PHY250

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Fall 2020

Fluids

Previous Chapters $\rightarrow$ Solid Objects, they maintain shape except for small amount of elastic deformation.

Now we are going to shifht our attention to materials that are very deformable and can flow.

We are going to examine both, Static Fluids and Dynamic Fluids.

Fluids

Ideal Gases,

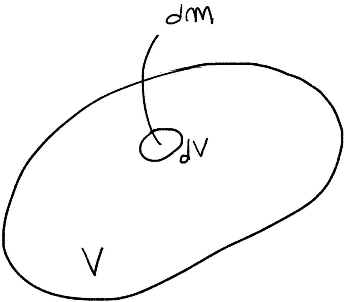
- They expand to occupy the whole volume of the container.
- they are compressible.
- Formed by N point particles that move in straight lines.
- No interaction between particles
- Elastic collisions, the kinetic energy is conserved.
- The mean kinetic energy is proportional to the temperature.
- There is no interaction between the particles.

Fluids

Liquids

- They take the shape of it container.
- They are not compressible (idealization).
- Their density is constant.
- There is interaction between the molecules $\rightarrow$ viscosity, superficial tension.

Density



$$\rho = \frac{dm}{dV} \qquad (1)$$

$$\rho = constant \rightarrow \rho = \frac{m}{V}$$

Then, the weight of an object is,

$$w = mg = \rho V g$$

Pressure

We define the pressure as,

$$P = \frac{F}{dA} \tag{2}$$

It is an scalar, its units are

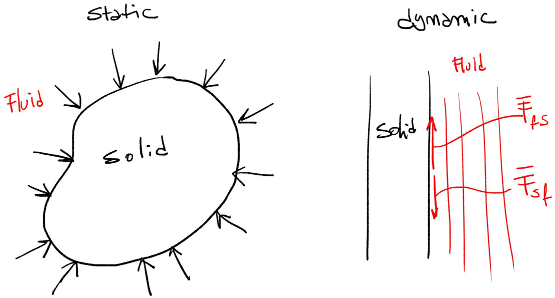
$$[P] = \frac{N}{m^2} = Pa \tag{3}$$

At sea level $\rightarrow P_0 = 1.013 \times 10^5 \frac{N}{m^2} = 1atm$

In a non-moving fluid, the pressure is the same in every direction at a given depth.

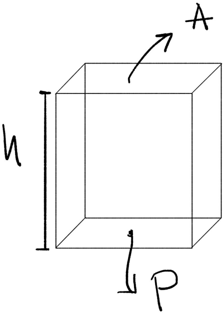
Pressure

In a fluid at rest, the force is perpendicular to any solid surface, otherwise the fluid would flow.



Pressure

How the pressure varies with depth h in a liquid?



P does not depends on A.

If the density is constant, the pressure at depth h due to the liquid itself is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho g V g}{A} \qquad (4)$$

$$V = Ah \rightarrow P = \rho g h \qquad (5)$$

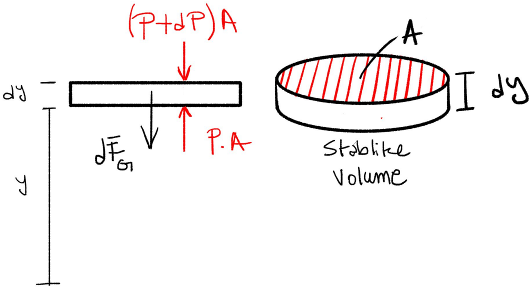
Pressure

What if

- There is additional pressure exerted at the surface?
- The density varies with depth?

Pressure

General Case



$$dF_G = (dm)g = \rho g dV = \rho g A dy \tag{6}$$

$$\sum F_y = 0 \rightarrow PA - (P + dP)A = \rho g A dy \tag{7}$$

Then,

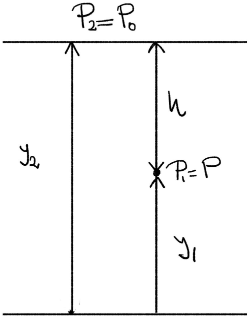
$$\frac{dP}{dy} = -\rho g \tag{9}$$

$\rightarrow$ The pressure decreases when y increases

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g dy \tag{10}$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g dy, \quad \rho = \rho(y) \tag{11}$$

If $\rho = constant \rightarrow \Delta P = -\rho g \Delta y$.
It is convenient to measure h from the surface,



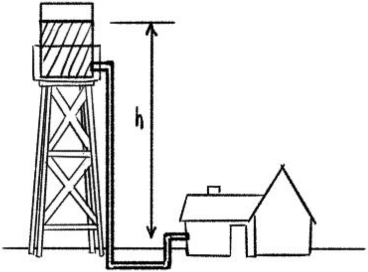
$$P_2 - P_1 = -\rho g(y_2 - y_1) \quad (12)$$

$$P_0 - P = -\rho g h \rightarrow P = P_0 + \rho g h \quad (13)$$

where P_0 is the pressure due to the atmosphere above.

Example

Pressure at a faucet The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.



