PHY250: Waves

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Digipen

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Itroduction

So far we have studied the Simple Harmonic Motion of a single particle...

Introduction

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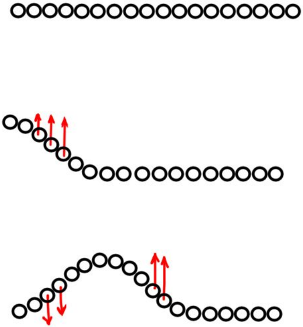

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- Mechanical Waves carry Energy as oscillation of matter, they does not carry matter.

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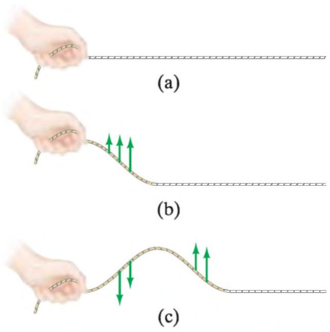

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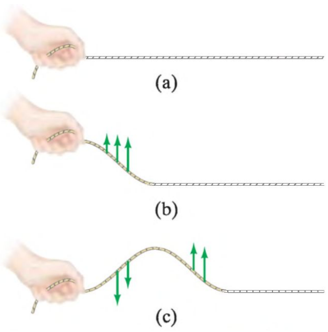

Single pulse formation: The hand pulls up and down on one end of the cord, each section of the cord is pulled up and down by the tension made by the adjacent section. The source of the traveling wave pulse is a disturbance, and cohesive forces between adjacent section of the cord cause the pulse to travel.

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1. In space: If you take a picture of the wave at a given instant of time, the wave will have the shape of a sine or a cosine.
2. In time: the up-down motion of an small segment of the cord at a certain position will be Simple Harmonic Motion.

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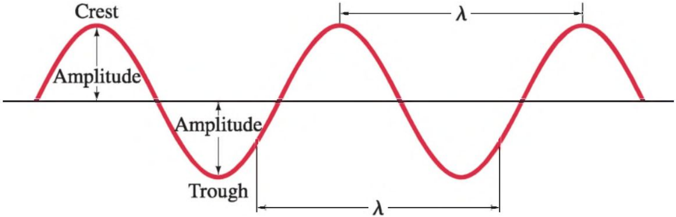

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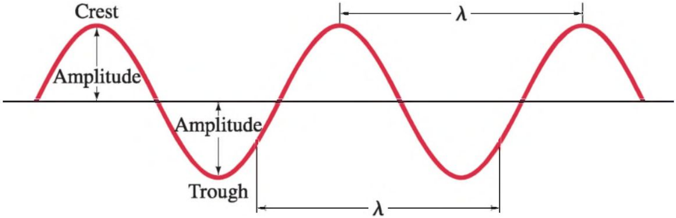

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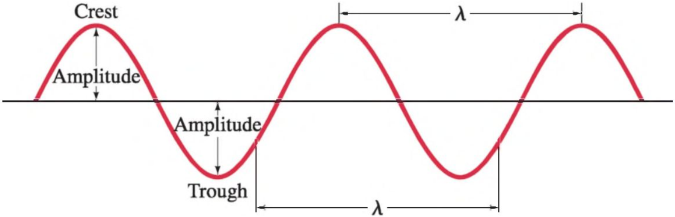

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$$\boxed{v = \lambda / T = \lambda f} \tag{1}$$

Types of waves

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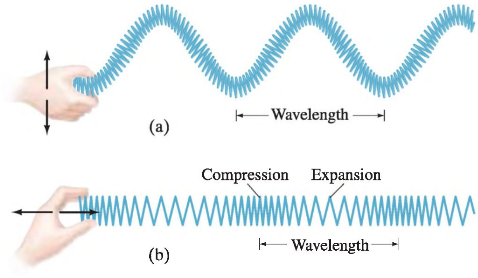

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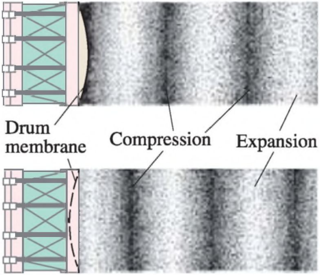

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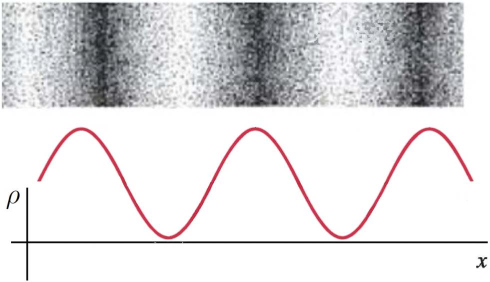

