PHY250: Waves 2D

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Digipen

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Waves in 2D

- Waves equation
- Doppler Effect

Wave Equation

$$\frac{\partial^2}{\partial t^2} D(t,x,y) = v^2 \left[\frac{\partial^2}{\partial x^2} D(t,x,y) + \frac{\partial^2}{\partial y^2} D(t,x,y) \right]$$

Wave Equation

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Assumptions

Wave Equation

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Assumptions

- Uniform density.

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Assumptions

- Uniform density.
- Uniform tension.

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- Uniform density.
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Assumptions

- Uniform density.
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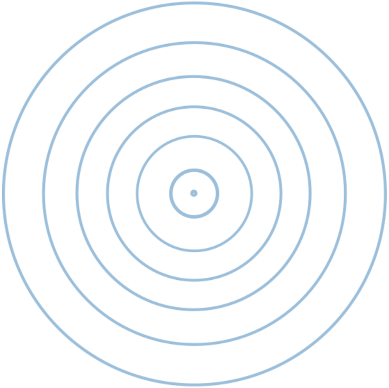
Solutions

Traveling waves:

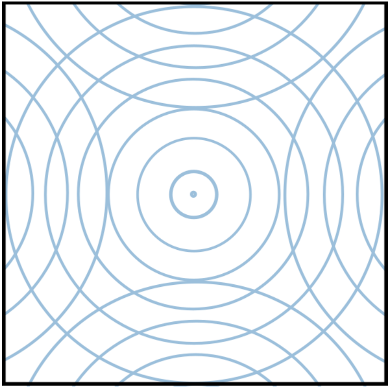
$$D(t, \vec{r}) = A \sin(\vec{k} \cdot \vec{r} - \omega t)$$



Solutions

Standing waves:

$$D(t,\vec{r}) = A_{nm} sin(\frac{n\pi x}{L_x}) sin(\frac{m\pi y}{L_y}) cos(\omega t)$$



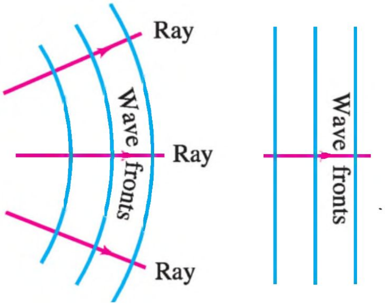
Fourier in 2D

Combination of Harmonics

$$D(t, \vec{r}) = \sum_m \sum_n A_{nm} sin(\frac{n\pi x}{L_x}) sin(\frac{m\pi y}{L_y}) cos(\omega t)$$

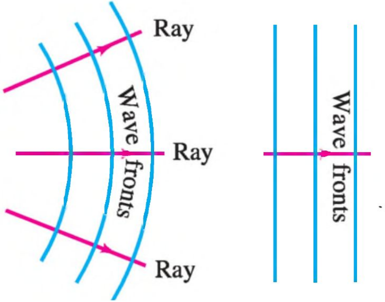
Waves in 2D

For a two- or three-dimensional wave, such as a water wave, we are concerned with wave fronts, by which we mean all the points along the wave forming the wave crest.



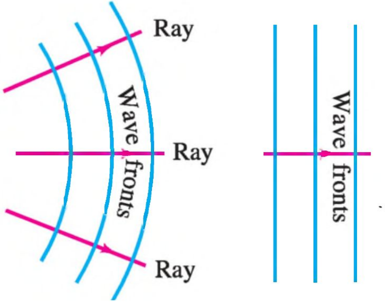
Waves in 2D

A line drawn in the direction of motion, perpendicular to the wave front, is called a ray.



Waves in 2D

Wave fronts far from the source have lost almost all their curvature and are nearly straight; they are then called plane waves.



