PHY115: Final Project

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1 Theory Questions

1.1 *Newton's Laws*

a) Newton's Laws are only considered valid when using an inertial frame of reference, which means that the frame of reference is always moving in linear motion at a constant speed (can be zero), unaffected by any external force.

b) Yes, it can.
The second law of motion states that the acceleration of an object is determined by the mass of that object and the force applied into it; while the first law states that an object won't change its own speed nor trajectory unless an external force acts on it.
Therefore, applying the equation of the second law of motion (F=m*a) on a body with a constant mass, we can see that if F (force) is equal to zero, the acceleration will also be 0, which coincides with the first law of motion.

c) No, it's not. Even if they are closely related, the normal force is not always the reaction pair of the weight.
(*W=m*g; N=-cos(α)*m*g*)
When calculating the normal, on a horizontal surface (with no acceleration), we can see that the magnitudes of weight and normal vectors are equivalent but in opposite directions. The thing is that, when the surface is inclined, that inclination is also a factor to consider, so the normal is no longer the exact opposite of the weight.

1.2 *Linear Momentum*

a) Under Newton's Laws, linear momentum is a vector, product of the mass and the velocity of a particle (or system of particles), whose value is conserved inside the system of reference. It reflects the quantity of motion of that particle (or system of particles) on a straight line. (P=m*v)

b) The change in the momentum of a body is equal to the net force that acts on it. (Δp=ΣF*Δt)

c) As it is stated in the second law, the linear momentum of a particle (or system of particles) is conserved as long as there is no external force acting on it.

d) A component of the momentum is conserved when the force applied to the particle (or system of particles) only affects one of the coordinate directions. For example, if the force is applied along the *x* axis, the *x* axis of the momentum vector will change while the *y* axis remains the same. Obviously if there is no force applied to the to the particle, all its components are conserved.

e) If a body is resting on a non-accelerating surface, with a zero linear motion in either of the axis, and other body collides with it exerting a force on the x axis, our first body will change its momentum on that axis (x), while keeping the y axis momentum the same as it was at the beginning. The change of the momentum on one of the axis, while the other is kept unbothered, leads to a change in the value of the linear momentum of the body as a whole.

f) Incredibles 2 (2018) - Elastigirl Saves The Train Scene (HD) (*min 3:16*)

1.3 *Angular Momentum*

a) Torque: The value of the tendency of a force to cause rotation.
Angular momentum: The product of the moment of inertia and the angular velocity of a particle, which determines the amount of rotation of said particle.

b) $\tau=\Delta L/\Delta t$
Torque is expressed as the variation of angular momentum over the variation of time.

c) $L=I*\omega$
This equation reflects that, just as the linear momentum ($p=m*v$), the angular momentum relates mass and velocity. But, in this case, it also takes into account the radius of the rotation ($I=m*r**2$).

d) $\tau=I*\alpha$
According to Newton's Second Law, the force on a body equals the mass times its acceleration. In the equation, we can see how the torque equals the mass times the acceleration, and just as in the previous one, it also takes into account the radius of the rotation.

e) The angular momentum of a body is conserved whenever its torque is equal to 0 over a given period of time.
This would happen for example to a body that, in space, receives an initial push by a force (can't be directed towards the centre of mass, wouldn't spin), and then is not affected by any other force. This would cause the body to float away in a continuous state of rotation.

f) "Gravity" continuous shot. Opening Scene. Space debris hits Explorer (*min 3:08*)

2 Test Your Understanding

2.1 *Explosion scale.*

- To convey the impression that the explosion is big, you can increase the velocity and the reach of the objects. A higher velocity with the same mass means a higher linear momentum (P=m*v).

2.2 *Solid or hollow?*

- You measure the treasured ball radius and mass. Then, with a material that can meet the conditions (the density of it would be the key), you make a solid sphere that has the same mass and radius as the client's. So you now have two balls that are exactly the same shape and the same mass.
  Now you can either make the roughness of the second ball's material match the client's, or, you cover both of them in the same material, keeping the form but augmenting the radius ever so slightly. So you get two balls with the same mass, same radius and same roughness.
  If you roll both balls down a slope you can have two different results:
  1. Both balls roll at the same speed, which means that the precious ball is solid.
  2. The treasured ball rolls slower than the second one, which would mean that it is hollow.
- The explanation of this comes in the distribution of mass of both balls. The mass distribution inside a body does not change the Angular momentum of the body itself, but it can change the moment of inertia:
  The further away from the centre the mass is distributed, the higher moment of inertia the body will have (I=m*r**2). If the angular momentum keeps the same, this can only mean that the angular velocity will decrease (L=I*ω).
  So, the solid ball, having its mass distributed uniformly (small r), has a low moment of inertia, and, therefore, a "high" angular velocity. And a hollow ball would have a bigger radius and low moment of inertia, and, thus, a lower angular velocity.

