

PHY250

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Digipen

Fall 2020

Wave Motion

Characteristic of Wave Motion

Energy transported by a wave

Mathematical description of a Wave

The Wave Equation

Superposition Principle

Reflection and transmission

Interference

Standing waves

Sound

Characteristics

Mathematical Description

Sources of Sound

Quality of Sound, and Noise; Superposition

Interference of Sound Waves

Doppler Effect

Introduction

So far we have studied the Simple Harmonic Motion of a single particle...



Introduction

What happens if the particle that is oscillating is part of a medium?



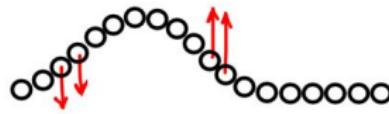
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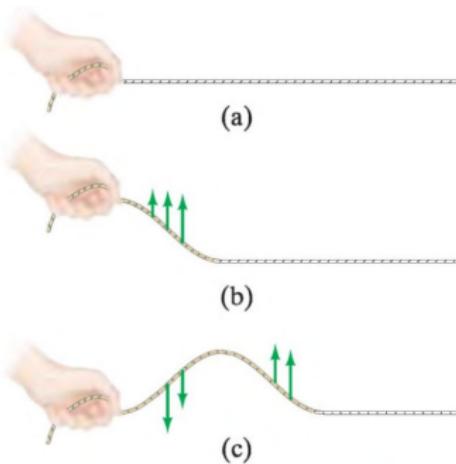
Mechanical Waves carry Energy as oscillation of matter, they does not carry matter.

Mechanical Waves

Example: How a wave is formed in a cord?

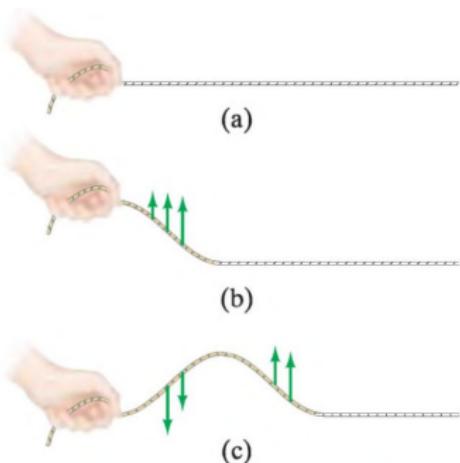
Mechanical Waves

Example: How a wave is formed in a cord?



Mechanical Waves

Example: How a wave is formed in a cord?



Single pulse formation: The hand pulls up and down on one end of the cord, each section of the cord is pulled up and down by the tension made by the adjacent section. The source of the traveling wave pulse is a disturbance, and cohesive forces between adjacent sections of the cord cause the pulse to travel.

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1. In space: If you take a picture of the wave in a given instant of time, the wave will have the shape of a sine or a cosine.
2. In time: the up-down motion of a small segment of the cord at a certain position will be Simple Harmonic Motion.

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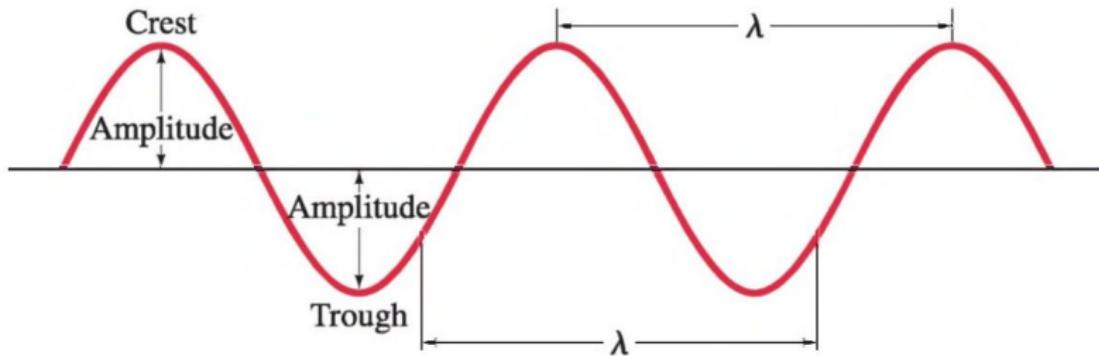
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Periodic Sinusoidal Waves

Picture of the wave at a certain time:

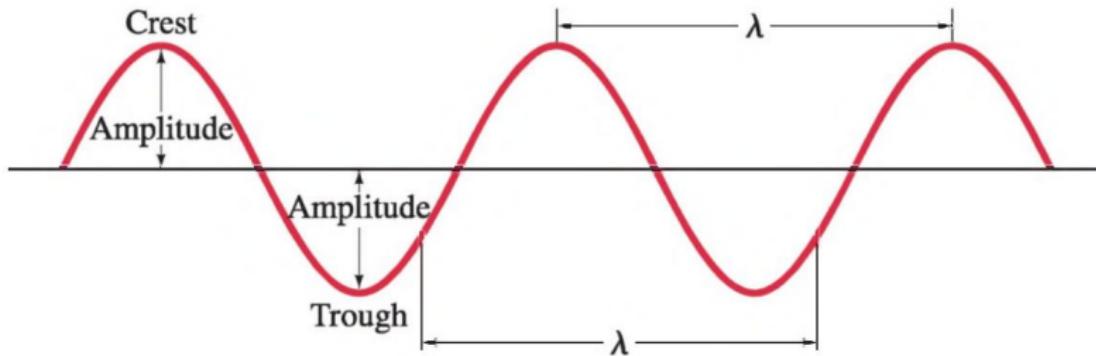
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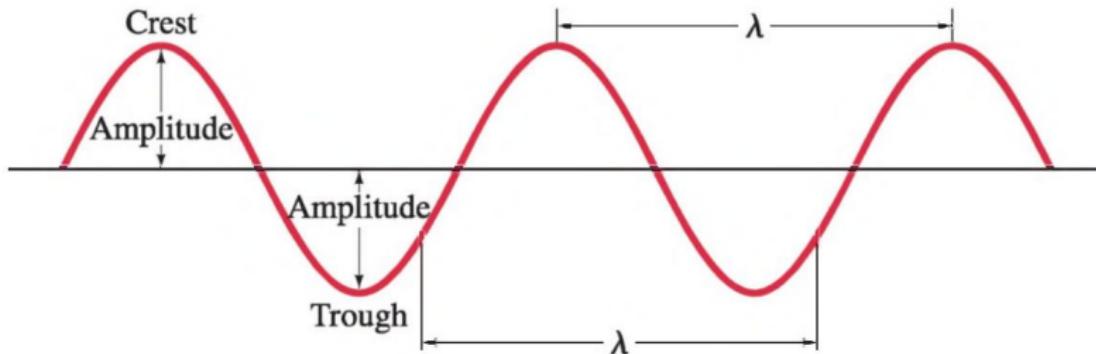
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$$v = \lambda/T = \lambda f$$

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Types of waves

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- ▶ **Transverse waves** The vibration up-down of the particles of the medium are in a direction transverse to the motion of the wave itself.

Types of waves

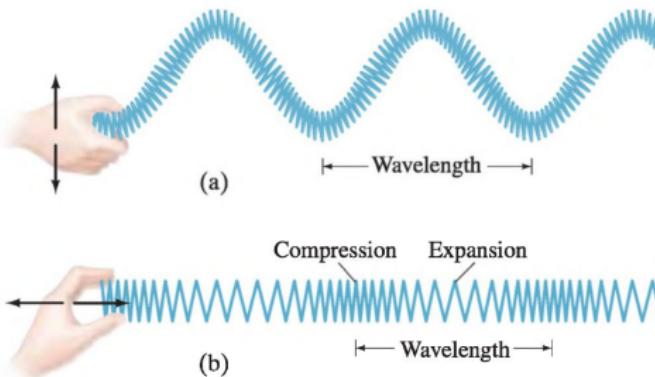
- ▶ **Transverse waves** The vibration up-down of the particles of the medium are in a direction transverse to the motion of the wave itself.
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Sound waves: example of longitudinal waves.

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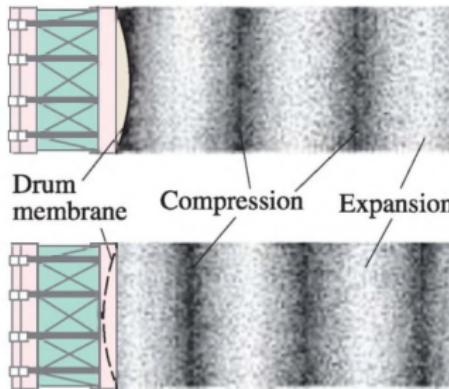
Sound waves: example of longitudinal waves.

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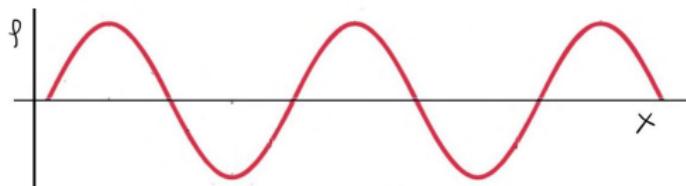


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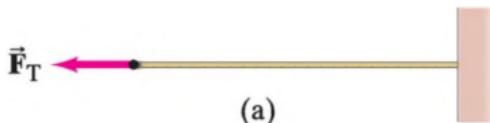
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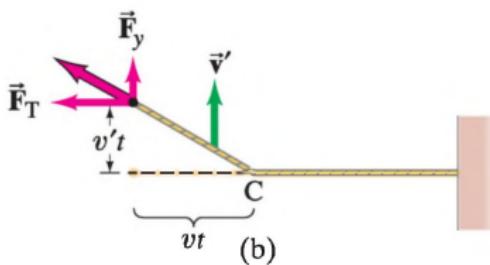
where, F_T is the tension on the cord and λ is the longitudinal density.

