# PHY250: STATIC FLUIDS

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Digipen

Fall 2022

We are going to consider a fluid as a system of particles.

# Fluids

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- Fluids $\rightarrow$ materials that are very deformable and can flow.

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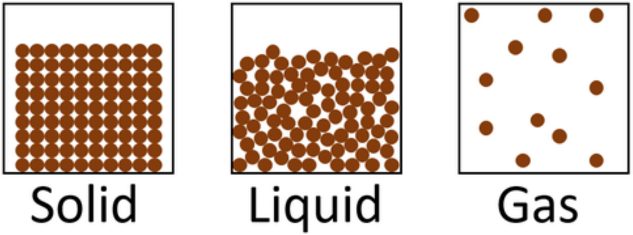
We are going to examine Static Fluids and Dynamic Fluids.



# Fluids

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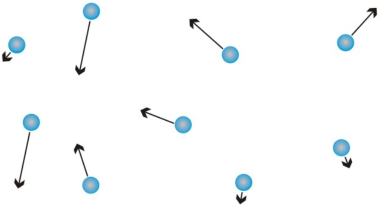

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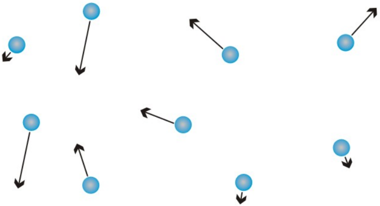

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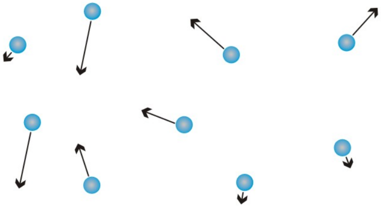

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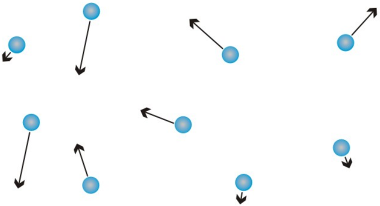

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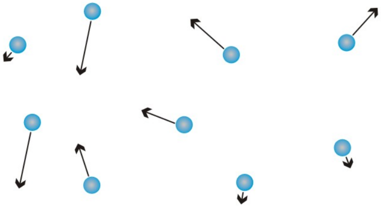

- Formed by N point particles that move in straight lines.
- They expand to occupy the whole volume of the container.
- They are compressible.
- There is no interaction between the particles.
- Elastic collisions, the kinetic energy is conserved.

# Fluids

## Liquids: Our Approach

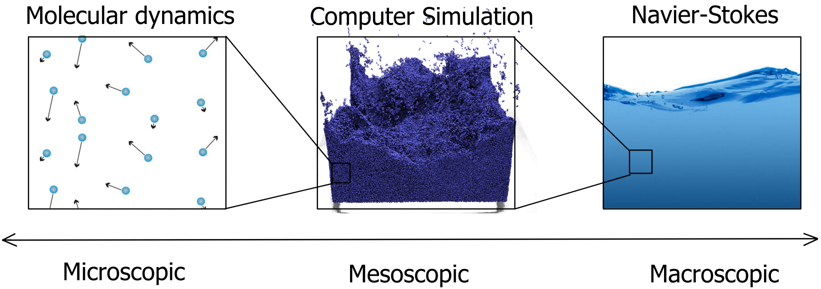


Figure adapted from https://www.mdpi.com/2076-3417/9/19/4041

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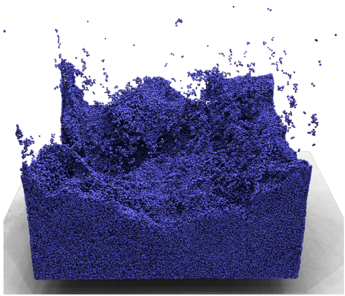

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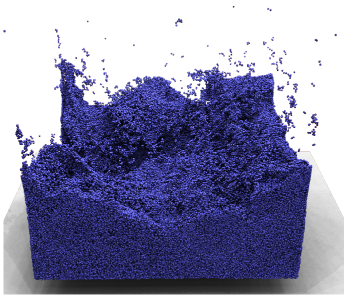

