

# PHY250: FLUIDS IN MOTION

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## Fluids Dynamic

Equation of continuity

Bernoulli's Equation

viscosity

Generalization

# Fluids in motion

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$$\rightarrow \nabla P + \rho \nabla \phi = \rho \vec{a}$$

*Dynamic Fluid*

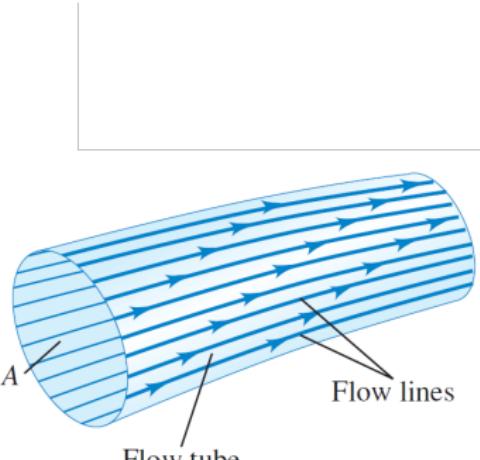
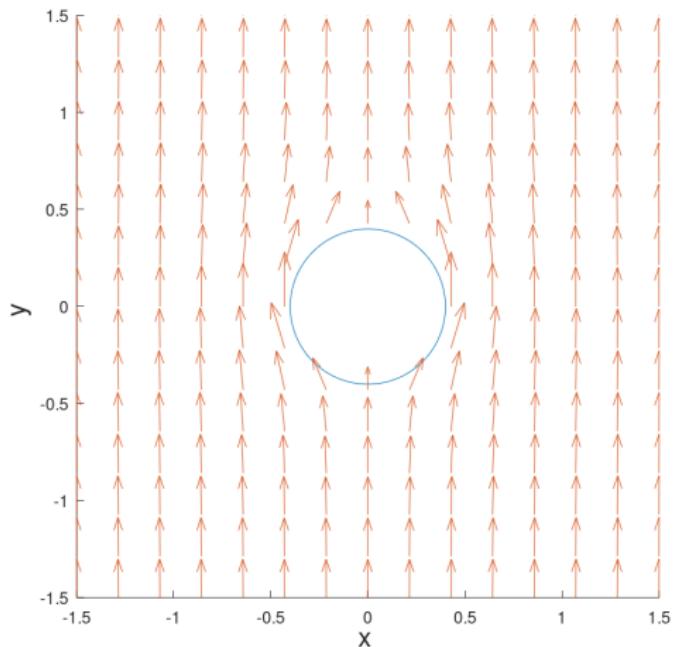
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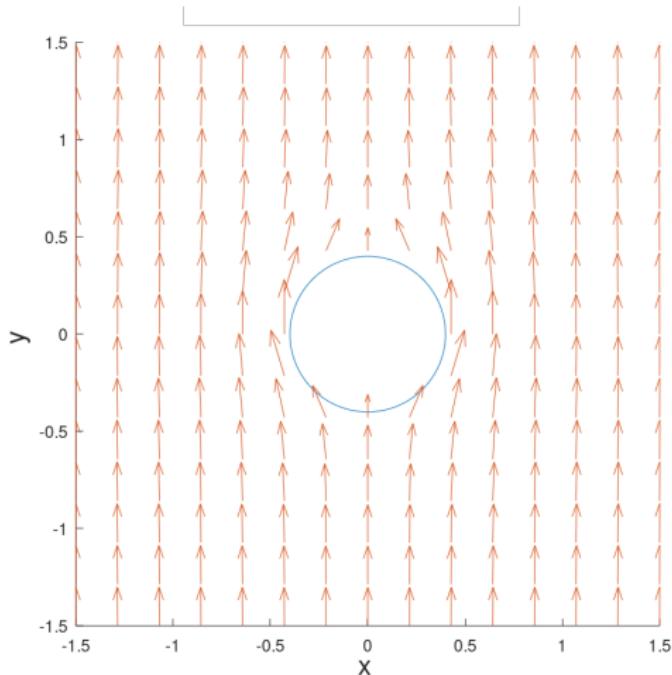
To describe the motion of a fluid, we need:

- ▶ An equation of state:  $P(\rho)$
- ▶ The velocity at every point



SEARS AND ZEMANSKY'S UNIVERSITY PHYSICS 13TH EDITION.

Streamlines: lines which are always tangent to the fluid velocity.



Obs: two streamlines can not cross each other. Why?

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- ▶ **Steady Flow:** at any one place in the fluid the velocity never changes → the streamlines are the fluid particles path.

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- ▶ **Non-Steady Flow:** At a given point, the velocity changes with time.

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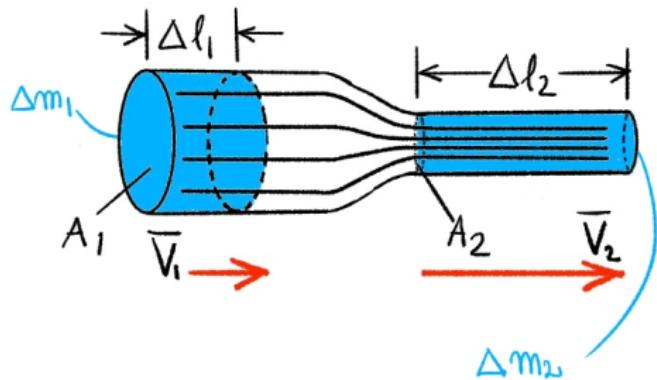
- ▶ The velocity of the flow << speed of sound in the Fluid.
- ▶  $\rho = \text{constant}$  (incompressible fluid)
- ▶ steady flow.

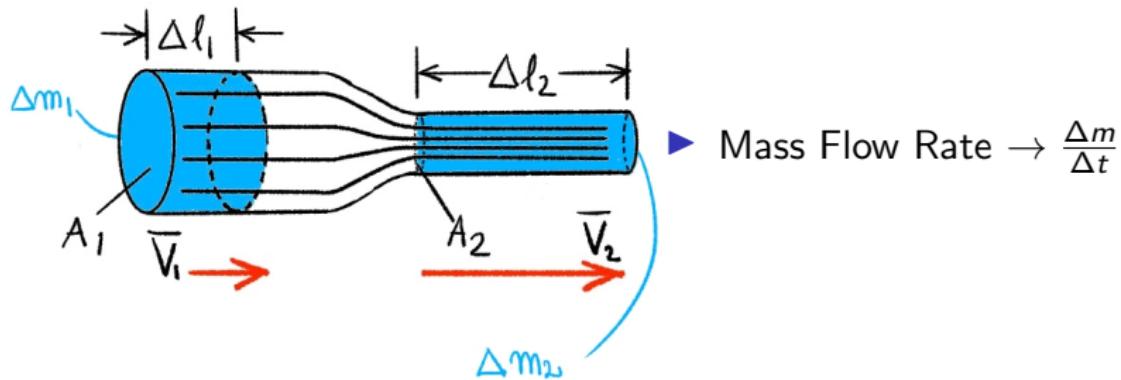
Steady flow:

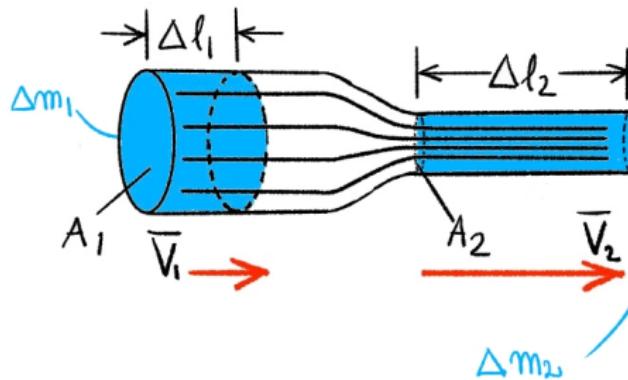
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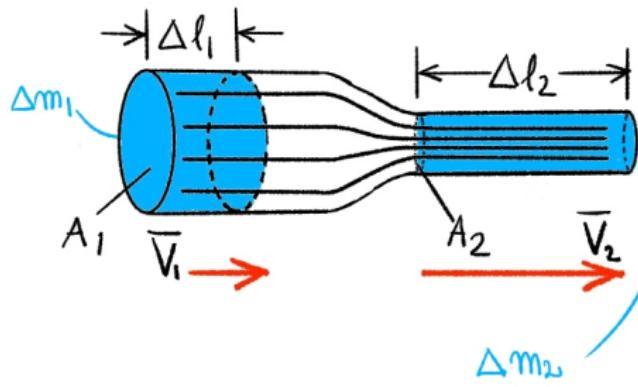
- ▶ At any one place in the fluid the velocity never changes.
- ▶ Fluid element moves in lines which are always tangent to the fluid velocity (streamlines).