Reflection and transmission

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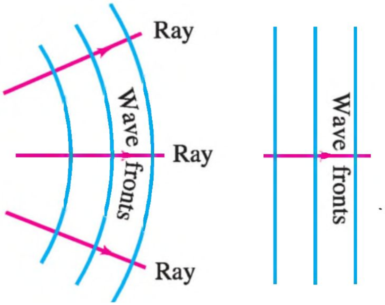

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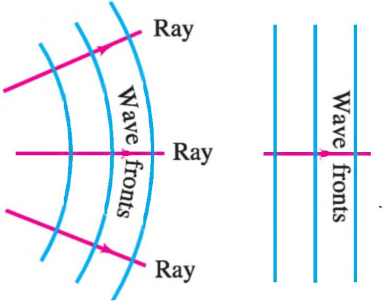

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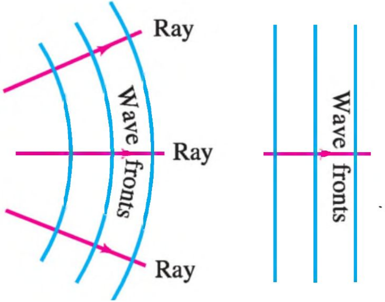
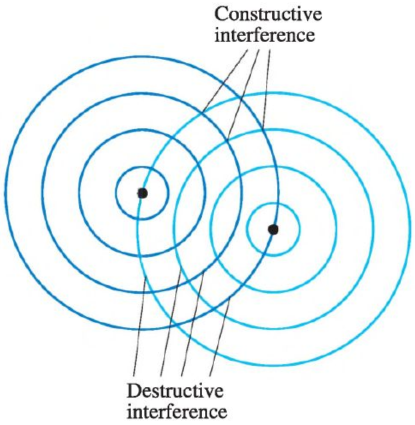


Interference

Two sources sending identical waves in a medium.

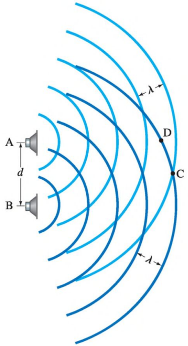
Interference

Two sources sending identical waves in a medium. Example two rocks thrown simultaneously in water:



Spatial interference

When two waves, with the same frequency, simultaneously pass through the same region of space, they interfere with one another. Interference also occurs with sound waves.



- Point C (same distance from each speaker)

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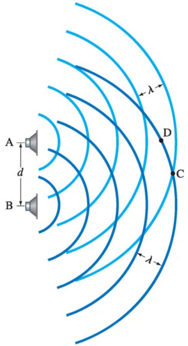

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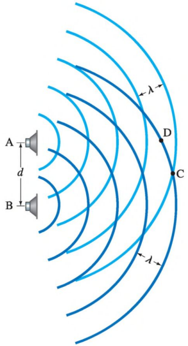
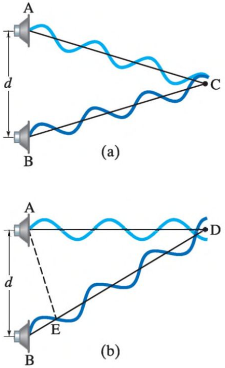


- Point C (same distance from each speaker) $\rightarrow$ loud sound (constructive interference).
- Point D, no sound or little sound (destructive interference).

Spacial interference



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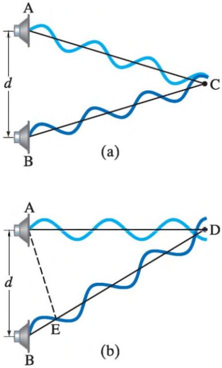

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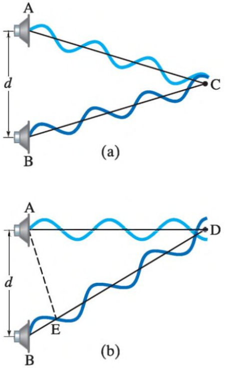

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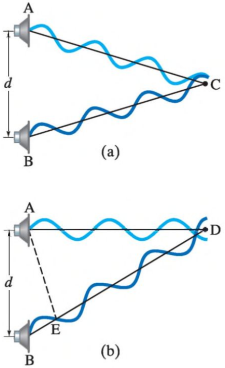

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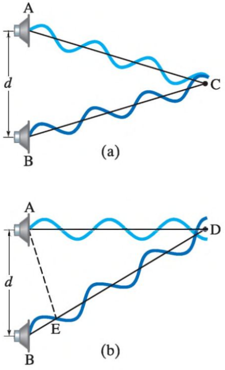
$\rightarrow$ destructive interference.
Then..