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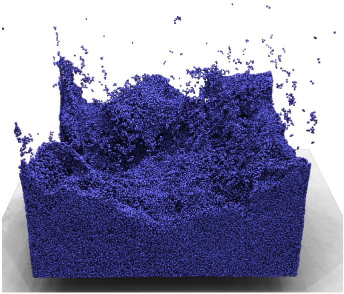

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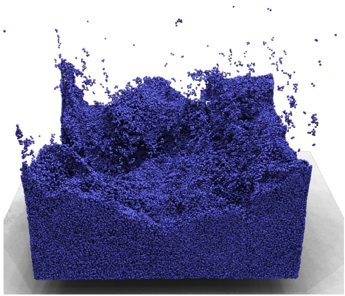

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- They are not compressible (idealization).
- Their density is constant.
- There is interaction between the particles $\rightarrow$ viscosity, superficial tension.

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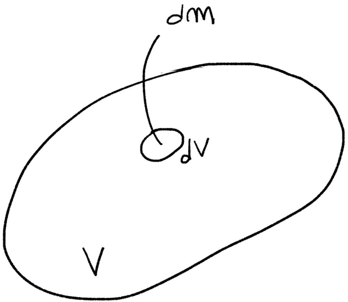

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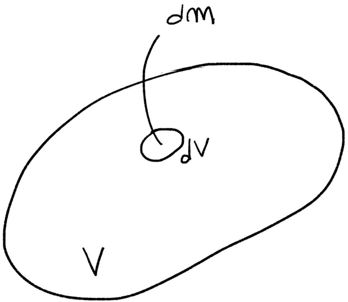


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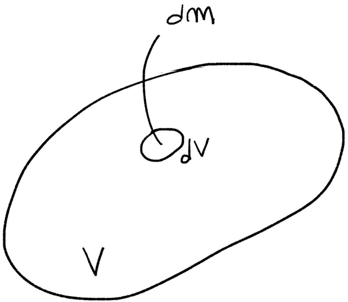


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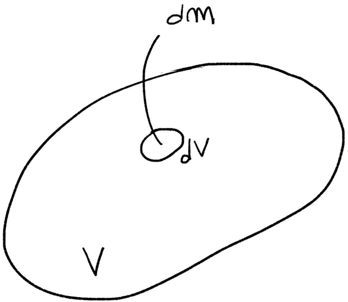


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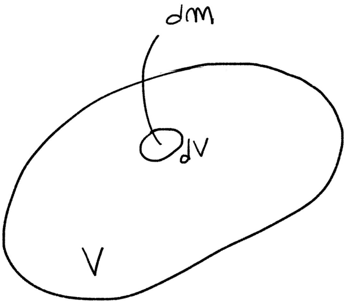


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$$w = mg = \rho V g$$

# Pressure

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At sea level $\rightarrow$ $\boxed{P_0 = 1.013 \times 10^5 \frac{N}{m^2} = 1atm}$

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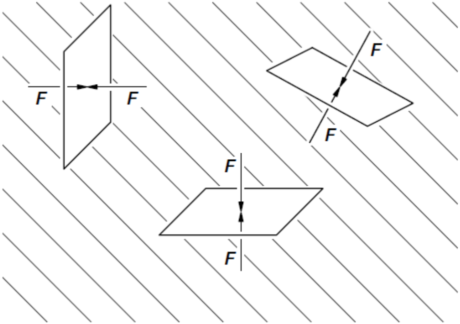


Figure: Figure from The Feynman Lectures vol II.

The pressure in a static liquid under the force of gravity

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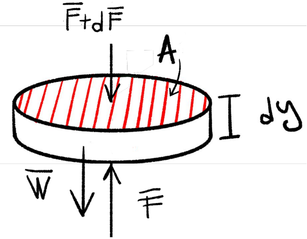

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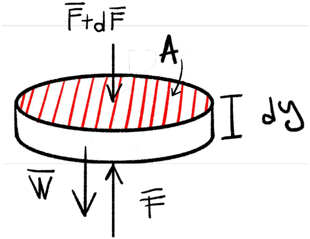


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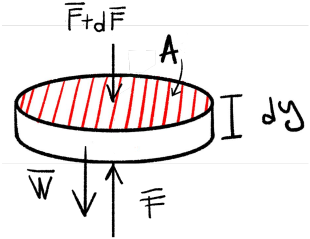


