PHY250: FLUIDS

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Fluids

- Solid Objects $\rightarrow$ maintain shape except for small amount of elastic deformation.
- Fluids $\rightarrow$ materials that are very deformable and can flow.

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We are going to examine Static Fluids and Dynamic Fluids.

Fluids

Ideal Gases

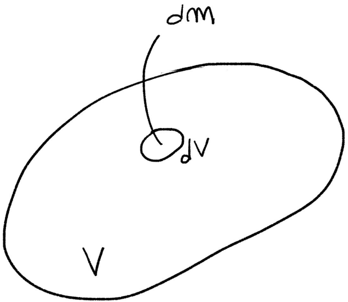
- Formed by N point particles that move in straight lines.
- They expand to occupy the whole volume of the container.
- They are compressible.
- There is no interaction between the particles.
- Elastic collisions, the kinetic energy is conserved.
- The temperature is proportional to the mean kinetic energy.

Fluids

Liquids

- They take the shape of its container.
- They are not compressible (idealization).
- Their density is constant.
- There is interaction between the molecules $\rightarrow$ viscosity, superficial tension.

Density



$$\rho = \frac{dm}{dV} \tag{1}$$

$$\rho = constant \rightarrow \rho = \frac{m}{V}$$

Then, the weight of an object is,

$$w = mg = \rho V g$$

Pressure

We define the pressure as,

$$P = \frac{F}{A} \tag{2}$$

It is an scalar, its units are

$$[P] = \frac{N}{m^2} = Pa \tag{3}$$

At sea level $\rightarrow$ $\boxed{P_0 = 1.013 \times 10^5 \frac{N}{m^2} = 1atm}$

In a fluid at rest,

- the pressure is the same in every direction at a given depth
- the force is perpendicular to any solid surface

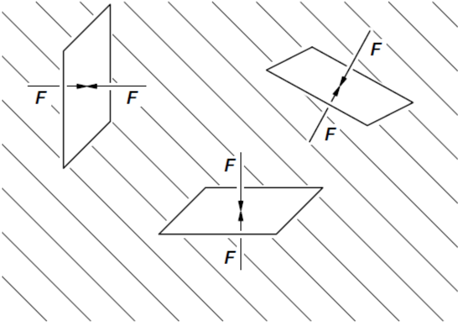
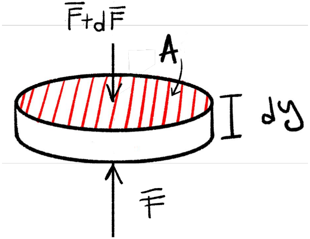


Figure: Figure from The Feynman Lectures vol II.

The pressure in a static liquid



$$Equilibrium \rightarrow \sum F_y = 0$$

$$\rightarrow dF = (dm)g = \rho g dV = \rho g A dy$$

$$\rightarrow PA - (P + dP)A = \rho g A dy$$

$$\rightarrow -dP = \rho g dy \tag{4}$$

Then,

$$\frac{dP}{dy} = -\rho g$$

$\rightarrow$ The pressure decreases when y increases

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g dy$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g dy, \quad \rho = \rho(y)$$

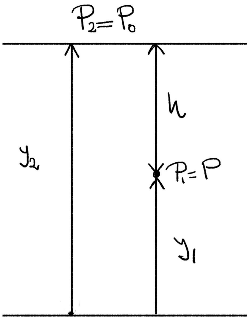


Figure: P_0 is the pressure due to the atmosphere above.

$\rho = constant \rightarrow \Delta P = -\rho g \Delta y$

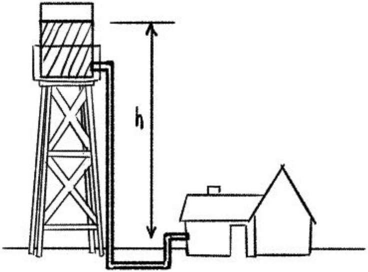
$$P_2 - P_1 = -\rho g (y_2 - y_1)$$

$$P_0 - P = -\rho g h$$

$$\rightarrow \boxed{P = P_0 + \rho g h} \qquad (5)$$

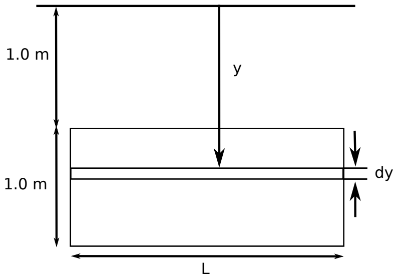
Example 1

Pressure at a faucet: The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.



Example 2

Force on aquarium window. Calculate the force due to water pressure exerted on a 1.0 m X 3.0 m aquarium viewing window whose top edge is 1.0 m below the water surface



Atmospheric Pressure

- Equation of Estate (EoS): relation between ρ and P
- Ideal gas: $\boxed{\rho \propto P}$

$$\rightarrow \frac{\rho}{\rho_0} = \frac{P}{P_0}$$

where ρ_0 and P_0 are the density and pressure at sea level.

Atmospheric Pressure

$$\rightarrow \frac{dP}{dy} = -\rho g = -\frac{\rho_0 P}{P_0} = -P\frac{\rho_0}{P_0}g$$

$$\rightarrow \frac{dP}{P} = -(\frac{\rho_0}{P_0})gdy$$

$$\rightarrow ln(P) - ln(P_0) = ln(\frac{P}{P_0}) = -(\frac{\rho_0}{P_0})gy$$

$$\rightarrow \boxed{P = P_0 e^{-\frac{\rho_0}{P_0}gy}} \tag{6}$$

Example 3

At what elevation is the air pressure equal to half the pressure at sea level?

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$$\frac{P_0}{2} = P_0 e^{-\frac{\rho_0}{P_0}gy}$$

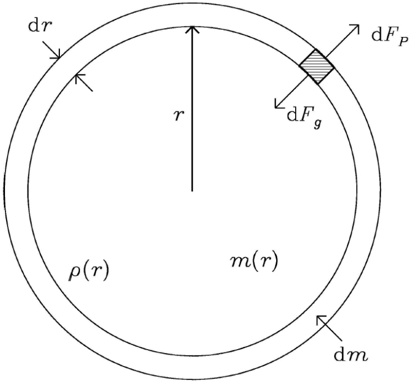
$$\rightarrow ln(\frac{1}{2}) = ln(e^{-\frac{\rho_0}{P_0}gy})$$

$$\rightarrow -ln(2) = -\frac{\rho_0}{P_0}gy$$

$$\frac{\rho_0}{P_0}g = 1.25 \times 10^{-4} m^{-1}$$

$$\rightarrow \boxed{y = 5550m}$$

Spherical geometry: Star Model



$$F_r = PA - (P + dP)A - dF = 0$$

$$dF = G\frac{dmM(r)}{r^2} = G\frac{\rho dvM(r)}{r^2}$$

$$\rightarrow AdP = -G\frac{\rho AdrM(r)}{r^2}$$

$$\rightarrow \boxed{\frac{dP}{dr} = -G\frac{\rho M(r)}{r^2}} \tag{7}$$

The equation for the mass is,

$$dM(r) = \rho(r)dV = \rho(r)4\pi r^2 dr \tag{8}$$

$$\rightarrow \frac{dM}{dr} = \rho(r)4\pi r^2 \tag{9}$$

Then, we have to solve the system:

$$\begin{cases} \frac{dP}{dr} = -G\frac{\rho M(r)}{r^2} \\ \\ \frac{dM}{dr} = \rho(r) 4\pi r^2 \end{cases} \tag{10}$$

With the conditions,

$$\begin{cases} \rho(r=0) = \rho_0 \\ \\ \rho(r=R) = 0 \end{cases} \tag{11}$$

We need the EoS: $P = P(\rho)$

Then,

- To model the structure of a Star, we need to find a microscopic model for the matter inside the star (an EoS).
- We need the central density of the star
- The mass and Radius of the star depends on the EoS

The pressure is a function of the EoS, and for certain conditions it may not be sufficient to withstand the gravitational attraction. Thus the structure equations imply there is a maximum mass that a star can have

Pascal's Principle

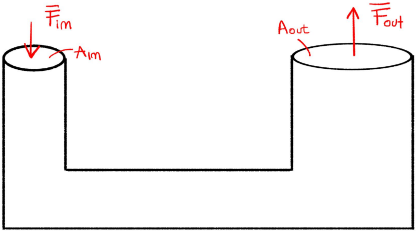
If an external pressure P_0 is applied to a confined fluid, the pressure at every point within the fluid increases by that amount P_0.