Velocity of transverse Waves

Proof:



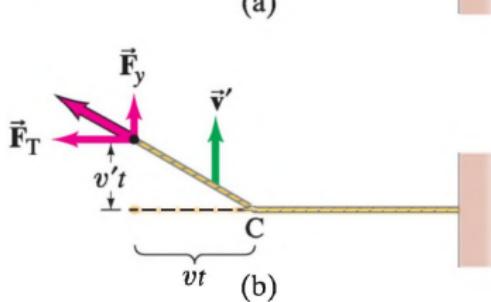
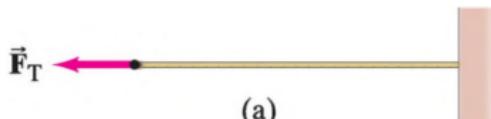
(a)



(b)

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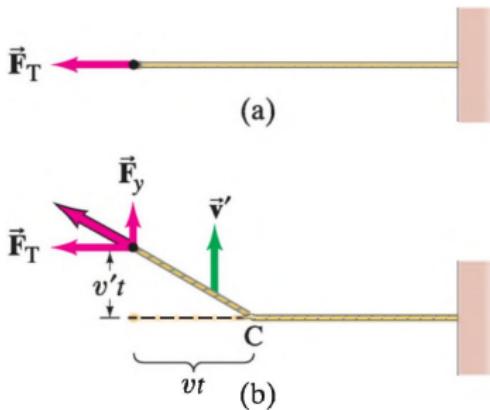
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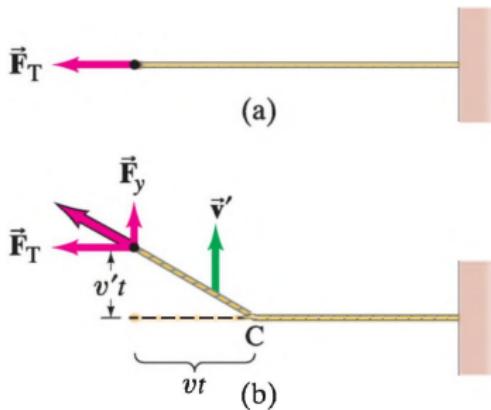


Small vertical displacement ($v't \ll vt$)

$$\frac{F_y}{\text{mass}} = \frac{v't}{vt} = \frac{v'}{v} \quad (3)$$

Velocity of transverse Waves

Proof:



Impulse:

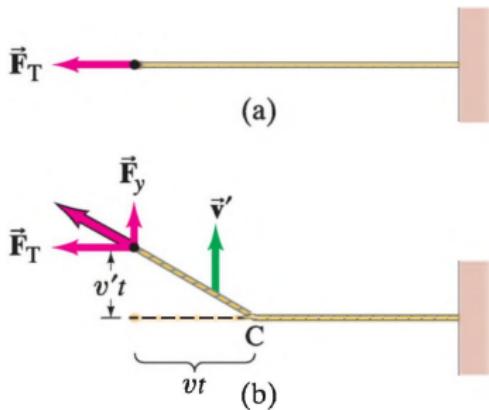
$$F_y t = \Delta P' = \Delta m v'$$

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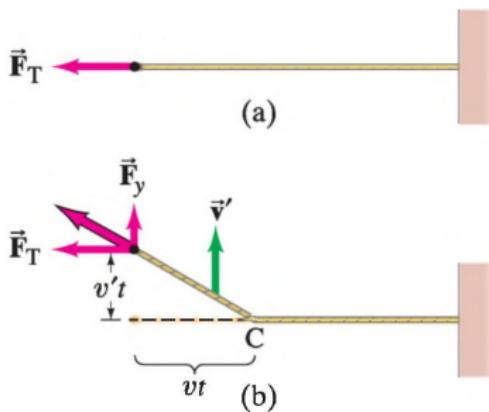
$$\Delta m = \mu x = \mu v t$$

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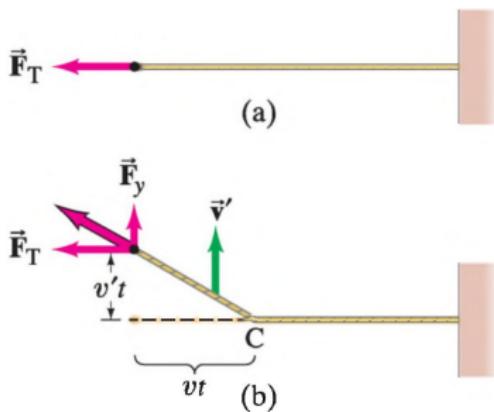
$$\rightarrow F_y t = \mu v t v' \rightarrow \frac{v'}{v} F_T = \mu v v'$$

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Impulse:

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Where, E and B are the elastic and bulk modulus, respectively.

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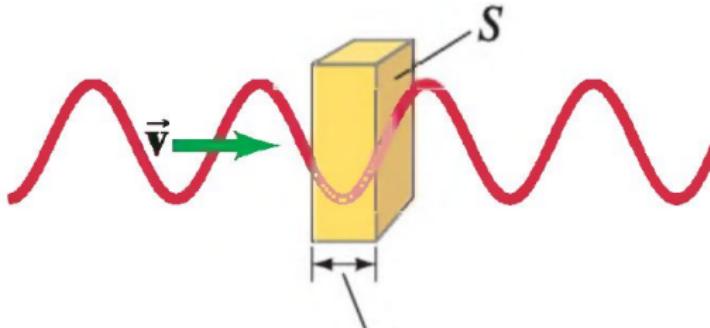
For three-dimensional waves traveling in an elastic medium, $m = \rho V$, and $V = S\ell$ and $\ell = vt$ (see the figure):

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$$\bar{P} = \frac{E}{t} \quad (7)$$

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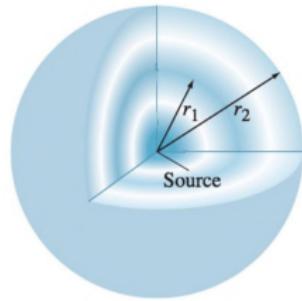
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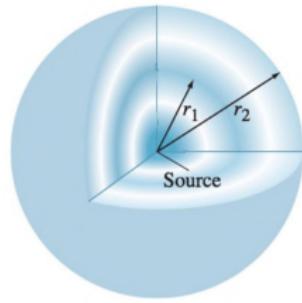
$$I = \frac{\bar{P}}{S} = \frac{E}{tS} = 2\pi^2 \rho v f^2 A^2 \quad (8)$$

Point source in an isotropic medium

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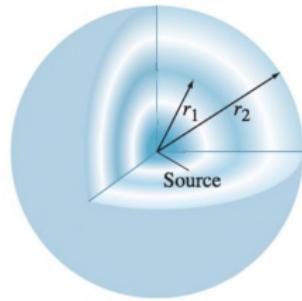


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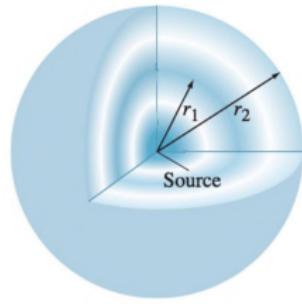
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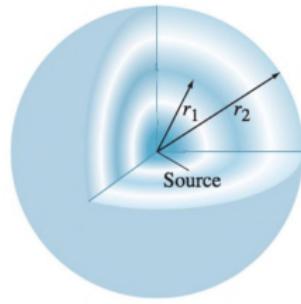


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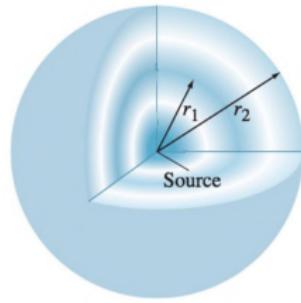
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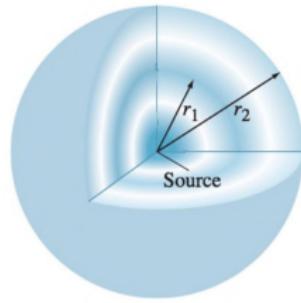
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wave is twice as far from the source \rightarrow amplitude is half as large

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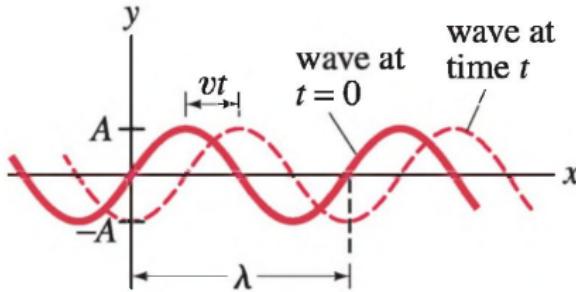
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$$v = \frac{2\pi}{k} \frac{\omega}{2\pi} = \frac{\omega}{k} \quad (15)$$

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$$D(x, t) = A \sin(kx + \omega t) \quad (16)$$

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The argument of the sine can also contain a phase ϕ determined by the value of D at $x = 0, t = 0$.

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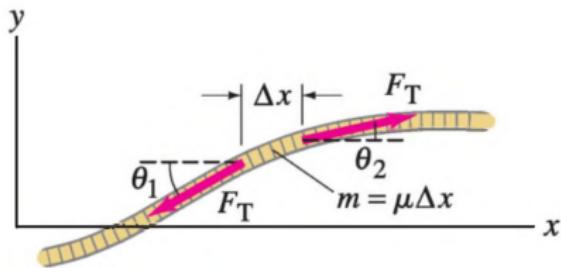
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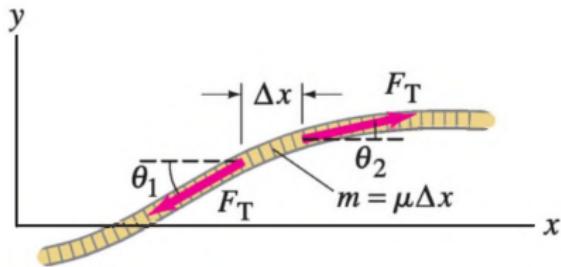
- ▶ The amplitude of the wave is small compared to the wavelength.
- ▶ The tension in the string does not vary during a vibration.

The Wave Equation



$$\sum F_y = ma_y$$

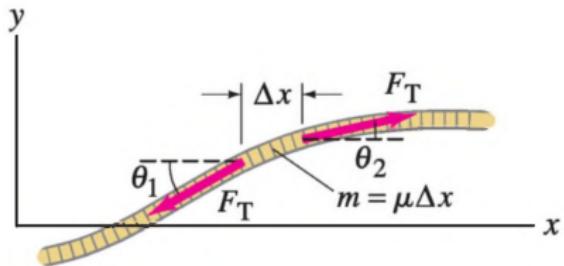
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$$F_T \sin \theta_2 - F_T \sin \theta_1 = (\mu \Delta x) \frac{\partial^2 D}{\partial t^2}$$

$$\sum F_y = ma_y$$

The Wave Equation

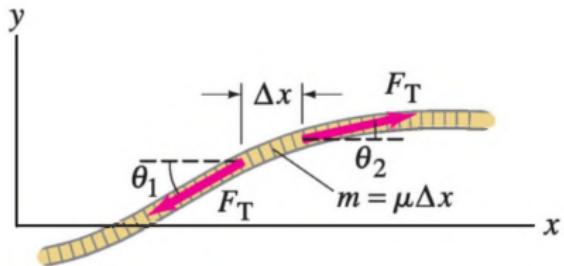


$$F_T \sin \theta_2 - F_T \sin \theta_1 = (\mu \Delta x) \frac{\partial^2 D}{\partial t^2}$$

where, $a_y = \frac{\partial^2 D}{\partial t^2}$ because the motion is vertical.

