

PHY115

Motion in More Than one Dimension

Digipen

Spring 2023

Motion in two or three dimensions

Projectile Motion

Circular Motion

Position and velocity Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (1)$$

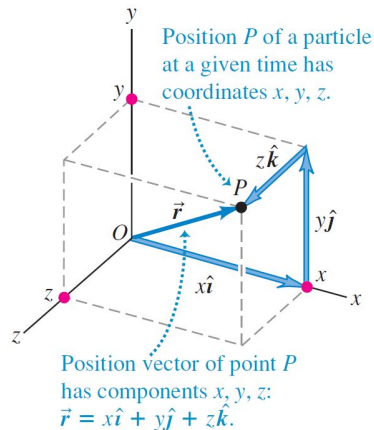


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Position and velocity Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (2)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \quad (3)$$

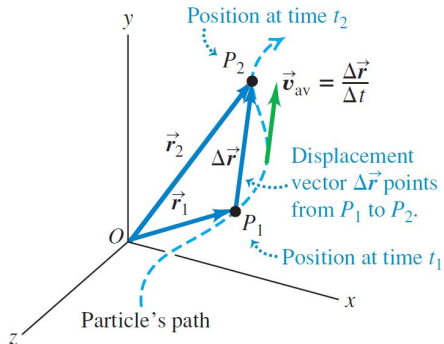


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Position and velocity Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (4)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \quad (5)$$

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \quad (6)$$

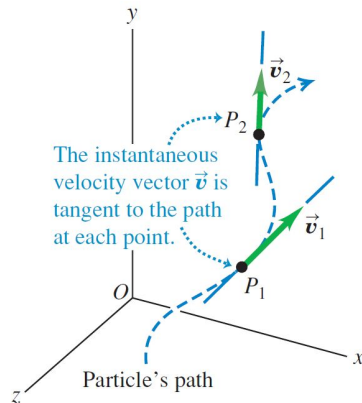


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

The Instantaneous velocity is,

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \quad (7)$$

where,

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt} \text{ and } v_z = \frac{dz}{dt} \quad (8)$$

Its magnitude is,

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (9)$$

When the motion is in the $x - y$ -plane

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \quad (10)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} \quad (11)$$

$$\tan\alpha = \frac{v_y}{v_x} \quad (12)$$

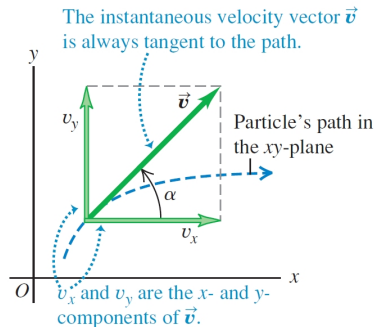


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EXERCISE 3.3

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy -plane. The rover, which we represent as a point, has x - and y -coordinates that vary with time:

$$x = (2.00 - 0.25t^2) \text{ m} \quad (13)$$

$$y = (t + 0.025t^3) \text{ m} \quad (14)$$

(a) Find the rover's coordinates and distance from the lander at $t = 2 \text{ s}$. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0 \text{ s}$ to $t = 2 \text{ s}$.

The Instantaneous acceleration is,

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} \quad (15)$$

where,

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt} \text{ and } a_z = \frac{dv_z}{dt} \quad (16)$$

Its magnitude is,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad (17)$$

Parallel and Perpendicular Components of Acceleration

A useful way to think about the acceleration is in terms of its component parallel to the path and its component perpendicular to the path.

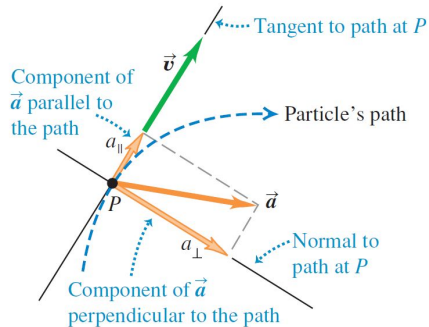
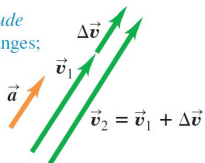


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

(a) Acceleration parallel to velocity

Changes only *magnitude*
of velocity: speed changes;
direction doesn't.



(b) Acceleration perpendicular to velocity

Changes only *direction* of
velocity: particle follows
curved path at constant
speed.