This means that an external pressure acting on a fluid is transmitted throughout the fluid.

$$P = \rho g h + P_0 \tag{12}$$

Example

Hydrostatic Lift



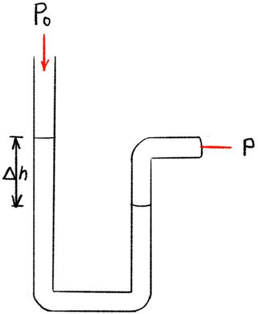
$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \tag{13}$$

$$\rightarrow F_{out} = \frac{A_{out}}{A_{in}} F_{in} \tag{14}$$

Manometer

The simplest device to measure the pressure is the open-tube Manometer, the Pressure is related to the difference Δh between the two levels of the liquid.

$$P = P_0 + \rho g \Delta h$$

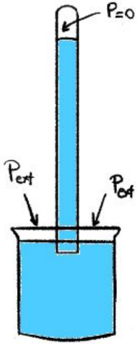


Manometer

Instead of calculating the product $\rho g \Delta h$, sometimes only the change in height Δh is specified. In fact, pressures are sometimes specified as so many "millimetres of mercury" (mm-Hg) or "mm of water" (mm-H20).

Barometer

A barometer is a glass tube completely filled with mercury and then inverted into a bowl of mercury. If the tube is long enough, the level of the mercury will drop, leaving a vacuum at the top of the tube.



$$P_{ext} = \rho g \Delta h + 0$$

Barometer

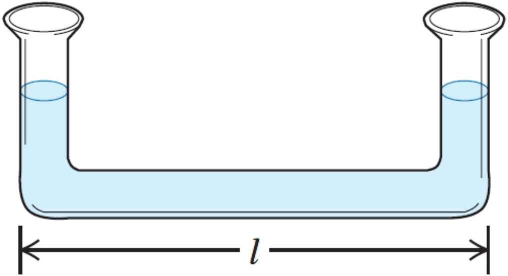
When $P_{ext} = P_0$, $\Delta h = 0.76cm$. That is, the atmospheric pressure can support a column of mercury only about 76 cm high.

If we replace the liquid by water, the column high would be $\sim 10m$

Example 4

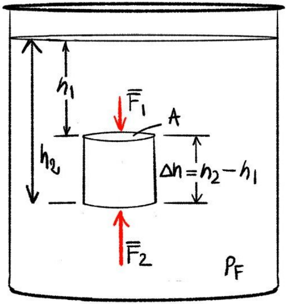
A U-shaped tube with a horizontal portion of length ℓ contains a liquid. What is the difference in height between the liquid columns in the vertical arms (a) if the tube has an acceleration a toward the right and (b) if the tube is mounted on a horizontal turntable rotating with an angular speed $\omega$with one of the vertical arms on the axis of rotation? (c) Explain why the difference in height does not depend on the density of the liquid or on the crosssectional area of the tube. Would it be the same if the vertical tubes did not have equal cross-sectional areas? Would it be the same if the horizontal portion were tapered from one end to the other? Explain.

Example 4



Arquimede's Principle

Consider a cylinder immersed in a liquid, the upward force exerted by the liquid is the **Buoyant Force**



$$
\begin{aligned}
F_B &= F_2 - F_1 \\
&= \rho_F g A (h_2 - h_1) \\
&= \rho_F g A \Delta h \\
&= m_f g
\end{aligned}
$$

Arquimede's Principle in general

For any irregular body...

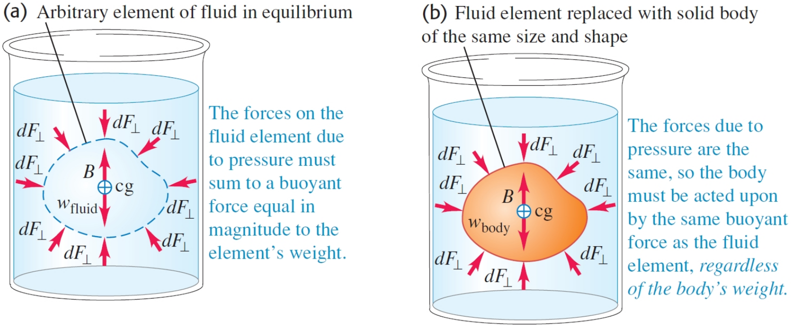


Figure: Figure from sears and zemansky's university physics 13th edition volume 1.

The buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.

Two pails of water. Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

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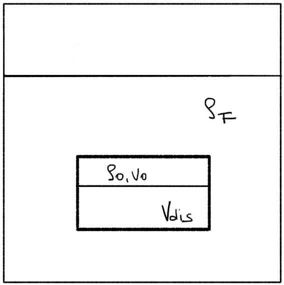
Answer Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood object; so the pails have the same weight.

Object in equilibrium

In this case, $F_B > mg$, then, the object is in equilibrium when

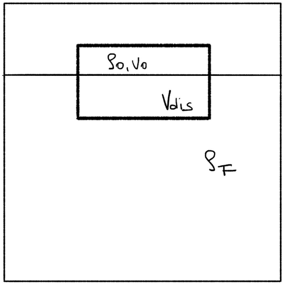


$$
\begin{aligned}
m_F g &= m_o g \\
\rho_F V_{dis} g &= \rho_O V_O g \\
\rightarrow \frac{V_{dis}}{V_O} &= \frac{\rho_O}{\rho_F}
\end{aligned}
$$

Where V_{dis}/V_O is the fraction of submerged Vol.

Floating objects

In this case, $F_B > mg$, then, the object is in equilibrium when



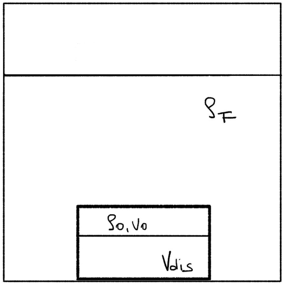
$$
\begin{aligned}
m_F g &> m_o g \\
\rho_F V_{dis} g &> \rho_O V_O g \\
\rightarrow \frac{V_{dis}}{V_O} &> \frac{\rho_O}{\rho_F}
\end{aligned}
$$

$$\rightarrow \boxed{1 > \frac{\rho_O}{\rho_F}}$$

Where V_{dis}/V_O is the fraction of submerged Vol.