Example

Force on aquarium window. Calculate the force due to water pressure exerted on a 1.0 m X 3.0 m aquarium viewing window whose top edge is 1.0 m below the water surface

Atmospheric Pressure

For an ideal gas $\rightarrow \rho \propto P$, then the relation between ρ and P is,

$$\frac{\rho}{\rho_0} = \frac{P}{P_0} \tag{14}$$

where ρ_0 and P_0 are the density and pressure at sea level.

$$\frac{dP}{dy} = -\rho g = -\frac{\rho_0 P}{P_0} = -P\frac{\rho_0}{P_0}g \tag{15}$$

$$\rightarrow \frac{dP}{P} = -(\frac{\rho_0}{P_0})gdy \tag{16}$$

$$\rightarrow ln(P) - ln(P_0) = ln(\frac{P}{P_0}) = -(\frac{\rho_0}{P_0})gy \tag{17}$$

$$\rightarrow P = P_0 e^{-\frac{\rho_0}{P_0}gy} \tag{18}$$

Example

At what elevation is the air pressure equal to half the pressure at sea level?

$$\frac{P_0}{2} = P_0 e^{-\frac{\rho_0}{P_0} gy}$$

$$\rightarrow ln(\frac{1}{2}) = ln(e^{-\frac{\rho_0}{P_0} gy})$$

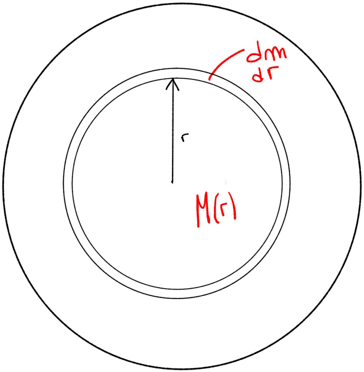
$$\rightarrow -ln(2) = -\frac{\rho_0}{P_0} gy$$

$$\frac{\rho_0}{P_0} g = 1.25 \times 10^{-4} m^{-1}$$

$$\rightarrow y = 5550m$$

Example

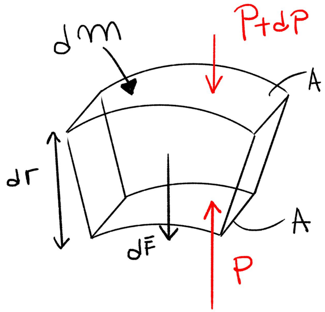
Star Model



Example

Star Model

Hydrostatic equilibrium, $\rightarrow \sum F_r = PA - (P + dP)A - dF = 0$



$$dF = G\frac{dmM(r)}{r^2} = G\frac{\rho dvM(r)}{r^2}$$

$$\rightarrow AdP = -G\frac{\rho AdrM(r)}{r^2}$$

$$\rightarrow \frac{dP}{dr} = -G\frac{\rho M(r)}{r^2} \qquad (19)$$

Example

The equation for the mass is,

$$dM(r) = \rho(r)dV = \rho(r)4\pi r^2 dr \tag{20}$$

$$\rightarrow \frac{dM}{dr} = \rho(r)4\pi r^2 \tag{21}$$

Example

Then, we have to solve a system of 2 differential equations,

$$\rightarrow \frac{dP}{dr} = -G\frac{\rho M(r)}{r^2} \tag{22}$$

$$\rightarrow \frac{dM}{dr} = \rho(r)4\pi r^2 \tag{23}$$

We need to integrate those equations for $r = 0 \rightarrow R$, we need a relation between the pressure and the density, the Equation of State (EoS) os the star,

$$P = P(\rho) \tag{24}$$

Example

To find the EoS, we need to model the mater inside the different depth of the star. Once we have modeled the mater inside the star, and find $P = P(\rho)$, we can solve the system,

$$\rightarrow \frac{dP(\rho)}{dr} = -G\frac{\rho M(r)}{r^2} \tag{25}$$

$$\rightarrow \frac{dM}{dr} = \rho(r)4\pi r^2 \tag{26}$$

With the conditions,

$$\rightarrow \rho(r=0) = \rho_0 \tag{27}$$

$$\rightarrow \rho(r=R) = 0 \leftarrow \textit{defines the radius of the Star} \tag{28}$$

Example

Then,

- To model the structure of a Star, we need to find a microscopic model for the matter inside the star (an EoS).
- We need the central density of the star
- The mass and Radius of the star depends on the EoS

The pressure is a function of the EoS, and for certain conditions it may not be sufficient to withstand the gravitational attraction. Thus the structure equations imply there is a maximum mass that a star can have

Pascal's Principle

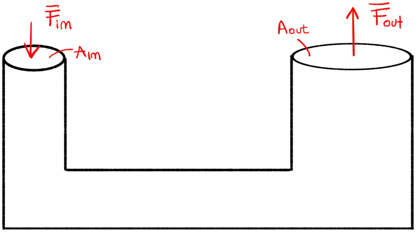
If an external pressure P_0 is applied to a confined fluid, the pressure at every point within the fluid increases by that amount P_0.