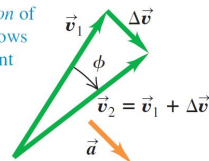
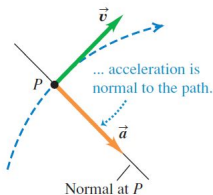


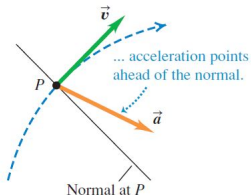
Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Velocity and acceleration vectors for a particle moving through a point P on a curved path with (a) constant speed, (b) increasing speed, and (c) decreasing speed.

(a) When speed is constant along a curved path ...



(b) When speed is increasing along a curved path ...



(c) When speed is decreasing along a curved path ...

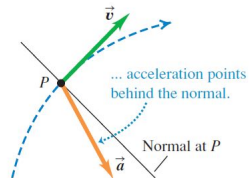


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Conceptual Example:

A skier moves along a ski-jump ramp. The ramp is straight from point A to point C and curved from point C onward. The skier speeds up as she moves downhill from point A to point E, where her speed is maximum. She slows down after passing point E. Draw the direction of the acceleration vector at each of the points B, D, E, and F.

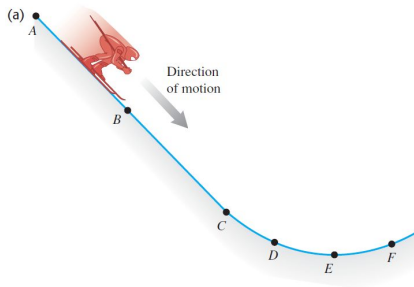


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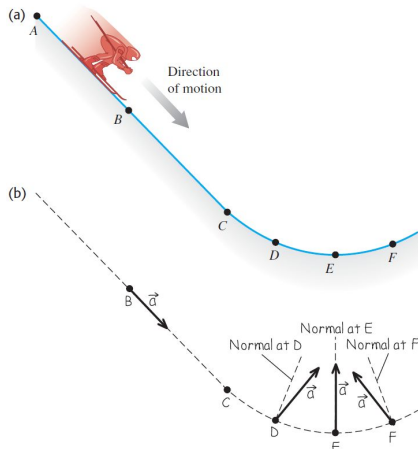


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Test Your Understanding

A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)

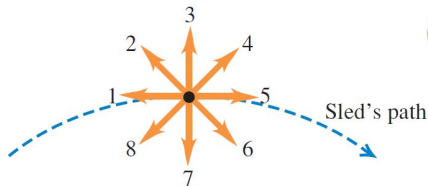


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Projectile Motion

A projectile is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

The path followed by a projectile is called its trajectory.

Projectile Motion

- ▶ we can treat the x - and y -coordinates separately.
- ▶ The x -component of acceleration is zero, and the y -component is constant and equal to $-g$
- ▶ So, projectile motion is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

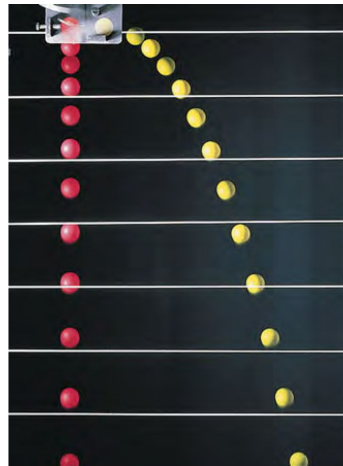


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

(18)

(19)

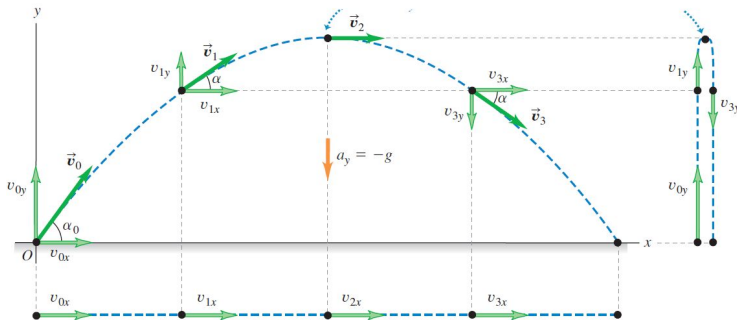
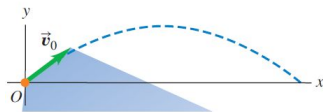


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.