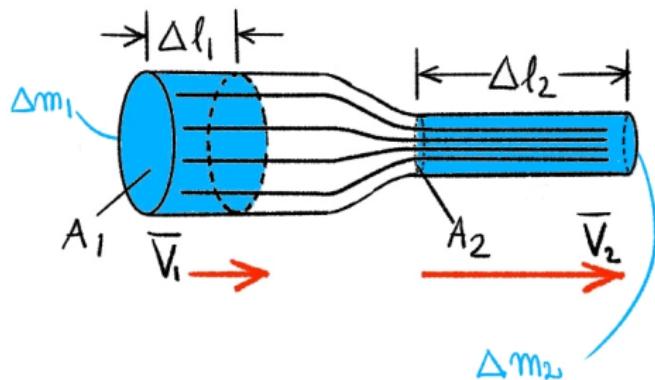


- ▶ Mass Flow Rate  $\rightarrow \frac{\Delta m}{\Delta t}$
- ▶ no fluid flows in or out the sides  $\rightarrow \Delta m_1 = \Delta m_2$

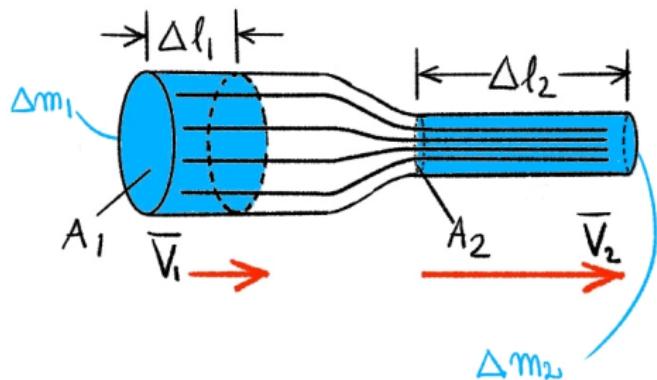


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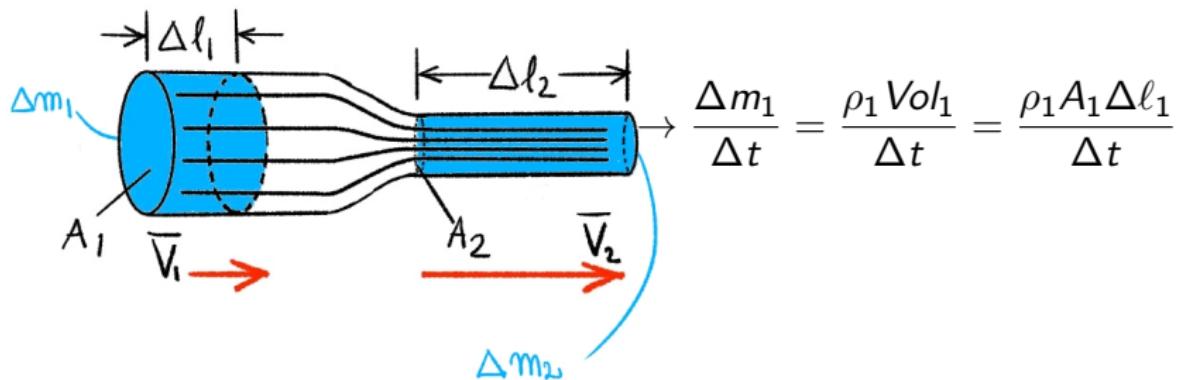
$$\rightarrow \frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \quad (1)$$



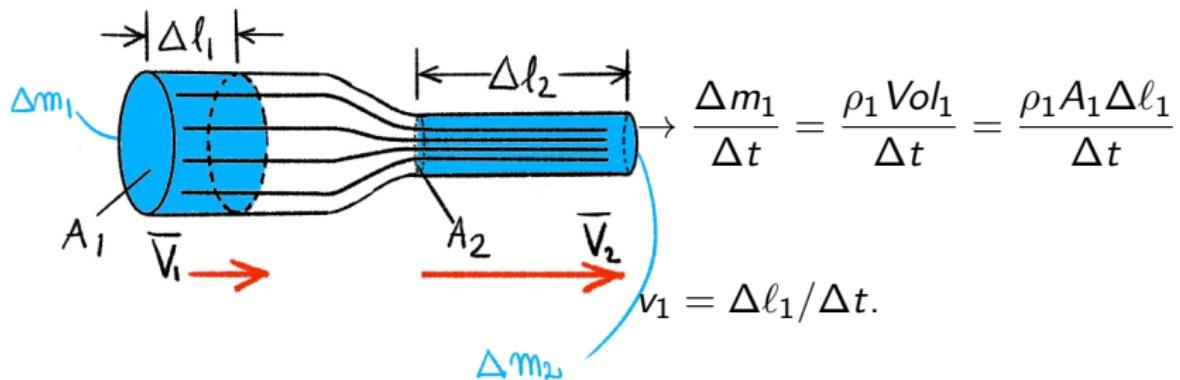
$$Vol_1 = A_1 \Delta l_1,$$



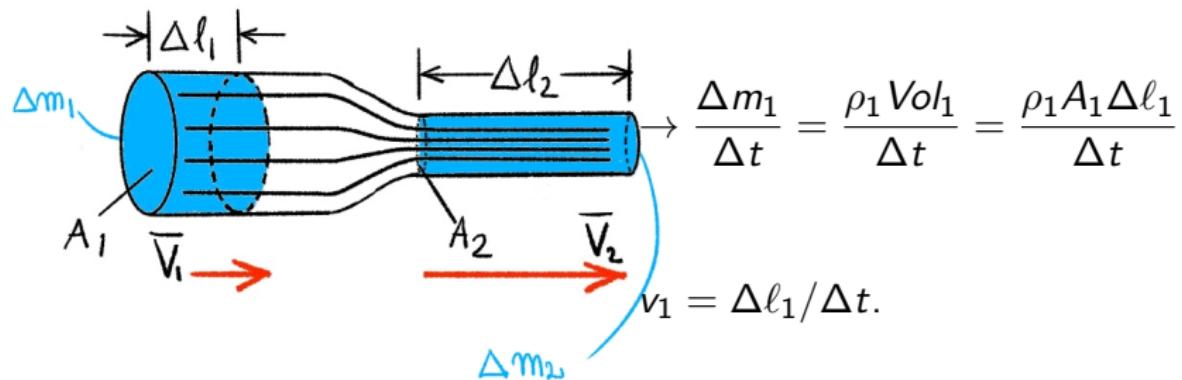
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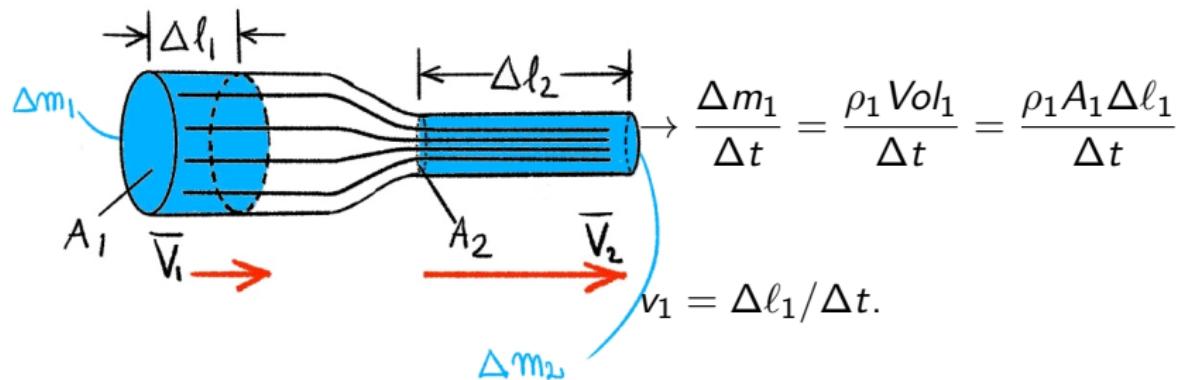


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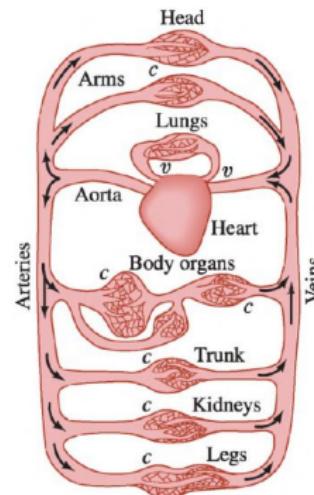
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$$Av = \text{constant}$$

## Example 1

**Blood flow.** In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about  $1.2\text{cm}$ , and the blood passing through it has a speed of about  $40\text{cm/s}$ . A typical capillary has a radius of about  $4 \times 10^{-4}\text{cm}$ , and blood flows through it at a speed of about  $5 \times 10^{-4}\text{m/s}$ .



Estimate the number of capillaries that are in the body.

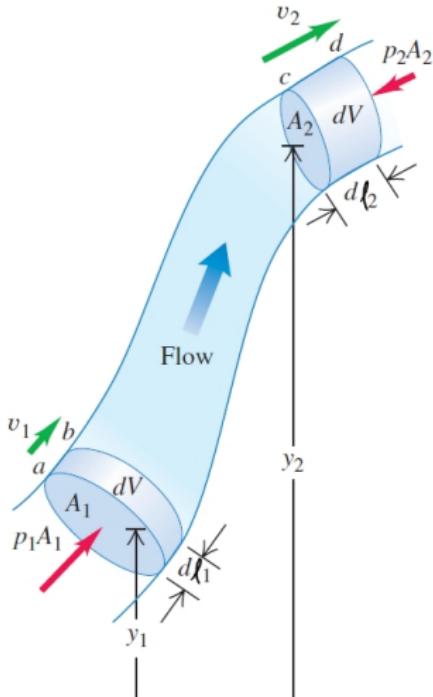
# Bernoulli's Principle

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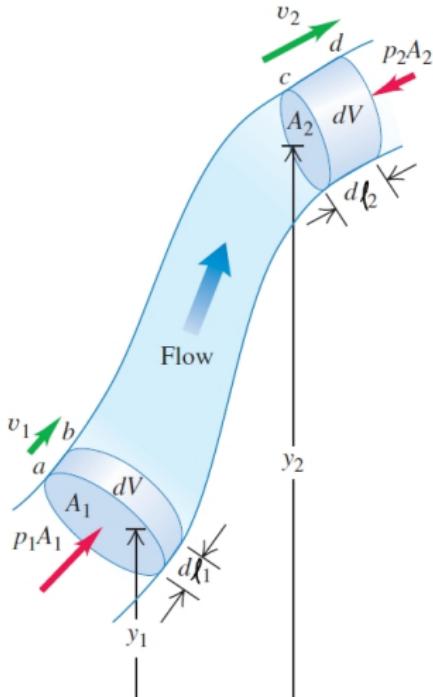
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The theorem of Bernoulli is in fact nothing more than a statement of the conservation of energy.