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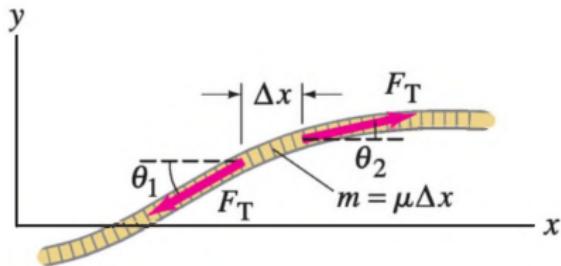
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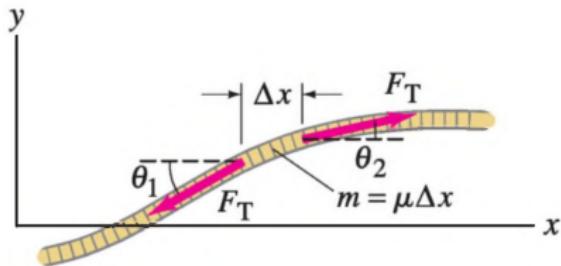
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The Wave Equation

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$$F_T(S_2 - S_1) = \mu\Delta x \frac{\partial^2 D}{\partial^2 t} \quad (17)$$

Or..

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$$\rightarrow v = \sqrt{\frac{F_T}{\mu}}$$

The Wave Equation

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This is the **one-dimensional** wave equation, and it can describe not only small amplitude waves on a stretched string, but also small amplitude longitudinal waves in gases, liquids, and elastic solids..

Superposition Principle

If D_1 and D_2 are solutions of

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}$$

Then, the sum $D_1 + D_2$ is also a solution..

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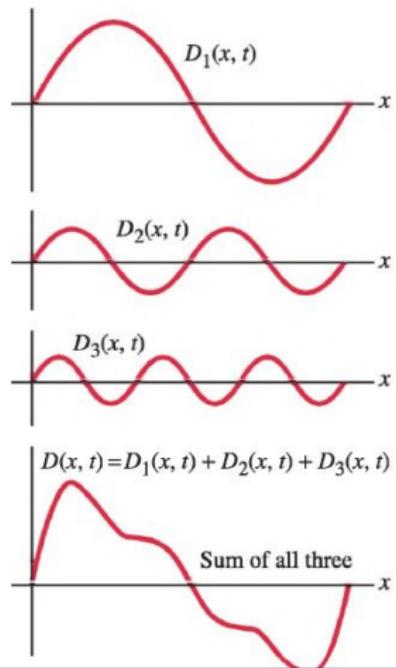
$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}$$

Then, the sum $D_1 + D_2$ is also a solution..

In general, when two or more waves pass through the same region of space at the same time, it is found that the displacement is the vector (or algebraic) sum of the separate displacements.

Superposition Principle

Example: In this case there are three waves present, on a stretched string, each of different amplitude and frequency. The resultant wave is not a simple sinusoidal wave and is called a composite (or complex) wave



Superposition Principle

It can be shown that any complex wave can be considered as being composed of many simple sinusoidal waves of different amplitudes, wavelengths, and frequencies. This is known as Fourier's theorem. A complex periodic wave of period T can be represented as a sum of pure sinusoidal terms whose frequencies are integral multiples of $f = 1/T$.

If the wave is not periodic, the sum becomes an integral (called a Fourier integral).

Superposition Principle

Any function that is integrable in the interval $(-L, L)$ can be expressed as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \quad (22)$$

where

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad (23)$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad (24)$$

Superposition Principle

One result of the superposition principle is that if two waves pass through the same region of space, they continue to move independently of one another. You may have noticed, for example, that the ripples on the surface of water (two-dimensional waves) that form from two rocks striking the water at different places will pass through each other.

Superposition Principle

In the previous analysis, we have assumed that the waves are a solution of the wave equation, that is

- ▶ The amplitude of the oscillation is small.

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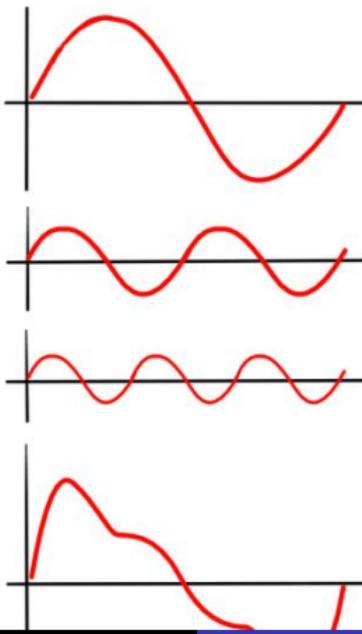
- ▶ The amplitude of the oscillation is small.
- ▶ The material in is the elastic regime, the restoring force acting on each piece of medium is proportional to the displacement.
- ▶ The velocity of the waves does not depend on the frequency.

Superposition Principle

When the restoring force is not proportional to the displacement for mechanical waves, the speed of sinusoidal waves depends on the frequency. The variation of speed with frequency is called **dispersion**. The different sinusoidal waves that compose a complex wave will travel with slightly different speeds in such a case. Consequently, a complex wave will change shape as it travels if the medium is “dispersive.”

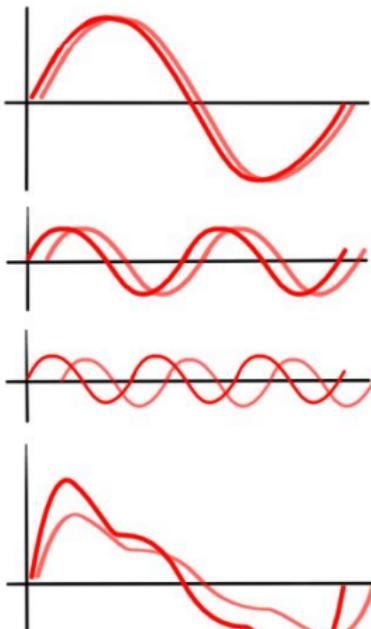
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Reflection and transmission

What happens when a wave strikes an obstacle, or comes to the end of the medium in which it is traveling?

Reflection and transmission

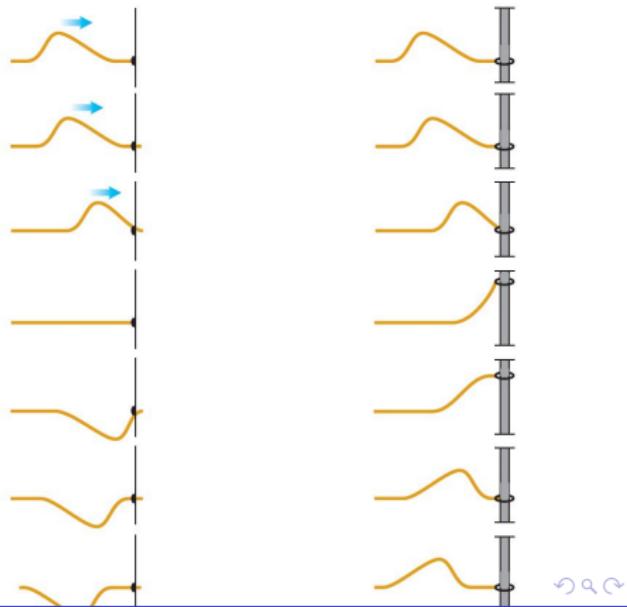
What happens when a wave strikes an obstacle, or comes to the end of the medium in which it is traveling?

At least a part of the wave is reflected.

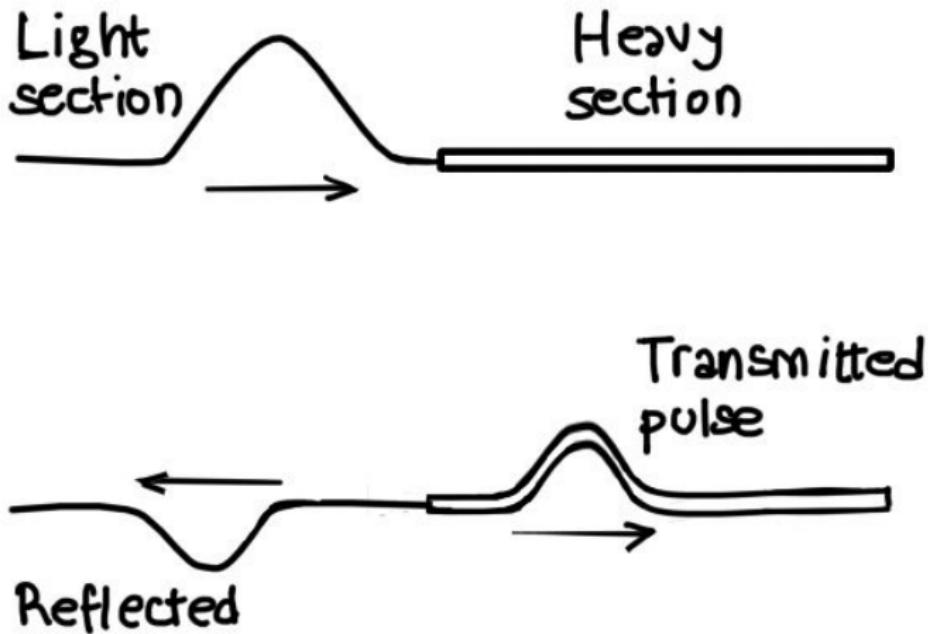
Reflection and transmission

Example:

A wave pulse traveling down a cord is reflected as shown in the figure. The reflected pulse returns inverted if the end of the cord is fixed; it returns right side up if the end is free.



Reflection and transmission

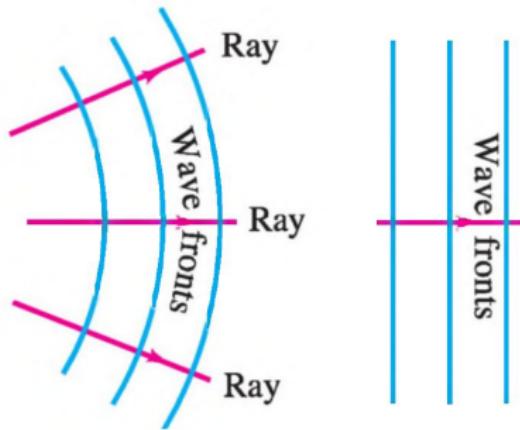


Reflection and transmission

Consider next a pulse that travels down a cord which consists of a light section and a heavy section. When the wave pulse reaches the boundary between the two sections, part of the pulse is reflected and part is transmitted. The heavier the second section of the cord, the less the energy that is transmitted.

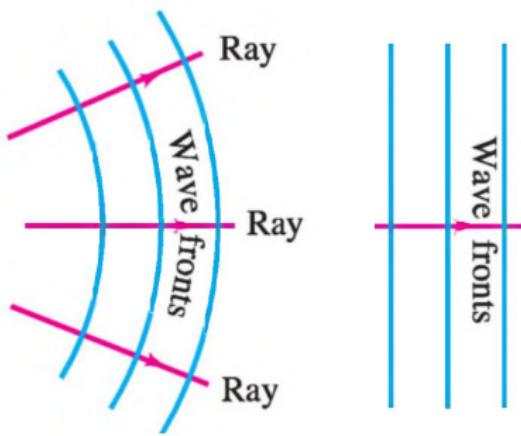
Reflection and transmission

For a two- or three-dimensional wave, such as a water wave, we are concerned with wave fronts, by which we mean all the points along the wave forming the wave crest.