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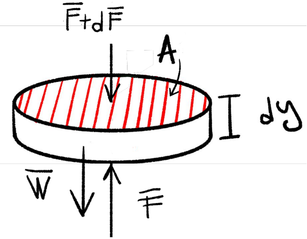


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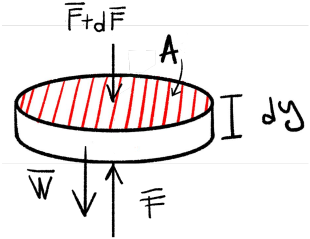


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$$\rightarrow -dP = \rho g dy \tag{4}$$

Then,

$$\frac{dP}{dy} = -\rho g$$

$\rightarrow$ The pressure decreases when y increases

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g dy$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g dy, \quad \rho = \rho(y)$$

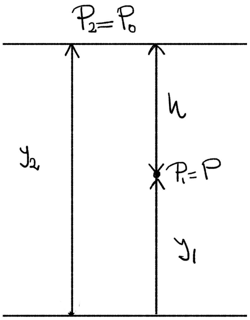



Figure: $P_0$ is the pressure due to the atmosphere above.

$\rho = constant \rightarrow \Delta P = -\rho g \Delta y$

$$P_2 - P_1 = -\rho g (y_2 - y_1)$$

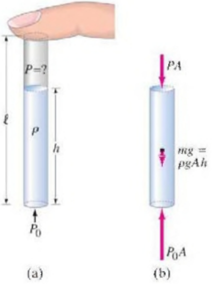
$$P_0 - P = -\rho g h$$

$$\rightarrow \boxed{P = P_0 + \rho g h} \qquad (5)$$

# Conceptual Example

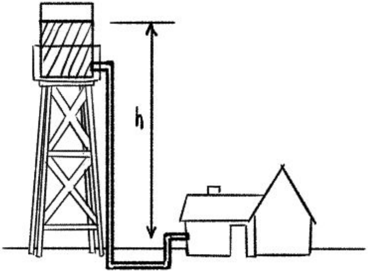
You insert a straw of length $\ell$ into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water.

How is $P$? greater than, equal to, or less than the atmospheric pressure $P_0$ outside the straw?



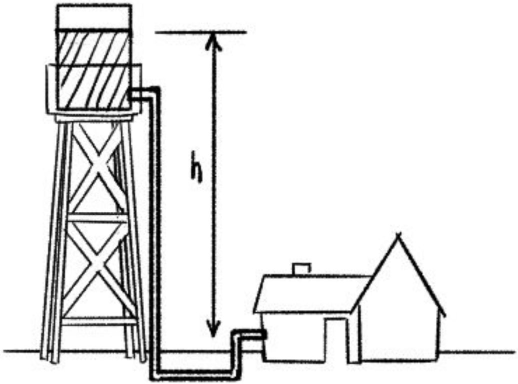
# Example 1

Pressure at a faucet: The surface of the water in a storage tank is 30 $m$ above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank (density of water:$\rho = 10^3\ kg/m^3$).



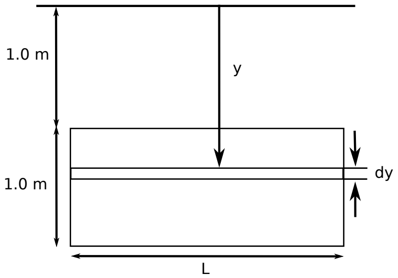
# Example 1

Solution in whiteboard



# Example 2

Force on aquarium window. Calculate the force due to water pressure exerted on a 1.0 m X 3.0 m aquarium viewing window whose top edge is 1.0 m below the water surface



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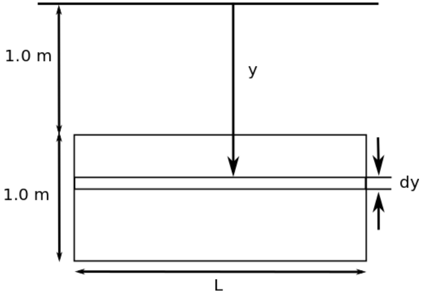

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we can solve the equation and find $P = P(y)$

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where $\rho_0$ and $P_0$ are the density and pressure at sea level.