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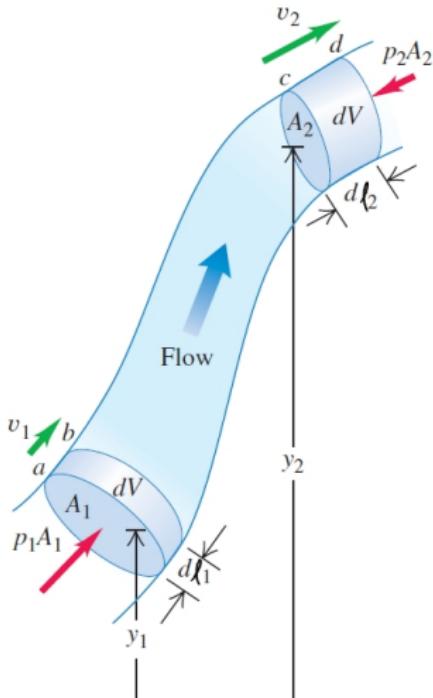
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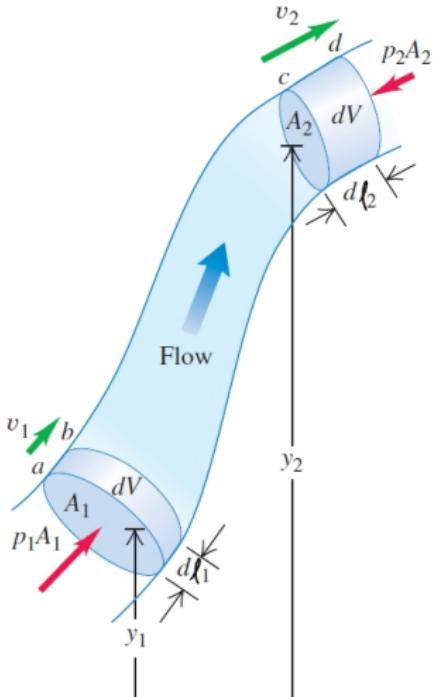


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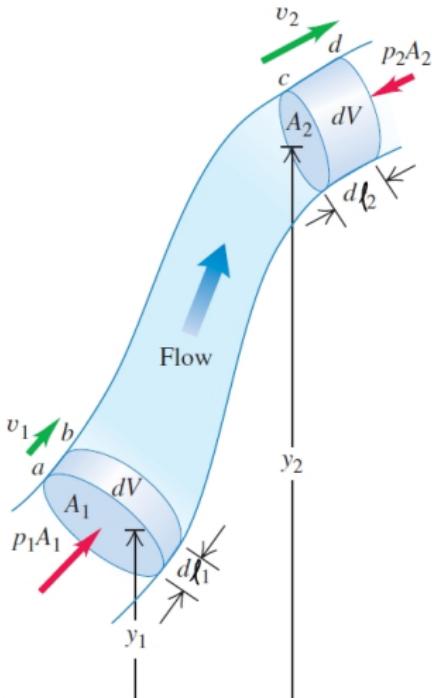
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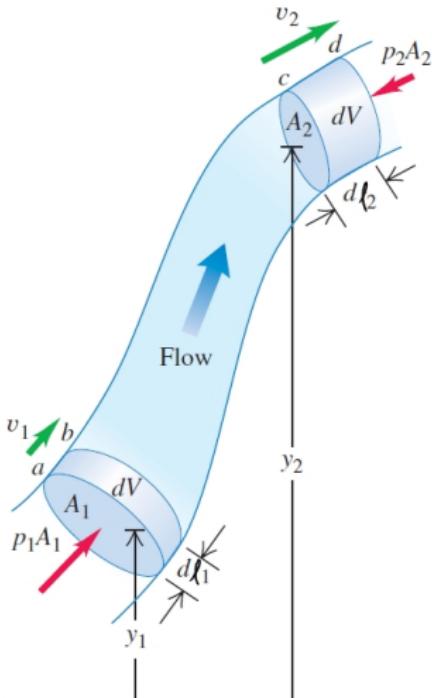
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$$\boxed{\frac{1}{2}\rho v^2 + P + \rho gy = \text{constant}} \quad (7)$$

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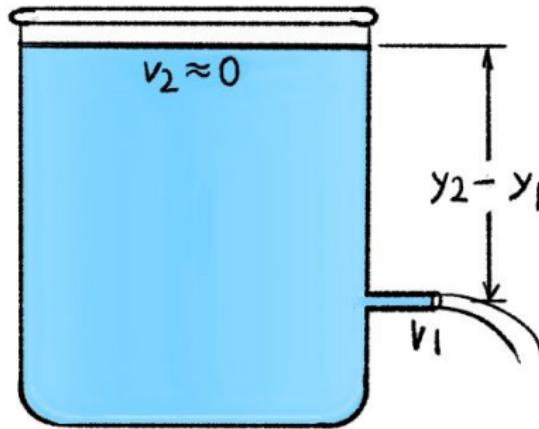
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$$P_2 - P_1 = -\rho g(y_2 - y_1) \leftarrow \text{Hydrostatic equation} \quad (8)$$

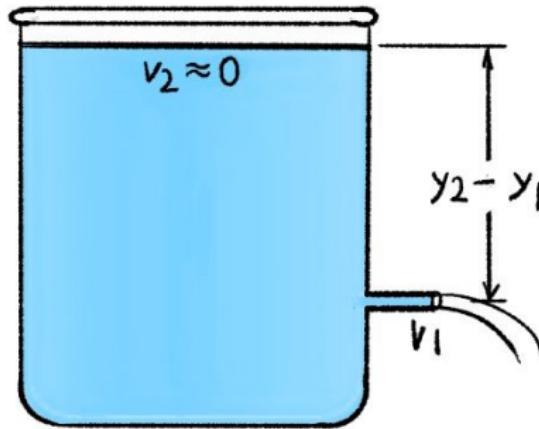
## Example 2

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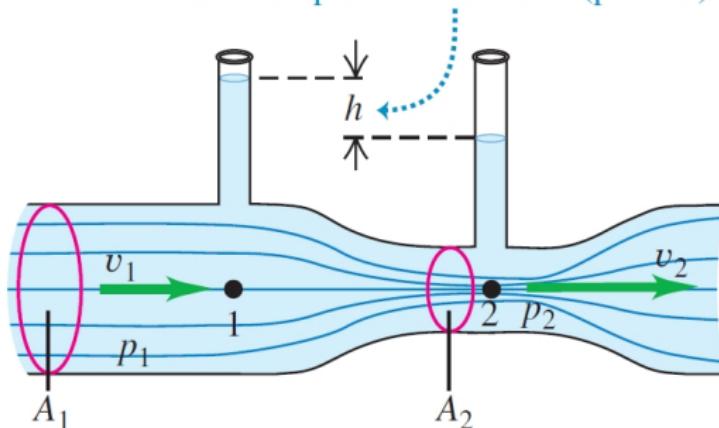


The liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height.

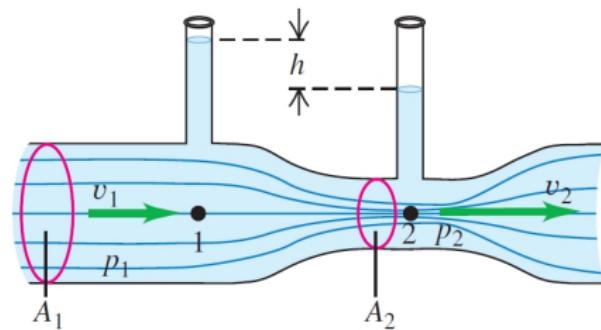
## Example 3

Figure 20 shows a Venturi meter, used to measure flow speed in a pipe. Derive an expression for the flow speed in terms of the cross-sectional areas and the difference in height  $h$  of the liquid levels in the two vertical tubes.

Difference in height results from reduced pressure in throat (point 2).



Solved in whiteboard



## Example 5

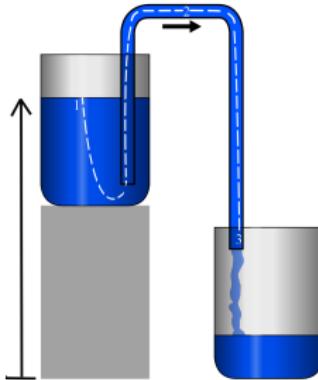
In 1993 the radius of Hurricane Emily was about 350 km. The wind speed near the center ("eye") of a hurricane, whose radius is 30 km, reaches about 200 km/h. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. (a) Estimate the wind speed at the rim of the hurricane. (b) Estimate the pressure difference at the earth's surface between the eye and the rim. Where is the pressure greater?