This means that an external pressure acting on a fluid is transmitted throughout the fluid.

$$P = \rho g h + P_0 \tag{29}$$

Example

Hydrostatic Lift



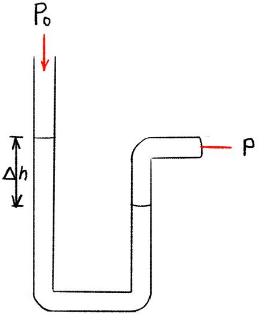
$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \tag{30}$$

$$\rightarrow F_{out} = \frac{A_{out}}{A_{in}} F_{in} \tag{31}$$

Manometer

The simplest device to measure the pressure is the open-tube Manometer, the Pressure is related to the difference Δh between the two levels of the liquid.

$$P = P_0 + \rho g \Delta h$$

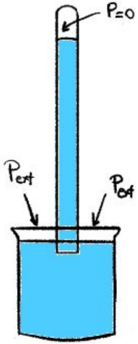


Manometer

Instead of calculating the product $\rho g \Delta h$, sometimes only the change in height Δh is specified. In fact, pressures are sometimes specified as so many "millimetres of mercury" (mm-Hg) or "mm of water" (mm-H20).

Barometer

A barometer is a glass tube completely filled with mercury and then inverted into a bowl of mercury. If the tube is long enough, the level of the mercury will drop, leaving a vacuum at the top of the tube.



$$P_{ext} = \rho g \Delta h + 0$$

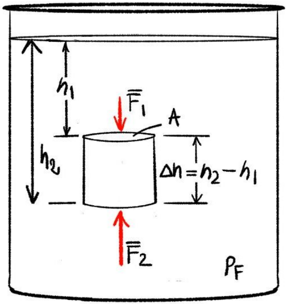
Barometer

When $P_{ext} = P_0$, $\Delta h = 0.76cm$. That is, the atmospheric pressure can support a column of mercury only about 76 cm high.

If we replace the liquid by water, the column high would be $\sim 10m$

Arquimede's Principle

Consider a cylinder immersed in a liquid, the upward force exerted by the liquid is the **Buoyant Force**

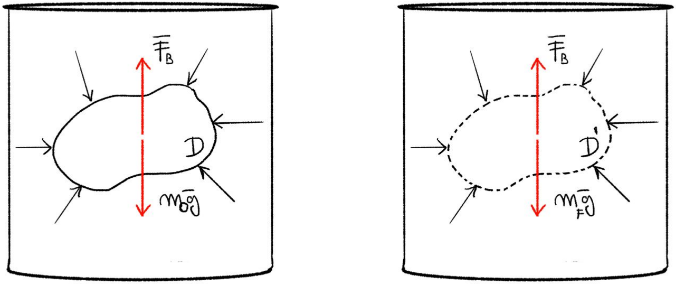


$$\begin{aligned} F_B &= F_2 - F_1 \\ &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \Delta h \\ &= m_f g \end{aligned}$$

This result means that: *the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.*

Arquimede's Principle in general

For any irregular body...



Two pails of water. Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

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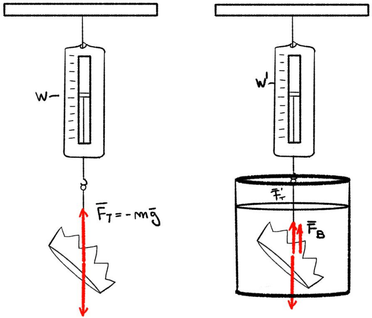
Answer Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood object; so the pails have the same weight.

Example

Recovering a submerged statue. A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0X10^4 cm^3$. How much force is needed to lift it?

Example

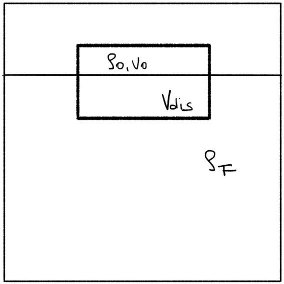
Archimedes: Is the crown gold? When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?



Example

Floating objects

In this case, $F_B > mg$, then, the object is in equilibrium when

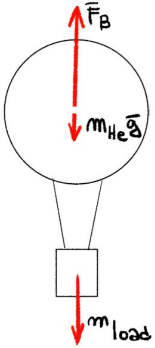


$$\begin{aligned} m_F g &= m_o g \\ \rho_F V_{dis} g &= \rho_O V_O g \\ \rightarrow \frac{V_{dis}}{V_O} &= \frac{\rho_O}{\rho_F} \end{aligned}$$

Where V_{dis}/V_O is the fraction of submerged Vol.

Example

Helium balloon. What volume V of helium is needed if a balloon is to lift a load of 180 *kg* (including the weight of the empty balloon)?



Summarizing Static Fluids

- The pressure on any object is perpendicular to the surface.
- If the only external force is the gravity, near earth, we have $\frac{dP}{dy} = -\rho g$
- the Arquimede's Principle is a consequence of the previous 2 items.

Generalization

If the external force is the gravity,

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Which is the equivalent expression for an arbitrary force?

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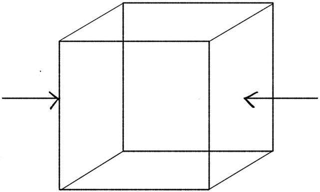
Which is the equivalent expression for an arbitrary force?