- $BD - AD = 2\lambda, 3\lambda, ... \rightarrow$ constructive interference

Spacial interference



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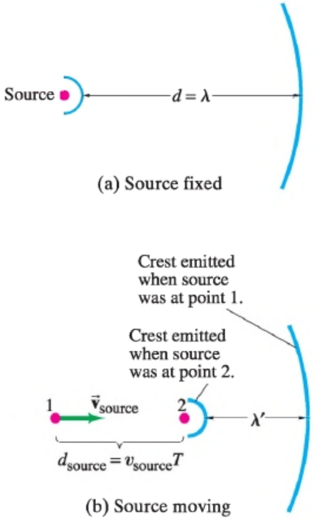
- $BD - AD = 2\lambda, 3\lambda, ... \rightarrow$ constructive interference
- $BD - AD = \lambda/2, 3\lambda/2, 5\lambda/2, ... \rightarrow$ destructive interference

Doppler Effect

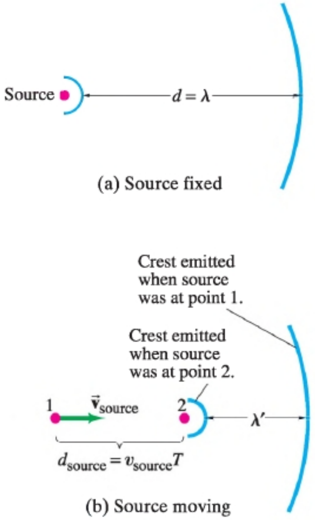
Change in Pitch when a source of sound is moving toward or moving away from the observer.

Doppler Effect



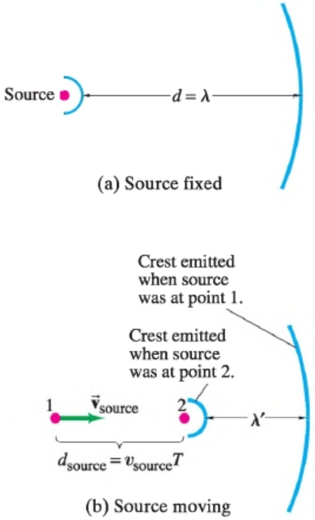
Source at rest

Doppler Effect



Source at rest $\rightarrow$ the wave travels at the velocity of sound in the air.

Doppler Effect



Source at rest $\rightarrow$ the wave travels at the velocity of sound in the air.

Its velocity is independent of the source velocity.

Doppler Effect

The wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_{src})T \tag{1}$$

Doppler Effect

Then, the wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_{src})T = (v_{snd} - v_{src})\frac{\lambda}{v_{snd}} \quad (2)$$

Doppler Effect

Then, the wavelength of a source traveling toward the observer is:

$$\lambda' = (v_{snd} - v_{src})T = (v_{snd} - v_{src})\frac{\lambda}{v_{snd}} = \lambda(1 - \frac{v_{src}}{v_{snd}}) \quad (3)$$

Doppler Effect

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$$\lambda' = (v_{snd} - v_{src})T = (v_{snd} - v_{src})\frac{\lambda}{v_{snd}} = \lambda(1 - \frac{v_{src}}{v_{snd}}) \qquad (3)$$

Where we consider that the source velocity is lower than the sound velocity.

Doppler Effect

Then, the shift in wavelength is,

Doppler Effect

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$$\Delta\lambda = -\lambda \frac{v_{src}}{v_{snd}}, \tag{4}$$

Doppler Effect

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$$\Delta\lambda = -\lambda \frac{v_{src}}{v_{snd}}, \tag{4}$$

proportional to the source velocity.

Doppler Effect

The frequency perceived by the observer is,

$$f' = \frac{v_{snd}}{\lambda'} = \frac{v_{snd}}{\lambda(1 - \frac{v_{src}}{v_{snd}})} \tag{5}$$

Doppler Effect

The frequency perceived by the observer is,

$$f' = \frac{f}{(1 - \frac{v_{src}}{v_{snd}})} \tag{6}$$

The denominator is less than 1, then the observed frequency is grater than the source frequency.