2.3 "2001 *space odyssey*"

- The space station rotates a quarter in 10 seconds (1:00-1:10), so the full rotation must be 40s.

$t = 40s$

$v = \frac{2\cdot\pi\cdot r}{t}$

$\omega = \frac{2\pi}{T}$

$F = m\cdot a_r$

$a_r = \frac{v^2}{r}$

- $\omega = \frac{2\pi}{40} = \frac{\pi}{20}$

consider that people inside the station experience an acceleration equivalent to that of gravity.

$v = \frac{2\pi\cdot r}{t} = \frac{2\pi}{t}\cdot r = \omega\cdot r$

$F = m\cdot\frac{v^2}{r} = m\cdot\frac{(\omega\cdot r)^{\cancel{2}}}{\cancel{r}} = m\cdot\omega^2\cdot r$

$\hookrightarrow \cancel{m}\cdot g = \cancel{m}\cdot\omega^2\cdot r$

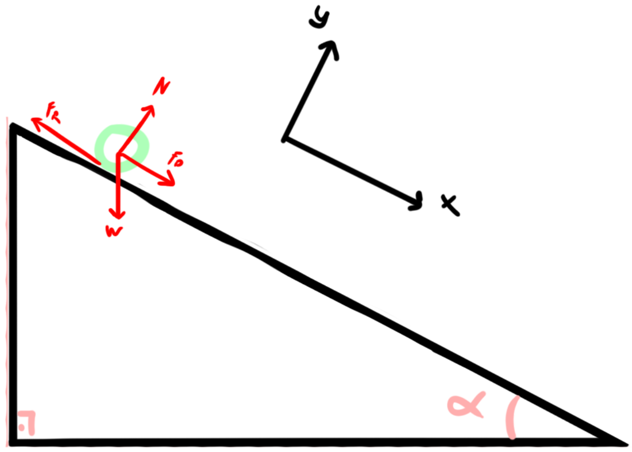
$\rightarrow r = \frac{g}{\omega^2} = \frac{9'8}{\left(\frac{\pi}{20}\right)^2}$

$\boxed{r = 397'58\ m}$

3 Octave Project

3.1 Theory

1.



$\alpha = 30°$

$Y_0 = 5\ cm \quad Y = 0\ cm$

$X_0 = 0\ cm \quad X = 10\ cm$

Metal + SKIN $\rightarrow \mu : 0'8 - 1'0$

$R = 5\ cm$

$$\left.\begin{array}{l}\tau = F_R R \\ \tau = I\alpha\end{array}\right] \rightarrow F_R R = I\alpha$$

$$F_D - F_R = m \cdot a$$
$$F_D = mg \sin\alpha$$
$$F_R = \mu N = \mu mg \cos\alpha$$

$$\alpha = \frac{a}{R}$$

$$I = \frac{2}{5} M R^2$$
$$I = \frac{1}{2} M \cdot (R_1^2 + R_2^2$$
$$I = \frac{1}{2} M R^2$$

2. The first equation for each object (the translation equation) is a statement of Newton's second law, which tells us that the net force acting on an object is equal to the mass of the object times its acceleration. In this case, the net force is the force of friction between the object and the incline minus the component of the object's weight that is parallel to the incline.

   The rotation equation for each object is based on Newton's second law of rotation, which tells us that the torque acting on an object is equal to its moment of inertia times its angular acceleration. In this case, the torque acting on the object is due to the force of friction between the object and the incline, which acts to rotate the object as it rolls down the incline. The moment of inertia depends on the shape and mass distribution of the object, and determines how easily it rotates. The angular acceleration determines how quickly the object rotates as it rolls down the incline.