In general,

$$\vec{F} = \vec{F}(x, y, z), \quad P = P(x, y, z) \tag{32}$$

Generalization

The resultant force on the x-direction due to the pressure of the liquid is,



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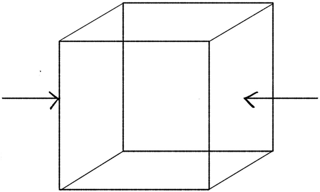

$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

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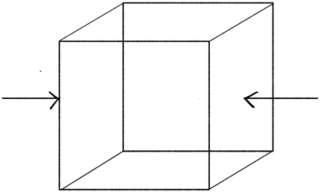

$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

and,

$$dP_x = \frac{\partial P}{\partial x} dx$$

Generalization

Then,

$$F_x = -\frac{\partial P}{\partial x} dxdydz$$

or,

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where f_x is the force per unit volume.

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The fluid is in hydrostatic equilibrium if,

$$f_x + f_x^e = 0$$

Generalization

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$$f_x^e = -\frac{\partial U}{\partial x} \rightarrow f_x^e = -\rho\frac{\partial \phi}{\partial x}$$

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Generalization

Then, for the 3 spacial directions...

$$-\frac{\partial P}{\partial x} = \rho\frac{\partial \phi}{\partial x}$$
$$-\frac{\partial P}{\partial y} = \rho\frac{\partial \phi}{\partial y}$$
$$-\frac{\partial P}{\partial z} = \rho\frac{\partial \phi}{\partial z}$$

Generalization

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$$
\begin{aligned}
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-\frac{\partial P}{\partial z} &= \rho\frac{\partial \phi}{\partial z}
\end{aligned}
$$

Using the nabla operator,

$$\nabla P = -\rho\nabla\phi \tag{33}$$

Summarizing Static Fluids

Then, the most general expression for hydrostatic equilibrium is,

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Another possibility which allows hydrostatic equilibrium is when $\rho = \rho(P)$.

Fluids in motion

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We can distinguish two main types of fluid flow:

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Fluids in motion

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If the density varies arbitrarily $\rightarrow$ Dynamic Fluid

We can distinguish two main types of fluid flow:

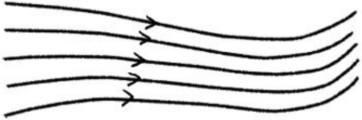
- Laminar Flow: Smooth flow $\rightarrow$ the neighbouring layers of the fluid slide by each other smoothly
- Turbulent Flow: Above a certain speed, the flow is characterized by erratic, small, whirlpool-like circles called eddy currents or eddies

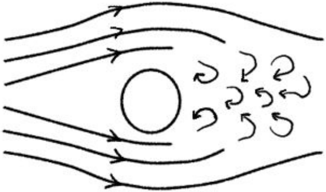
https://www.youtube.com/watch?v=WG-YCpAGgQQ

Fluids in motion

In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another.

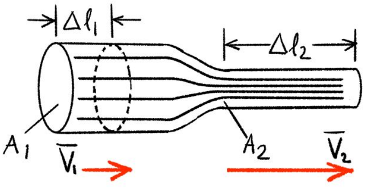
In a turbulent Flow, eddies absorb a great deal of energy, and although a certain amount of internal friction called viscosity is present even during streamline flow, it is much greater when the flow is turbulent.





Fluids in motion

Definition of Mass Flow Rate $\rightarrow \frac{\Delta m}{\Delta t}$



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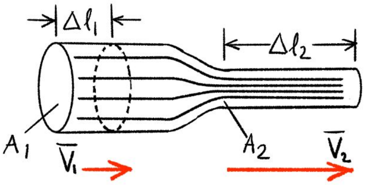
Volume of fluid passing point 1 in time $\Delta t \rightarrow A_1 \Delta l_1$,



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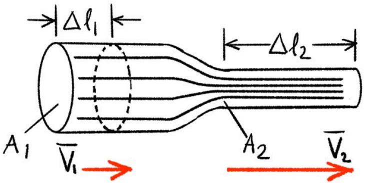

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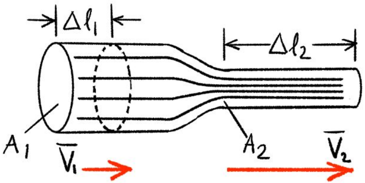

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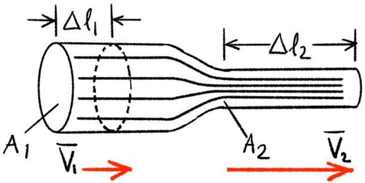

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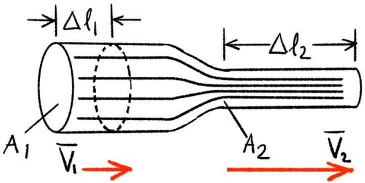

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thought A_2 :

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incompressible, $\rho_2 = \rho_1$,

$$A_1 v_1 = A_2 v_2 \tag{37}$$

Since no fluid flows in or out the sides, the flow rates through A_1 and A_2 must be equal, $\Delta m_1/\Delta t = \Delta m_2/\Delta t$. Then,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \tag{36}$$

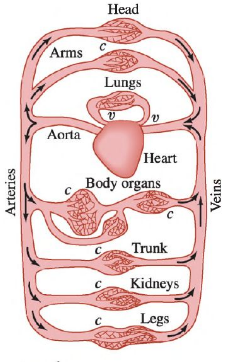
This is called the **equation of continuity**. If the fluid is

incompressible, $\rho_2 = \rho_1$,

$$A_1 v_1 = A_2 v_2 \tag{37}$$

Av is the *volume rate of flow*

Blood flow. In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about $1.2cm$, and the blood passing through it has a speed of about $40cm/s$. A typical capillary has a radius of about $4 \times 10^{-4}cm$, and blood flows through it at a speed of about $5 \times 10^{-4}m/s$.