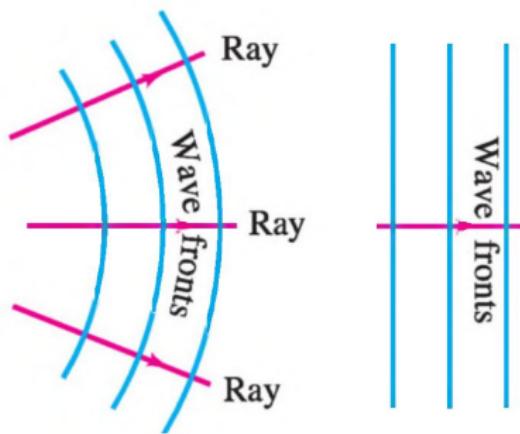
Reflection and transmission

A line drawn in the direction of motion, perpendicular to the wave front, is called a ray.



Reflection and transmission

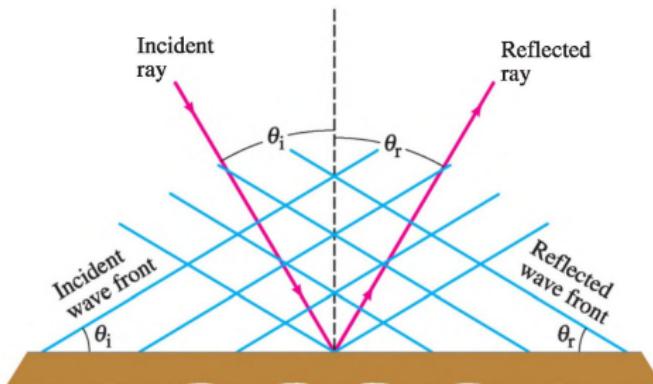
Wave fronts far from the source have lost almost all their curvature and are nearly straight; they are then called plane waves.



Reflection and transmission

For reflection of a two- or three-dimensional plane wave, the angle that the incoming or incident wave makes with the reflecting surface is equal to the angle made by the reflected wave. This is the **law of reflection**:

the angle of reflection equals the angle of incidence.



Interference

Interference refers to what happens when two waves pass through the same region of space at the same time. In the region where they overlap, the resultant displacement is the algebraic sum of their separate displacements.

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Interference

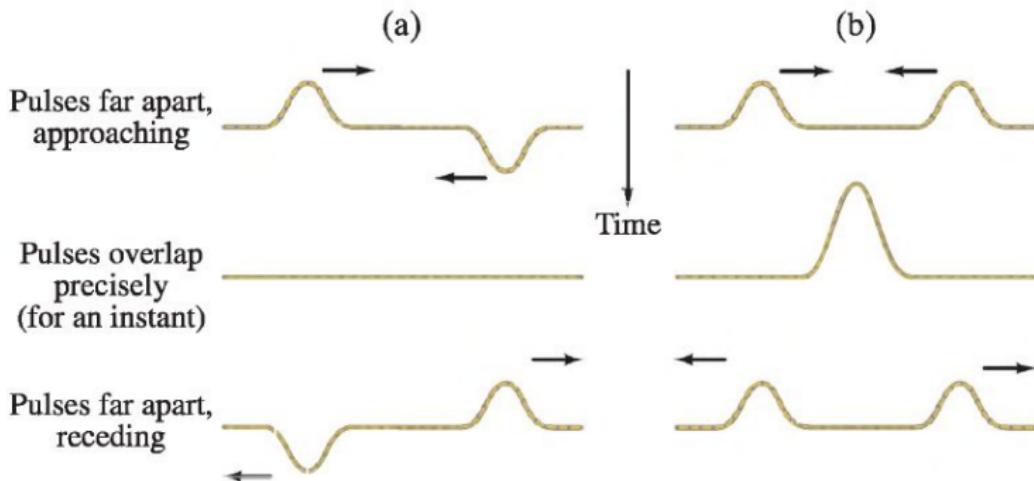
Interference refers to what happens when two waves pass through the same region of space at the same time. In the region where they overlap, the resultant displacement is the algebraic sum of their separate displacements.

We can have:

- ▶ **Destructive Interference** the two waves have opposite displacements at the instant they pass one another, and they add to zero.
- ▶ **Constructive Interference** they produce a resultant displacement that is greater than the displacement of either separate pulse,

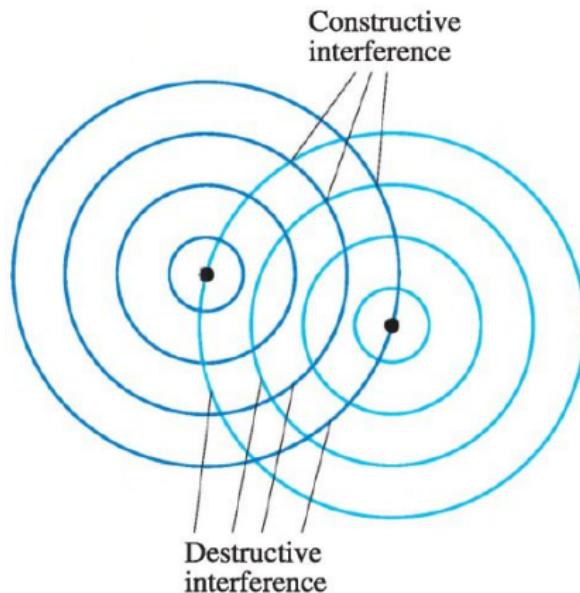
Interference

Example two pulses in a cord:



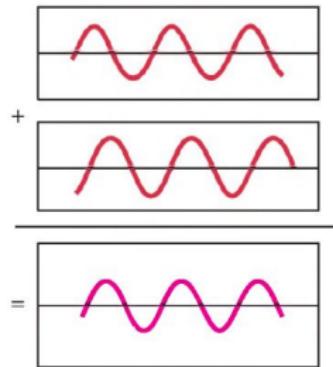
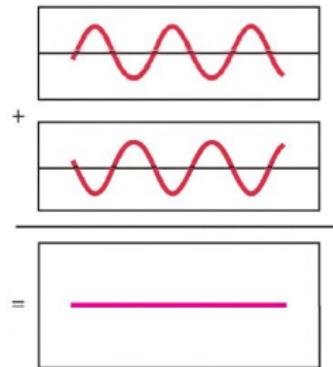
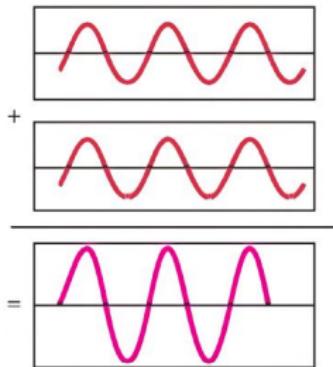
Interference

Example two rocks thrown simultaneously in water:



Interference

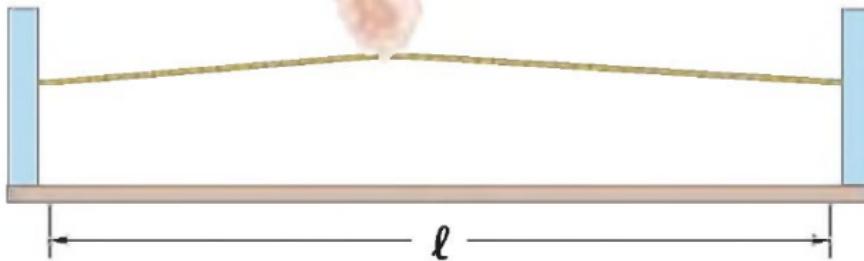
To summarize, the interference pattern of two equal waves can be constructive (waves in phase, phase 0 degree), destructive (out of phase, phase 180 degree), or partially destructive (other angles or different amplitudes).



Resonance

What happens when a string is poked?

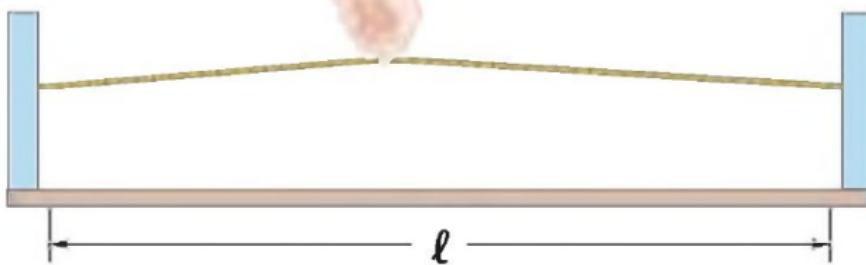
(a)



Resonance

What happens when a string is poked?

(a)



The initial pulse is reflected in both extremes generating waves that travel in both directions that are reflected in the extremes and interfere.

Resonance

A standing wave can be considered to consist of two traveling waves that move in opposite directions.

$$D(x, t) = D_1(x, t) + D_2(x, t)$$

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$$D(x, t) = D_1(x, t) + D_2(x, t)$$

$$D_1(x, t) = A \sin(kx - \omega t), \quad D_2(x, t) = A \sin(kx + \omega t)$$

$$\rightarrow D(x, t) = 2A \sin(kx) \cos(\omega t) \tag{25}$$

Resonance

If the string is fixed at its two ends,

$$D(x = 0, t) = D(x = \ell, t) = 0$$

then,

$$k\ell = n\pi \rightarrow k = \frac{n\pi}{\ell} \quad (26)$$

Resonance

All particles of the string vibrate with the same frequency,

$$f = \frac{\omega}{2\pi}$$

but the amplitude depends on x ,

$$\text{amplitude} = 2A\sin(kx)$$

The amplitude has a maximum, equal to $2A$, when

$$kx = \frac{(2n+1)\pi}{2} \rightarrow x = \frac{(2n+1)\pi}{k} \quad (27)$$

Resonance

The wavelengths of the standing waves bear a simple relationship to the length ℓ of the string:

$$\lambda_n = \frac{2\ell}{n}, \quad n = 1, 2, 3, \dots \quad (28)$$

Fundamental frequency: $n = 1 \rightarrow \lambda_1$

Harmonics $\rightarrow \lambda_n$

Resonance

The frequency f of each vibration is,

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$$v = \sqrt{\frac{F_T}{\mu}}$$

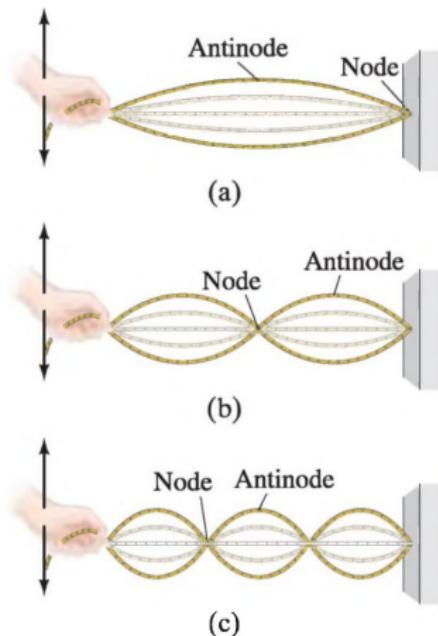
Resonance

The frequencies at which standing waves are produced are the **natural frequencies** or **resonant frequencies** of the cord.