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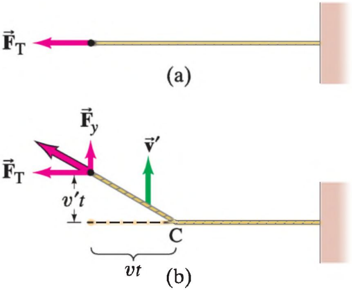
where, F_T is the tension on the cord and μ is the longitudinal density.

Proof:



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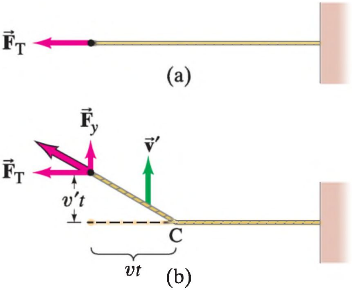

Small vertical displacement ($v't \ll vt$)

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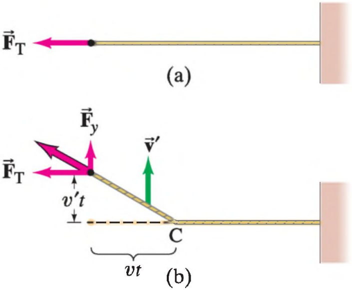

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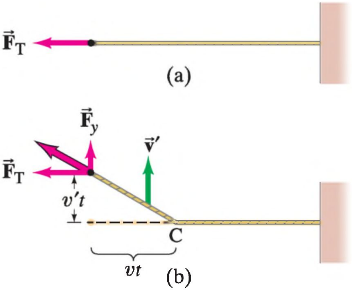

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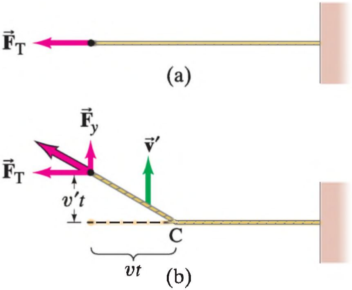

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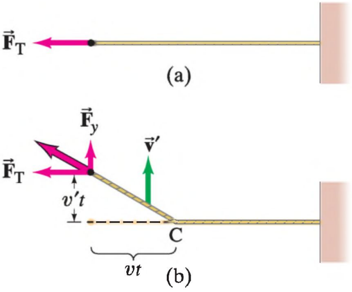

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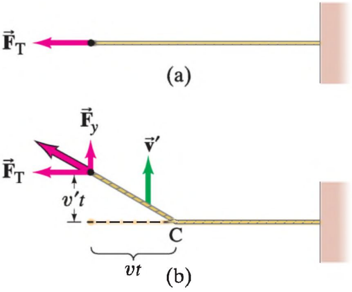

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Where, E and B are the elastic and bulk modulus, respectively.

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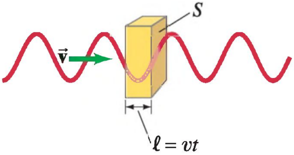

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$$\overline{P} = \frac{E}{t} \tag{6}$$

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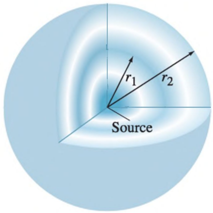

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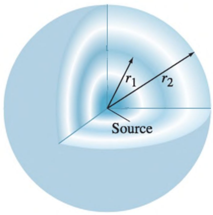

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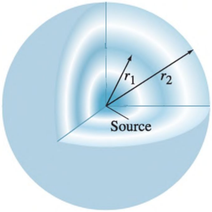

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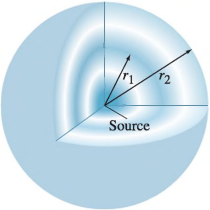

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If the power output P is constant $\rightarrow I \propto \frac{1}{r^2}$

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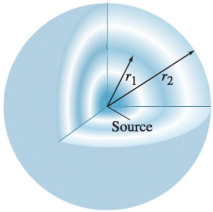

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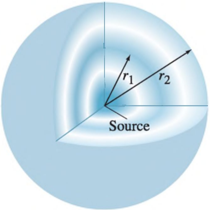

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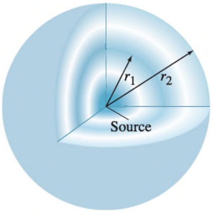

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wave is twice as far from the source $\rightarrow$ amplitude is half as large.