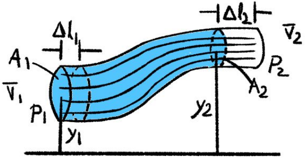
Estimate the number of capillaries that are in the body.

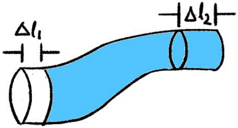
Bernoulli's Principle

Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

To proof this we are going to assume,

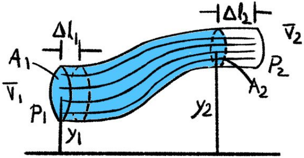
- The flow is steady and laminar
- The fluid is incompressible
- The viscosity is small enough to be ignored
- The fluid is flowing in a tube of variable cross section

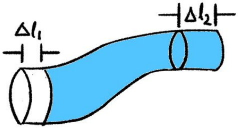




The work done at A_1 is,

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1 \tag{38}$$





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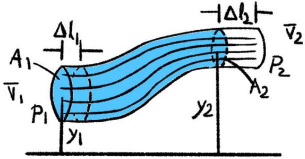

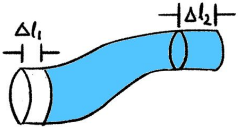

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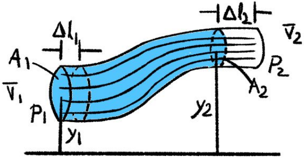

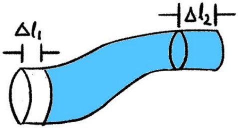

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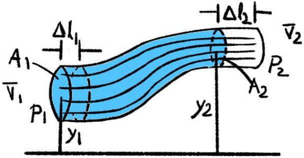

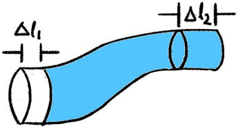

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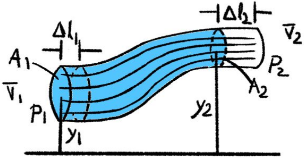

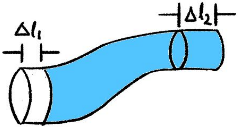

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$$\begin{aligned} W &= W_1 + W_2 + W_3 \\ &= P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg(y_2 - y_1) \end{aligned}$$

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$$\frac{1}{2}\rho v^2 + P + \rho g y = constant \tag{41}$$

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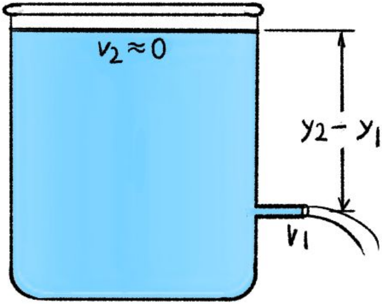
Bernoulli's equation is an expression of the law of energy

conservation, since we derived it from the work-energy principle.

Example

Calculate the velocity, v_1, of a liquid flowing out of a spigot at the bottom of a reservoir.



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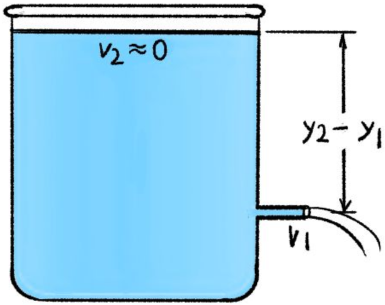
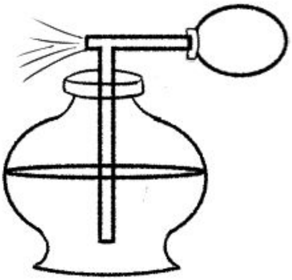


The liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height.

Example

Perfume Atomizer



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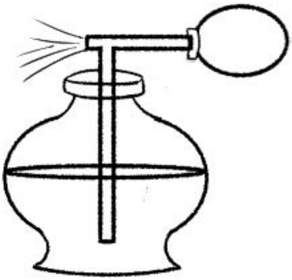
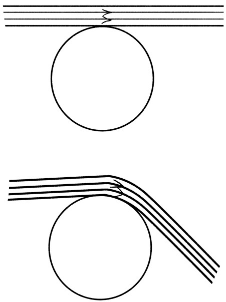


The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top.

Example

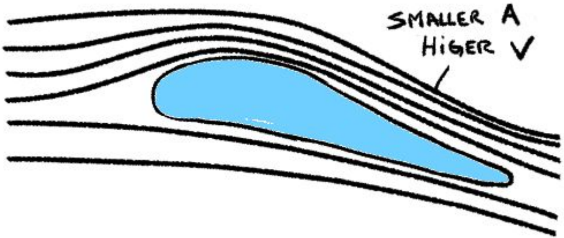
Coanda effect



https://www.youtube.com/watch?v=NvzXKZNJ7ZU&t=4s

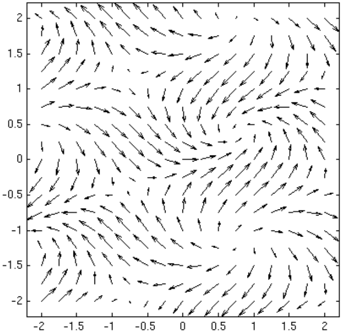
Example

Airplane Wings and Dynamic Lift



Generalization

To fully describe a fluid, we need the velocity vector field, that is $\vec{v}(x, y, z, t)$.