When a string is plucked, only standing waves corresponding to resonant frequencies persist for long.

Resonance

Example: if you vibrate a cord at just the right frequency, the cord simply appears to have segments that oscillate up and down in a fixed pattern. The points of destructive interference are called **nodes**. Points of constructive interference are called **antinodes**. The nodes and antinodes remain in fixed positions for a particular frequency.

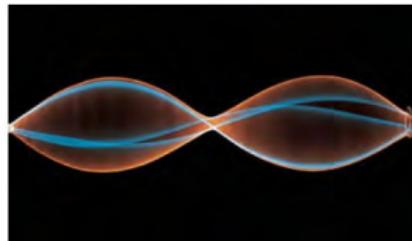


Resonance

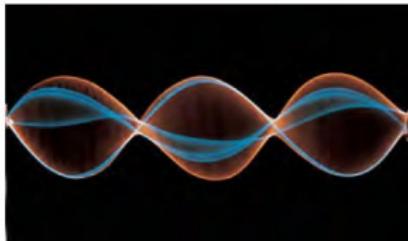
(a) String is one-half wavelength long.



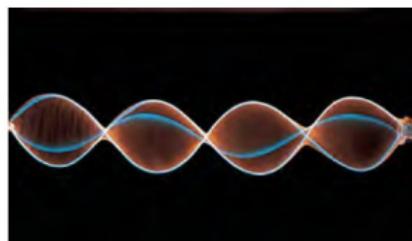
(b) String is one wavelength long.



(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



Resonance

The term “standing” wave is also meaningful from the point of view of energy. Since the string is at rest at the nodes, no energy flows past these points. Hence the energy is not transmitted down the string but “stands” in place in the string.

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Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane or an object made of metal or wood.

Questions

1. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?

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4. Is energy always conserved when two waves interfere? Explain.
5. When a standing wave exists on a string, the vibrations of incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed? Explain.
6. Can the amplitude of the standing waves be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?

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Is an interpretation of our brain of a physical sensation that stimulate our ears, that is, a longitudinal wave.

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Sound

Is an interpretation of our brain of a physical sensation that stimulate our ears, that is, a longitudinal wave.

We must consider...

- ▶ Source → vibrating object.
- ▶ Needs mater to spread.
- ▶ The energy is transferred as longitudinal waves.
- ▶ Detection → ears, microphone, etc.

Sound Speed

The velocity of the propagation of sound in a medium is,

$$v = \sqrt{\frac{B}{\rho}} \quad (31)$$

where B is the Bulk modulus, defined by

$$\Delta P = -B \frac{\Delta V}{V}$$

It relates the fractional change of volume due to a change in pressure. The minus sign means that the volume decreases if the pressure increases.

Sound Speed

TABLE 16–1 Speed of Sound in Various Materials (20°C and 1 atm)

Material	Speed (m/s)
Air	343
Air (0°C)	331
Helium	1005
Hydrogen	1300
Water	1440
Sea water	1560
Iron and steel	≈ 5000
Glass	≈ 4500
Aluminum	≈ 5100
Hardwood	≈ 4000
Concrete	≈ 3000

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To describe the sound, we have to consider two aspects,

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The audible range by humans is 20 Hz to 20000 Hz

Pressure Waves

A sound wave is a longitudinal wave described by,

Pressure Waves

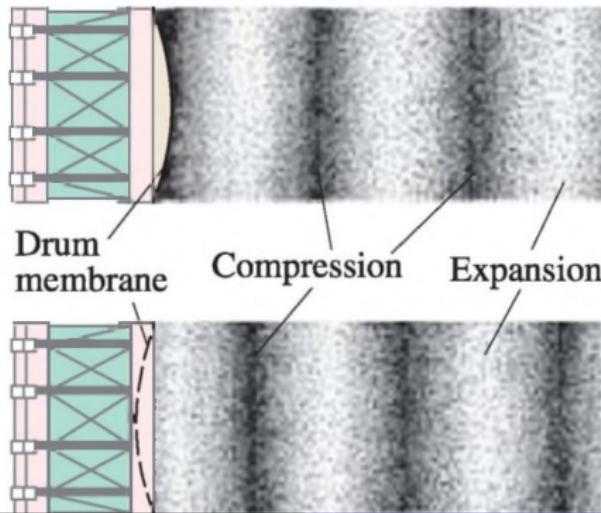
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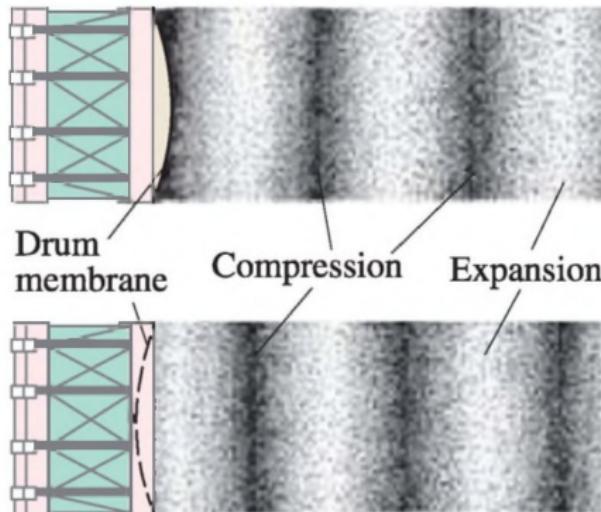
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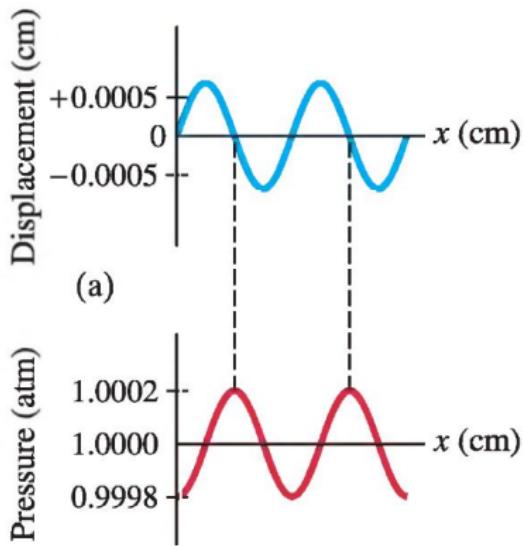
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Pressure Waves

The displacement and pressure are $\frac{\pi}{2}$ out of phase.



Pressure Waves

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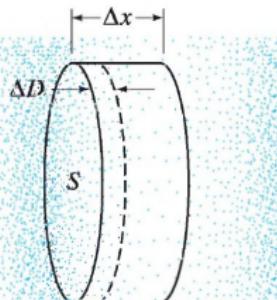
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Pressure Waves

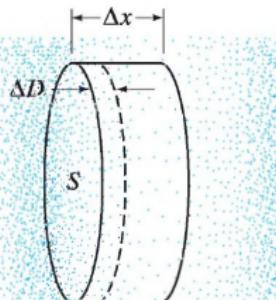
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The variation in volume due to the wave is: $\Delta V = S\Delta D$



Pressure Waves

If we know the displacement function, $D(x, t)$, we can find the pressure wave using the definition of Bulk modulus,

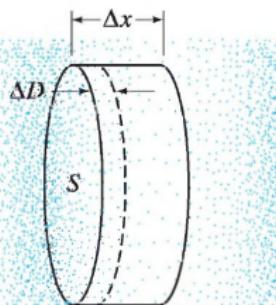
$$\Delta P = -B \frac{\Delta V}{V}$$

Let's consider a longitudinal wave crossing a certain volume element in a fluid,

The volume is, $V = S\Delta x$.

The variation in volume due to the wave is: $\Delta V = S\Delta D$

The variation of pressure is..



$$\Delta P = -B \frac{S\Delta D}{S\Delta x} \quad (32)$$

Pressure Waves

Taking the limit for $\Delta x \rightarrow 0$

$$\Delta P = -B \frac{\partial D}{\partial x} \quad (33)$$

And...

$$\frac{\partial D}{\partial x} = kA \cos(kx - \omega t) \quad (34)$$

Then,

$$\Delta P = -BkA \cos(kx - \omega t) \quad (35)$$

where the pressure amplitude is,

$$\Delta P_M = BkA \quad (36)$$

Pressure Waves

Using the relations,

$$v = \sqrt{\frac{B}{\rho}}, \quad k = \frac{2\pi f}{v}$$

$$\Delta P_M = BkA = 2\pi v \rho f A \quad (37)$$

Intensity of sound

We are going to define a new measurement unit that relates the intensity with loudness.

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Then, we are going to define this new unit in log scale.

Decibel

We are going to define one decibel (1 dB) as,

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where \log is in base 10, and I_0 is the intensity of a chosen reference level.

$$I_0 = 10^{-12} \frac{W}{m^2}, \text{ minimum audible intensity} \quad (39)$$

Decibel

Example:

What is the level of a sound whose intensity is $I = 10^{-10} \frac{W}{m^2}$?

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At the threshold of hearing? $I = 10^{-12} \frac{W}{m^2}$?

$$\beta = 10\log \left(\frac{10^{-12}}{10^{-12}} \right) = 10\log 1 = 0 \quad (41)$$

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An increase in I by a factor 10^2 is equivalent to an increase in 20 dB and so on...

Decibel

TABLE 16–2
Intensity of Various Sounds

Source of the Sound	Sound Level (dB)	Intensity (W/m²)
Jet plane at 30 m	140	100
Threshold of pain	120	1
Loud rock concert	120	1
Siren at 30 m	100	1×10^{-2}
Truck traffic	90	1×10^{-3}
Busy street traffic	80	1×10^{-4}
Noisy restaurant	70	1×10^{-5}
Talk, at 50 cm	65	3×10^{-6}
Quiet radio	40	1×10^{-8}
Whisper	30	1×10^{-9}
Rustle of leaves	10	1×10^{-11}
Threshold of hearing	0	1×10^{-12}

Decibel

Conceptual example:

A trumpeter plays at a sound level of 75 dB . Three equally loud trumpet players join in. What is the new sound level?

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Decibel

Conceptual example:

A trumpeter plays at a sound level of 75 dB. Three equally loud trumpet players join in. What is the new sound level?

$$\beta = 10 \log \frac{4I_1}{I_0} = 10 \log(4) + 10 \log \frac{I_1}{I_0} = 6.0 \text{ dB} + 75 \text{ dB} = 81 \text{ dB}$$

Sources of Sound

The source of any sound is a vibrating object.

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In musical instruments, standing waves are produced and the source vibrates at its natural resonant frequencies.

The vibrating source is in contact with the air (or other medium) and pushes on it to produce sound waves that travel outward.

The frequencies of the waves are the same as those of the source, but the speed and wavelengths can be different.