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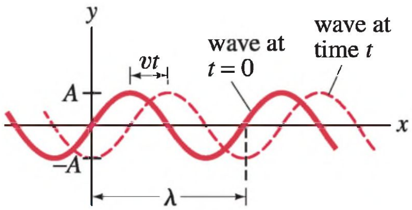

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$$\boxed{v = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}} \tag{14}$$

Wave traveling to the left:

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$$D(x,t) = A sin(kx + \omega t) \tag{15}$$

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The argument of the sine can also contain a phase ϕ determined by the value of D at $x = 0, \ t = 0$.

The Wave Equation

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- The tension in the string does not vary during a vibration.

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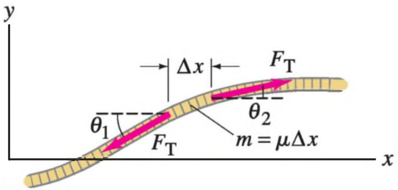
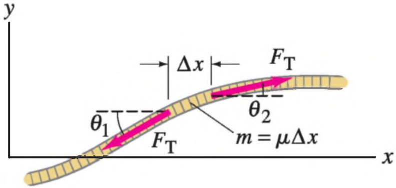


The Wave Equation



$$Newton \rightarrow \sum F_y = ma_y$$

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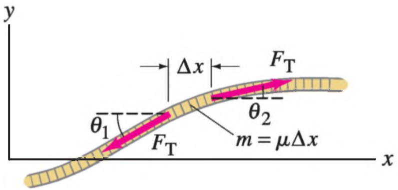

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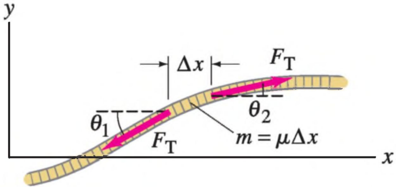

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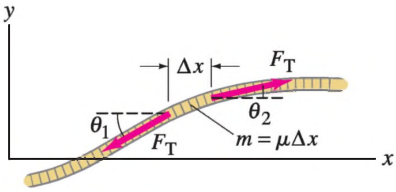

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$$sin\theta \approx tan\theta = \frac{\partial D}{\partial x} = S$$

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$$\rightarrow \frac{\partial^2 D}{\partial x^2} = \frac{\mu}{F_T}\frac{\partial^2 D}{\partial^2 t} \tag{16}$$

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$$\rightarrow v = \sqrt{\frac{F_T}{\mu}}$$

The Wave Equation

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$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial^2 t}} \tag{17}$$

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This is the **one-dimensional** wave equation, and it can describe not only small amplitude waves on a stretched string, but also small amplitude longitudinal waves in gases, liquids, and elastic solids.

Superposition Principle

If D_1 and D_2 are solutions of

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial^2 t}$$

Then, $D_1 + D_2$ is a solution.

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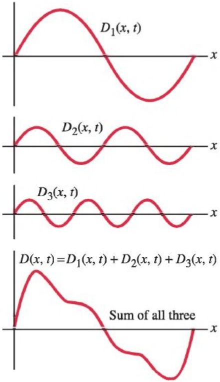
When two or more waves pass through the same region of space at the same time, the displacement is the vector sum of the separate displacements.

Superposition Principle

Sum of three sinusoidal Waves

Superposition Principle

Sum of three sinusoidal Waves
$\rightarrow$ it is not sinusoidal.



Superposition Principle

The shape changes if the velocity of the waves depends on the frequency.

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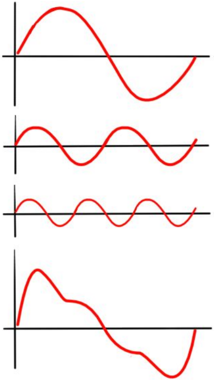
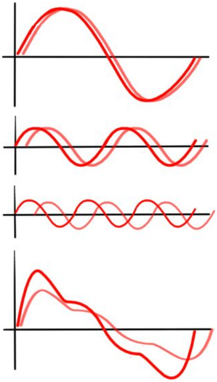


Superposition Principle

Dispersion



Superposition Principle

Fourier's Theorem:

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Any complex periodic wave = sum of simple sinusoidal waves

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If the wave is not periodic, the sum becomes an integral (called a Fourier integral).