## Example 6

Solved in whiteboar

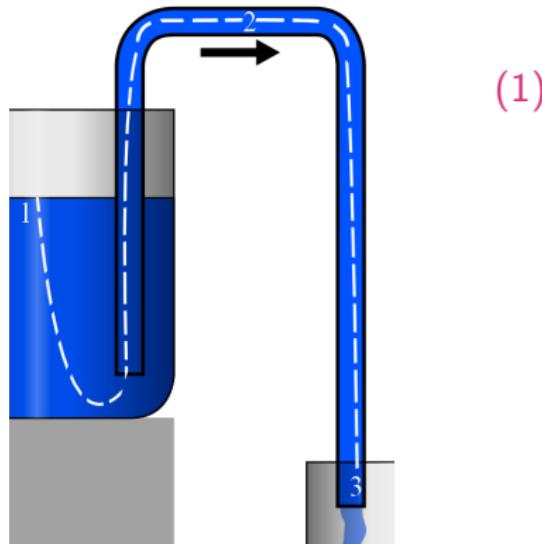
## Example 7

A siphon is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density  $\rho$  and let the atmospheric pressure be  $P_{atm}$ . Assume that the cross-sectional area of the tube is the same at all points along it.

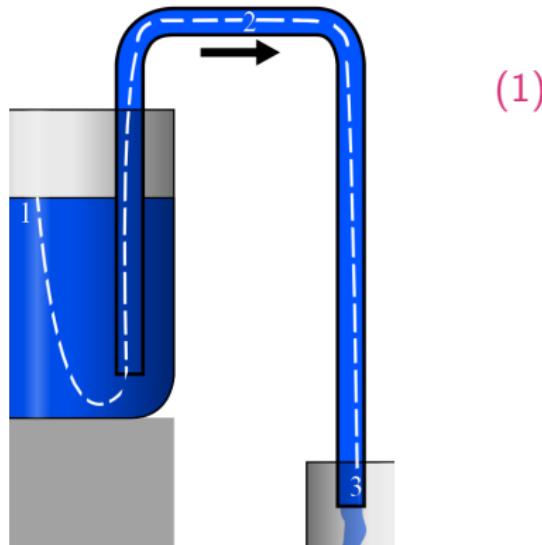


## Example 7

1. If the lower end of the siphon is at a distance  $h$  below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.)
2. A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height  $H$  that the high point of the tube can have if flow is still to occur?



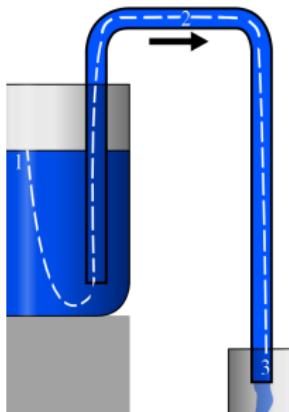
$$P_0 + \rho gh = P_0 + \frac{1}{2} \rho v^2$$



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$$\rightarrow v_3 = \boxed{\sqrt{2gh}}$$

(2)

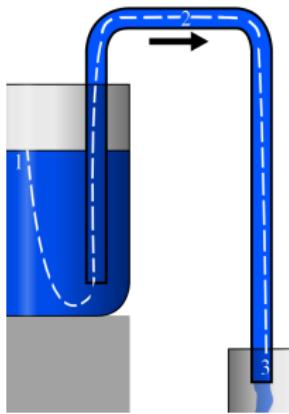
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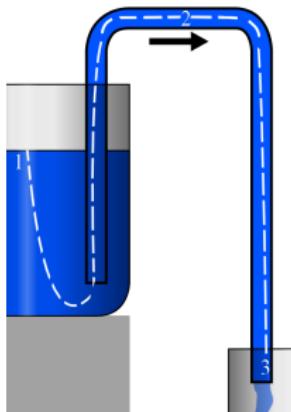
$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g(H + h) = P_0 + \frac{1}{2}\rho v_3^2$$



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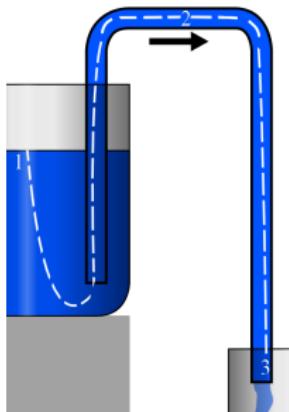
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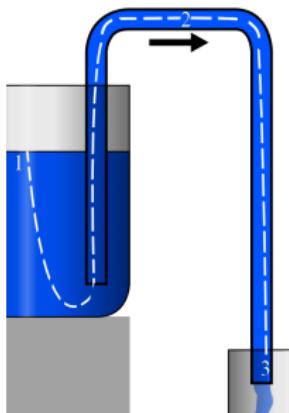


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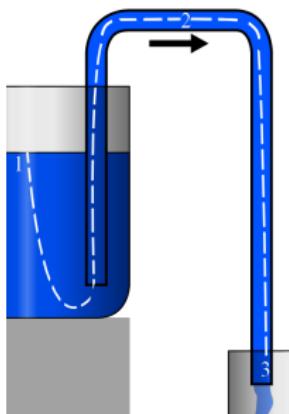
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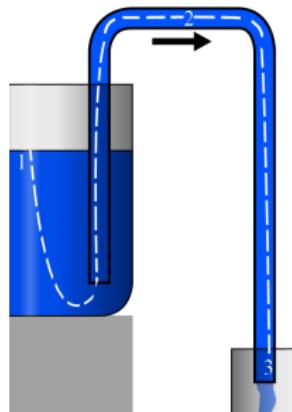
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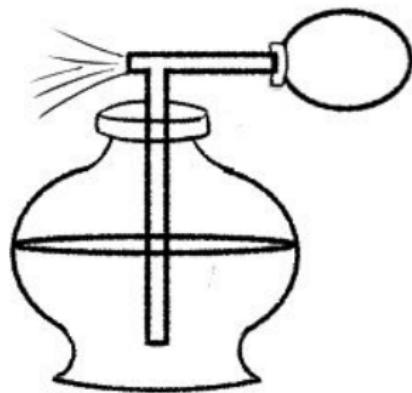
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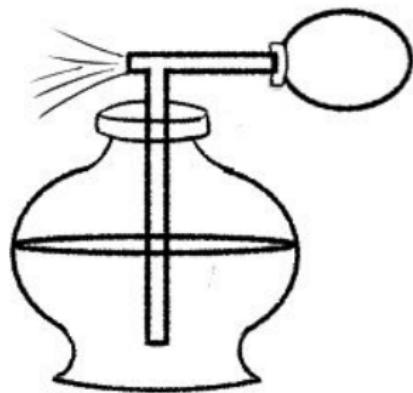
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$$\min P_2 = 0 \rightarrow H_{max} = \boxed{\frac{P_0}{\rho g} - h}$$

## Conceptual example: How does a Perfume Atomizer work?



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The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top.

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- ▶ In gases, collisions between the molecules. *it increases with T*

# Viscosity

Real Fluids have certain amount of internal friction called **viscosity**. It is a frictional force between adjacent layers of fluid.

The origin of the viscosity is,

- ▶ In fluids, electrical cohesive forces between the molecules. *it decreases with T*
- ▶ In gases, collisions between the molecules. *it increases with T*

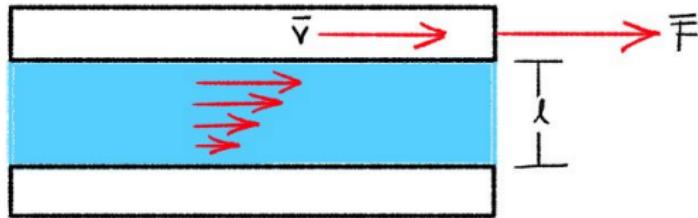
The viscosity can be expressed quantitatively by a *coefficient of viscosity*  $\eta$

# Viscosity

How do we calculate it?

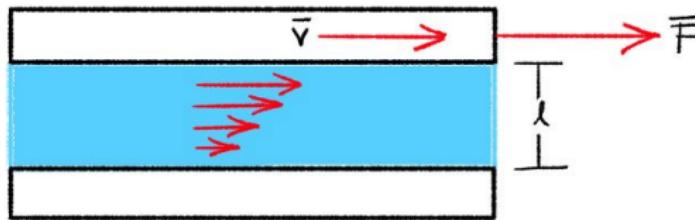
# Viscosity

How do we calculate it?



# Viscosity

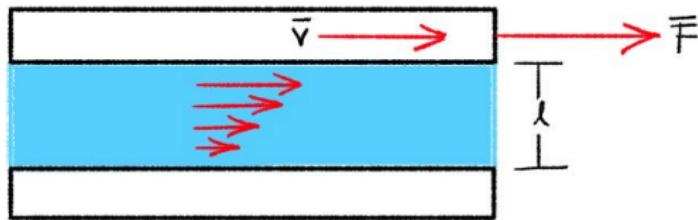
How do we calculate it?



$$F = \propto A \frac{v}{\ell} \quad (9)$$

# Viscosity

How do we calculate it?

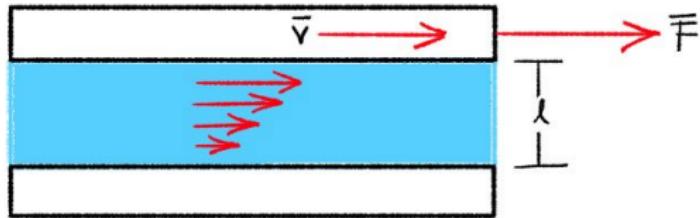


$$F = \propto A \frac{v}{\ell} \quad (9)$$

$$F = \eta A \frac{v}{\ell} \rightarrow \eta = \boxed{\frac{F\ell}{Av}} \quad (10)$$

# Viscosity

How do we calculate it?