→ https://www.youtube.com/watch?v=VJ2ZDLQk3IQ

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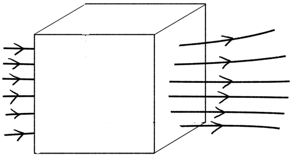

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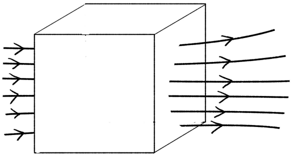

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$$\rho v_x dydz$$

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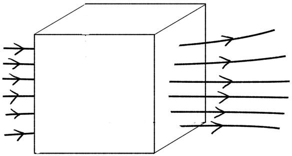

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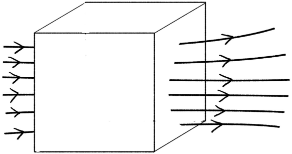

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The difference between the mass flow rate entering through A_{X2} and A_{X1} is,

$$\rho \frac{\partial v_x}{\partial x} dydzdx$$

Generalization

If we do the same for y and z,

$$\rho\frac{\partial v_y}{\partial y}dydzdx$$

$$\rho\frac{\partial v_z}{\partial z}dydzdx$$

Then, the difference between the total flow entering and exiting the volume is

$$\rho(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z})dydzdx$$

Generalization

If the entering flux is equatl to the exiting flux

$$\rho(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z})dydzdx = 0$$

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or using the nabla operator,

$$\nabla \cdot \vec{v} = 0$$

Generalization

If the density is not constant,

$$\nabla \cdot (\rho \vec{v}) = 0$$

If the entering flux is not equal to exiting flux,

$$\nabla \cdot (\rho \vec{v}) = -\frac{\partial \rho}{\partial t}$$

This is the Hydrodynamic Equation of Continuity.

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We suppose that the liquid is "thin" the viscosity is unimportant, so we will omit f_{vis} $\rightarrow$ *Dry Water Flow*

What is the acceleration?

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To obtain the acceleration we must calculate,

$$\frac{\Delta \vec{v}}{\Delta t}$$

for an actual displacement of an element of fluid. That is,

$$\frac{\Delta \vec{v}}{\Delta t} = v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}$$

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This equation can be expressed as,

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$\rightarrow$ `https://mathinsight.org/curl_idea`

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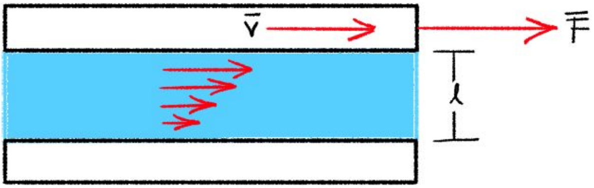
The origin of the viscosity is,

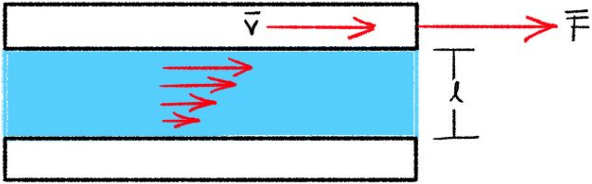
- In fluids, electrical cohesive forces between the molecules.
- In gases, collisions between the molecules

The viscosity can be expressed quantitatively by a *coefficient of viscosity* η

Viscosity



Viscosity



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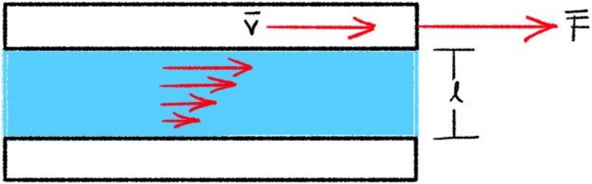
Viscosity



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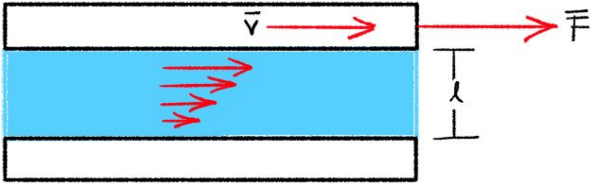
Viscosity



$$F =\propto A\frac{v}{\ell} \tag{43}$$

$$F = \eta A\frac{v}{\ell} \rightarrow \eta = \frac{F\ell}{Av} \tag{44}$$

The unit of η is $N \cdot s/m^2$

Viscosity

TABLE 13–3
Coefficients of Viscosity

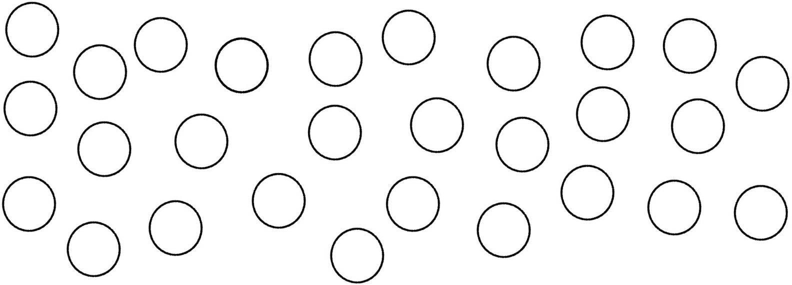
Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s)[†]
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

[†] 1 Pa·s = 10 P = 1000 cP.