Equally Tempered Chromatic Scale

The pitch of a pure sound is determined by the frequency.

TABLE 16–3 Equally Tempered Chromatic Scale[†]

Note	Frequency (Hz)
C	262
C [#] or D [♭]	277
D	294
D [#] or E [♭]	311
E	330
F	349
F [#] or G [♭]	370
G	392
G [#] or A [♭]	415
A	440
A [#] or B [♭]	466
B	494
C'	524

Stringed Instruments

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Possible frequencies for standing waves:

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When a finger is placed on the string of a guitar or violin, the effective length of the string is shortened. So its fundamental frequency, and pitch, is higher since the wavelength of the fundamental is shorter.

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In pianos and harps the strings are of different lengths. For the lower notes the strings are not only longer, but heavier as well.

Piano Strings

Example:

The highest key on a piano corresponds to a frequency about 150 times that of the lowest key. If the string for the highest note is 5.0 cm long, how long would the string for the lowest note have to be if it had the same mass per unit length and was under the same tension?

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$$\rightarrow \frac{\ell_L}{\ell_H} = \frac{f_H}{f_L} = 150 \rightarrow \ell_L = \frac{f_H}{f_L} = 150\ell_H = 750 \text{ cm} \quad (45)$$

Piano Strings

The longer strings of lower frequency are made heavier, of higher mass per unit length, so even on grand pianos the strings are less than 3 m long.

Example

Frequencies and wavelengths in the violin:

A 0.32 m long violin string is tuned to play A above middle C at 440 Hz. (a) What is the wavelength of the fundamental string vibration, and (b) what are the frequency and wavelength of the sound wave produced? (c) Why is there a difference?

Sound Amplification

Stringed instruments would not be very loud if they relied on their vibrating strings to produce the sound waves since the strings are too thin to compress and expand much air. Stringed instruments therefore make use of a kind of mechanical amplifier known as a sounding board (piano) or sounding box (guitar, violin), which acts to amplify the sound by putting a greater surface area in contact with the air.

When the strings are set into vibration, the sounding board or box is set into vibration as well. Since it has much greater area in contact with the air, it can produce a more intense sound wave. On an electric guitar, the sounding box is not so important since the vibrations of the strings are amplified electronically.

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The air within the tube vibrates with a variety of frequencies, but only frequencies that correspond to standing waves will persist.

Wind Instruments

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In terms of $D(x, t)$:

- ▶ The air at the closed end of a tube is a displacement node.
- ▶ The air near the open end of a tube there will be an antinode.

Modes of vibration for an open tube

The exact position of the antinode near the open end of a tube depends on the diameter of the tube.

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Small diameter compared to the length → the antinode occurs very close to the end.

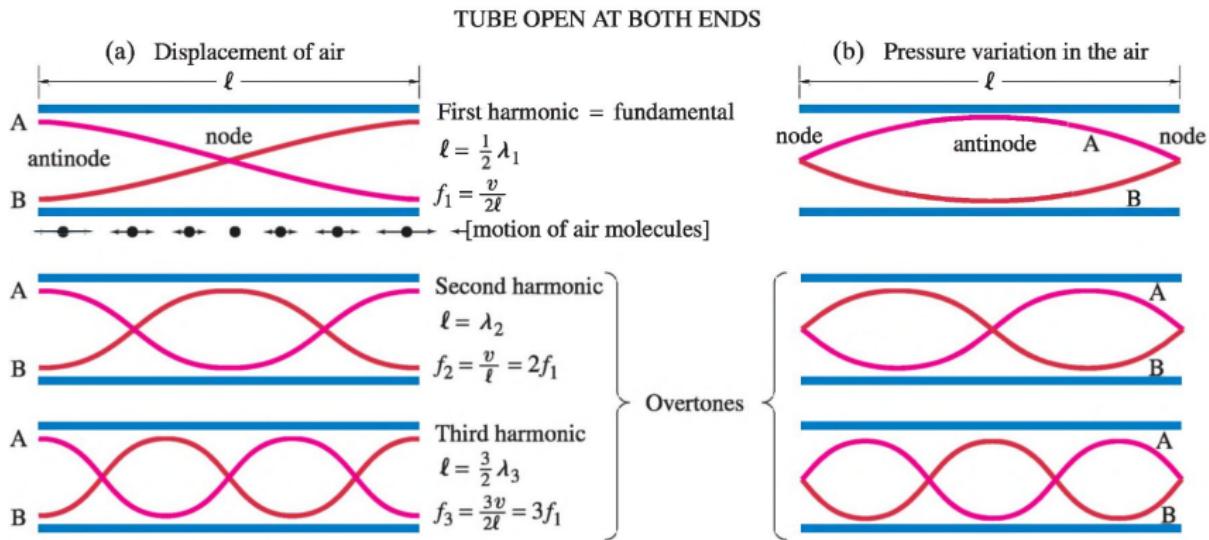
Modes of vibration for an open tube

The exact position of the antinode near the open end of a tube depends on the diameter of the tube.

Small diameter compared to the length → the antinode occurs very close to the end.

The position of the antinode may also depend slightly on the wavelength and other factors.

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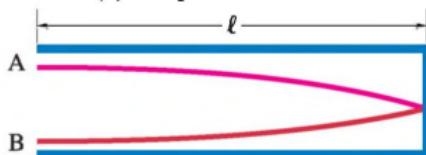
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- ▶ $f_n = nv/\lambda = nv/2\ell$.

Modes of vibration for a tube closed at one end

TUBE CLOSED AT ONE END

(a) Displacement of air



First harmonic = fundamental

$$\ell = \frac{1}{4} \lambda_1$$

$$f_1 = \frac{v}{4\ell}$$

Third harmonic

$$\ell = \frac{3}{4} \lambda_3$$

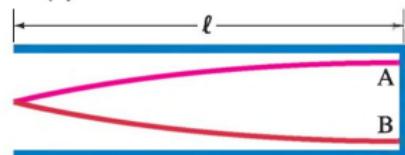
$$f_3 = \frac{3v}{4\ell} = 3f_1$$

Fifth harmonic

$$\ell = \frac{5}{4} \lambda_5$$

$$f_5 = \frac{5v}{4\ell} = 5f_1$$

(b) Pressure variation in the air



Overtones

Modes of vibration for a tube closed at one end

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- ▶ Distance between a node and the nearest antinode $\lambda/4$
- ▶ Only the odd harmonics are present in a closed tube: the overtones have frequencies equal to $3f_1, 5f_1, 7f_1, \dots$
- ▶ There is no way for waves with $2f_1, 4f_1, 6f_1$

Modes of vibration for a tube closed at one end

In terms of the pressure in the air:

- ▶ Air in a wave compressed → higher pressure.

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Modes of vibration for a tube closed at one end

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- ▶ Air in a wave compressed → higher pressure.
- ▶ Wave expansion (or rarefaction) → lower pressure.
- ▶ The open end of a tube is open to the atmosphere → the pressure remains at the outside atmospheric pressure (node).
- ▶ If a tube has a closed end → the pressure alternates to be above or below atmospheric pressure (antinode).

Modes of vibration for a tube closed at one end

Example:

Organ pipes. What will be the fundamental frequency and first three overtones for a 26 cm long organ pipe at 20C if it is (a) open and (b) closed?

Modes of vibration for a tube closed at one end

Example:

A flute is designed to play middle C (262 Hz) as the fundamental frequency when all the holes are covered. Approximately how long should the distance be from the mouthpiece to the far end of the flute? (This is only approximate since the antinode does not occur precisely at the mouthpiece.) Assume the temperature is 20°C.

modes of vibration for an open tube

Example:

To see why players of wind instruments “warm up” their instruments (so they will be in tune), determine the fundamental frequency of the flute when all holes are covered and the temperature is 10°C instead of 20°C.

Quality of sound

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The quality of a sound depends on the presence of overtones, their number and their relative amplitudes. Generally, when a note is played on a musical instrument, the fundamental as well as overtones are present simultaneously.

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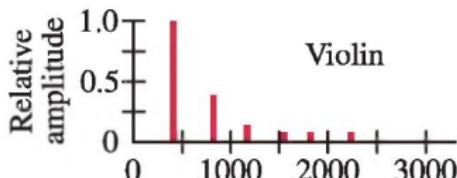
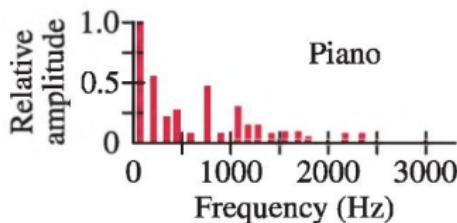
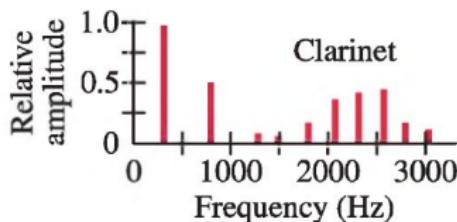
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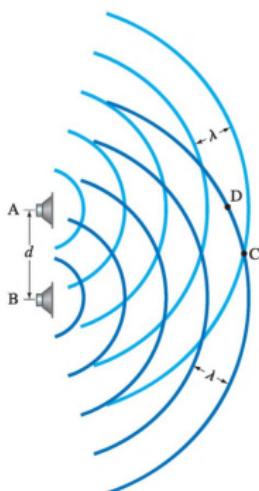
Normally, more than two overtones are present. [Any complex wave can be analyzed into a superposition of sinusoidal waves of appropriate amplitudes, wavelengths, and frequencies (Fourier)]

Sound spectra for different instruments



Spacial interference

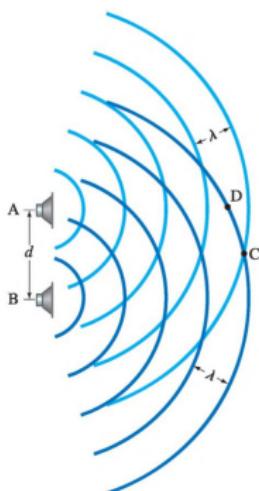
When two waves, with the same frequency, simultaneously pass through the same region of space, they interfere with one another. Interference also occurs with sound waves.



- ▶ Point C (same distance from each speaker)
 \rightarrow loud sound (constructive interference).

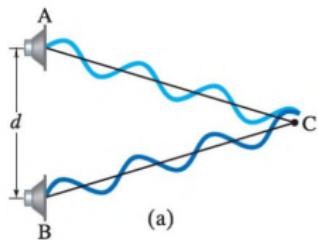
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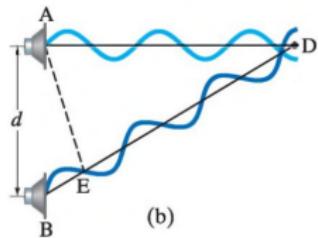


- ▶ Point C (same distance from each speaker)
rightarrow loud sound (constructive interference).
- ▶ Point D, no sound or little sound (destructive interference).