Sank object

In this case, $F_B < mg$, then, the object is in equilibrium when



$$\begin{aligned} m_F g &< m_o g \\ \rho_F V_{dis} g &< \rho_O V_O g \\ \rightarrow \frac{V_{dis}}{V_O} &< \frac{\rho_O}{\rho_F} \end{aligned}$$

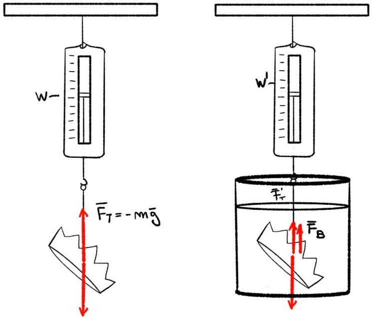
$$\rightarrow \boxed{1 < \frac{\rho_O}{\rho_F}}$$

Example 5

Recovering a submerged statue. A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 cm^3$. How much force is needed to lift it?

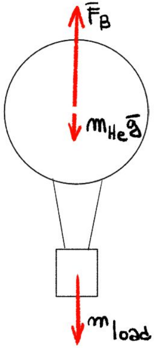
Example 6

Archimedes: Is the crown gold? When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?



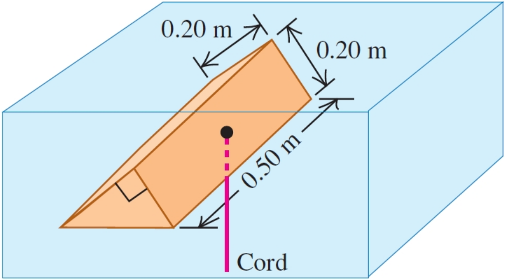
Example 7

Helium balloon. What volume V of helium is needed if a balloon is to lift a load of 180 *kg* (including the weight of the empty balloon)?

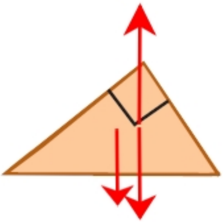


Example 8

Suppose a piece of styrofoam, $\rho = 180\ kg/m^3$ is held completely submerged in water. (a) What is the tension in the cord? Find this using Archimedes's principle. (b) Use $p = p_0 + \rho g h$ to calculate directly the force exerted by the water on the two sloped sides and the bottom of the styrofoam; then show that the vector sum of these forces is the buoyant force.



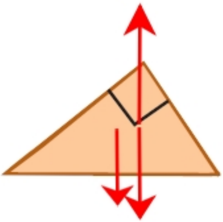
Example 8



a)

$$F_B - T - W = 0$$

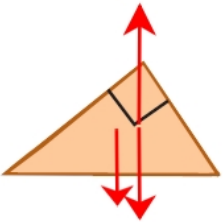
Example 8



a)

$$
\begin{aligned}
F_B - T - W &= 0 \\
\rightarrow T &= (\rho_w - \rho_o)Vg
\end{aligned}
$$

Example 8



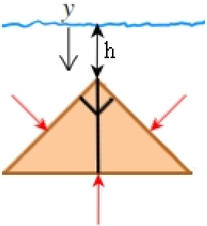
a)

$$\begin{aligned} F_B - T - W &= 0 \\ \rightarrow T &= (\rho_w - \rho_o)Vg \\ \rightarrow T &= \frac{1}{2}a^2\ell(\rho_w - \rho_o) \end{aligned}$$

Example 8

b) sides:

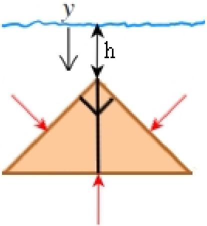
$$dF = PdA$$



Example 8

b) sides:

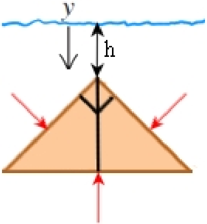
$$\begin{aligned} dF &= PdA \\ dA &= \ell dr \end{aligned}$$



Example 8

b) sides:

$$
\begin{aligned}
dF &= PdA \\
dA &= \ell dr \\
&= \ell \frac{2}{\sqrt{2}} dy
\end{aligned}
$$



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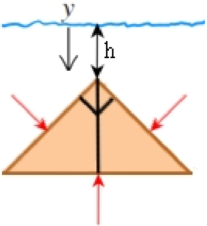
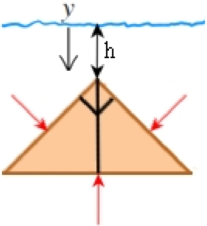


$$
\begin{aligned}
dF &= PdA \\
dA &= \ell dr \\
&= \ell \frac{2}{\sqrt{2}} dy \\
\rightarrow dF &= \frac{2}{\sqrt{2}} \rho g y \ell dy + P_0 \frac{2}{\sqrt{2}} \ell dy
\end{aligned}
$$

Example 8



b) sides:

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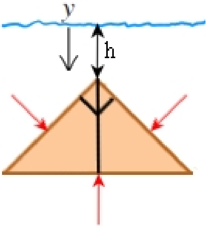
\begin{aligned}
dF &= PdA \\
dA &= \ell dr \\
&= \ell \frac{2}{\sqrt{2}} dy \\
\to dF &= \frac{2}{\sqrt{2}} \rho g y \ell dy + P_0 \frac{2}{\sqrt{2}} \ell dy \\
\to F &= \frac{2}{\sqrt{2}} \ell \int_h^{h+\sqrt{2}/2a} (P_0 + y\rho g) dy
\end{aligned}
$$

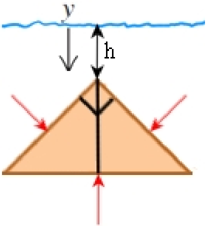
Example 8

b) sides:



$$
\begin{aligned}
dF &= PdA \\
dA &= \ell dr \\
&= \ell \frac{2}{\sqrt{2}} dy \\
\rightarrow dF &= \frac{2}{\sqrt{2}} \rho g y \ell dy + P_0 \frac{2}{\sqrt{2}} \ell dy \\
\rightarrow F &= \frac{2}{\sqrt{2}} \ell \int_h^{h+\sqrt{2}/2a} (P_0 + y \rho g) dy \\
&= \boxed{P_0 a \ell + \rho g a h \ell + \frac{\rho g}{2\sqrt{2}} a^2 \ell}
\end{aligned}
$$

Example 8

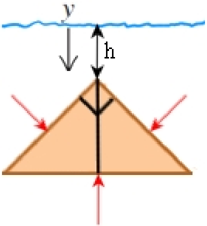


b) bottom:

$$F = (\rho g h + P_0) a\sqrt{2}\ell$$

Example 8

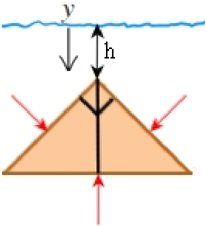
Total:



$$F_B = F_{Bo} - 2F_S\frac{\sqrt{2}}{2}$$

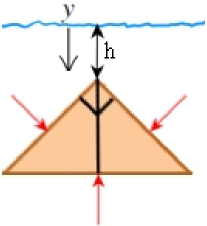
Example 8

Total:



$$F_B = F_{Bo} - 2F_S\frac{\sqrt{2}}{2}$$
$$= \rho g\frac{a^2\ell}{2}$$

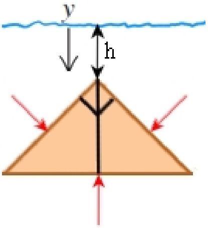
Example 8



Total:

$$\begin{aligned} F_B &= F_{Bo} - 2F_S\frac{\sqrt{2}}{2} \\ &= \rho g \frac{a^2\ell}{2} \\ &= \rho g V \end{aligned}$$

Example 8



Total:

$$\begin{aligned} F_B &= F_{Bo} - 2F_S\frac{\sqrt{2}}{2} \\ &= \rho g\frac{a^2\ell}{2} \\ &= \rho g V \\ &= \boxed{gm_w} \quad \checkmark \end{aligned}$$

Generalization

- The pressure on any object is perpendicular to the surface.
- If the only external force is the gravity, near earth, we have $\frac{dP}{dy} = -\rho g$
- The Arquimede's Principle is a consequence of the previous 2 items.