$$v_{x0} = ? \quad v_{y0} = ?$$

$$v_{x0} = v_0 \cos \alpha_0, \quad v_{y0} = v_0 \sin \alpha_0$$

$$\tan \alpha_0 = \frac{v_{x0}}{v_{y0}}$$

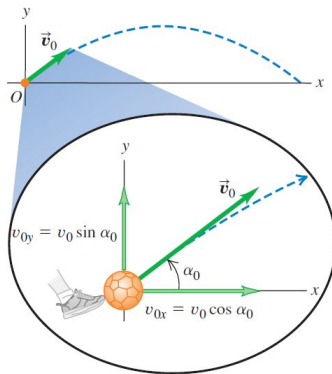


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

$$x = (v_0 \cos \alpha_0)t \quad (\text{projectile motion})$$

$$y = (v_0 \sin \alpha_0)t - \frac{1}{2}gt^2 \quad (\text{projectile motion})$$

$$v_x = v_0 \cos \alpha_0 \quad (\text{projectile motion})$$

$$v_y = v_0 \sin \alpha_0 - gt \quad (\text{projectile motion})$$

Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Motion Path?

Motion Path?

$$y(x) = (\tan\alpha_0)x - \frac{g}{2v_0^2 \cos^2\alpha_0}x^2$$

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PARABOLA

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PARABOLA

$$\rightarrow y(x) = bx - cx^2 \quad \text{parabola}$$

What if we include air resistance?

What if we include air resistance?

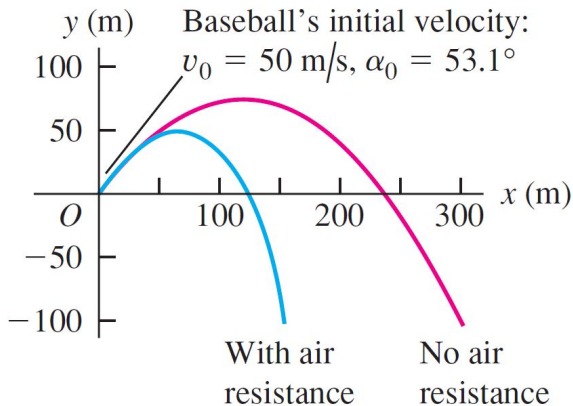


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HEIGHT AND RANGE OF A PROJECTILE

HEIGHT AND RANGE OF A PROJECTILE

A batter hits a baseball so that it leaves the bat at speed $v_0 = 37 \text{ m/s}$ at an angle $\alpha_0 = 53.1^\circ$. Find:

1. the time for the highest point, and its height h at this times
2. the horizontal range R

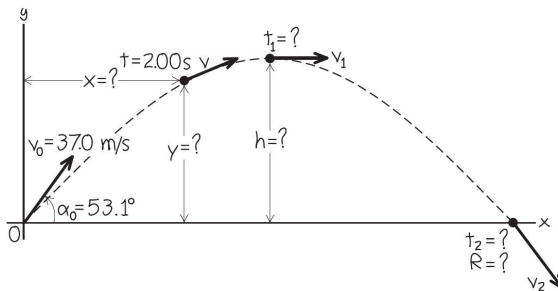


Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 \sin \alpha_0}{g}$$

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$$h = v_{0y}t - \frac{1}{2}gt^2$$

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$$\rightarrow h = \frac{v_0^2 \sin^2 \alpha_0}{2g}$$

HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 \sin \alpha_0}{g}$$

$$h = v_{0y}t - \frac{1}{2}gt^2$$

$$\rightarrow h = \frac{v_0^2 \sin^2 \alpha_0}{2g}$$

EXTRA-CREDIT: PROOF IT

RANGE

$$t_f = 2 \frac{v_0 \sin \alpha_0}{g}$$

$$R = v_{0x} t = 2v_{0x} \frac{v_0 \sin \alpha_0}{g} = 2v_0 \cos \alpha \frac{v_0 \sin \alpha_0}{g}$$

$$\rightarrow R = \frac{v_0^2 \sin 2\alpha_0}{g}$$

VARIATION OF RANGE WITH INITIAL INCLINATION

A 45° launch angle gives the greatest range;
other angles fall shorter.

