PHY115

Dynamics of Rotation

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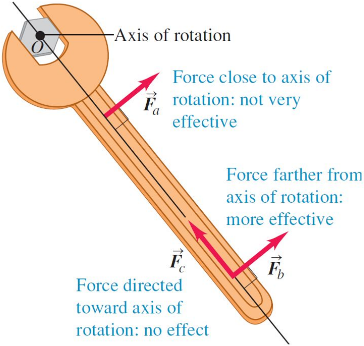
Spring 2023

Torque
- Torque and Angular Acceleration

Angular Momentum

Torque

- Direction and magnitud of forces $\rightarrow$ affect Translational Motion.
- The point of appliclation $\rightarrow$ affects Rotational Motion.



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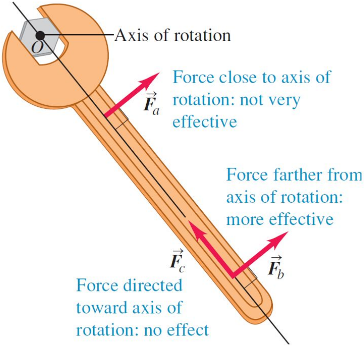
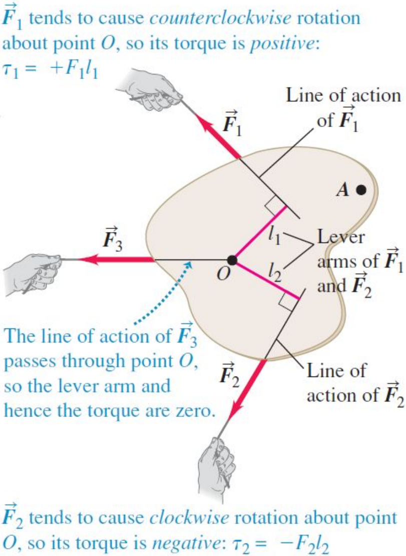
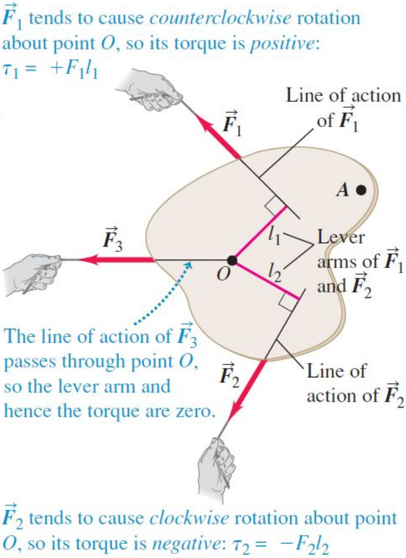


The quantitative measure of the tendency of a force to cause or change a body's rotational motion is called torque;



$$\tau = F\ell$$

ℓ : *lever arm*



$$\tau = F\ell$$

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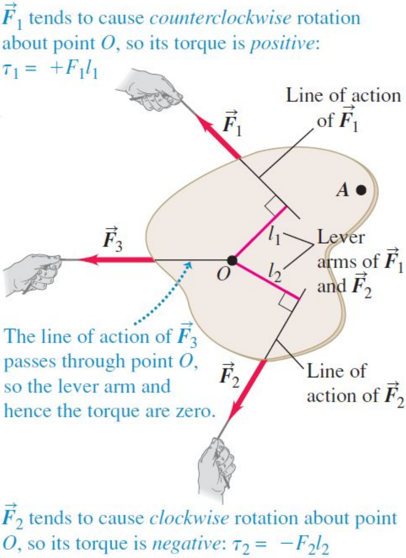
$$\tau_1 = +F_1\ell_1 \ (counterclockwise)$$



$$\tau = F\ell$$

ℓ : *lever arm*

$$\tau_1 = +F_1\ell_1 \ (counterclockwise)$$

$$\tau_2 = -F_2\ell_2 \ (clockwise)$$

EXAMPLE: what is the torque?

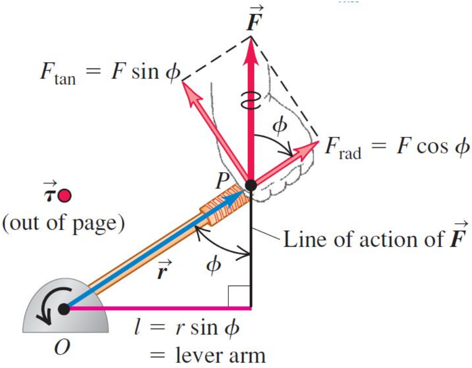


Figure: Image from Sears and Zemansky's University Physics With Modern Physics 13th edition.

EXAMPLE: what is the torque?

$$\boxed{\tau = F\ell = rFsin\Phi} \tag{1}$$

TORQUE AS A VECTOR

- Angular velocity $\rightarrow$ vector

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- Angular acceleration $\rightarrow$ vector

TORQUE AS A VECTOR

- Angular velocity $\rightarrow$ vector
- Angular acceleration $\rightarrow$ vector
- Torque $\rightarrow$ also a vector

TORQUE AS A VECTOR

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- Magnitude: $\tau = F\ell = rFsin\Phi$
- Direction: Right Hand Rule.
  - $\vec{\tau}$ perpendicular to the plane of $\vec{r}$ and $\vec{F}$
  - $(\cdot) \rightarrow \vec{\tau}$ points out the screen.
  - $(\times) \rightarrow \vec{\tau}$ points into the screen.