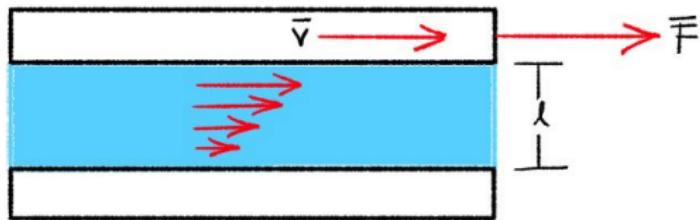
$$F = \propto A \frac{v}{\ell} \quad (9)$$

$$F = \eta A \frac{v}{\ell} \rightarrow \eta = \boxed{\frac{F\ell}{Av}} \quad (10)$$

The unit of  $\eta$  is  $N \cdot s/m^2 = Pa \cdot s$

# Viscosity

How do we calculate it?



There is always a thin boundary layer of fluid near the surface, in which the fluid is nearly at rest with respect to the surface. That's why dust particles can cling to a fan blade even when it is rotating rapidly

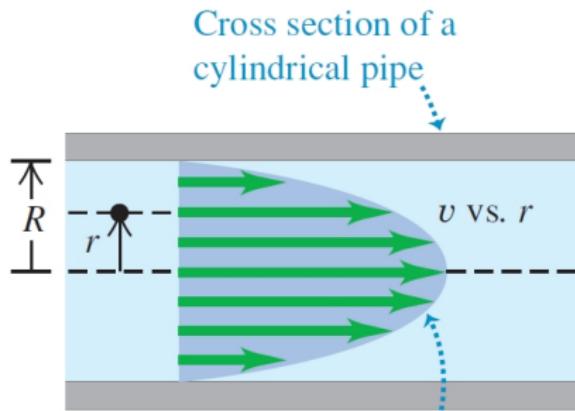
**TABLE 13–3**  
**Coefficients of Viscosity**

Fluid (temperature in °C)	Coefficient of Viscosity, $\eta$ (Pa · s) <sup>†</sup>
Water (0°)	$1.8 \times 10^{-3}$
(20°)	$1.0 \times 10^{-3}$
(100°)	$0.3 \times 10^{-3}$
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	$1.2 \times 10^{-3}$
Engine oil (30°) (SAE 10)	$200 \times 10^{-3}$
Glycerine (20°)	$1500 \times 10^{-3}$
Air (20°)	$0.018 \times 10^{-3}$
Hydrogen (0°)	$0.009 \times 10^{-3}$
Water vapor (100°)	$0.013 \times 10^{-3}$

<sup>†</sup> 1 Pa · s = 10 P = 1000 cP.

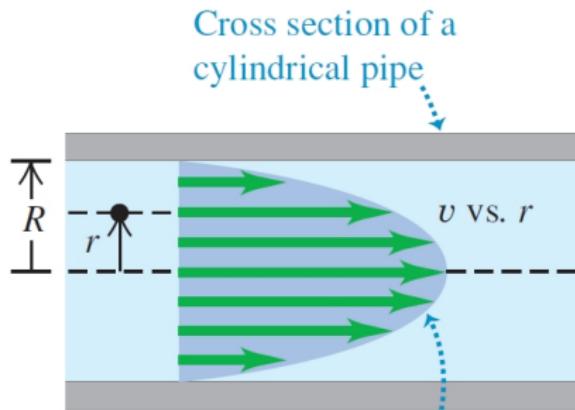
## Viscous fluid in a cylindrical pipe:

Viscous fluid in a cylindrical pipe: "The motion is like a lot of concentric tubes sliding relative to one another".



The velocity profile for viscous fluid flowing in the pipe has a parabolic shape.

Viscous fluid in a cylindrical pipe: "The motion is like a lot of concentric tubes sliding relative to one another".



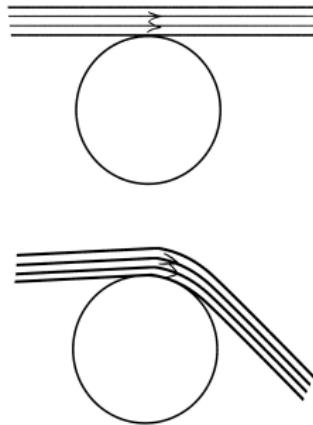
The velocity profile for viscous fluid flowing in the pipe has a parabolic shape.

A pressure difference  $\propto L/R^4$  is required to sustain the motion.

- ▶ This simple relationship,  $\propto L/R^4$ , explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure.

- ▶ This simple relationship,  $\propto L/R^4$ , explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure.
- ▶ That's why you have to keep squeezing a tube of toothpaste or a packet of ketchup to keep the fluid coming out of its container.

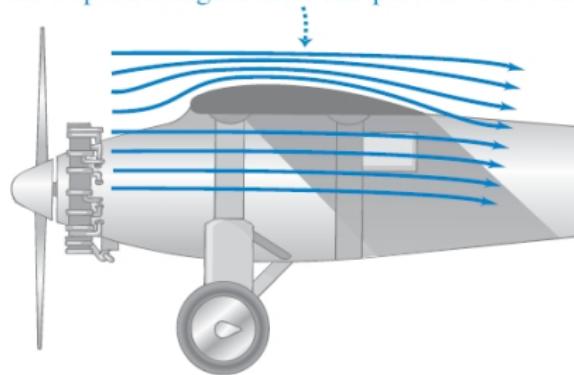
## Coanda effect



<https://www.youtube.com/watch?v=NvzXKZNJ7ZU&t=4s>

## Airplane Wings and Dynamic Lift

Flow lines are crowded together above the wing, so flow speed is higher there and pressure is lower.



# Surface Tension and Capillarity

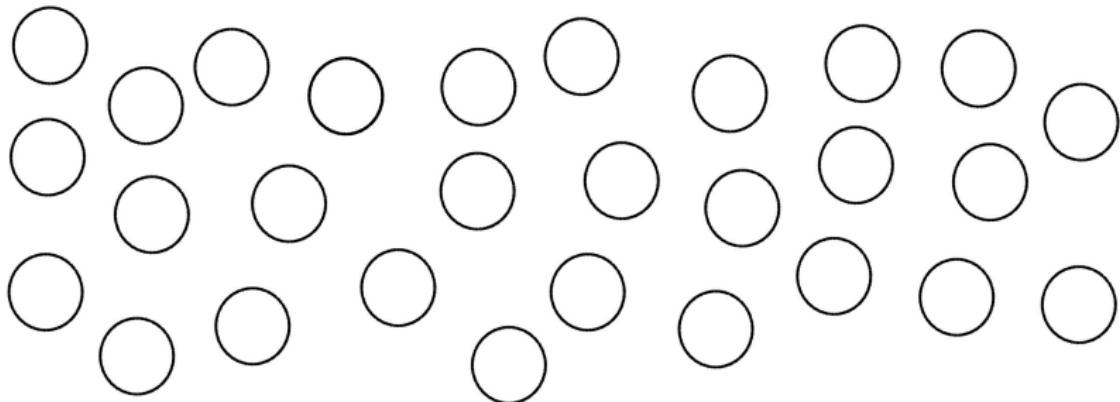
# Surface Tension and Capillarity



Why a body with a density several times the density of water cans rest *atop* the water surface?

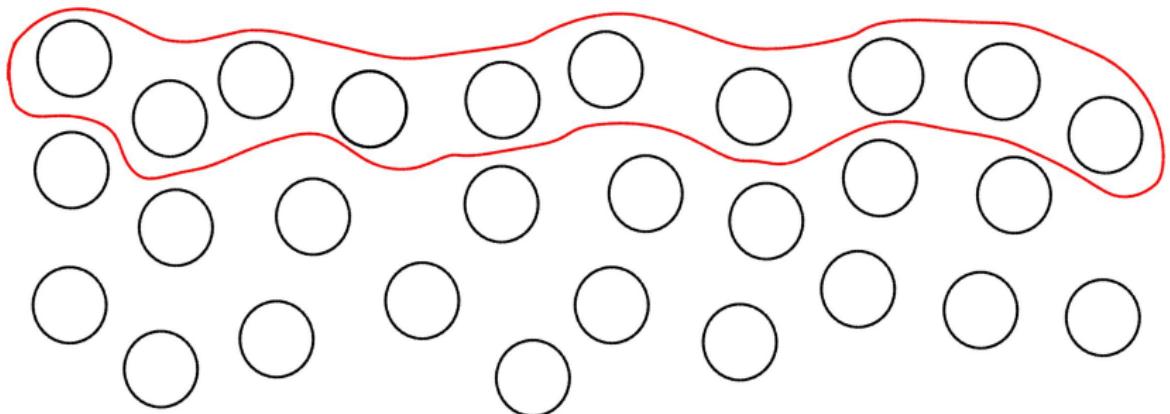
# Surface Tension and Capillarity

The particles that make up the liquid are in constant random motion.



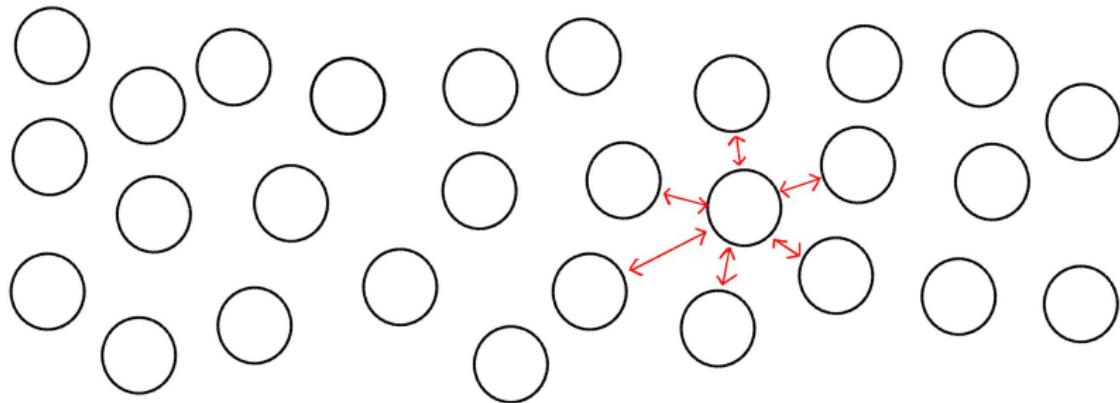
# Surface Tension and Capillarity

Do the particles at the surface form a random irregular surface?