Superposition Principle

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\left(\frac{2n\pi x}{P}\right) + \sum_{n=1}^{\infty} b_n sin\left(\frac{2n\pi x}{P}\right) \tag{18}$$

Superposition Principle

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where $f(x)$ integrable in the interval $(-P/2, P/2)$

$$a_0 = \frac{2}{P}\int_{-P/2}^{P/2} f(x)dx \tag{19}$$

$$a_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)cos\left(\frac{2n\pi x}{P}\right)dx \tag{20}$$

$$b_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)sin\left(\frac{2n\pi x}{P}\right)dx \tag{21}$$

Interference

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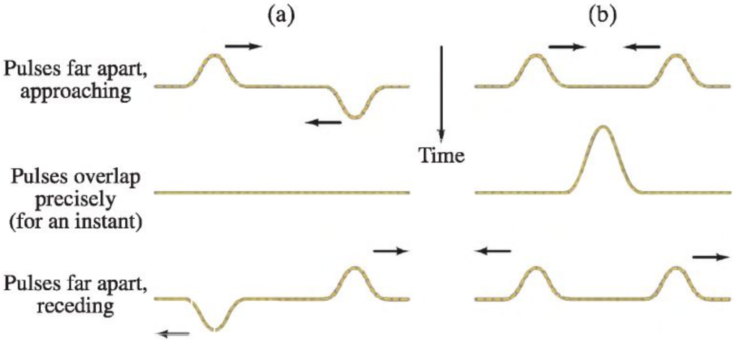
- **Destructive Interference** the two waves have opposite displacements and they add to zero.
- **Constructive Interference** they produce a resultant displacement that is greater than the displacement of either separate pulse,

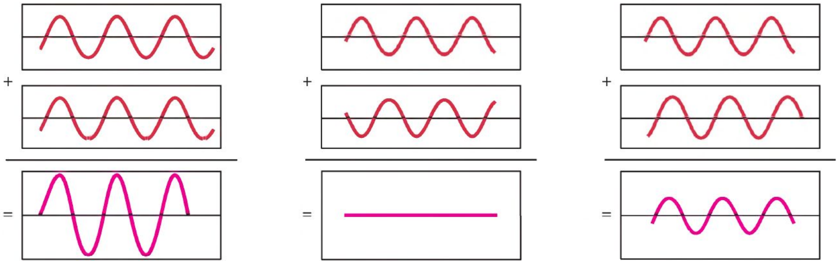
Interference

Example two pulses in a cord:



Interference

The interference pattern of two equal waves can be constructive (waves is phase, phase 0 degree), destructive (out of phase, phase 180 degree), partially destructive (other angles or different amplitudes).

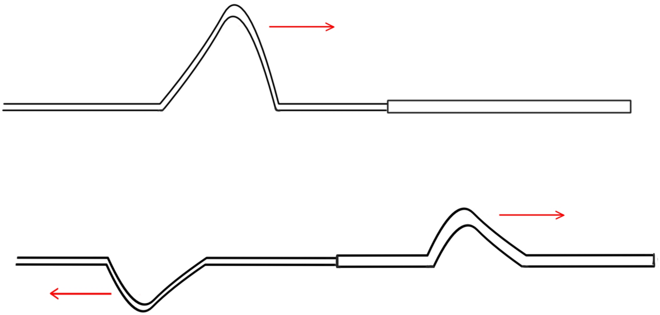


Reflection and transmission

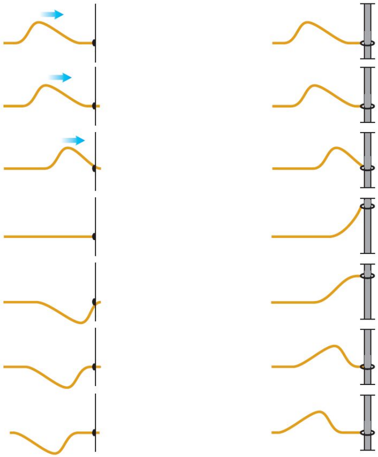
What happens when a wave strikes an obstacle, or comes to the end of the medium in which it is traveling?

