PHY250: Review and Introduction

Anabela R. Turlione

Digipen Institute

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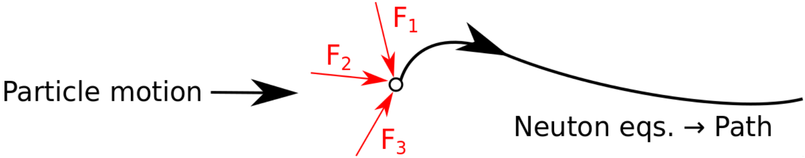
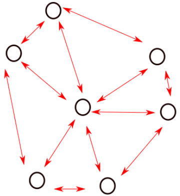
In PHY 200 you studied:



In PHY 200 you studied:

System of particles →



→ CM

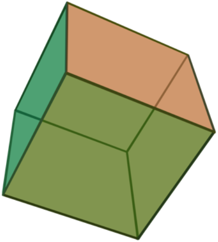
→ Torque

→ Angular Momentum

→ Inertia Moment

In PHY 200 you studied:

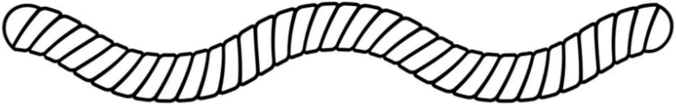
Continuous systems
constant distances
↓
Rigid bodies



CM
Particle Motion
↓
Rot + Trans
↑
Torque
Angular Momentum
Inertia

In PHY250 we are going to study fluids, we can think of them as systems of particles.

We also are going to study mechanics waves, energy traveling in a medium. We can think of them as the collective motion of the particles in a continuous system.



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- this time, the distance between their particles may vary
- we are going to study how the energy is propagated
- finally, we are going to study the nature of light and how it interacts with different mediums

So far, you have been describing the motion of a particle, using the equations of Newton:

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$$\sum_i \vec{F}_i = m\vec{a} \tag{1}$$

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Where

$$E_k = \frac{1}{2} m v^2 \tag{4}$$

is the kinetic energy.

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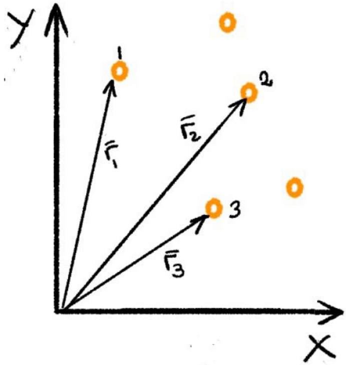
$$E = U + K = constant \tag{7}$$

So, when a force is conservative, that is, there is a function $U(x)$ such that

$$F = -\frac{U(x)}{dx} \tag{8}$$

then, the energy of the particle is constant.

You also studied systems of N particles,:



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$$\vec{\tau}_i = \vec{r}_i \times \vec{F}_i \rightarrow \vec{\tau} = \sum_i^N \vec{\tau}_i \quad (Torque) \tag{11}$$

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And you found the following expressions that relates the motion of the center of mass with the total external force:

$$\vec{P} = M\frac{d}{dt}(\vec{R}_{CM}) = M\vec{V_{CM}} \tag{15}$$

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So the center of mass moves as a particle acted by the total external force.

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where **I** is the **Moment of Inertia**

$$I = \sum_{i}^{N} m_i r_i^2 \tag{19}$$

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$$Tanslation\ of\ \ CM\ +\ Rotation\ Around\ \ CM$$

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