# Surface Tension and Capillarity

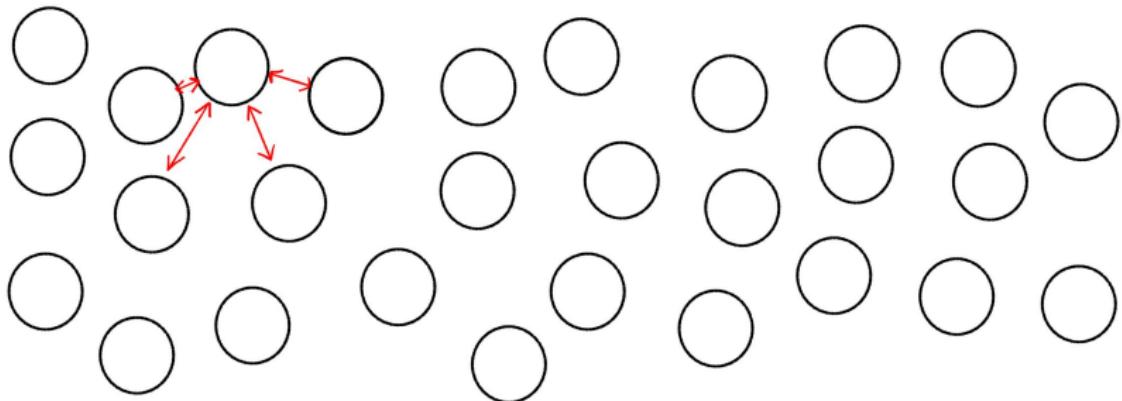
Molecules in a liquid are attracted by neighboring molecules.



Molecules in the interior are equally attracted in all directions.

# Surface Tension and Capillarity

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Molecules in the interior are equally attracted in all directions.

# Surface Tension and Capillarity

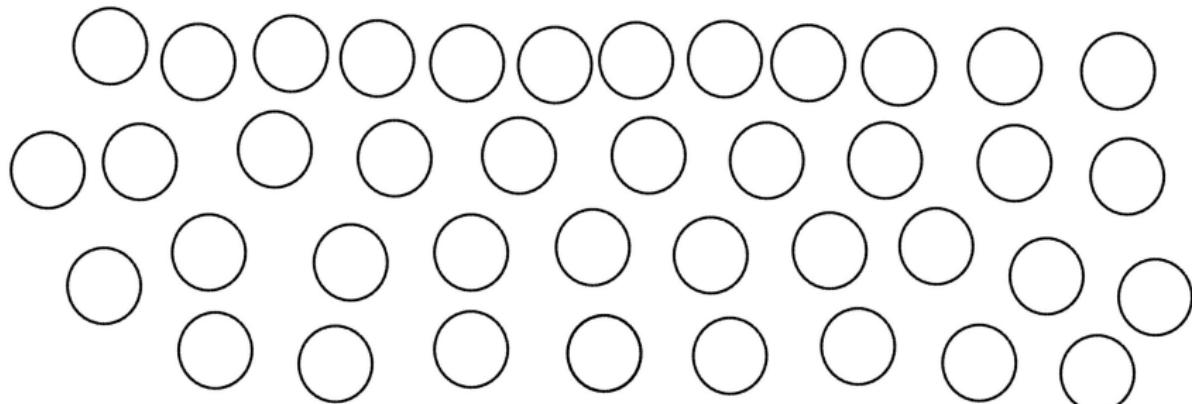
Surface molecules form a much smoother surface.

# Surface Tension and Capillarity

Surface molecules form a much smoother surface. Surface  
Molecules are compressed and minimizes the surface →

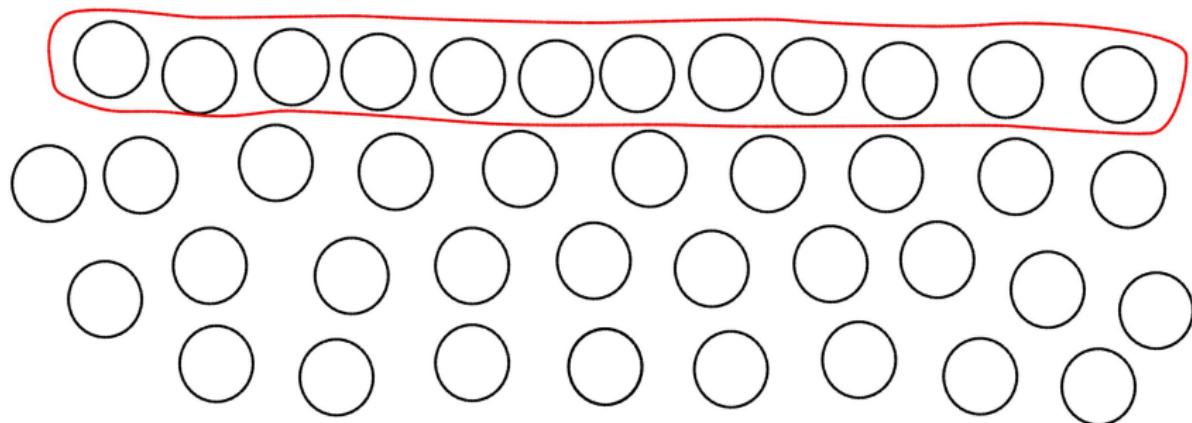
# Surface Tension and Capillarity

Surface molecules form a much smoother surface. Surface Molecules are compressed and minimizes the surface → Higher energy at the surface.



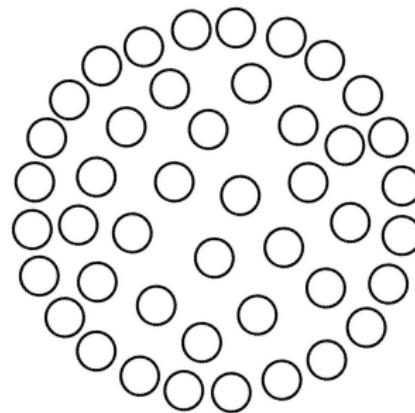
# Surface Tension and Capillarity

Liquid surface  $\leftrightarrow$  Stretched membrane under tension



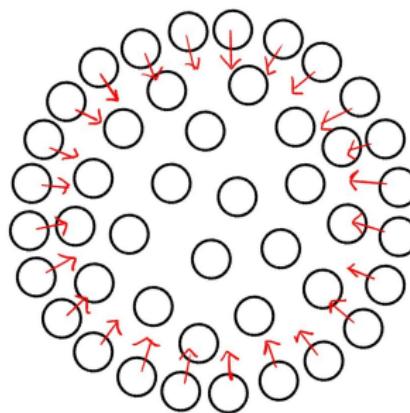
# Surface Tension and Capillarity

Liquid not confined in a container.



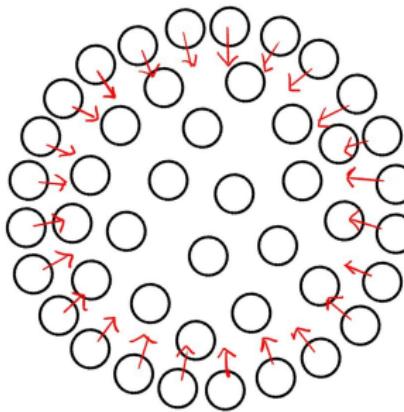
# Surface Tension and Capillarity

Liquid not confined in a container.



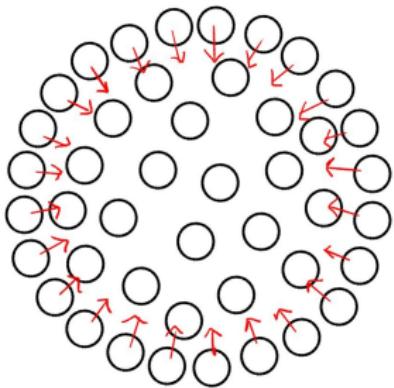
# Surface Tension and Capillarity

Liquid not confined in a container.



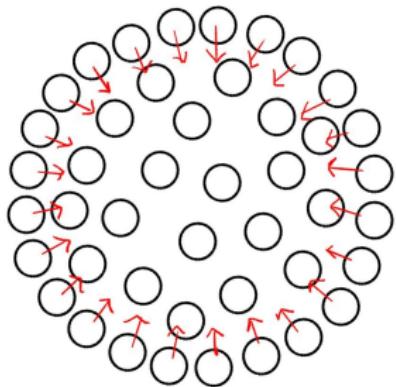
The ratio of surface area to volume is  $4\pi r^2/(4\pi r^3/3) = 3/r$

# Surface Tension and Capillarity



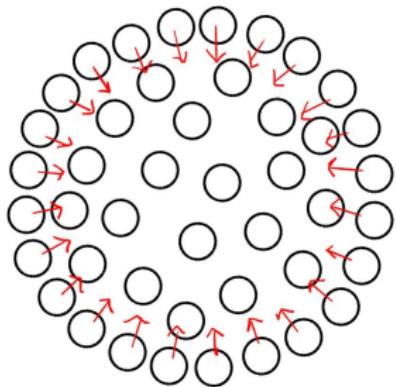
- ▶ The ratio of surface area to volume is  $4\pi r^2/(4\pi r^3/3) = 3/r$

# Surface Tension and Capillarity



- ▶ The ratio of surface area to volume is  $4\pi r^2/(4\pi r^3/3) = 3/r$
- ▶ Large quantities of liquid → surface tension << pressure forces.