3. ACCELERATION

$$F_R R = I \frac{a}{R}$$

$$F_R = I \cdot \frac{a}{R^2}$$

$$Mg \sin\alpha - I \frac{a}{R^2} = Ma$$

$$Mg \sin\alpha - \boxed{I} \frac{a}{R^2} = Ma$$

$$\boxed{\frac{2}{5} MR^2} \qquad \boxed{MR^2} \qquad \boxed{\frac{1}{2} MR^2}$$

$$x \to \cancel{M}g \sin\alpha - \frac{2}{5} \cancel{M} \frac{a}{\cancel{R^2}} = \cancel{M}a \qquad a = \boxed{\frac{5}{7} g \sin\alpha}$$

$$y \to \cancel{M}g \sin\alpha - \cancel{MR^2} \frac{a}{\cancel{R^2}} = \cancel{M}a \qquad a = \boxed{\frac{1}{2} g \sin\alpha}$$

$$z \to \cancel{M}g \sin\alpha - \frac{1}{2} \cancel{MR^2} \frac{a}{\cancel{R^2}} = Ma \qquad a = \boxed{\frac{4}{6} g \sin\alpha}$$

ANGULAR ACCELERATION

$$\alpha = a \cdot t \cdot R$$

$$\alpha = \frac{5}{7} g \sin\alpha \cdot t \cdot 0'5 \qquad \alpha = \frac{1}{2} g \sin\alpha \cdot t \cdot 0'5 \qquad \alpha = \frac{4}{6} \cdot 980 \sin\alpha\, t$$

FRICTION COEFFICIENT

$$\cancel{g} \cdot (\sin\alpha - \mu \cos\alpha) = \frac{5}{7} \cancel{g} \sin\alpha \qquad \cancel{g} \cdot (\sin\alpha - \mu \cos\alpha) = \frac{1}{2} \cancel{g} \sin\alpha \qquad \cancel{g} \cdot (\sin\alpha - \mu \cos\alpha) = \frac{4}{6} \cancel{g} \sin\alpha$$



4. Respecting the angle we have chosen for the slope, we should be using the Highest friction coefficient.This because if we take another lower, the object that need more will slip. That is why we have chosen this material that fits on this coefficient perfectly.

5.

$$\cancel{g} \cdot (\sin\alpha - \mu \cos\alpha) = \frac{5}{7} \cancel{g} \sin\alpha$$
$$\sin\alpha - \mu \cos\alpha = \frac{5}{7} \sin\alpha$$
$$\frac{2}{7} \sin\alpha = \mu \cos\alpha$$
$$\mu = \frac{2}{7} \tan\alpha$$
$$\boxed{\mu = 0'165}$$

$$g \cdot (\sin\alpha - \mu \cos\alpha) = \frac{1}{2} \cancel{g} \sin\alpha$$
$$\sin 30 - \mu \cos 30 = \frac{1}{2} \sin 30$$
$$\frac{1}{2} \sin 30 = \mu \cos 30$$
$$\mu = \frac{1}{2} \tan 30$$
$$\boxed{\mu = 0{,}289}$$

$$\cancel{g} \cdot (\sin\alpha - \mu \cos\alpha) = \frac{4}{6} \cancel{g} \sin\alpha$$
$$\sin 30 - \mu \cos\alpha = \frac{4}{6} \sin 30$$
$$\frac{2}{6} \sin 30 = \mu \cos 30$$
$$\mu = \frac{2}{6} \tan 30$$
$$\boxed{\mu = 0{,}192}$$

6. For the objects not to slide, the three of them have to have a value of the coefficient of friction over that value we obtained in the last exercise. For all of them to fulfill that condition, we have to choose the biggest value (from the hollow cylinder), being μ = 0.289. As we have chosen a combination of materials with a friction value way over that, we will have no problem with that.

Skin	Metals	Clean and Dry	0.8 - 1.0

7. For the equations of motion of the object, we have to divide them in two groups:

- Translation (½*a*t^2) (in 3 dimensions the vertical plane is z, so z will take the values of y, and y is now 0):
  x = 0.5*((5/7)*g*cos(θ))*t^2
  y = 0
  z = -0.5*((5/7)*g*sin(θ))*t^2
  x = 0.5*((1/2)*g*cos(θ))*t^2
  y = 0
  z = -0.5*((1/2)*g*sin(θ))*t^2
  x = 0.5*((2/3)*g*cos(θ))*t^2
  y = 0
  z = -0.5*((2/3)*g*sin(θ))*t^2

- Rotation (just one axis rotation):
  x = 0
  y = 0.5*((5*g*sin(θ))/(7*5))*t^2
  z = 0
  x = 0
  y = 0.5*((g*sin(θ))/(2*5))*t^2
  z = 0
  x = 0
  y = 0.5*((2*g*sin(θ))/(3*5))*t^2
  z = 0

3.2 Practice

*video file attached next to this pdf