Generalization

If the external force is the gravity,

$$\frac{dP}{dy} = -\rho g$$

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Which is the equivalent expression for an arbitrary force?

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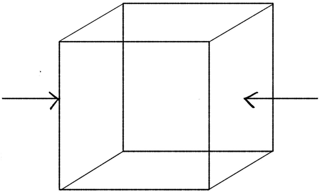
Which is the equivalent expression for an arbitrary force?

In general,

$$\vec{F} = \vec{F}(x, y, z), \quad P = P(x, y, z) \tag{15}$$

Generalization

The resultant force on the x-direction due to the pressure of the liquid is,



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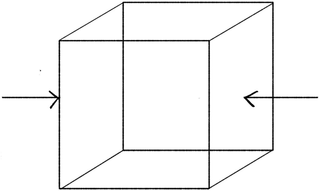
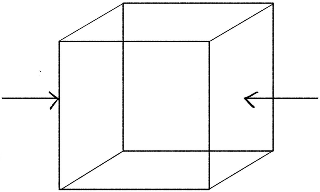


$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

Generalization

The resultant force on the x-direction due to the pressure of the liquid is,



$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

and,

$$dP_x = \frac{\partial P}{\partial x}dx$$

Generalization

Then,

$$F_x = -\frac{\partial P}{\partial x} dxdydz$$

or,

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where f_x is the force per unit volume.

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The fluid is in hydrostatic equilibrium if,

$$f_x + f_x^e = 0$$

Generalization

Then,

$$-\frac{\partial P}{\partial x} = -f_x^e$$

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If the external force is conservative,

$$f_x^e = -\frac{\partial U}{\partial x} \rightarrow f_x^e = -\rho \frac{\partial \phi}{\partial x}$$

Generalization

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$$\to -\frac{\partial P}{\partial x} = \rho\frac{\partial \phi}{\partial x}$$

Generalization

Then, for the 3 spacial directions...

$$
\begin{aligned}
-\frac{\partial P}{\partial x} &= \rho\frac{\partial \phi}{\partial x}\\
-\frac{\partial P}{\partial y} &= \rho\frac{\partial \phi}{\partial y}\\
-\frac{\partial P}{\partial z} &= \rho\frac{\partial \phi}{\partial z}
\end{aligned}
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-\frac{\partial P}{\partial z} &= \rho\frac{\partial \phi}{\partial z}
\end{aligned}
$$

Using the nabla operator,

$$\nabla P = -\rho\nabla\phi \tag{16}$$

Generalization

Then, the most general expression for hydrostatic equilibrium is,

$$\boxed{\nabla P + \rho \nabla \phi = 0} \tag{17}$$

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Another possibility which allows hydrostatic equilibrium is when $\rho = \rho(P)$.

Fluids in motion

In general, the equation $\nabla P + \rho \nabla \phi = 0$ has no solution.

Fluids in motion

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$$\rightarrow \nabla P + \rho \nabla \phi = \rho \vec{a}$$

$$\boxed{\textit{Dynamic Fluid}}$$

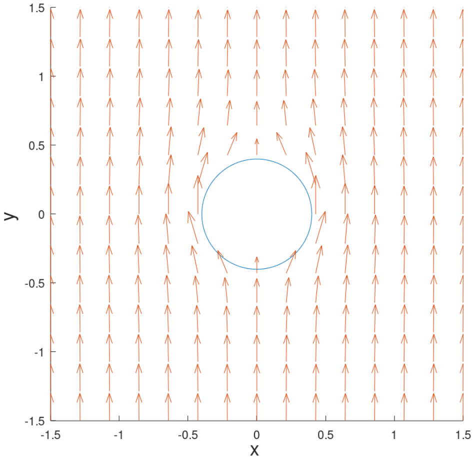
Fluids in motion

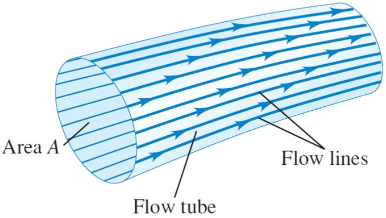
To describe the motion of a fluid, we need:

Fluids in motion

To describe the motion of a fluid, we need:

- An equation of state: $P(\rho)$
- The velocity at every point





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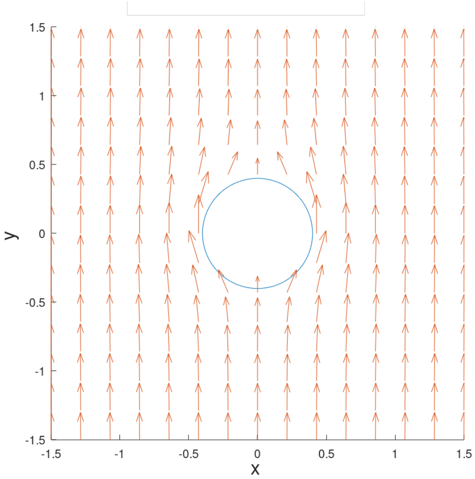
Streamlines: lines which are always tangent to the fluid velocity.



Obs: two streamlines can not cross each other. Why?

We can distinguish two main types of fluid flow:

- Steady Flow: at any one place in the fluid the velocity never changes $\rightarrow$ the streamlines are the fluid particles path.

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- Steady Flow: at any one place in the fluid the velocity never changes $\rightarrow$ the streamlines are the fluid particles path.
- Non-Steady Flow: At a given point, the velocity changes with time.

We are going to consider:

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- The velocity of the flow $<<$speed of sound in the Fluid.
- $\rho = constant$ (insompressible fluid)
- steady flow.

Steady flow:

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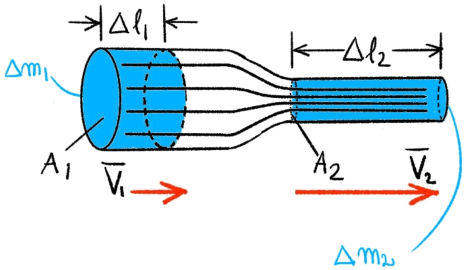
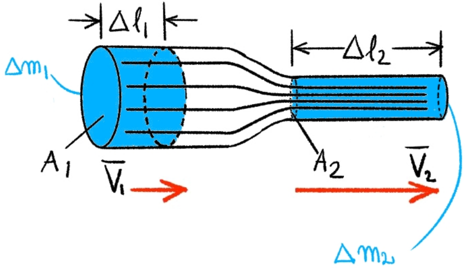
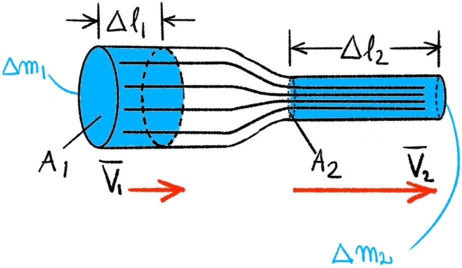
Steady flow:

- At any one place in the fluid the velocity never changes.
- Fluid element moves in lines which are always tangent to the fluid velocity (streamlines).