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$$\rightarrow \boxed{P = P_0 e^{-\frac{\rho_0}{P_0}gy}} \tag{6}$$

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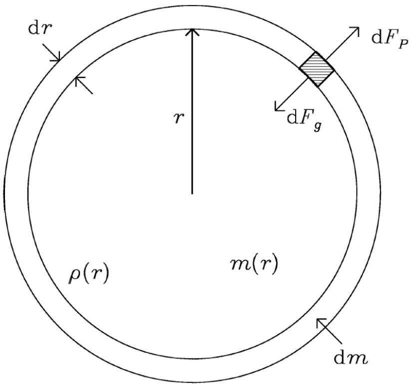
$$\rightarrow \boxed{y = 5550m}$$

## Spherical geometry: Star Model



$$F_r = PA - (P + dP)A - dF = 0$$

$$dF = G\frac{dmM(r)}{r^2} = G\frac{\rho dvM(r)}{r^2}$$

$$\rightarrow AdP = -G\frac{\rho AdrM(r)}{r^2}$$

$$\rightarrow \boxed{\frac{dP}{dr} = -G\frac{\rho M(r)}{r^2}} \tag{7}$$

The equation for the mass is,

$$dM(r) = \rho(r)dV = \rho(r)4\pi r^2 dr \tag{8}$$

$$\rightarrow \frac{dM}{dr} = \rho(r)4\pi r^2 \tag{9}$$

So if we know $\rho(r)$, we can solve the eqs. (7) and (9)

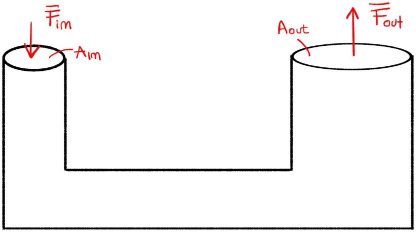
# Pascal's Principle

If an external pressure $P_0$ is applied to a confined fluid, the pressure at every point within the fluid increases by that amount $P_0$.

This means that an external pressure acting on a fluid is transmitted throughout the fluid.

$$P = \rho g h + P_0 \tag{10}$$

# Hydrostatic Lift




$$P_{in} = P_{out}$$

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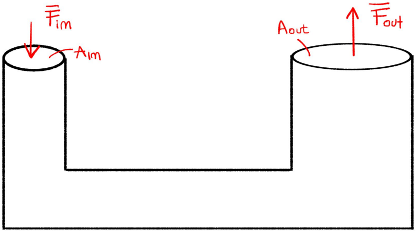


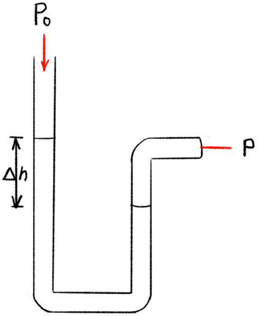

$$P_{in} = P_{out} \rightarrow \frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \tag{11}$$

$$\rightarrow F_{out} = \frac{A_{out}}{A_{in}} F_{in} \tag{12}$$

# Manometer

The simplest device to measure the pressure is the open-tube Manometer, the Pressure is related to the difference $\Delta h$ between the two levels of the liquid.

$$P = P_0 + \rho g \Delta h$$

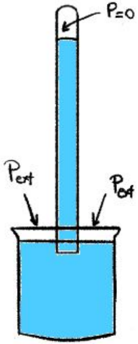


# Manometer

Instead of calculating the product $\rho g \Delta h$, sometimes only the change in height $\Delta h$ is specified. In fact, pressures are sometimes specified as so many "millimetres of mercury" (mm-Hg) or "mm of water" (mm-H20).

# Barometer

A barometer is a glass tube completely filled with mercury and then inverted into a bowl of mercury. If the tube is long enough, the level of the mercury will drop, leaving a vacuum at the top of the tube.




$$P_{ext} = \rho g \Delta h + 0$$

# Barometer

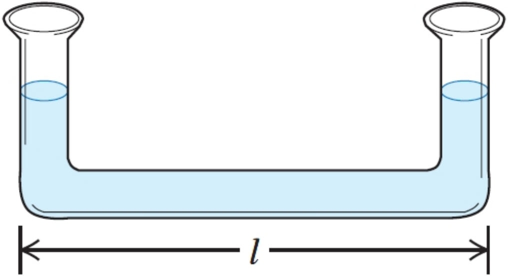
When $P_{ext} = P_0$, $\Delta h = 0.76cm$. That is, the atmospheric pressure can support a column of mercury only about 76 cm high.