Doppler Effect

If the source is moving away the observer,

$$\lambda' = (v_{snd} + v_{src})T = \lambda(1 + \frac{v_{src}}{v_{snd}}) \qquad (7)$$

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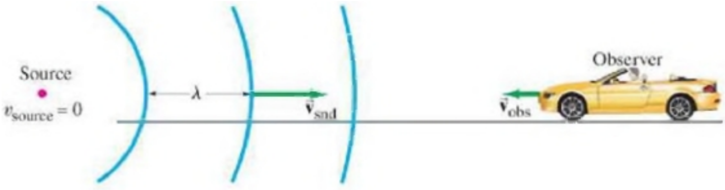
$$f' = \frac{f}{(1 + \frac{v_{src}}{v_{snd}})} \tag{8}$$

The observed frequency in this case is lower than the source frequency.

Doppler Effect

What happens if the source is at rest and the observer is moving toward the source?



Doppler Effect

What is the frequency perceived by the observer that is moving toward the source?

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$$f' = \frac{v_{snd} + v_{obs}}{\lambda} \tag{9}$$

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$$f' = \frac{v_{snd} + v_{obs}}{\lambda} = f\left(1 + \frac{v_{obs}}{v_{sound}}\right) \tag{10}$$

Doppler Effect

What is the frequency perceived by the observer that is moving toward the source?

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Quantitatively the change in frequency is different than for the case of a moving source.

Doppler Effect

- Fixed source and a moving observer $\rightarrow \lambda$ does change, but the velocity of the crest respect to the observer changes.

Doppler Effect

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Doppler Effect

If the observer is moving away from the source, the velocity of the crests respect to the observer is decreased, and the frequency is,

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If the observer is moving away from the source, the velocity of the crests respect to the observer is decreased, and the frequency is,

$$f' = \frac{v_{snd} - v_{obs}}{\lambda} = f\left(1 - \frac{v_{obs}}{v_{sound}}\right) \tag{11}$$

Doppler Effect

When a sound wave is reflected from a moving obstacle, the frequency of the reflected wave will, because of the Doppler effect, be different from that of the incident wave.

Doppler Effect

Example: Two Doppler shifts

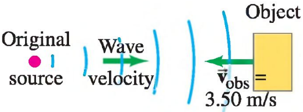
Doppler Effect

Example: Two Doppler shifts

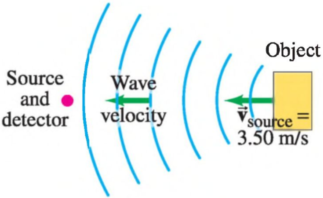
A 5000 Hz sound wave is emitted by a stationary source. This sound wave reflects from an object moving 3.50 m/s toward the source. What is the frequency of the wave reflected by the moving object as detected by a detector at rest near the source?

Doppler Effect

Example:



(a)



Doppler Effect

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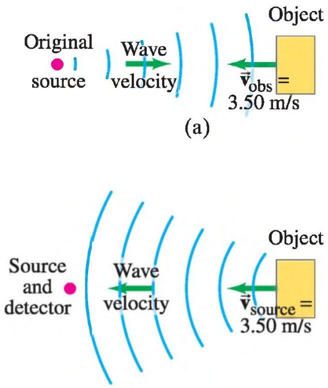
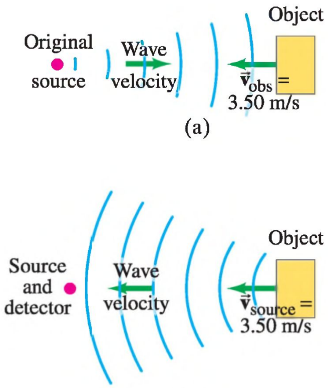


The frequency detected by the moving object (obs) is:

$$f' = f\left(1 + \frac{v_{obs}}{v_{sound}}\right)$$

Doppler Effect

Example:



The frequency detected by the moving object (obs) is:

$$f' = f\left(1 + \frac{v_{obs}}{v_{sound}}\right) = 5051\ Hz$$

Doppler Effect

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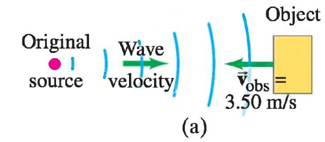
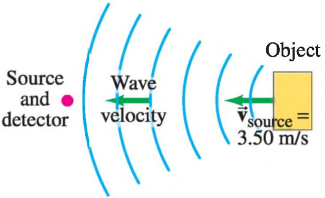


