PHY 250

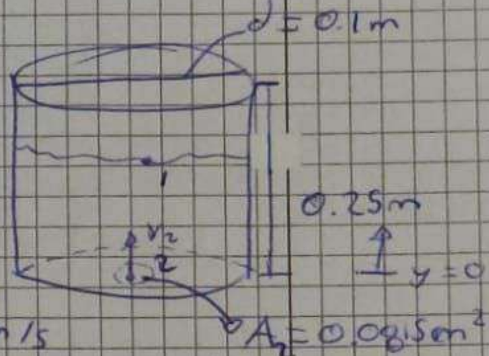
HW 2

Exercise 1.-

25 cm = 0.25 m

10 cm = 0.1 m

1.5 cm² = 0.00015 m²



$Fr = 2.4 \times 10^{-4}\ m^3/s$

Flow rate, not a force

$Fr = A_1 v_1 = A_2 v_2 = const$

$Fr = A_1 \cdot v_1$

$$v_1 = \frac{Fr}{\pi \cdot r_1^2} = \frac{2.4\times 10^{-4}\ m^3/s}{\pi\cdot(0.1\ m)^2} = 7.63\times 10^{-3}\ m/s$$

$Fr = A_2 \cdot v_2$

$$v_2 = \frac{Fr}{A_2} = \frac{2.4\times10^{-4}\ m^3/s}{0.00015\ m^2} = 1.6\ m/s$$

Bernoulli

$y_2 = 0$

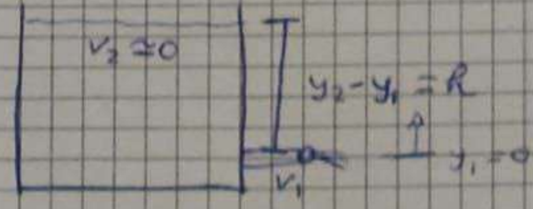
$$\frac{1}{2}\cdot \rho \cdot v_1^2 + \rho\cdot g\cdot y_1 + \cancel{P} = \frac{1}{2}\rho\cdot v_2^2 + \cancel{\rho \cdot g \cdot y_2} + \cancel{P}$$

$$\rho\cdot g\cdot y_1 = \frac{1}{2}\rho\cdot v_2^2 - \frac{1}{2}\rho\cdot v_1^2$$

$$y_1 = \frac{\frac{1}{2}\cancel{\rho}\cdot v_2^2 - \frac{1}{2}\cancel{\rho}\cdot v_1^2}{\cancel{\rho}\cdot g} = \frac{\frac{1}{2}\cdot(1.6\ m/s)^2 - \frac{1}{2}\cdot(7.63\times10^{-3}\ m/s)^2}{9.8\ m/s^2}$$

$$\boxed{y_1 = 0.13\ m}$$

Exercise 2.- v_1?



Bernoulli

$$\frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} + \cancel{P_0} = \frac{1}{2}\rho v_2^2 + \rho g h + \cancel{P_0} \qquad y_1 = 0$$

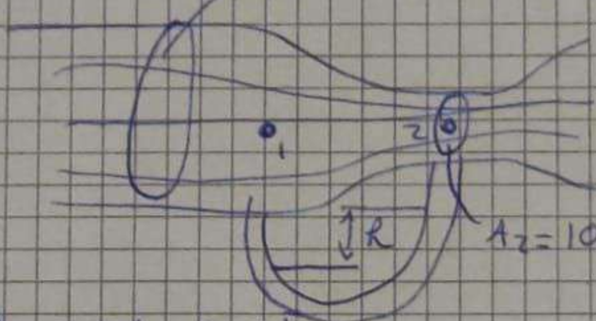
$$\frac{1}{2} v_1^2 = \cancel{\frac{1}{2} v_2^2} + g h \qquad v_2 \approx 0$$

$$\frac{1}{2} v_1^2 = g h$$

$$\boxed{v_1 = \sqrt{2 g h}}$$

Exercise 3.-

$A_1 = 40 cm^2 = 4 \times 10^{-3} m^2$ $\qquad \rho_{water} \simeq 1000 kg/m^3$



$A_2 = 10 cm^2 = 1 \times 10^{-3} m^2$

a) $A_1 \cdot v_1 = A_2 \cdot v_2 = F_r$

$$\boxed{v_1 = \frac{F_r}{A_1} = \frac{6 \times 10^{-3} m^3/s}{4 \times 10^{-3} m^2} = 1.5 m/s}$$

$$\boxed{v_2 = \frac{F_r}{A_2} = \frac{6 \times 10^{-3} m^3/s}{10^{-3} m^2} = 6 m/s}$$

b) Bernoulli $\qquad y_1 = y_2$

$$\frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} + P_1 = \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2} + P_2$$

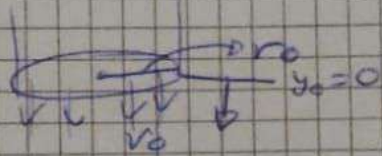
$$P_2 - P_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$\boxed{P_2 - P_1 = \frac{1}{2} \cdot 1000 kg/m^3 ((1.5 m/s)^2 - (6 m/s)^2) = -16875}$$

c) $\Delta P = \rho_m \cdot g \cdot \Delta h$

$$\boxed{h = \frac{\Delta P}{\rho_m \cdot g}}$$

Exercise 4 -



a) Kinematic formula for free falling

$$v^2 = v_0^2 + 2g\cdot y$$

$$v = \sqrt{v_0^2 + 2gy}$$

$$A_0 v_0 = A_1 v_1$$

$$(\pi r_0^2)\cdot v_0 = \pi r_1^2 \cdot v_1$$

$$r_1 = \sqrt{\frac{v_0 \cdot r_0^2}{v_1}}$$

$$\boxed{r_1 = \sqrt{\frac{v_0 \cdot r_0^2}{\sqrt{v_0^2 + 2gy}}}}$$

r_1 and v_1 are the radius and velocity at any arbitrary point in the free fall.

b) $v_0 = 1.2\,m/s$ $\quad r = \frac{r_0}{2}$ $\quad y?$

$$r^2 = \left(\frac{\cancel{r_0^2}}{4}\right)^2 = \left(\frac{v_0 \cdot \cancel{r_0^2}}{\sqrt{v_0^2 + 2gy}}\right)^2$$

$$\frac{1}{16} = \frac{v_0^2}{v_0^2 + 2gy} \Leftrightarrow v_0^2 + 2gy = 16 v_0^2 \Leftrightarrow y = \frac{16 v_0^2 - v_0^2}{2g}$$

$$\boxed{y = \frac{16 \cdot (1.2\,m/s)^2 - (1.2\,m/s)^2}{2 \cdot 9.8\,m/s^2} = 1.1\,m}$$