If we replace the liquid by water, the column high would be $\sim 10m$

# Example 4

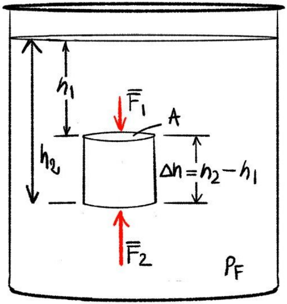
A U-shaped tube with a horizontal portion of length $\ell$ contains a liquid. What is the difference in height between the liquid columns in the vertical arms (a) if the tube has an acceleration a toward the right and (b) if the tube is mounted on a horizontal turntable rotating with an angular speed $\omega$with one of the vertical arms on the axis of rotation? (c) Explain why the difference in height does not depend on the density of the liquid or on the crosssectional area of the tube. Would it be the same if the vertical tubes did not have equal cross-sectional areas? Would it be the same if the horizontal portion were tapered from one end to the other? Explain.

# Example 4



## Archimedes' Principle

Consider a cylinder immersed in a liquid, the upward force exerted by the liquid is the **Buoyant Force**




$$
\begin{aligned}
F_B &= F_2 - F_1 \\
&= \rho_F g A(h_2 - h_1) \\
&= \rho_F g A \Delta h \\
&= m_f g
\end{aligned}
$$

# Archimedes' Principle in general

For any irregular body...

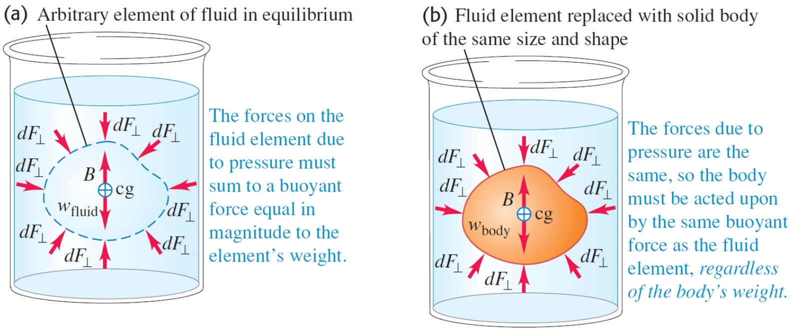



Figure: Figure from sears and zemansky's university physics 13th edition volume 1.

*The buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.*

**Two pails of water**. Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

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**Answer** Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood object; so the pails have the same weight.

## Object in equilibrium

The object is in equilibrium when

$$m_F g = m_o g$$

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$$\begin{aligned} m_F g &= m_o g \\ \rightarrow \rho_F V_{dis} g &= \rho_O V_O g \end{aligned}$$

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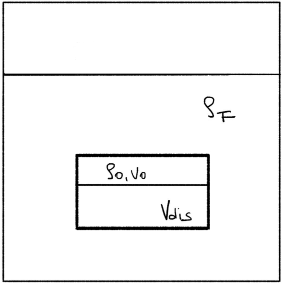
$$
\begin{aligned}
m_F g &= m_o g \\
\rightarrow \rho_F V_{dis} g &= \rho_O V_O g \\
\rightarrow \frac{V_{dis}}{V_O} &= \frac{\rho_O}{\rho_F}
\end{aligned}
$$

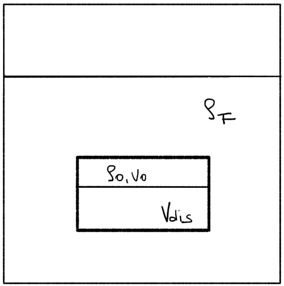
Where $V_{dis}/V_O$ is the fraction of submerged Vol.

