = 0.025 \text{ s}$

T : period
 A : amplitude

Now, substitute all the values in The equation:

$$y(x, t) = A \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)$$

$$y(x, t) = 0.03 \text{ m} \cos 2\pi \left(\frac{x}{0.25 \text{ m}} - \frac{t}{0.025 \text{ s}} \right)$$

$$d) a_{\max} = 4H^2 g^2 A$$

$$a_{\max} = 4H^2 (40 \text{ Hz})^2 (0.03 \text{ m})^2$$

$$a_{\max} = 1893 \text{ m/s}^2$$

e) The acceleration due to gravity always have a vertical component only that is: $a_y = -g$. Here, "g" is the acceleration due to gravity.

The acceleration due to gravity don't have any horizontal component only, that is: $a_x = 0$.

Therefore, the transverse wave is nearly horizontal, because the effect of force of gravity is ignored.

Hence, the answer is **yes**.

④ Given That a string that lies along the +X-axis has a free end at $x=0$.

a) We can derive a wave function for the standing wave by adding wave functions.

$$y_1(x,t) = A \cos(kx + \omega t) \text{ and } y_2(x,t) = A \cos(kx - \omega t)$$

$$\Rightarrow y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= A \cos(kx + \omega t) + A \cos(kx - \omega t)$$

$$= A [\cos(kx + \omega t) + \cos(kx - \omega t)]$$

$$= A [\cos(kx) \cos(\omega t) - \sin(kx) \sin(\omega t) + \cos(kx) \cos(\omega t) + \sin(kx) \sin(\omega t)]$$

$$= 2A \cos(kx) \cos(\omega t)$$

b) If $x=0$, the antinode for the standing wave at its free end is: $y(0,t) = 2A \cos(0) \cos(\omega t)$

$$y(0,t) = 2A \cos(\omega t)$$

So, $x=0$ is an antinode

- c). The maximum displacement from the above standing wave is.

$$y(0,t) = 2A \cos(\omega t)$$

$$\boxed{A_{sw} = 2A} \text{ when } t=0$$

- Maximum velocity is:

$$y(0,t) = 2A \cos(\omega t)$$

$$v_y = \frac{dy}{dt} = -2A\omega \sin(\omega t) = 2A\omega \sin(\omega t) = \omega A_{sw} \sin(\omega t)$$

~~$v_{y \max} = \omega A_{sw}$~~ $\boxed{v_{y \max} = \omega A_{sw}}$ when the object reaches the maximum amplitude.

- ~~Maximum acceleration is:~~

~~$a_y = \frac{dv_y}{dt}$~~
 ~~$a_y = a$~~

- Maximum acceleration is:

$$a_y = \frac{d}{dt}(-2A\omega \sin(\omega t))$$

$$a_y = -2A\omega^2 \cos(\omega t) = 2A\omega^2 \cos(\omega t)$$

$$\boxed{a_{y \max} = 2A\omega^2} \text{ when } t=0$$