Δl_1
Δl_2
Δm_1
A_1
$\overline{V}_1$
A_2
$\overline{V}_2$
Δm_2



- Mass Flow Rate $\rightarrow \frac{\Delta m}{\Delta t}$



- Mass Flow Rate $\rightarrow \frac{\Delta m}{\Delta t}$
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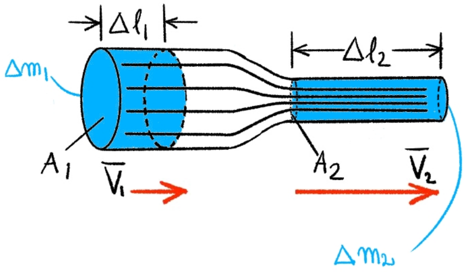

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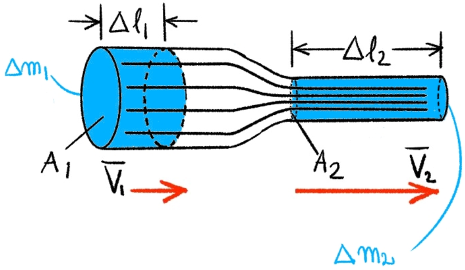
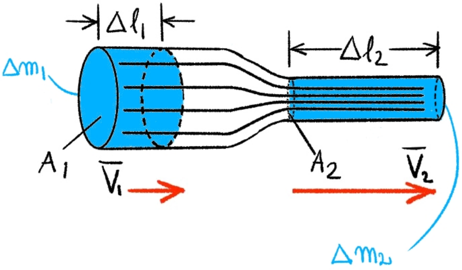
$$\rightarrow \frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \tag{19}$$


Δl1
Δl2
Δm1
A1
V1
A2
V2
Δm2

$$Vol_1 = A_1 \Delta\ell_1,$$



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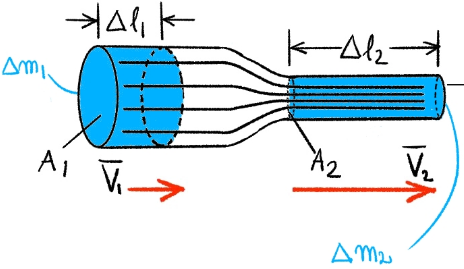

$$\to \frac{\Delta m_1}{\Delta t} = \frac{\rho_1 Vol_1}{\Delta t} = \frac{\rho_1 A_1 \Delta \ell_1}{\Delta t}$$

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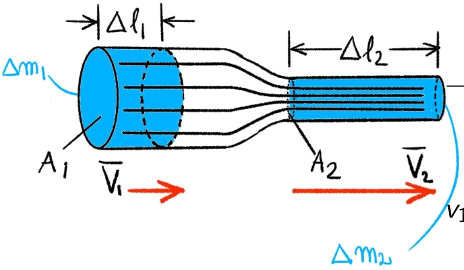

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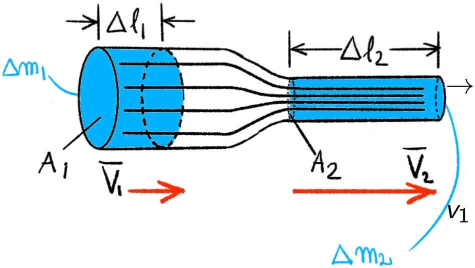

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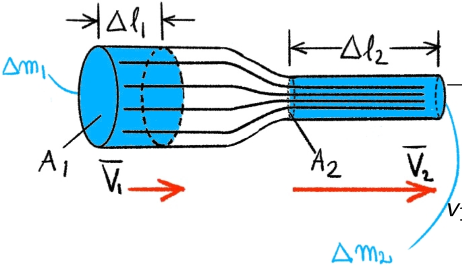

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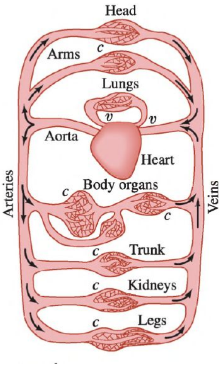
Av is the *volume rate of flow*

$$\boxed{Av = constant}$$

Example 1

Blood flow. In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about $1.2cm$, and the blood passing through it has a speed of about $40cm/s$. A typical capillary has a radius of about $4 \times 10^{-4}cm$, and blood flows through it at a speed of about $5 \times 10^{-4}m/s$.



Estimate the number of capillaries that are in the body.

Bernoulli's Principle

Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

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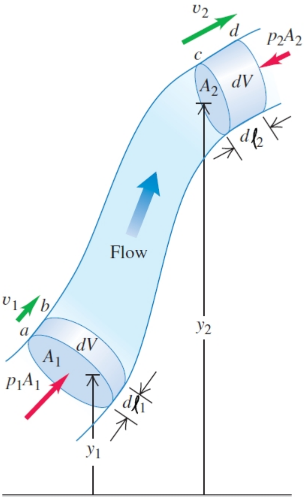
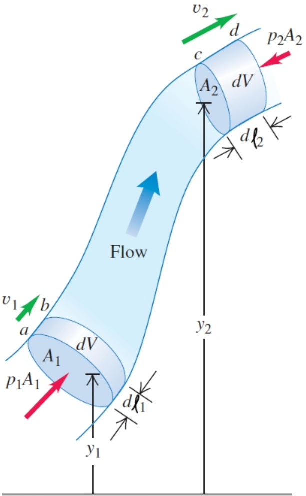
The theorem of Bernoulli is in fact nothing more than a statement of the conservation of energy.



The work done at A_1 is,

$$W_1 = F_1 \Delta \ell_1 = P_1 A_1 \Delta \ell_1 \quad (22)$$



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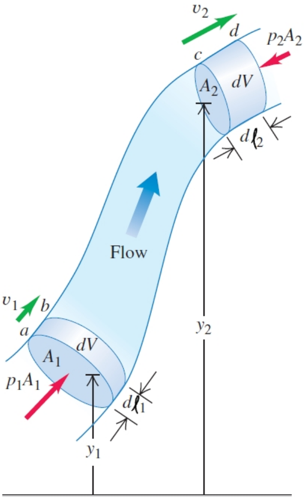

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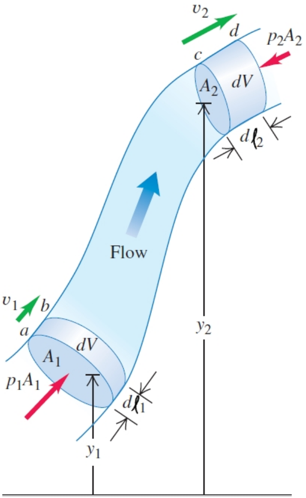

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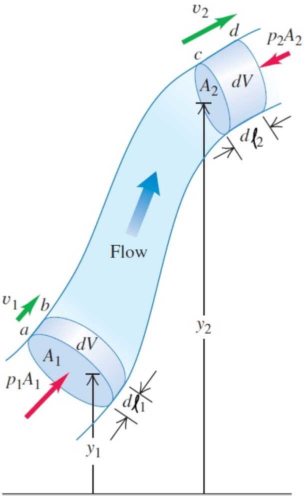