Spacial interference

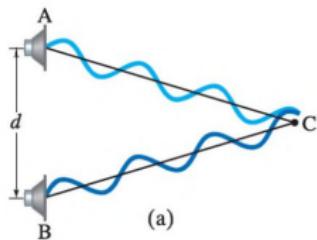


(a)



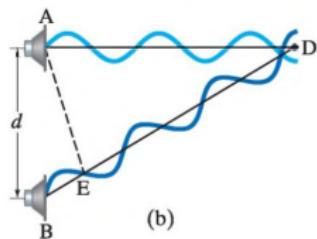
(b)

Spacial interference



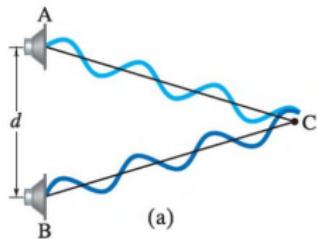
(a)

$ED = AD$, If $BE = \lambda/2$ the two waves will be exactly out of phase when they reach D

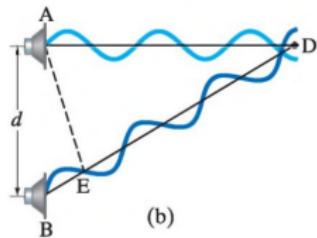


(b)

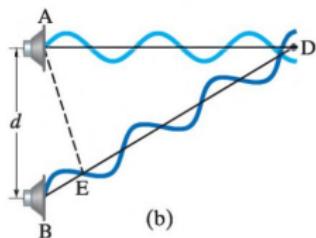
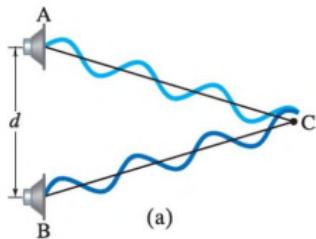
Spacial interference



$ED = AD$, If $BE = \lambda/2$ the two waves will be exactly out of phase when they reach D
→ destructive interference.



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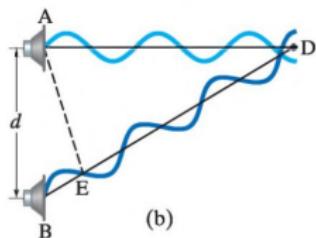
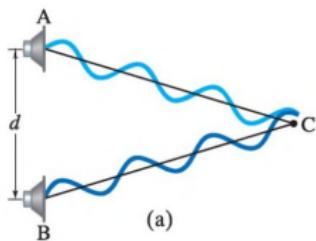


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- ▶ $BD - AD = 2\lambda, 3\lambda, \dots \rightarrow$ constructive interference
- ▶ $BD - AD =$
 $\lambda/2, 3\lambda/2, 5\lambda/2, \dots \rightarrow$ destructive interference

Example

Two loudspeakers are 1.00 m apart. A person stands 4.00 m from one speaker. How far must this person be from the second speaker to detect destructive interference when the speakers emit an 1150-Hz sound? Assume the temperature is 20°C .

Beats—Interference in Time

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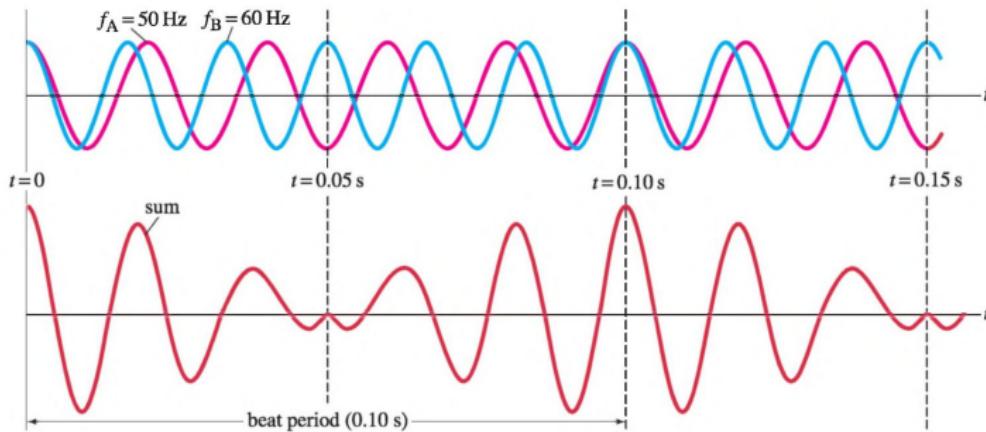
Beats—Interference in Time

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The sound level at a given position alternately rises and falls in time.

The regularly spaced intensity changes are called beats.

Beats, Interference in Time



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At a fixed point in space:

$$D_1 = A \sin(2\pi f_1 t)$$

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$$D = \left[2A\cos2\pi\left(\frac{f_1 - f_2}{2}\right)t \right] \sin2\pi\left(\frac{f_1 + f_2}{2}\right)t \quad (46)$$

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→ two beats occur per cycle. Then the beat frequency is $f_1 - f_2$.

Example

A tuning fork produces a steady 400 Hz tone. When this tuning fork is struck and held near a vibrating guitar string, twenty beats are counted in five seconds. What are the possible frequencies produced by the guitar string?

Doppler Effect

Change in Pitch when a source of sound is moving toward or moving away from the observer.

Doppler Effect

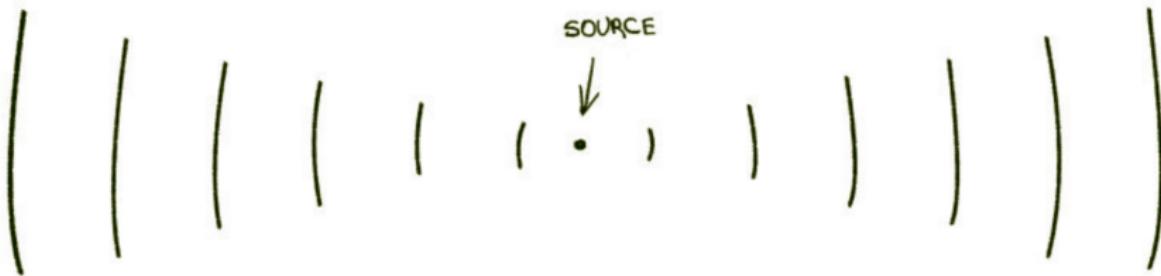
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Doppler Effect

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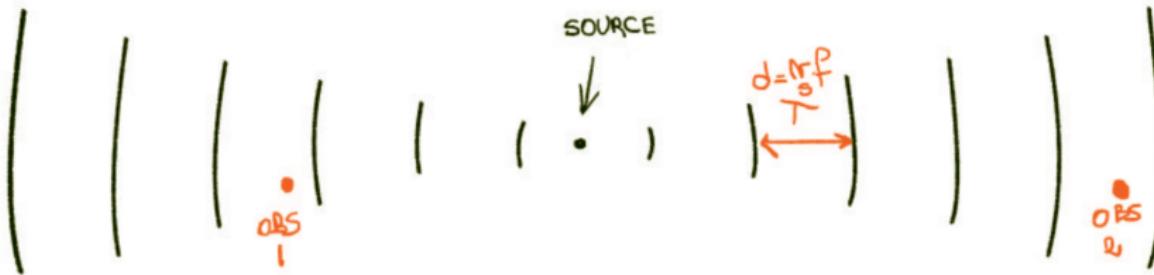
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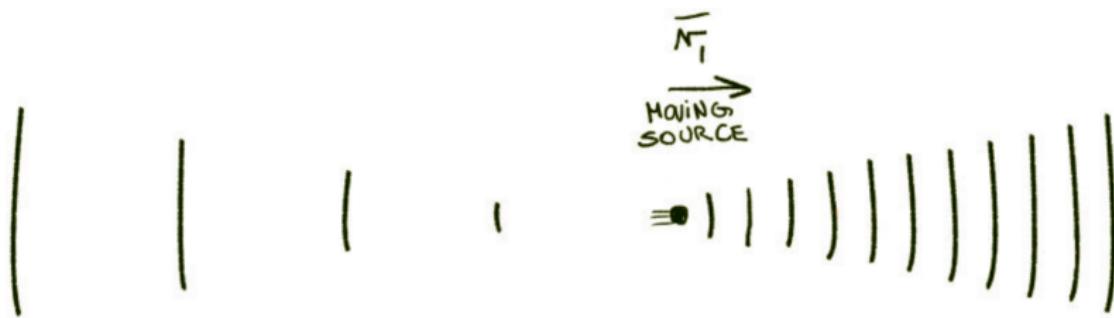


Doppler Effect

Source in motion → the wave travels at the velocity of sound in the air (its velocity is independent of the source velocity).

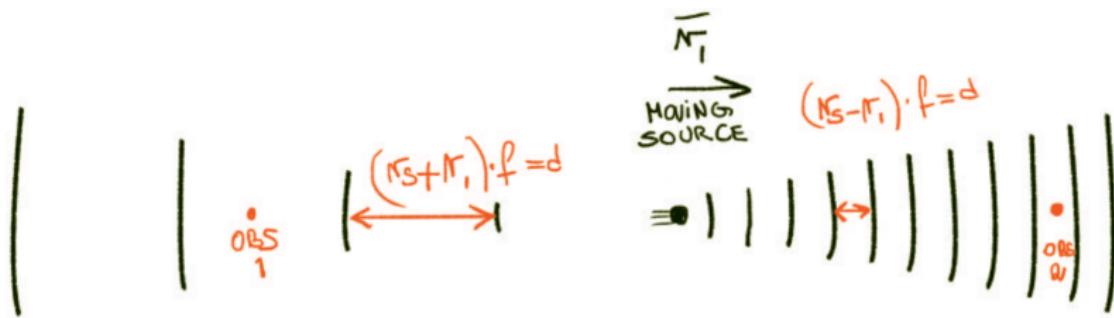
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Doppler Effect

The wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_1)f \quad (47)$$

Doppler Effect

Then, the wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_1)f = (v_{snd} - v_{source})\frac{\lambda}{v_{snd}} \quad (48)$$

Doppler Effect

Then, the wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_{source})f = (v_{snd} - v_{source})\frac{\lambda}{v_{snd}} = \lambda\left(1 - \frac{v_{source}}{v_{snd}}\right) \quad (49)$$

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Where we consider that the source velocity is lower than the sound velocity.

Doppler Effect

Then, the shift in wavelength is,

Doppler Effect

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$$\Delta\lambda = -\lambda \frac{v_{source}}{v_{snd}}, \quad (50)$$

Doppler Effect

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proportional to the source velocity.

Doppler Effect

The frequency perceived by the observer is,

$$f' = \frac{v_{snd}}{\lambda'} = \frac{v_{snd}}{\lambda \left(1 - \frac{v_{source}}{v_{snd}}\right)} \quad (51)$$

Doppler Effect

The frequency perceived by the observer is,

$$f' = \frac{f}{\left(1 - \frac{v_{source}}{v_{snd}}\right)} \quad (52)$$

The denominator is less than 1, then the observed frequency is greater than the source frequency.