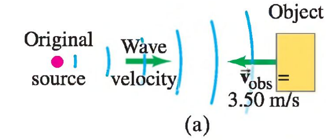


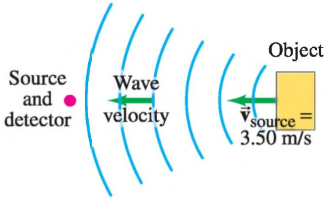
The detected frequency is,

$$f'' = \frac{f'}{(1 - \frac{v_{source}}{v_{snd}})}$$

Doppler Effect

Example:





The frequency emitted by the "new" source is,

$$f'' = \frac{f'}{(1 - \frac{v_{source}}{v_{snd}})} = 5103\ HZ$$

Doppler Effect

We can summarize both effects in a single equation:

$$f' = f\left(1 + \frac{v_{obs}}{v_{snd}}\right)$$

$$f'' = \frac{f'}{(1 - \frac{v_{source}}{v_{snd}})}$$

$$f'' = \frac{f}{(1 - \frac{v_{source}}{v_{snd}})}\left(1 + \frac{v_{obs}}{v_{snd}}\right) \tag{12}$$

Doppler Effect

Then, the frequency perceived by an observer when observer and source approach each other is,

$$f' = f\,\frac{v_{snd} + v_{obs}}{v_{snd} - v_{source}} \tag{13}$$

Doppler Effect

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$$f' = f \frac{v_{snd} + v_{obs}}{v_{snd} - v_{source}} \tag{13}$$

And the frequency perceived by an observer when observer and source move apart is,

$$f' = f \frac{v_{snd} - v_{obs}}{v_{snd} + v_{source}} \tag{14}$$

Doppler Effect

We can summarize all cases in a single equation:

$$f' = f \frac{v_{snd} \mp v_{obs}}{v_{snd} \mp v_{source}} \tag{15}$$

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The upper signs in numerator and denominator applies if source and/or observer move toward each other; the lower signs applies if they are moving apart.

Doppler Effect

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Doppler Effect

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- measuring the beats frequency, we can obtain the value of the moving object.
- For example, ultrasonic waves reflected from red blood cells can be used to determine the velocity of blood flow.
- the technique can be used to detect the movement of the chest of a young fetus and to monitor its heartbeat.

Doppler Effect

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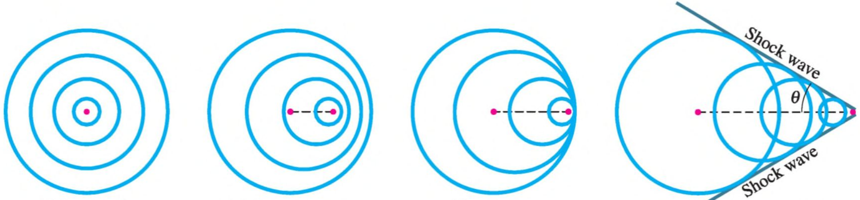

(a) $v_{obj} = 0$ (b) $v_{obj} < v_{snd}$ (c) $v_{obj} = v_{snd}$ (d) $v_{obj} > v_{snd}$

Shock Waves

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A very clear explanation:
https://www.youtube.com/watch?v=If-yK7sQE8Q

Questions

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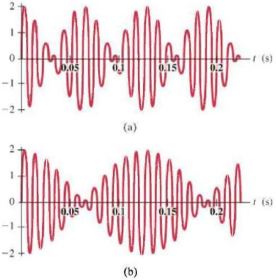
- How will the air temperature in a room affect the pitch of organ pipes?
- Is there a Doppler shift if the source and observer move in the same direction, with the same velocity? Explain.

Questions

Consider the two waves shown in the figure. Each wave can be thought of as a superposition of two sound waves with slightly different frequencies. In which of the waves, (a) or (b), are the two component frequencies farther apart? Explain.



Questions

The figure shows various positions of a child on a swing moving toward a person on the ground who is blowing a whistle. At which position, A through E, will the child hear the highest frequency for the sound of the whistle? Explain your reasoning.