# Surface Tension and Capillarity



- ▶ The ratio of surface area to volume is  $4\pi r^2/(4\pi r^3/3) = 3/r$
- ▶ Large quantities of liquid → surface tension << pressure forces.
- ▶ Small quantities of liquid → surface tension >> pressure forces.

# Surface Tension and Capillarity

How do we calculate the surface tension?

# Surface Tension and Capillarity

How do we calculate the surface tension?

## Surface Tension (definition)

Force per unit length that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

# Surface Tension and Capillarity

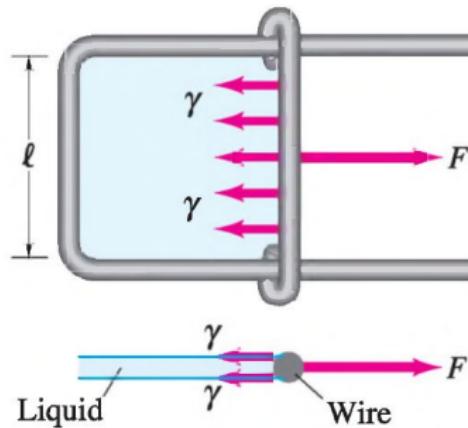
How do we calculate the surface tension?

## Surface Tension (definition)

Force per unit length that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\gamma = \frac{F}{\ell} \quad (11)$$

# Surface Tension and Capillarity



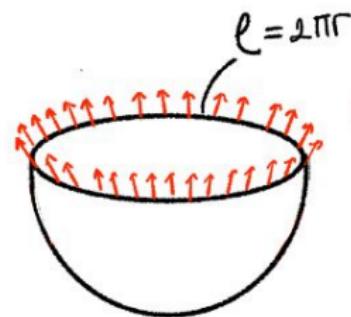
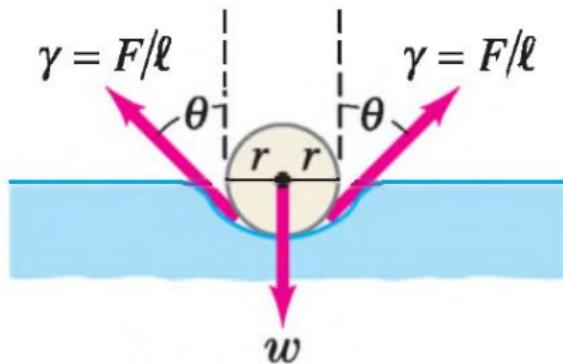
$$2\gamma l = F \rightarrow \gamma = \frac{F}{2l}$$

**TABLE 13–4**  
**Surface Tension of Some Substances**

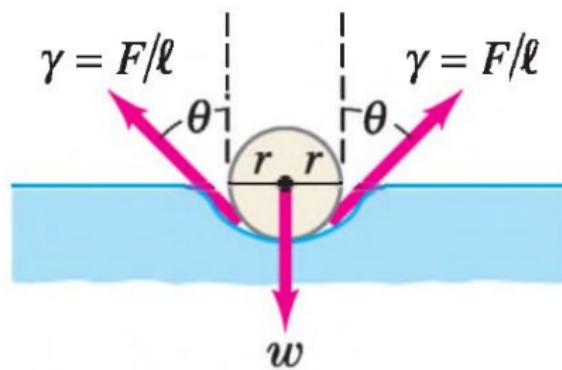
Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059

## Example 8

**Insect walks on water.** The base of an insect's leg is approximately spherical in shape, with a radius of about  $2.0 \times 10^{-5} m$ . The 0.0030 g mass of the insect is supported equally by its six legs. Estimate the angle  $\theta$  for an insect on the surface of water. Assume the water temperature is 20°C.

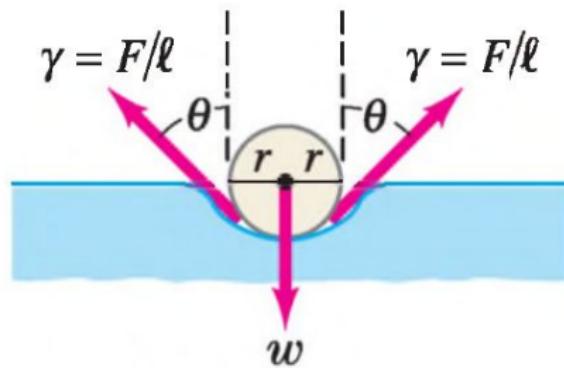


# Surface Tension and Capillarity



$$F \simeq (\gamma \cos\theta)\ell = 2\pi r\gamma \cos\theta$$

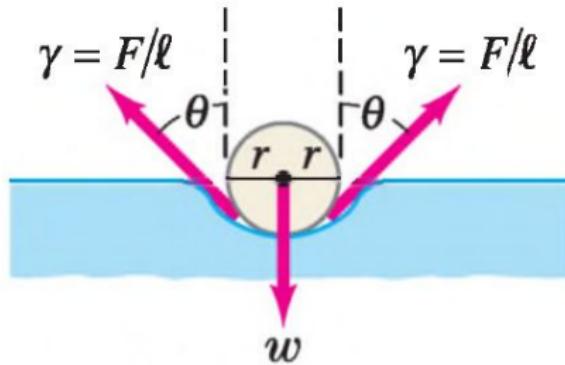
# Surface Tension and Capillarity



$$F \simeq (\gamma \cos\theta)\ell = 2\pi r \gamma \cos\theta$$

$$2\pi r \gamma \cos\theta \simeq \frac{1}{6}mg$$

# Surface Tension and Capillarity

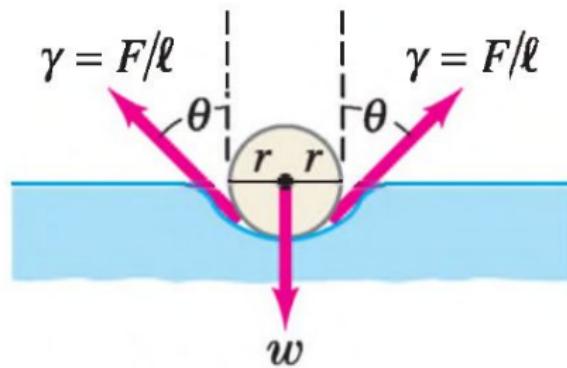


$$F \simeq (\gamma \cos\theta)\ell = 2\pi r \gamma \cos\theta$$

$$2\pi r \gamma \cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow \cos\theta \simeq \frac{1}{6} \frac{mg}{2\pi r \gamma}$$

# Surface Tension and Capillarity



$$F \simeq (\gamma \cos\theta)\ell = 2\pi r\gamma \cos\theta$$

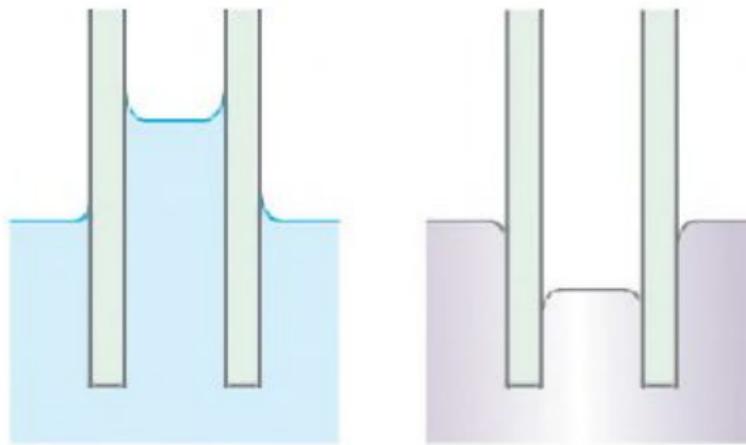
$$2\pi r\gamma \cos\theta \simeq \frac{1}{6}mg$$

$$\rightarrow \cos\theta \simeq \frac{1}{6} \frac{mg}{2\pi r\gamma}$$

$$\rightarrow \cos\theta \simeq 0.54 \rightarrow \theta \simeq 57^\circ$$

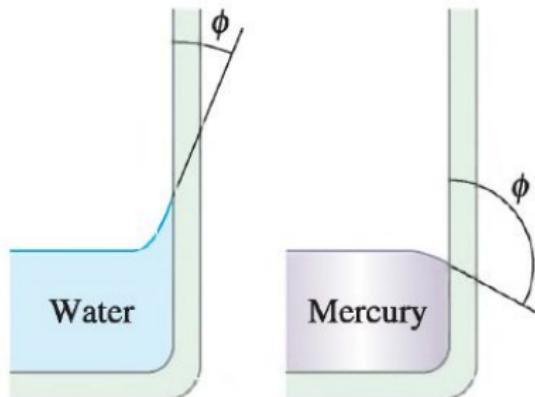
# Surface Tension and Capillarity

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called capillarity



# Surface Tension and Capillarity

Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. Cohesion refers to the force between molecules of the same type, whereas adhesion refers to the force between molecules of different types.