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We can conclude that the angular momentum of a body can be expressed as:

$$L = I \cdot \vec{\omega} \tag{3}$$

Here, I is a number that depends on how the mass is distributed with respect to the axis of rotation."

When the mass is distributed far away from the axis of rotation, I is bigger and the rotation is slower:

$$I \cdot \vec{\omega} = constant \tag{4}$$

When the mass is distributed closer to the axis of rotation, I is smaller and the rotation is faster.

$$I \cdot \vec{\omega} = constant \tag{5}$$

Then, what is I?

Inertia Momentum

$$I = \sum m_i r_i^2 \tag{6}$$

Moments of Inertia of Various Bodies

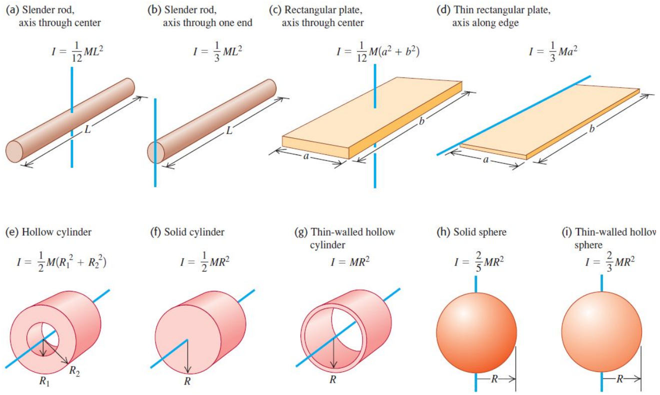

Figure: Image from Sears and Zemansky's University Physics With Modern Physics 13th edition.

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$$\frac{\Delta \vec{L}}{\Delta t} = \vec{\tau} \tag{7}$$

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Just as Newton's second law says that the net force on a particle equals the particle's mass times its acceleration, the net torque on a rigid body equals the body's moment of inertia about the rotation axis times its angular acceleration.

Some observations. . .

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- All the previous expressions are only valid for planar rotations, general 3D rotations are much more complex.
- The real definition for the angular momentum of a particle is $\vec{L} = \vec{r} \times \vec{p}$.
- The total angular momentum of a system is the sum of the angular momentum of the particles.
- It can be shown that for the case of planar motion, the total angular takes the form $\vec{L} = I\omega$

Test Your Understanding of the Section

Rank the magnitudes of the following forces that act during the motion, in order from largest to smallest magnitude. (i) the tension force (magnitude T_1) in the horizontal part of the string; (ii) the tension force (magnitude T_2) in the vertical part of the string; (iii) the weight $m_2 g$ of the hanging object.

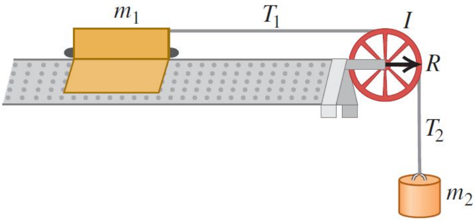
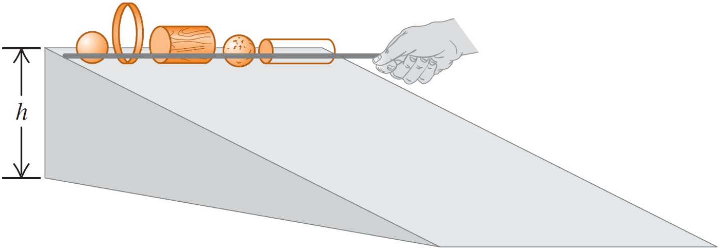


Figure: Image from Sears and Zemansky.

Race of the rolling bodies

What shape should a body have to reach the bottom of the incline first?



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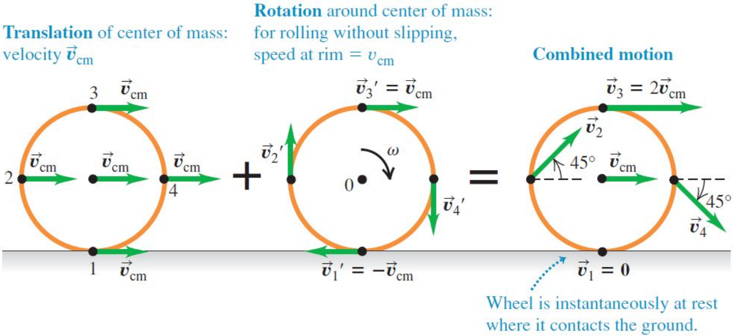
Rolling Without Slipping condition

The motion of a rolling wheel is the sum of the translational motion of the center of mass plus the rotational motion of the wheel around the center of mass.



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Test Your Understanding of Section

If the polar ice caps were to completely melt due to global warming, the melted ice would redistribute itself over the earth. This change would cause the length of the day (the time needed for the earth to rotate once on its axis) to

1. increase;
2. decrease;
3. remain the same.

(Hint: Use angular momentum ideas. Assume that the sun, moon, and planets exert negligibly small torques on the earth.)

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3. Experienced cooks can tell whether an egg is raw or hardboiled by rolling it down a slope (taking care to catch it at the bottom). How is this possible? What are they looking for?