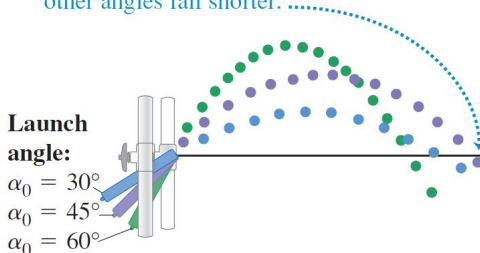


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

A classic Physics problem: "The zookeeper and the monkey"

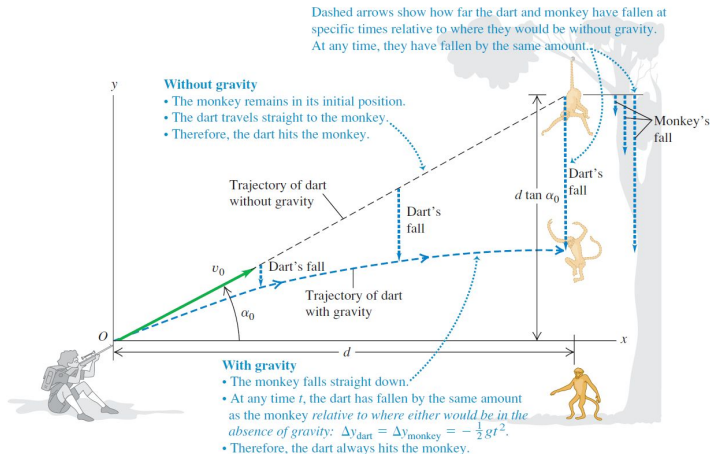


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DART:

$$x_d = v_{0x}t$$

$$y_d = v_{0y}t - \left(\frac{1}{2}\right)gt^2$$

MONKEY:

$$x_m = D$$

$$y_m = -\left(\frac{1}{2}\right)gt^2 + H$$

$$\rightarrow \tan \alpha = \frac{H}{D}$$

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The dart is going to hit the monkey as long the zookeeper points right to the monkey at the beginning.

$$\rightarrow \tan \alpha = \frac{H}{D}$$

The dart is going to hit the monkey as long the zookeeper points right to the monkey at the begining.

EXTRA-CREDIT: PROOF IT

Suppose the tranquilizer dart has a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.

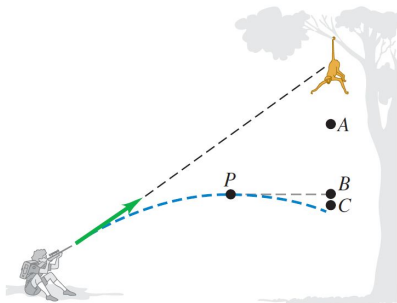


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Uniform Circular Motion

- ▶ Motion in a circle with constant speed.
- ▶ There is no component of acceleration parallel (tangent) to the path.
- ▶ The acceleration vector is perpendicular to the path and hence directed inward.
- ▶ This causes the direction of the velocity to change without changing the speed

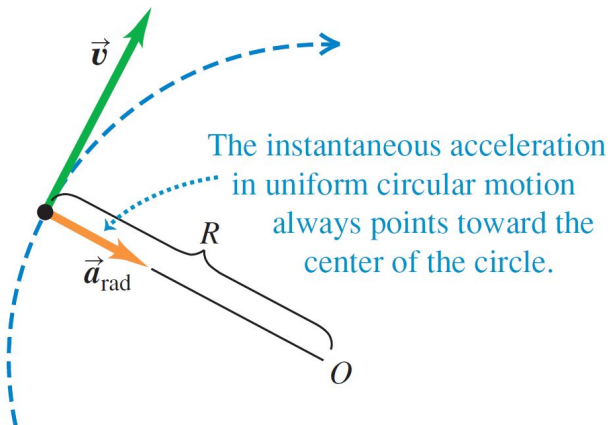


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Magnitude of \vec{a} ?

$$\Delta\phi R = \Delta s$$

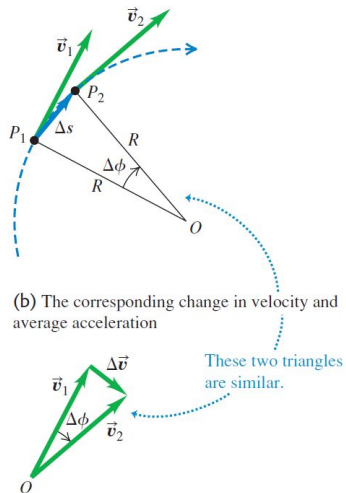


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Magnitude of \vec{a} ?

$$\Delta\phi R = \Delta s$$

$$\Delta\phi v_1 = |\Delta\vec{v}| \rightarrow \frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R}$$