# Fluids in motion

We can distinguish two main types of fluid flow:

- ▶ Laminar Flow: Smooth flow → the neighbouring layers of the fluid slide by each other smoothly

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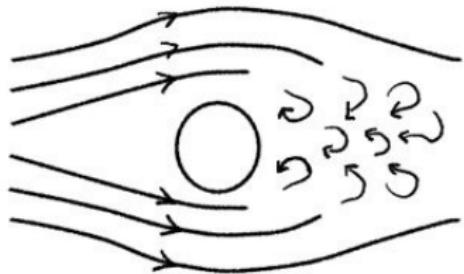
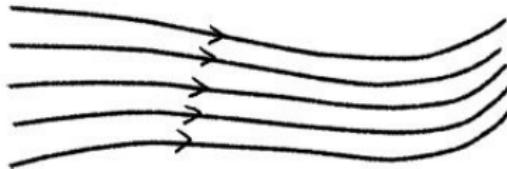
- ▶ Laminar Flow: Smooth flow → the neighbouring layers of the fluid slide by each other smoothly
- ▶ Turbulent Flow: Above a certain speed, the flow is characterized by erratic, small, whirlpool-like circles called eddy currents or eddies

<https://www.youtube.com/watch?v=WG-YCpAGgQQ>

## Fluids in motion

In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another.

In a turbulent Flow, eddies absorb a great deal of energy, and although a certain amount of internal friction called viscosity is present even during streamline flow, it is much greater when the flow is turbulent.



How do we know if the flow is laminar or turbulent?

How do we know if the flow is laminar or turbulent? → *Reynold's number*

How do we know if the flow is laminar or turbulent? → *Reynold's number*

The equations that describe the motion of the elements of fluids are:

$$\rho \vec{a} = -\nabla P - \rho \nabla \phi + \vec{f}_{vis}$$
 Navier Stokes eq.

We can write an equivalent dimensionless equation, so we can get rid of the units:

$$\frac{d\vec{v}'}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v}'$$

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$$\frac{d\vec{v}'}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v}'$$

$$R_e = \frac{\rho v D}{\eta}$$

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$$R_e = \frac{\rho v D}{\eta}$$

$v$  : characteristic velocity

$D$  : characteristic dimension

$\rho$  : characteristic density

We can write an equivalent dimensionless equation, so we can get rid of the units:

$$\underbrace{\frac{d\vec{v}'}{dt'}}_{acceleration} = \underbrace{-\nabla' P'}_{pressure} - \underbrace{-\nabla' \phi'}_{gravity} + \overbrace{\frac{1}{R_e} \nabla'^2 \vec{v}'}^{viscosity}$$

We can write an equivalent dimensionless equation, so we can get rid of the units:

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Now we can compare acceleration with pressure with gravity.

$$R_e \gg 1$$

$$\frac{d\vec{v}'}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v}'$$

$R_e \gg 1$

$$\frac{d\vec{v}'}{dt'} = -\nabla' P' - \nabla' \phi' + \frac{1}{R_e} \nabla'^2 \vec{v}'$$

Viscosity is not important

$$R_e \ll 1$$

$$R_e \frac{d\vec{v}'}{dt'} = -\frac{1}{\eta} \nabla P - \frac{\rho}{\eta} \nabla \phi + \nabla'^2 \vec{v}'$$

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We no longer have time in our equation!!!

$$R_e \ll 1$$

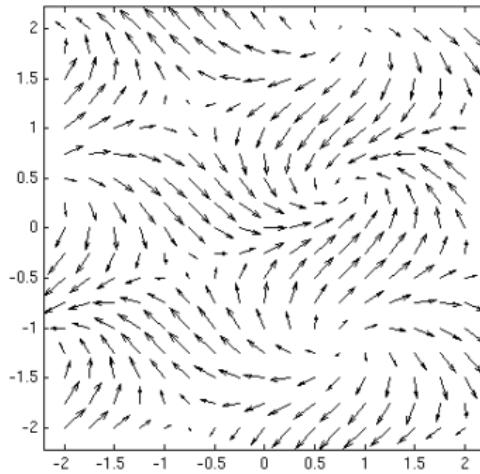
$$R_e \frac{d\vec{v}'}{dt'} = -\frac{1}{\eta} \nabla P - \frac{\rho}{\eta} \nabla \phi + \nabla'^2 \vec{v}'$$

We no longer have time in our equation!!!

You can go forward in time and then do the same thing backward in time and "undo" the process.

## Generalization

To fully describe a fluid, we need the velocity vector field, that is  $\vec{v}(x, y, z, t)$ .

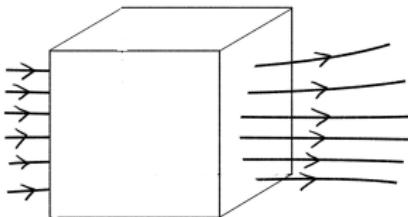


# Generalization

Which are the equations  
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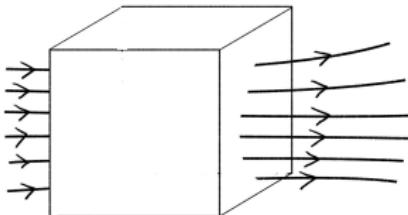


# Generalization

The mass flow rate passing  $A_{X_1}$  is

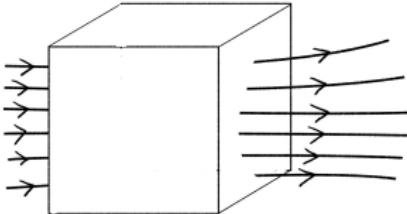
Which are the equations  
that describe the velocity  
field?

$$\rho v_x dy dz$$



## Generalization

Which are the equations that describe the velocity field?



The mass flow rate passing  $A_{x_1}$  is

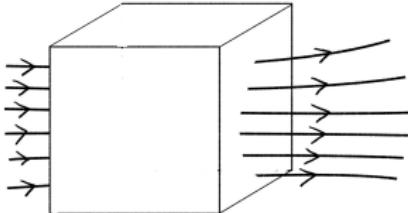
$$\rho v_x dy dz$$

The mass flow rate passing  $A_{x_2}$  is

$$\rho \left( v_x + \frac{\partial v_x}{\partial x} dx \right) dy dz$$

## Generalization

Which are the equations that describe the velocity field?



The mass flow rate passing  $A_{x_1}$  is

$$\rho v_x dy dz$$

The mass flow rate passing  $A_{x_2}$  is

$$\rho \left( v_x + \frac{\partial v_x}{\partial x} dx \right) dy dz$$

The difference between the mass flow rate entering through  $A_{x_2}$  and  $A_{x_1}$  is,

$$\rho \frac{\partial v_x}{\partial x} dy dz dx$$

## Generalization

If we do the same for  $y$  and  $z$ ,

$$\rho \frac{\partial v_y}{\partial y} dy dz dx$$

$$\rho \frac{\partial v_z}{\partial z} dy dz dx$$

Then, the difference between the total flow entering and exiting the volume is

$$\rho \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) dy dz dx$$

# Generalization

If the entering flux is equal to the exiting flux

$$\rho \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) dy dz dx = 0$$

## Generalization

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## Generalization

If the entering flux is equal to the exiting flux

$$\rho \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) dy dz dx = 0$$

Then,

$$\left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0$$

or using the nabla operator,

$$\nabla \cdot \vec{v} = 0$$

## Generalization

If the density is not constant,

$$\nabla \cdot (\rho \vec{v}) = 0$$

If the entering flux is not equal to exiting flux,

$$\nabla \cdot (\rho \vec{v}) = -\frac{\partial \rho}{\partial t}$$

This is the Hydrodynamic Equation of Continuity.

## Generalization

Then we have one equation for 3 unknowns.

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We will get our next equation from Newton's law which tells us how the velocity changes because of the forces:

$$\rho \vec{a} = -\nabla P - \rho \nabla \phi + \vec{f}_{vis}$$

## Generalization

Then we have one equation for 3 unknowns.

We will get our next equation from Newton's law which tells us how the velocity changes because of the forces:

$$\rho \vec{a} = -\nabla P - \rho \nabla \phi + \vec{f}_{vis}$$

We suppose that the liquid is “thin” the viscosity is unimportant, so we will omit  $f_{vis} \rightarrow Dry\ Water\ Flow$

What is the acceleration?

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IT IS NOT  $\vec{a} = \frac{\partial \vec{v}}{\partial t}$ !!!!

What is the acceleration?

IT IS NOT  $\vec{a} = \frac{\partial \vec{v}}{\partial t}$ !!!!

To obtain the acceleration we must calculate,

$$\frac{\Delta \vec{v}}{\Delta t}$$

for an actual displacement of an element of fluid. That is,

$$\frac{\Delta \vec{v}}{\Delta t} = v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}$$

Using the nabla,

$$\vec{a} = (\vec{v} \cdot \nabla) \vec{v} + \frac{\partial \vec{v}}{\partial t}$$

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Then, the second Newton's Law takes the form,

$$(\vec{v} \cdot \nabla) \vec{v} + \frac{\partial \vec{v}}{\partial t} = -\frac{\nabla P}{\rho} - \nabla \phi$$

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This equation can be expressed as,

$$\Omega \times \vec{v} + \frac{1}{2} \nabla v^2 + \frac{\partial \vec{v}}{\partial t} = -\frac{\nabla P}{\rho} - \nabla \phi$$

Using the nabla,

$$\vec{a} = (\vec{v} \cdot \nabla) \vec{v} + \frac{\partial \vec{v}}{\partial t}$$

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→ [https://mathinsight.org/curl\\_idea](https://mathinsight.org/curl_idea)