## Object in equilibrium



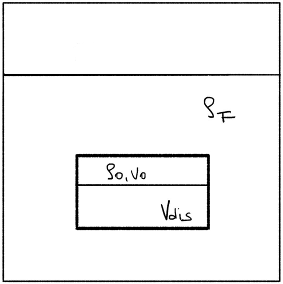

If $\rho_O/\rho_F = 1$

## Object in equilibrium




If $\rho_O/\rho_F = 1 \rightarrow V_o = V_{disp}$

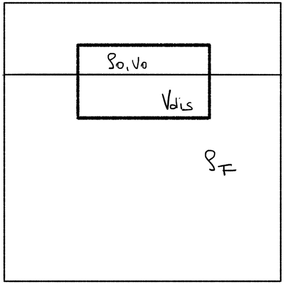
## Object in equilibrium




If $\rho_O/\rho_F = 1 \rightarrow V_o = V_{disp}$

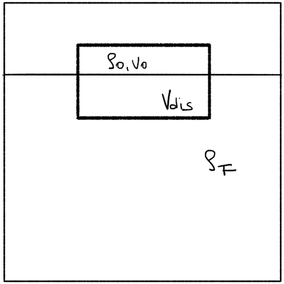
Then, the object is completely submerged and in equilibrium

## Floating objects



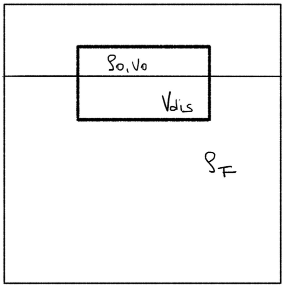

If $\rho_O / \rho_F < 1$

## Floating objects




If $\rho_O/\rho_F < 1 \rightarrow V_o > V_{disp}$

## Floating objects

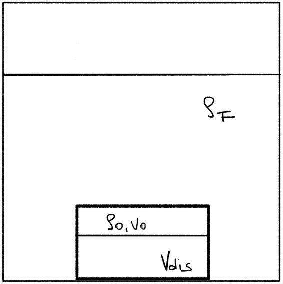



If $\rho_O/\rho_F < 1 \rightarrow V_o > V_{disp}$

Then, the object floats

## Sank object

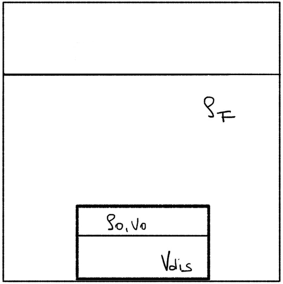
In this case, $F_B < mg$, then, the object is in equilibrium when




If $\rho_O / \rho_F > 1$

## Sank object

In this case, $F_B < mg$, then, the object is in equilibrium when




If $\rho_O/\rho_F > 1 \rightarrow$ *no equilibrium*

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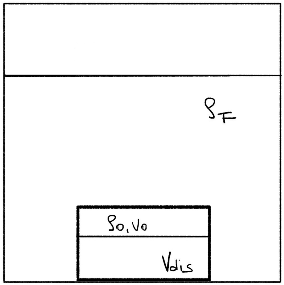



If $\rho_O/\rho_F > 1 \rightarrow$ *no equilibrium*

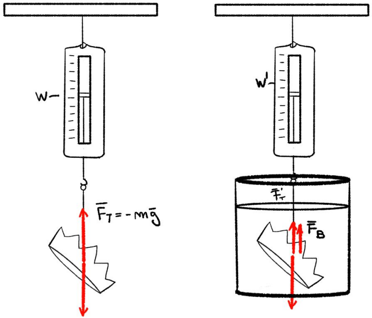
The, the object sinks.

# Example 5

**Recovering a submerged statue**. A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 cm^3$. How much force is needed to lift it?

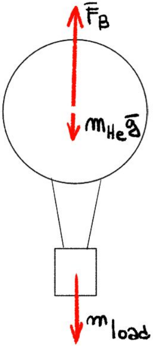
# Example 6

**Archimedes:** Is the crown gold? When a crown of mass 14.7 $kg$ is submerged in water, an accurate scale reads only 13.4 $kg$. Is the crown made of gold?



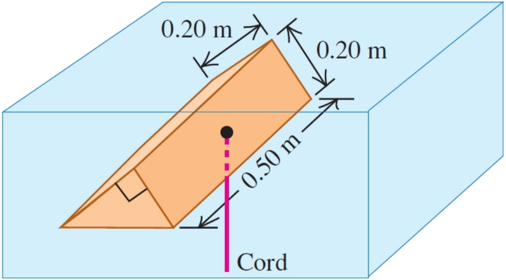
# Example 7

Helium balloon. What volume V of helium is needed if a balloon is to lift a load of 180 *kg* (including the weight of the empty balloon)?