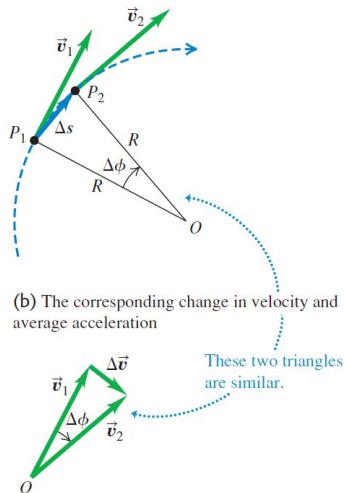


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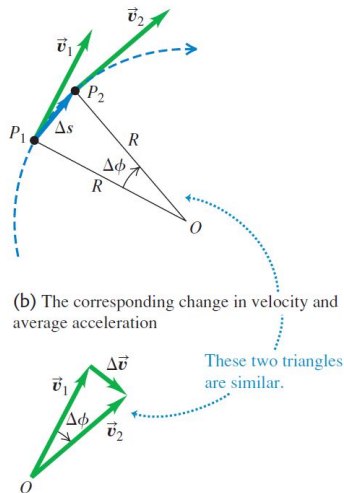


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$$\Delta\vec{v} = v_1 \frac{\Delta s}{R} \rightarrow a_{av} = \frac{v_1}{R} \frac{\Delta s}{\Delta t}$$

Taking the limit for $\Delta t \rightarrow 0$

$$a = \frac{v^2}{R}$$

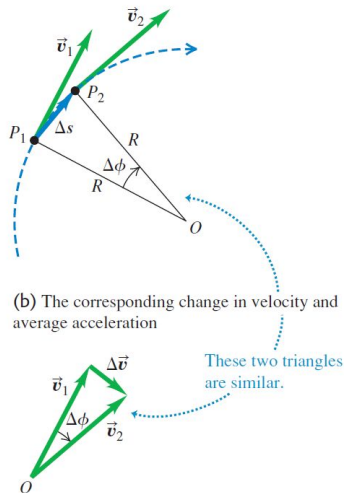


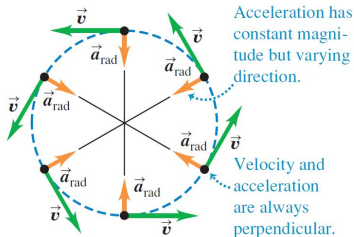
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Uniform Motion vs. Projectile Motion

1. The magnitude of acceleration is the same at all times.
2. In uniform circular motion the direction of \mathbf{a} always points toward the center of the circle.

Uniform Motion vs. Projectile Motion

(a) Uniform circular motion



(b) Projectile motion

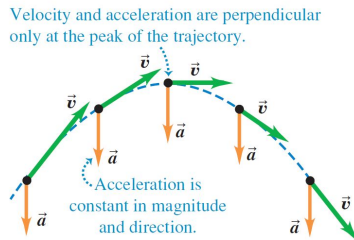
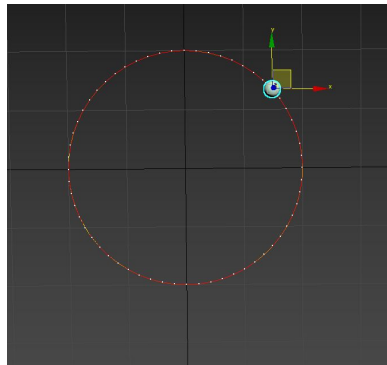
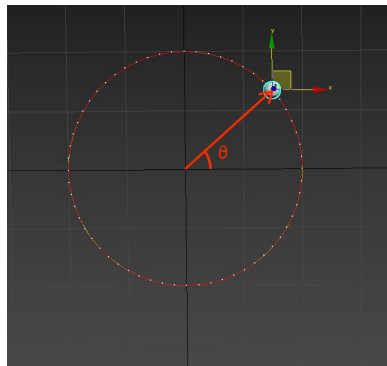


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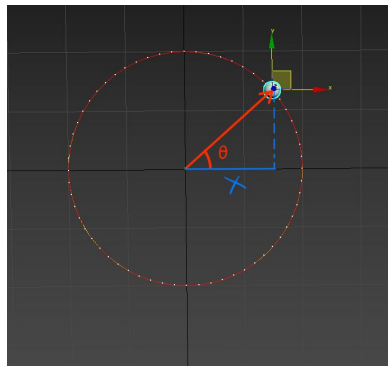
Coordinates in circular motion?



