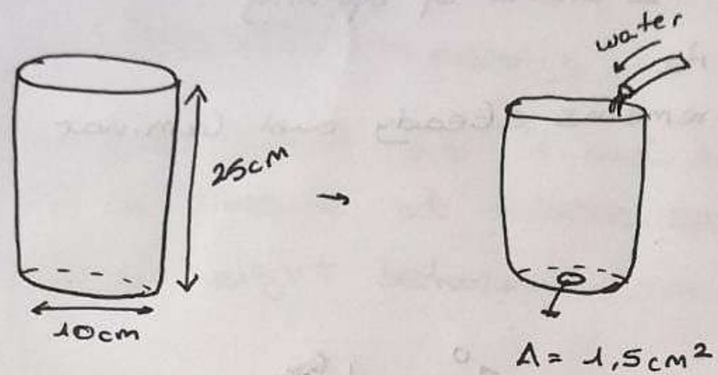
PHY 250

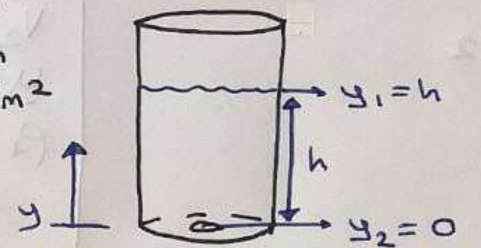
1.



rate of : $2,4 \cdot 10^{-4}\ m^3/s$

How high will the water rise?

→ Height = 0,25m
Radius = 0,05m
$1,5cm^2 = 1,5 \cdot 10^{-4} m^2$



Using: $R_V = A_1 \cdot V_1$

$$V_1 = \frac{R_V}{A_1}$$

$$V_1 = \frac{2,4 \cdot 10^{-4} m^3/s}{\pi \cdot 0,05^2 m}$$

$$\boxed{v_1 = 0,03055\, m/s}$$

speed at which the water gets into the bucket.

Knowing that $R_V = A_1 V_1 = $ const.

Rate of exit $A_2 V_2 = R_V$

$$V_2 = \frac{R_V}{A_2} = \frac{2,4 \cdot 10^{-4} m^3/s}{1,5 \cdot 10^{-4} m^2}$$

$$\boxed{V_2 = 1,6 m/s}$$

speed at which the water leaves the bucket.

Applying Bernoulli's eq.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2$$

$P_1 \to$ Patm, $y_1 \to h$, $P_2 \to$ Patm, $y_2 \to 0$

$$P_{atm} + \rho g h + \frac{1}{2}\rho v_1^2 = P_{atm} + 0 + \rho \frac{1}{2}\rho v_2 \Rightarrow \rho g h = \frac{1}{2}\rho (v_2^2 - v_1^2)$$

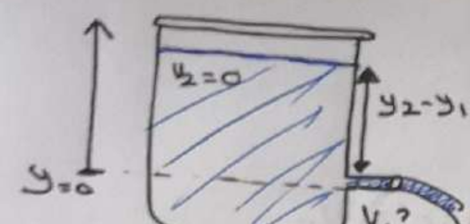
$$h = \frac{\frac{1}{2}(v_2^2 - v_1^2)}{g}$$

$$h = \frac{1}{2 \cdot 9,8}(1,6^2 - 0,03055^2)$$

$$\boxed{h = 0,13056m \simeq 13cm}$$

height the water will get to

2.



$h = y_2 - y_1$

A_1 and A_2 areas of opening

$A_1 << A_2$

Flow remains steady and laminar

Knowing:

$$R_v = A \cdot V = const$$

And

$$P_1 + \rho_1 g \cancel{y_1}^{0} + \frac{1}{2}\rho v_1^2 = P_2 + \rho_2 g y_2 + \frac{1}{2}\cancel{v_2^2}^{0}$$

$$\cancel{P_{atm}} + \frac{1}{2}\rho v_1^2 = \cancel{P_{atm}} + \rho g y_2$$

they cancel each other

$$\frac{1}{2}\rho v_1^2 = \rho g y_2$$

$$v_1^2 = \frac{2\cancel{\rho} g y_2}{\cancel{\rho}} = \boxed{V_1 = \sqrt{2g\,y_2}}$$

We know that

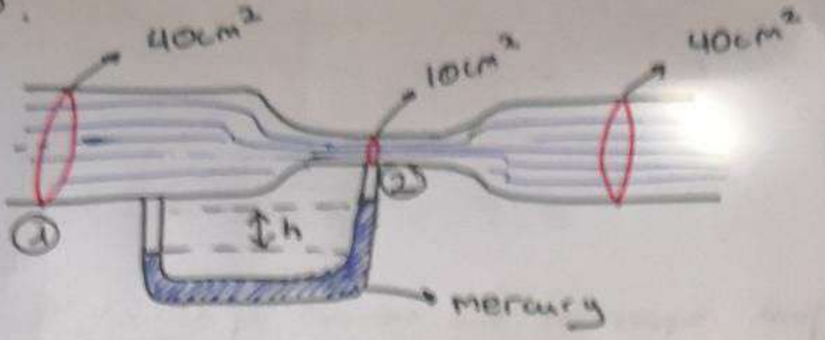
$P_1 = P_2 = P_{atm}$

$V_2 = 0\,m/s$

$y_1 = 0 \quad V_1 = ?$

$\rho_1 = \rho_2 \quad g = 9.8\,m/s$

3.