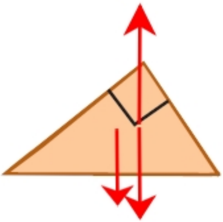


# Example 8

Suppose a piece of styrofoam, $\rho = 180\ kg/m^3$ is held completely submerged in water. (a) What is the tension in the cord? Find this using Archimedes's principle. (b) Use $p = p_0 + \rho g h$ to calculate directly the force exerted by the water on the two sloped sides and the bottom of the styrofoam; then show that the vector sum of these forces is the buoyant force.



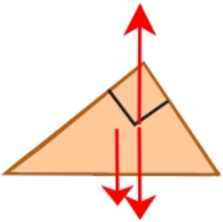
# Example 8



a)

$$F_B - T - W = 0$$

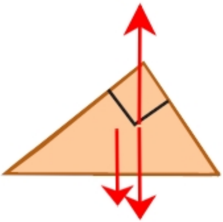
# Example 8



a)

$$\begin{aligned} F_B - T - W &= 0 \\ \rightarrow T &= (\rho_w - \rho_o)Vg \end{aligned}$$

# Example 8



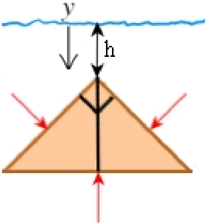
a)

$$
\begin{aligned}
F_B - T - W &= 0 \\
\rightarrow T &= (\rho_w - \rho_o)Vg \\
\rightarrow T &= \frac{1}{2}a^2\ell(\rho_w - \rho_o)g
\end{aligned}
$$

# Example 8

b) sides:

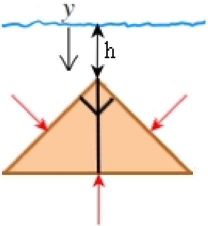
$$dF = PdA$$



# Example 8

b) sides:

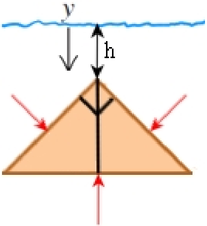
$$\begin{aligned} dF &= PdA \\ dA &= \ell dr \end{aligned}$$



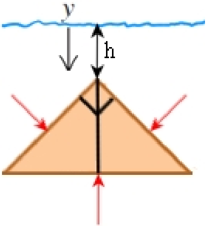
# Example 8

b) sides:

$$
\begin{aligned}
dF &= PdA \\
dA &= \ell dr \\
&= \ell \frac{2}{\sqrt{2}} dy
\end{aligned}
$$



# Example 8




b) sides:

$$\begin{aligned}
dF &= PdA \\
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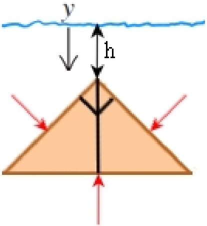
&= \ell \frac{2}{\sqrt{2}} dy \\
\rightarrow dF &= \frac{2}{\sqrt{2}} \rho g y \ell dy
\end{aligned}$$

# Example 8

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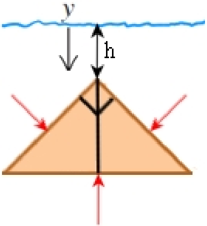
\rightarrow F &= \frac{2}{\sqrt{2}} \ell \rho g \int_{h}^{h+\sqrt{2}/2a} y dy
\end{aligned}
$$

# Example 8

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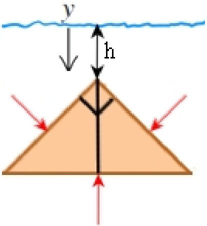
&= \boxed{\rho g a h \ell + \frac{\rho g}{2\sqrt{2}} a^2 \ell}
\end{aligned}
$$

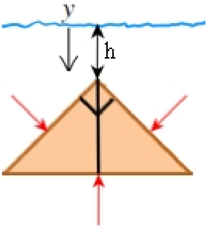
# Example 8




b) bottom:

$$F_{Bo} = \rho g(h + \frac{\sqrt{2}}{2}a)a\sqrt{2}\ell$$

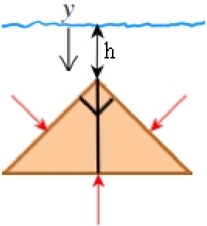
# Example 8




Total:

$$F_B = F_{Bo} - 2F_S\frac{\sqrt{2}}{2}$$

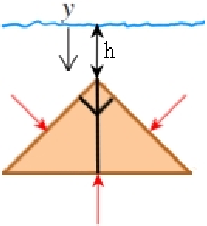
# Example 8




Total:

$$
\begin{aligned}
F_B &= F_{Bo} - 2F_S\frac{\sqrt{2}}{2} \\
&= \rho g \frac{a^2\ell}{2}
\end{aligned}
$$