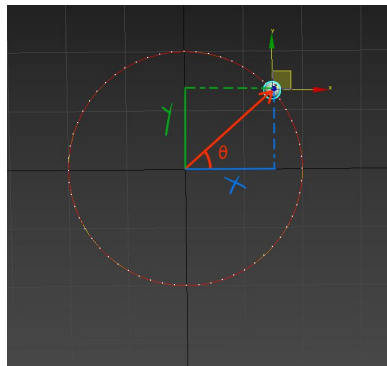
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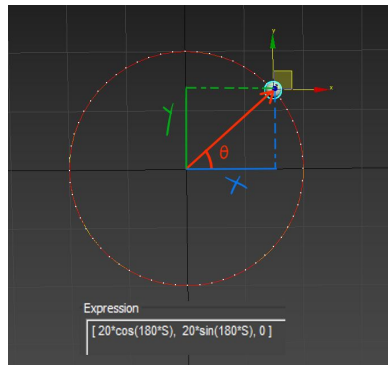
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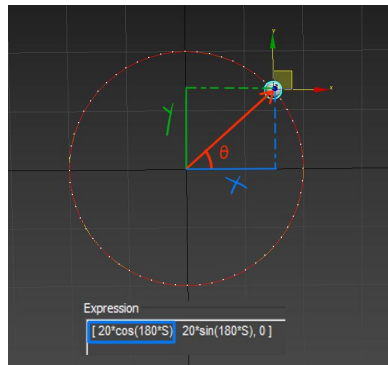


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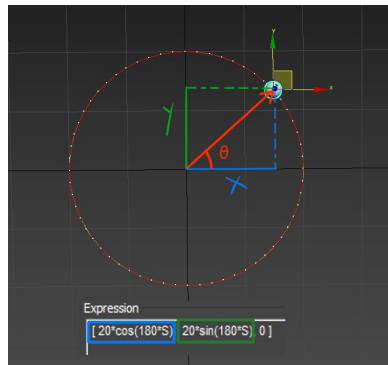
$$x = R \cos \theta(t)$$



Coordinates in circular motion?

$$x = R \cos \theta(t)$$

$$y = R \sin \theta(t)$$



What is $\theta(t)$?

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Let's define the angular velocity:

$$\omega = \frac{d\theta}{dt} \quad (20)$$

ω counts how many turns per unit time.

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$$\omega = \frac{d\theta}{dt} \quad (20)$$

ω counts how many turns per unit time.

Let's define the period of rotation:

$$T = \frac{2\pi}{\omega} \quad (21)$$

T counts how long a single turn takes.

Example, what is the angular velocity if the Space station in 2001:
A Space Odyssey?

<https://www.youtube.com/watch?v=0ZoSYsNADtY>

When ω is constant,

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (22)$$

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Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

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Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

$$\rightarrow v = \frac{\Delta S}{\Delta t} = R \frac{\Delta\theta}{\Delta t}$$

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Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

$$\rightarrow v = \frac{\Delta S}{\Delta t} = R \frac{\Delta\theta}{\Delta t}$$

$$\rightarrow v = R\omega$$

Nonuniform Circular Motion

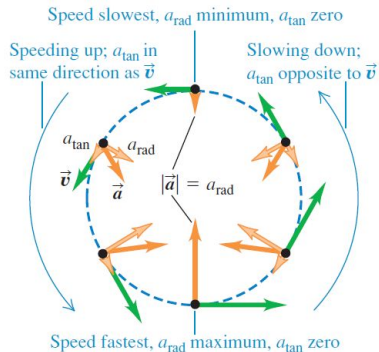


Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Nonuniform Circular Motion

1. The speed varies.
2. the radial component of acceleration is $a_{rad} = \frac{v^2}{R}$.
3. v changes $\rightarrow a$ changes.
4. ω also changes.
5. There is also a tangential component, $a_{tan} = \frac{d|\vec{v}|}{dt}$

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Let's define the angular acceleration:

$$\alpha = \frac{d\omega}{dt} \quad (23)$$

quantity	spacial	angular
coordinates	x	θ
velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Motion with constant angular accerelation:

$$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \quad (24)$$

Test your understanding of the section:

What is the difference between $\frac{d|\vec{v}|}{dt}$ and $|\frac{d\vec{v}}{dt}|$

Test your understanding of the section:

Suppose that the particle in the previous figure experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i) $\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great?