Surface Tension and Capillarity

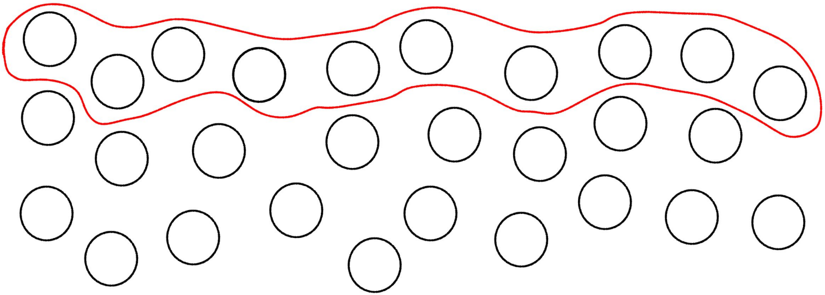
Surface Tension and Capillarity

The particles that make up the liquid are in constant random motion.



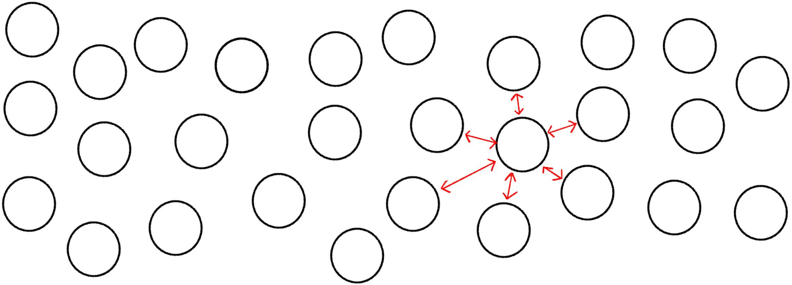
Surface Tension and Capillarity

Do the particles at the surface form a random surface?



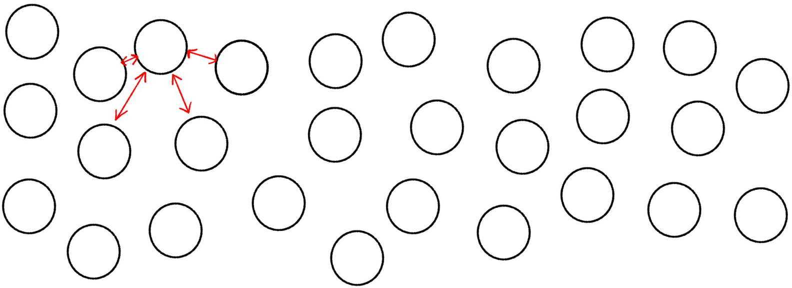
Surface Tension and Capillarity

Intermolecular attractions influence the surface.



Surface Tension and Capillarity

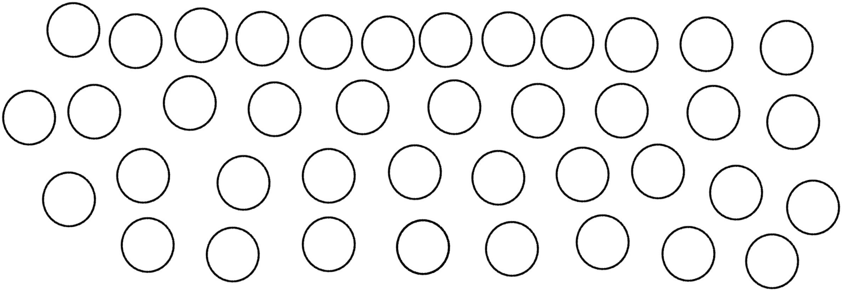
Intermolecular attractions influence the surface.



Surface Tension and Capillarity

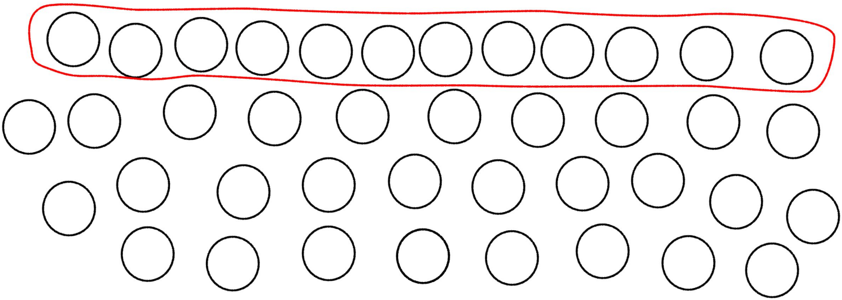
Surface molecules form a much smoother surface.

Surface Molecules are compressed $\rightarrow$ Higher energy at the surface.