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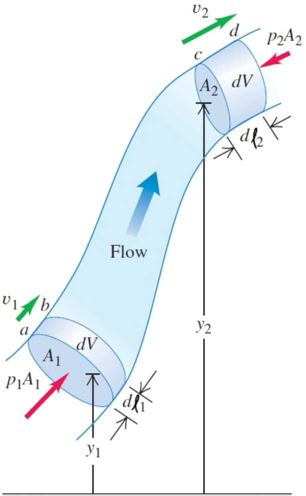
$$W_3 = -mg(y_2 - y_1) \quad (24)$$



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Reordering the expression,

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This is **Bernoulli's equation**. Since points 1 and 2 can be any two points along a tube of flow, Bernoulli's equation can be written as

$$\boxed{\frac{1}{2}\rho v^2 + P + \rho g y = constant} \tag{25}$$

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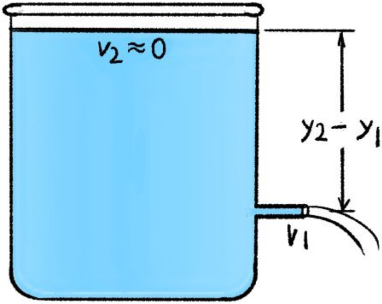
$$P_2 - P_1 = -\rho g (y_2 - y_1) \leftarrow Hydrostatic\ equation \qquad (26)$$

Example 2

Calculate the velocity, v_1, of a liquid flowing out of a spigot at the bottom of a reservoir.



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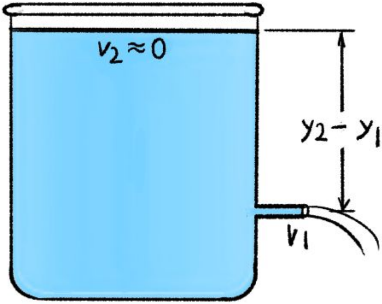


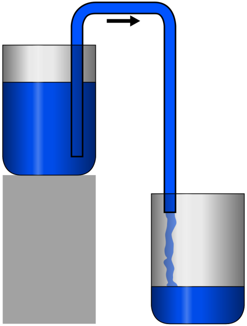
The liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height.

Example 3

The wind speed near the center ("eye") of a hurricane, whose radius is 30 km, reaches about 200 km/h. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. (a) Estimate the wind speed at the rim of the hurricane. (b) Estimate the pressure difference at the earth's surface between the eye and the rim. Where is the pressure greater?

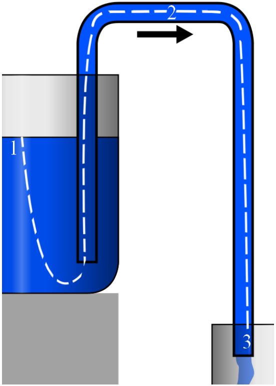
Example 4

A siphon is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density ρ and let the atmospheric pressure be P_{atm}. Assume that the cross-sectional area of the tube is the same at all points along it.



Example 4

1. If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.
2. A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?



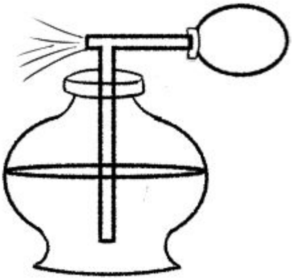
In a streamline:

$$P_0 + \rho g h = P_0 + \frac{1}{2}\rho v_3^2$$
$$\rightarrow v_3 = \boxed{\sqrt{2gh}}$$

$$P_2 + \rho g(H + h) = P_0$$
$$\rightarrow H = \frac{P_0 - P_2}{\rho g} - h$$

$$P_2\ min = 0 \ \rightarrow H_{max} = \boxed{\frac{P_0}{\rho g} - h}$$

How does a Perfume Atomizer work?



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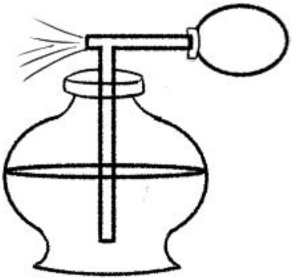
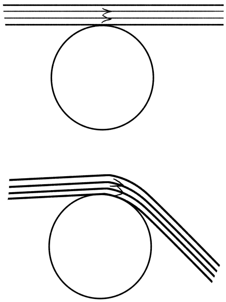


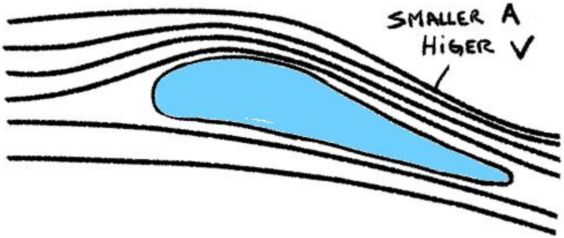
The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top.

Coanda effect



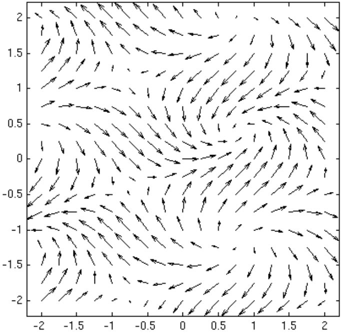
https://www.youtube.com/watch?v=NvzXKZNJ7ZU&t=4s

Airplane Wings and Dynamic Lift



Generalization

To fully describe a fluid, we need the velocity vector field, that is $\vec{v}(x, y, z, t)$.



→ https://www.youtube.com/watch?v=VJ2ZDLQk3IQ

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Which are the equations that describe the velocity field?

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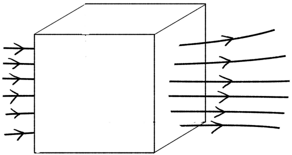

Generalization

The mass flow rate passing A_{X1} is

$$\rho v_x dydz$$

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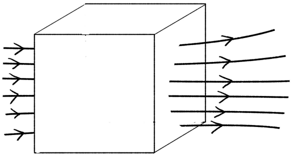

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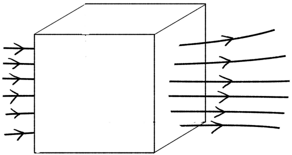

The mass flow rate passing A_{X1} is

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The mass flow rate passing A_{x2} is

$$\rho(v_x + \frac{\partial v_x}{\partial x} dx) dydz$$

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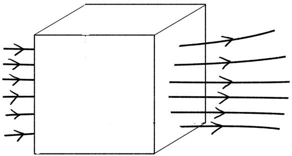

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The mass flow rate passing A_{x2} is

$$\rho(v_x + \frac{\partial v_x}{\partial x}dx)dydz$$

The difference between the mass flow rate entering through A_{X2} and A_{X1} is,

$$\rho\frac{\partial v_x}{\partial x}dydzdx$$

Generalization

If we do the same for y and z,

$$\rho\frac{\partial v_y}{\partial y}dydzdx$$

$$\rho\frac{\partial v_z}{\partial z}dydzdx$$

Then, the difference between the total flow entering and exiting the volume is

$$\rho(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z})dydzdx$$

Generalization

If the entering flux is equatl to the exiting flux

$$\rho(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z})dydzdx = 0$$

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Then,

$$(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}) = 0$$

or using the nabla operator,

$$\nabla \cdot \vec{v} = 0$$

Generalization

If the density is not constant,

$$\nabla \cdot (\rho \vec{v}) = 0$$

If the entering flux is not equal to exiting flux,

$$\nabla \cdot (\rho \vec{v}) = -\frac{\partial \rho}{\partial t}$$

This is the Hydrodynamic Equation of Continuity.

Generalization

Then we have one equation for 3 unknowns.