Doppler Effect

If the source is moving away the observer,

$$\lambda' = (v_{snd} + v_{source})f = \lambda \left(1 + \frac{v_{source}}{v_{snd}}\right) \quad (53)$$

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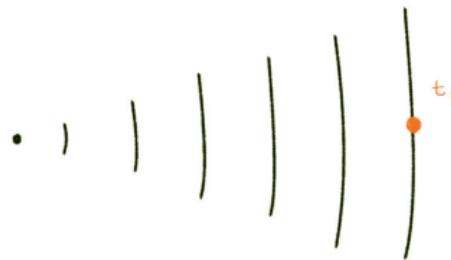
The observed frequency in this case is higher than the source frequency.

Doppler Effect

What happens if the source is at rest and the observer is moving toward the source?

Doppler Effect

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Doppler Effect

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Doppler Effect

What is the frequency perceived by the observer that is moving toward the source?

$$f' = \frac{v_{snd} + v_{obs}}{\lambda} \quad (55)$$

Doppler Effect

What is the frequency perceived by the observer that is moving toward the source?

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Doppler Effect

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Doppler Effect

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- ▶ Fixed source and a moving observer → λ is not changed, but the velocity of the crest respect to the observer changes.
- ▶ Moving source and fixed observer → λ changes, but the velocity of the crest respect to the observer does not change.

Doppler Effect

If the observer is moving away from the source, the velocity of the crests respect to the observer is decreased, and the frequency is,

$$f' = \frac{v_{snd} - v_{obs}}{\lambda} = f \left(1 - \frac{v_{obs}}{v_{sound}} \right) \quad (57)$$

Doppler Effect

When a sound wave is reflected from a moving obstacle, the frequency of the reflected wave will, because of the Doppler effect, be different from that of the incident wave.

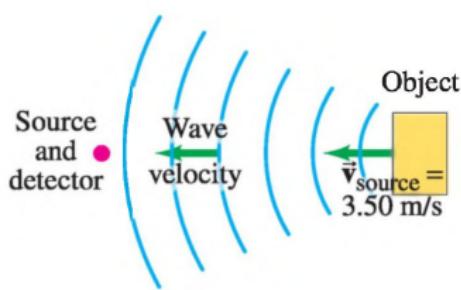
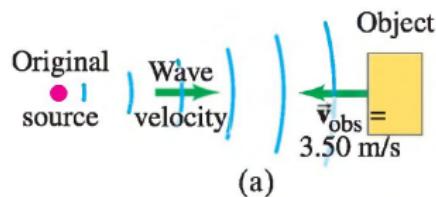
Doppler Effect

Example:

Two Doppler shifts. A 5000 Hz sound wave is emitted by a stationary source. This sound wave reflects from an object moving 3.50 m/s toward the source. What is the frequency of the wave reflected by the moving object as detected by a detector at rest near the source?

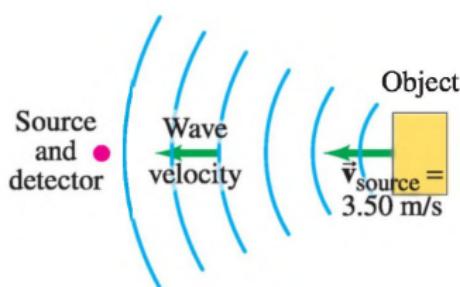
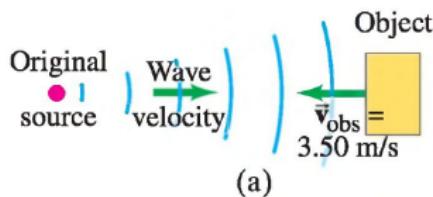
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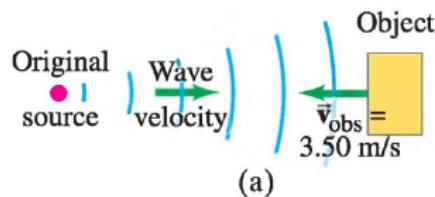


The frequency detected by the observer is:

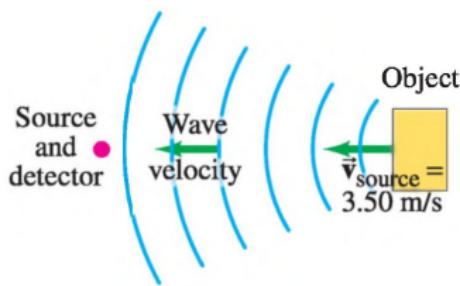
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Doppler Effect

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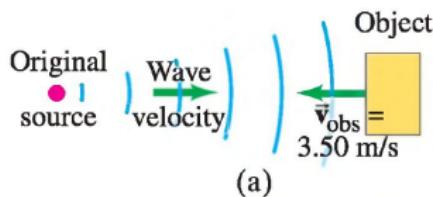


(a)

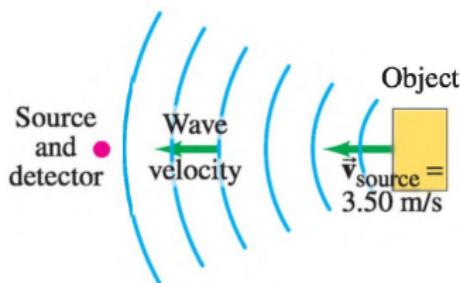


Doppler Effect

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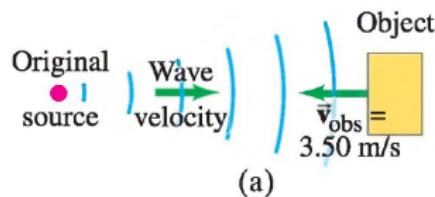
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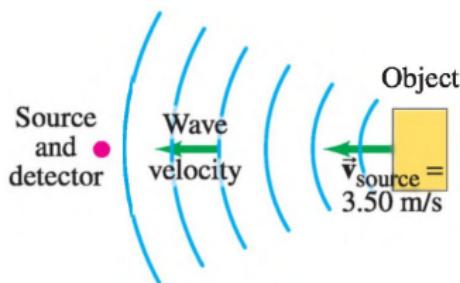
$$f' = f \left(1 + \frac{v_{obs}}{v_{sound}} \right) = 5051 \text{ Hz}$$

Doppler Effect

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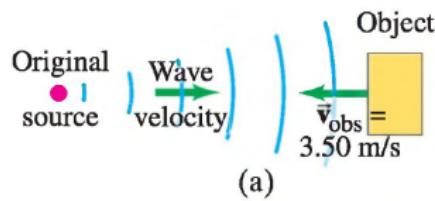
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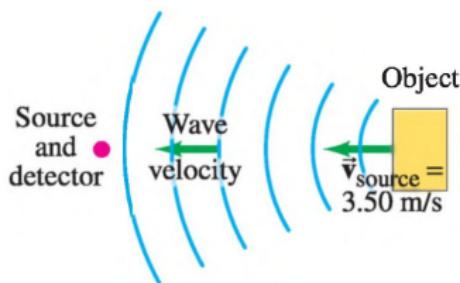
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Doppler Effect

Example:



The frequency emitted by the "new" source is,



$$f'' = \frac{f'}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)} = 5103 \text{ Hz}$$

Doppler Effect

We can summarize both effects in a single equation:

$$f' = f \left(1 + \frac{v_{obs}}{v_{snd}} \right)$$

$$f' = \frac{f}{\left(1 - \frac{v_{source}}{v_{snd}} \right)}$$

$$f' = \frac{f}{\left(1 - \frac{v_{source}}{v_{snd}} \right)} \left(1 + \frac{v_{obs}}{v_{snd}} \right) \quad (58)$$

Doppler Effect

Then, the frequency perceived by an observer when observer and source approach each other is,

$$f' = f \frac{v_{snd} + v_{obs}}{v_{snd} - v_{source}} \quad (59)$$

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$$f' = f \frac{v_{snd} + v_{obs}}{v_{snd} - v_{source}} \quad (59)$$

And the frequency perceived by an observer when observer and source move apart is,

$$f' = f \frac{v_{snd} - v_{obs}}{v_{snd} + v_{source}} \quad (60)$$

Doppler Effect

We can summarize all cases in a single equation:

$$f' = f \frac{v_{snd} \mp v_{obs}}{v_{snd} \mp v_{source}} \quad (61)$$

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The upper signs in numerator and denominator apply if source and/or observer move toward each other; the lower signs apply if they are moving apart.

Doppler Effect

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Doppler Effect

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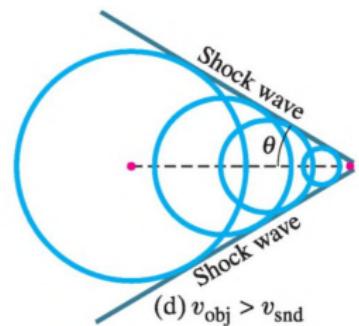
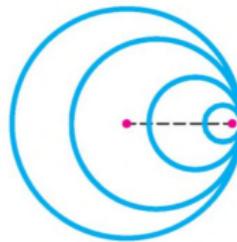
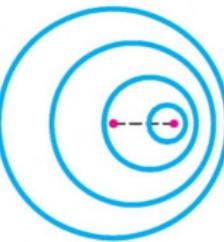
- ▶ measuring the beats frequency, we can obtain the value of the moving object.
- ▶ For example, ultrasonic waves reflected from red blood cells can be used to determine the velocity of blood flow.
- ▶ the technique can be used to detect the movement of the chest of a young fetus and to monitor its heartbeat.

Doppler Effect

What happens if the source travels at a velocity equal or higher than the sound velocity?

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A very clear explanation:

<https://www.youtube.com/watch?v=If-yK7sQE8Q>

Questions

- ▶ Two tuning forks oscillate with the same amplitude, but one has twice the frequency. Which (if either) produces the more intense sound?

Questions

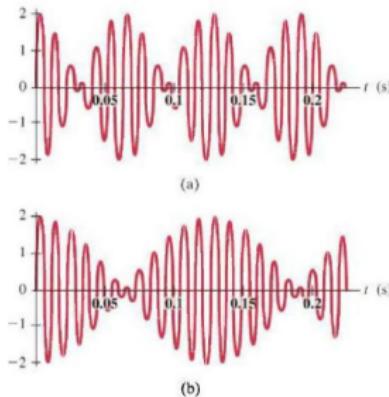
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Questions

- ▶ Two tuning forks oscillate with the same amplitude, but one has twice the frequency. Which (if either) produces the more intense sound?
- ▶ How will the air temperature in a room affect the pitch of organ pipes?
- ▶ Is there a Doppler shift if the source and observer move in the same direction, with the same velocity? Explain.

Questions

Consider the two waves shown in the figure. Each wave can be thought of as a superposition of two sound waves with slightly different frequencies. In which of the waves, (a) or (b), are the two component frequencies farther apart? Explain.



Questions

The figure shows various positions of a child on a swing moving toward a person on the ground who is blowing a whistle. At which position, A through E, will the child hear the highest frequency for the sound of the whistle? Explain your reasoning.