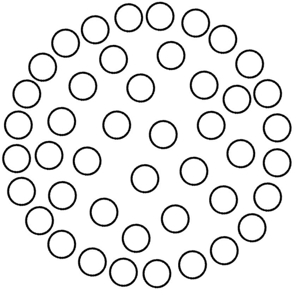
Surface Tension and Capillarity

Liquid surface $\leftrightarrow$ Stretched membrane under tension



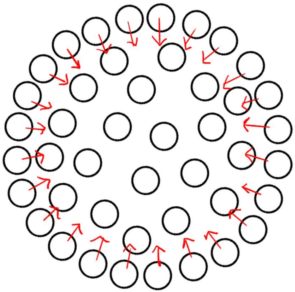
Surface Tension and Capillarity

Liquid not confined in a container.



Surface Tension and Capillarity

Liquid not confined in a container.



Surface Tension and Capillarity

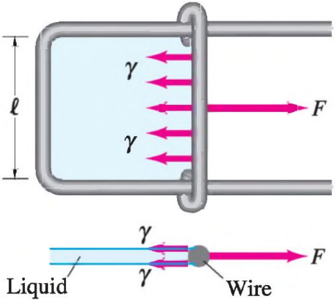
Liquid surface $\leftrightarrow$ Stretched membrane under tension

Surface Tension

Force per unit lenght that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\gamma = \frac{F}{\ell} \tag{45}$$

Surface Tension and Capillarity



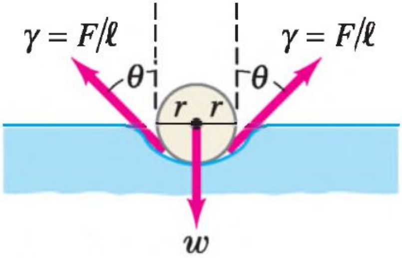
$$2\gamma\ell = F \rightarrow \gamma = \frac{F}{2\ell}$$

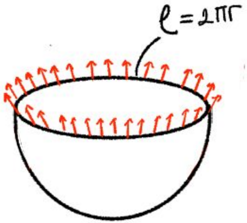
TABLE 13–4
Surface Tension of Some Substances

Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059

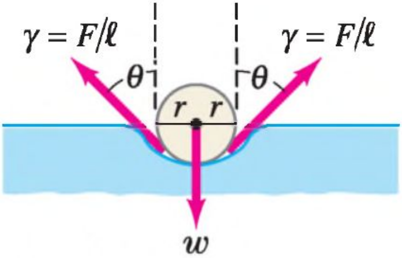
Surface Tension and Capillarity

Insect walks on water. The base of an insect's leg is approximately spherical in shape, with a radius of about $2.0 \times 10^{-5} m$. The 0.0030 g mass of the insect is supported equally by its six legs. Estimate the angle for an insect on the surface of water. Assume the water temperature is 20°C.



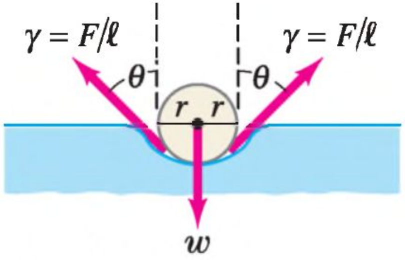


Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

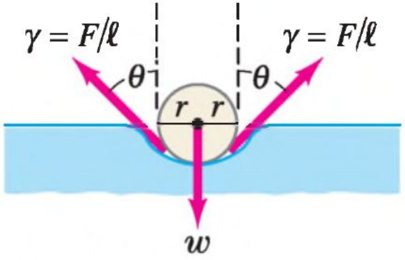
Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

Surface Tension and Capillarity

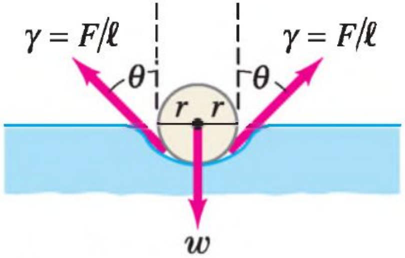


$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

Surface Tension and Capillarity



$$F \simeq (\gamma cos\theta)\ell = 2\pi r\gamma cos\theta$$

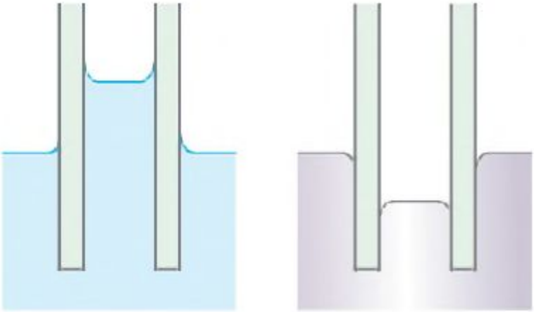
$$2\pi r\gamma cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow cos\theta \simeq \frac{1}{6}\frac{mg}{2\pi r\gamma}$$

$$\rightarrow cos\theta \simeq 0.54 \rightarrow \theta \simeq 57$$

Surface Tension and Capillarity

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called capillarity



Surface Tension and Capillarity

Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. Cohesion refers to the force between molecules of the same type, whereas adhesion refers to the force between molecules of different types.