Reflection and transmission

Change of medium



Reflection and transmission



Resonance

String fixed at its two ends,

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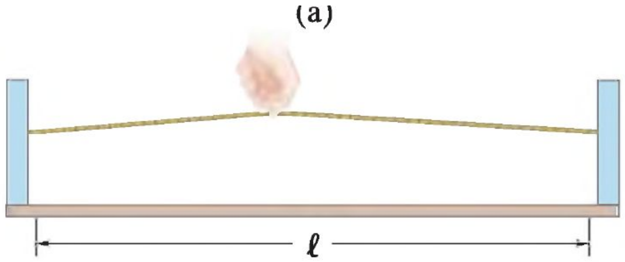
String fixed at its two ends, What happens when the string is pocked?



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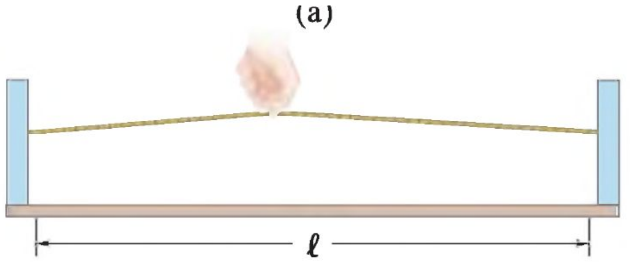

The initial pulse generates two traveling waves that are reflected in both extremes.

Resonance

$$D(x,t) = D_1(x,t) + D_2(x,t)$$

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$$\rightarrow \boxed{D(x,t) = 2Asin(kx)cos(\omega t)}$$

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$$\rightarrow \boxed{D(x,t) = 2Asin(kx)cos(\omega t)}\quad \textit{Standing Wave} \tag{22}$$

Resonance

If the string is fixed at its two ends,

$$D(x=0,t)=D(x=\ell,t)=0$$

then,

$$k\ell = n\pi \rightarrow k = \frac{n\pi}{\ell} \tag{23}$$

$$\lambda_n = \frac{2\ell}{n},\ \ n = 1\ ,2\ ,3, \ldots \tag{24}$$

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All particles of the string vibrate with the same frequency:

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Fundamental frequency: $n = 1 \rightarrow f_1$

Harmonics $\rightarrow f_n$

Resonance

The amplitude of the motion depends on x,

$$amplitude = 2Asin(kx)$$

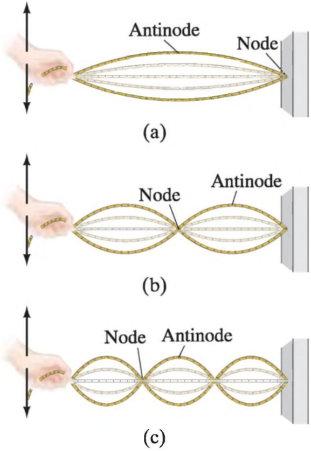
The amplitude has a maximum, equal to $2A$, when

$$kx = \frac{(2n+1)\pi}{2} \rightarrow x = \frac{(2n+1)\pi}{k} \tag{27}$$

Resonance

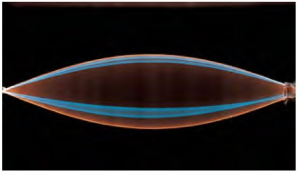
The frequencies at which standing waves are produced are the **natural frequencies** or **resonant frequencies** of the cord.
When a string is pocked, only standing waves corresponding to resonant frequencies persist for long.

Resonance

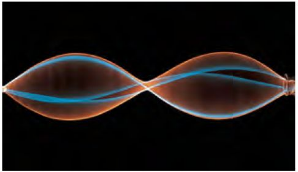


Resonance

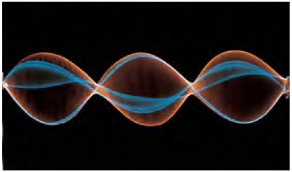
(a) String is one-half wavelength long.



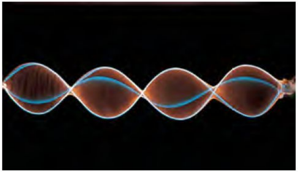
(b) String is one wavelength long.



(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



Resonance

- The term "standing" wave is also meaningful from the point of view of energy. Since the string is at rest at the nodes, no energy flows past these points. Hence the energy is not transmitted down the string but "stands" in place in the string.

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- Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane or an object made of metal or wood.

Questions

1. Describe how you could estimate the speed of water waves across the surface of a pond.

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2. Will any function of $(x - vt)$ represent a wave motion?
3. When a standing wave exists on a string, the vibrations of incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed?
4. Can the amplitude of the standing waves be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?