# Example 8




Total:

$$
\begin{aligned}
F_B &= F_{Bo} - 2F_S\frac{\sqrt{2}}{2} \\
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&= \rho g V
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$$

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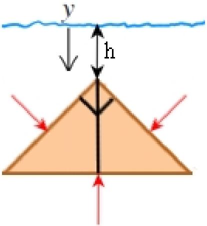


Total:

$$
\begin{aligned}
F_B &= F_{Bo} - 2F_S\frac{\sqrt{2}}{2} \\
&= \rho g\frac{a^2\ell}{2} \\
&= \rho g V \\
&= \boxed{g m_w} \quad \checkmark
\end{aligned}
$$

## Generalization

- The pressure on any object is perpendicular to the surface.
- If the only external force is the gravity, near earth, we have $\frac{dP}{dy} = -\rho g$
- The Arquimede's Principle is a consequence of the previous 2 items.

## Generalization

If the external force is the gravity,

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Which is the equivalent expression for an arbitrary force?

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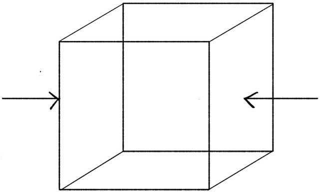
Which is the equivalent expression for an arbitrary force?

In general,

$$\vec{F} = \vec{F}(x, y, z), \quad P = P(x, y, z) \tag{13}$$

## Generalization

The resultant force on the x-direction due to the pressure of the liquid is,



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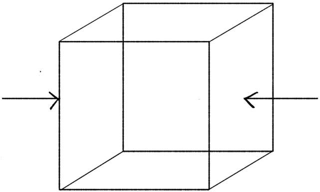

$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

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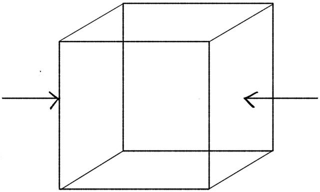

$$F_x = Pdydz - (P + dP_x)dydz = -dP_x dydz$$

and,

$$dP_x = \frac{\partial P}{\partial x} dx$$

## Generalization

Then,

$$F_x = -\frac{\partial P}{\partial x} dxdydz$$

or,

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where $f_x$ is the force per unit volume.

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where $f_x$ is the force per unit volume.

The fluid is in hydrostatic equilibrium if,

$$f_x + f_x^e = 0$$

## Generalization

Then,

$$-\frac{\partial P}{\partial x} = -f_x^e$$

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If the external force is conservative,

$$f_x^e = -\frac{\partial U}{\partial x} \rightarrow f_x^e = -\rho \frac{\partial \phi}{\partial x}$$

## Generalization

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If the external force is conservative,

$$f_x^e = -\frac{\partial U}{\partial x} \rightarrow f_x^e = -\rho\frac{\partial \phi}{\partial x}$$

$$\rightarrow -\frac{\partial P}{\partial x} = \rho\frac{\partial \phi}{\partial x}$$

## Generalization

Then, for the 3 spacial directions...

$$
\begin{aligned}
-\frac{\partial P}{\partial x} &= \rho \frac{\partial \phi}{\partial x} \\
-\frac{\partial P}{\partial y} &= \rho \frac{\partial \phi}{\partial y} \\
-\frac{\partial P}{\partial z} &= \rho \frac{\partial \phi}{\partial z}
\end{aligned}
$$

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-\frac{\partial P}{\partial z} &= \rho\frac{\partial \phi}{\partial z}
\end{aligned}
$$

Using the nabla operator,

$$\nabla P = -\rho\nabla\phi \tag{14}$$

## Generalization

Then, the most general expression for hydrostatic equilibrium is,

$$\boxed{\nabla P + \rho \nabla \phi = 0} \tag{15}$$

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If the density is constant, the equation has a solution,

$$\boxed{P + \rho \phi = constant} \tag{16}$$

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Another possibility which allows hydrostatic equilibrium is when $\rho = \rho(P)$.