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We suppose that the liquid is "thin" the viscosity is unimportant, so we will omit f_{vis} $\rightarrow$ *Dry Water Flow*

What is the acceleration?

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$$IT\ IS\ NOT\ \vec{a} = \frac{\partial \vec{v}}{\partial t}!!!!$$

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To obtain the acceleration we must calculate,

$$\frac{\Delta \vec{v}}{\Delta t}$$

for an actual displacement of an element of fluid. That is,

$$\frac{\Delta \vec{v}}{\Delta t} = v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}$$

Using the nabla,

$$\vec{a} = (\vec{v} \cdot \nabla)\vec{v} + \frac{\partial \vec{v}}{\partial t}$$

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This equation can be expressed as,

$$\Omega \times \vec{v} + \frac{1}{2}\nabla v^2 + \frac{\partial \vec{v}}{\partial t} = -\frac{\nabla P}{\rho} - \nabla \phi$$

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Is the curl of $\vec{v}$ and is called vorticity.

Physical interpretation of the curl

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The curl is the micro-circulation per unit area of $\vec{v}$ and has the same direction than the normal to the surface

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$$\nabla \times \vec{v} = \left(\lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint_S \vec{v} \cdot d\vec{r} \right) \cdot \hat{n}$$

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$\rightarrow$ `https://mathinsight.org/curl_idea`

Viscosity

Real Fluids have certain amount of internal friction called **viscosity**. It is a frictional force between adjacent layers of fluid.

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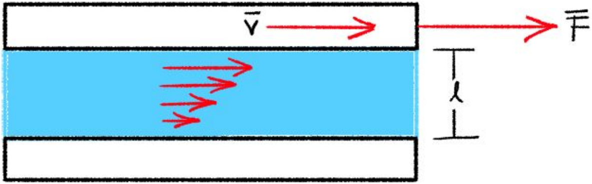
The origin of the viscosity is,

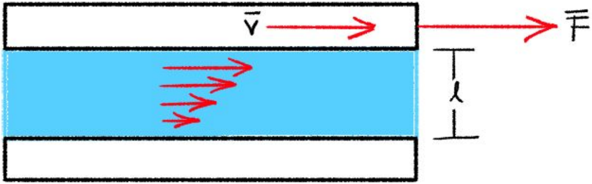
- In fluids, electrical cohesive forces between the molecules.
- In gases, collisions between the molecules

The viscosity can be expressed quantitatively by a *coefficient of viscosity* η

Viscosity

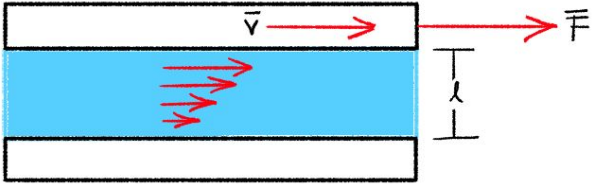


Viscosity



$$F = \propto A\frac{v}{\ell} \tag{27}$$

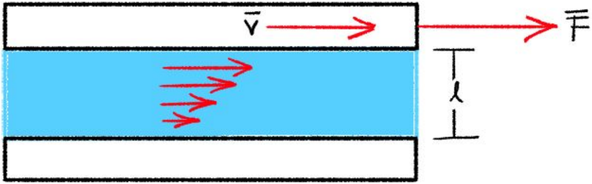
Viscosity



$$F = \propto A\frac{v}{\ell} \tag{27}$$

$$F = \eta A\frac{v}{\ell} \rightarrow \eta = \frac{F\ell}{Av} \tag{28}$$

Viscosity



$$F = \propto A\frac{v}{\ell} \tag{27}$$

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The unit of η is $N \cdot s/m^2$

Viscosity

TABLE 13–3
Coefficients of Viscosity

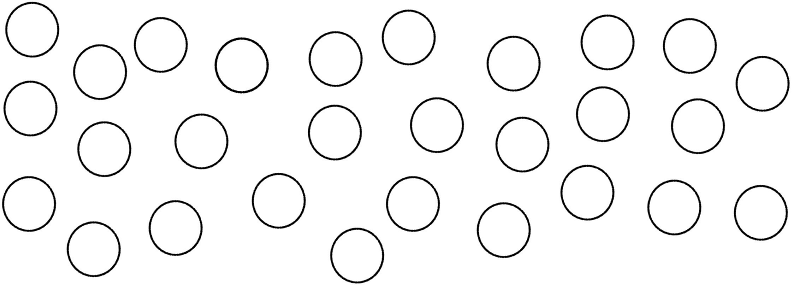
Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s)[†]
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

[†] 1 Pa·s = 10 P = 1000 cP.

Surface Tension and Capillarity

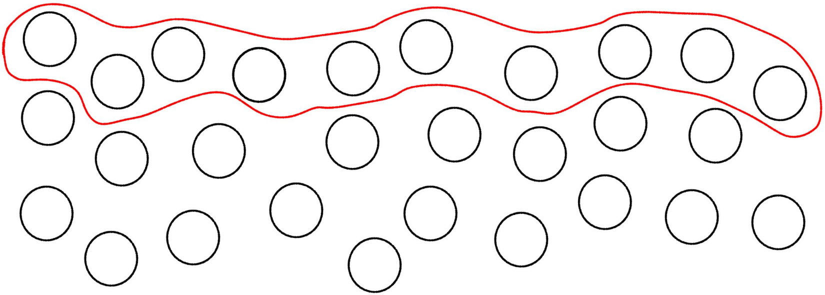
Surface Tension and Capillarity

The particles that make up the liquid are in constant random motion.



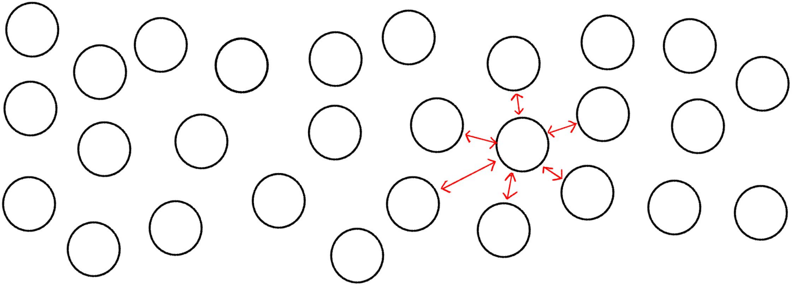
Surface Tension and Capillarity

Do the particles at the surface form a random surface?



Surface Tension and Capillarity

Intermolecular attractions influence the surface.