Discharge from the pipe
$6 \cdot 10^{-3} m^3/s$

a) flow speed at 1 and 2
b) diff of pressure at thease sections
c) diff of height between mercury columns

We know

$A_1 = 40cm^2 = 0{,}04m^2$

$A_2 = 10cm^2 = 0{,}01m^2$

$\rho_w = 1 \cdot 10^3 kg/m^3$

$\rho_m = 13{,}6 \cdot 10^3 kg/m^3$

Discharge $= 6 \cdot 10^{-3} m/s$

a)

we know $A_1 \cdot V_1 = \text{const} = Q = A_2 \cdot V_2$

$$0{,}04 \cdot V_1 = Q = \boxed{V_1 = \frac{6 \cdot 10^{-3}}{0{,}04} = 1{,}5 m/s}$$

$$0{,}01 \cdot V_2 = Q = \boxed{V_2 = \frac{6 \cdot 10^{-3}}{0{,}01} = 6 m/s}$$

b)

we know

they cancel

$$P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1}^{\,0} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}^{\,0} \qquad y_2 = y_1 \quad \rho_2 = \rho_1$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

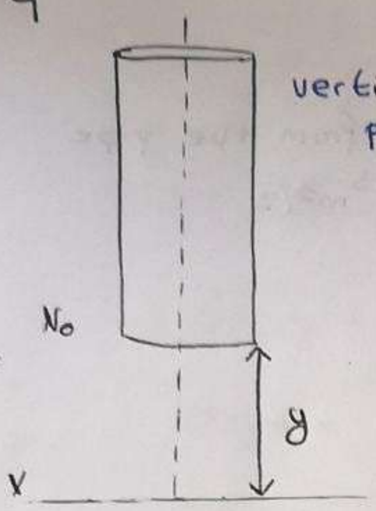
$$\boxed{\Delta P = \frac{1}{2}\rho\left(1{,}5^2 - 6^2\right) = \frac{1}{2} \cdot 10^3 \cdot \left(1{,}5^2 - 6^2\right) = -16{,}875 Pa}$$

c)

we know

$$\Delta P = \rho g \Delta h \rightarrow \boxed{\Delta h = \frac{\Delta P}{\rho g} = \frac{16{,}875}{13{,}6 \times 10^3 \cdot 9{,}8} = 0{,}127 m}$$

4



V_0 is just speed at the moment of leaving the pipe. Once liquid leaves → Free fall

We know that

$\underline{V \cdot A} = \underline{V_0 A_0}$

a) Find eq. for speed of the liquid in terms of "y". Find eq. for radius of stream as function of y.

b) $V_0 = 1{,}2 \, m/s$ → at which "y" will $r' = \frac{r}{2}$?

a) velocity formula → $V = V_0 + V_{ff}$ → velocity gained with the fall

$$V_{ff} = t * \underset{g}{a} = \sqrt{2gy}$$

$$\boxed{V = V_0 + \sqrt{2gy}}$$

Knowing the $V = V_0 A_0$ → $V_0 + \sqrt{2gy} \cdot \pi r'^2 = V_0 (\pi r_0^2)$

$$\boxed{r' = \sqrt{\frac{V_0 (\pi r_0^2)}{(V_0 + \sqrt{2gy}) \cdot \pi}}}$$

$$\boxed{r' = \sqrt{\frac{V_0 r_0^2}{V_0 + \sqrt{2gy}}}}$$

b) $r' = \frac{r}{2}$

$$\frac{r}{2} = \sqrt{\frac{1{,}2 \cdot r^2}{1{,}2 + \sqrt{2gy}}} \qquad \frac{r^2}{4} = \frac{1{,}2 \, r^2}{1{,}2 + \sqrt{2gy}}$$

$$\frac{\cancel{r^2}}{4} \cdot 1{,}2 + \sqrt{2gy} = 1{,}2 \, \cancel{r^2}$$

$$\frac{1{,}2 + \sqrt{2gy}}{4} = 1{,}2$$

→ next page

$$\frac{1,2+\sqrt{2gy}}{4} = 1,2$$

$$1,2+\sqrt{2gy} = 4,8$$

$$\sqrt{2gy} = 3,6$$

$$2gy = 3,6^2 \rightarrow y = \frac{3,6^2}{2 \cdot 9,8} \Rightarrow \boxed{y = 0,66\,m}$$