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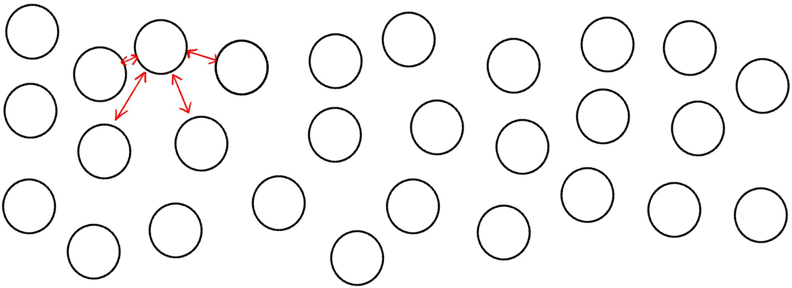
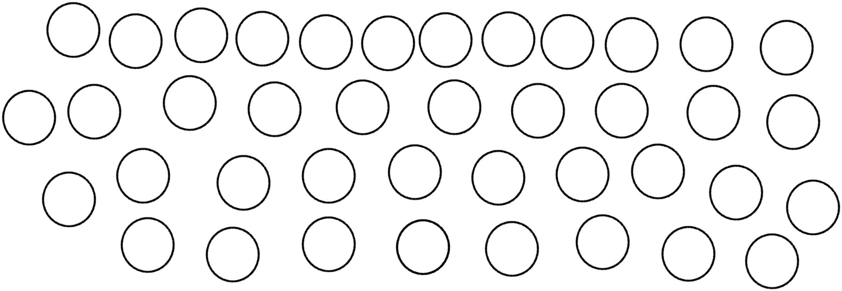


Surface Tension and Capillarity

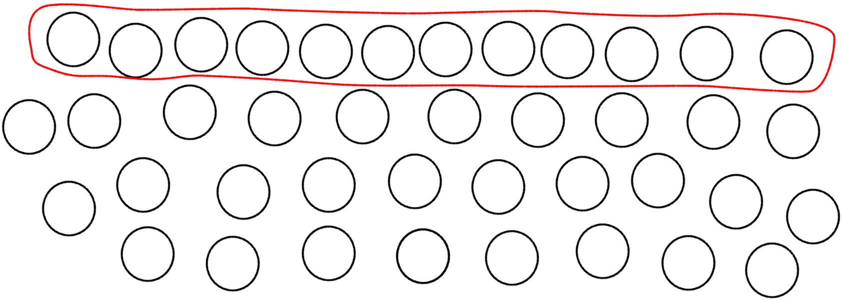
Surface molecules form a much smoother surface.

Surface Molecules are compressed $\rightarrow$ Higher energy at the surface.



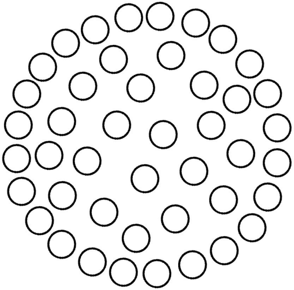
Surface Tension and Capillarity

Liquid surface $\leftrightarrow$ Stretched membrane under tension



Surface Tension and Capillarity

Liquid not confined in a container.



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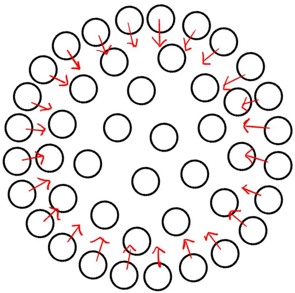


Surface Tension and Capillarity

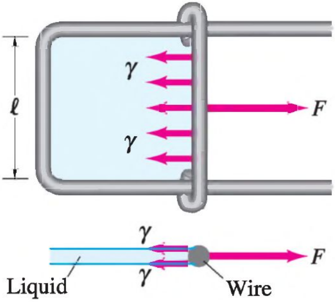
Liquid surface $\leftrightarrow$ Stretched membrane under tension

Surface Tension

Force per unit lenght that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\gamma = \frac{F}{\ell} \tag{29}$$

Surface Tension and Capillarity



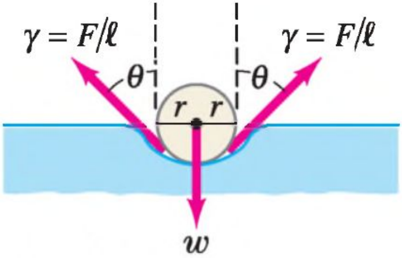
$$2\gamma\ell = F \rightarrow \gamma = \frac{F}{2\ell}$$

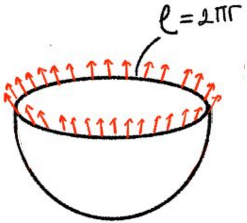
TABLE 13–4
Surface Tension of Some Substances

Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059

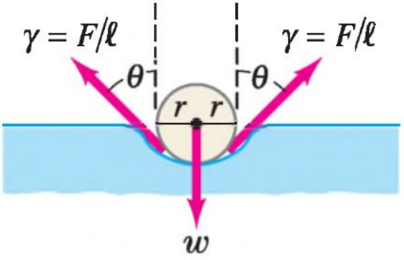
Surface Tension and Capillarity

Insect walks on water. The base of an insect's leg is approximately spherical in shape, with a radius of about $2.0 \times 10^{-5} m$. The 0.0030 g mass of the insect is supported equally by its six legs. Estimate the angle for an insect on the surface of water. Assume the water temperature is 20°C.



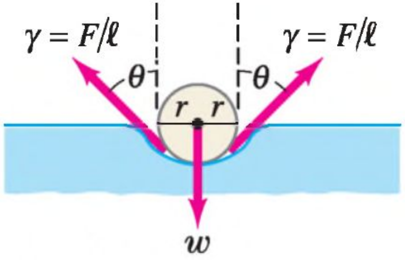


Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

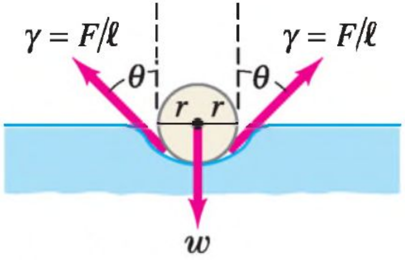
Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

Surface Tension and Capillarity



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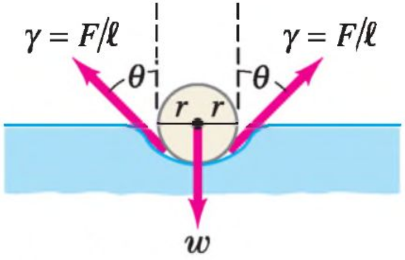
$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

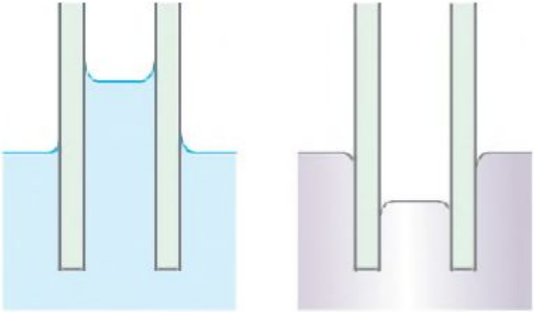
$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

$$\rightarrow cos\theta \simeq 0.54 \rightarrow \theta \simeq 57$$

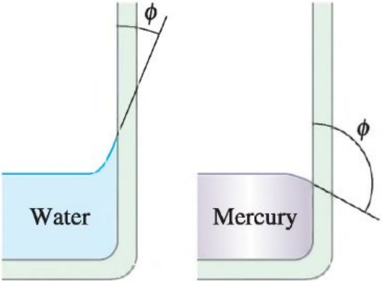
Surface Tension and Capillarity

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called capillarity



Surface Tension and Capillarity

Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. Cohesion refers to the force between molecules of the same type, whereas adhesion refers to the force between molecules of different types.



Fluids in motion

We can distinguish two main types of fluid flow:

- Laminar Flow: Smooth flow $\rightarrow$ the neighbouring layers of the fluid slide by each other smoothly

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- Laminar Flow: Smooth flow $\rightarrow$ the neighbouring layers of the fluid slide by each other smoothly
- Turbulent Flow: Above a certain speed, the flow is characterized by erratic, small, whirlpool-like circles called eddy currents or eddies

https://www.youtube.com/watch?v=WG-YCpAGgQQ

Fluids in motion

In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another.

In a turbulent Flow, eddies absorb a great deal of energy, and although a certain amount of internal friction called viscosity is present even during streamline flow, it is much greater when the flow is turbulent.