PHY115

Motion in More Than one Dimension

Digipen

Spring 2023

Motion in two or three dimensions
- Projectile Motion
- Circular Motion

Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (1)$$

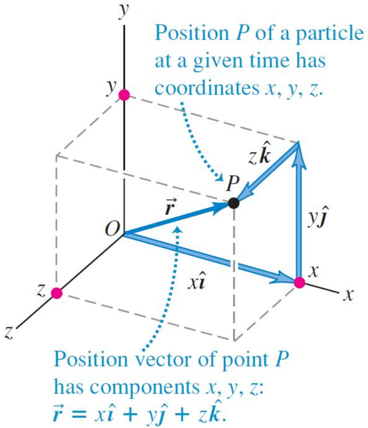
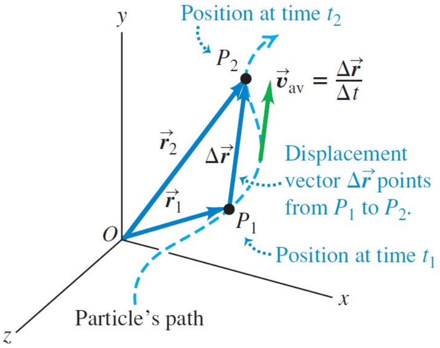


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (2)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t} \qquad (3)$$



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Position and velocity Vectors

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (4)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t} \qquad (5)$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t} \qquad (6)$$

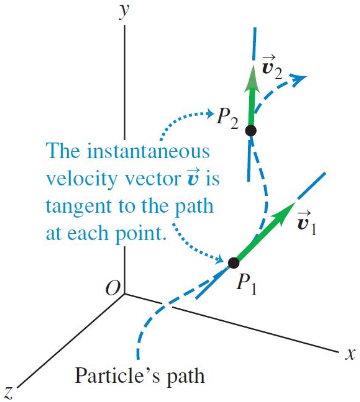


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

The Instantaneous velocity is,

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{7}$$

where,

$$v_x = \frac{dx}{dt},\ \ v_y = \frac{dy}{dt}\ and\ v_z = \frac{dz}{dt} \tag{8}$$

Its magnitude is,

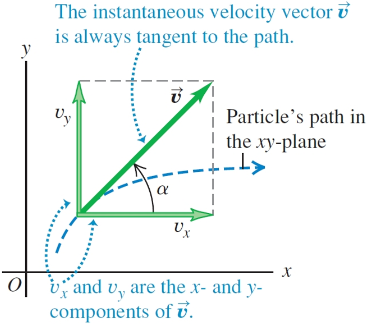
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{9}$$

When the motion is in the $x-y$-plane

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \qquad (10)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} \qquad (11)$$

$$tan\alpha = \frac{v_y}{v_x} \qquad (12)$$



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EXERCISE 3.3

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = (2.00 - 0.25t^2)\ m \tag{13}$$

$$y = (t + 0.025t^3)\ m \tag{14}$$

(a) Find the rover's coordinates and distance from the lander at $t = 2\ s$. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0\ s$ to $t = 2\ s$.

The Instantaneous acceleration is,

$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} \tag{15}$$

where,

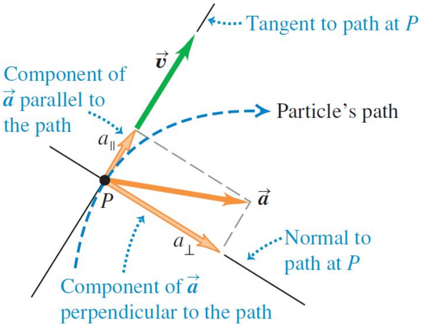
$$a_x = \frac{dv_y x}{dt}, \;\; a_y = \frac{dv_y}{dt} \;\; and \;\; a_z = \frac{dv_z}{dt} \tag{16}$$

Its magnitude is,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{17}$$

Parallel and Perpendicular Components of Acceleration

A useful way to think about the acceleration is in terms of its component parallel to the path and its component perpendicular to the path.



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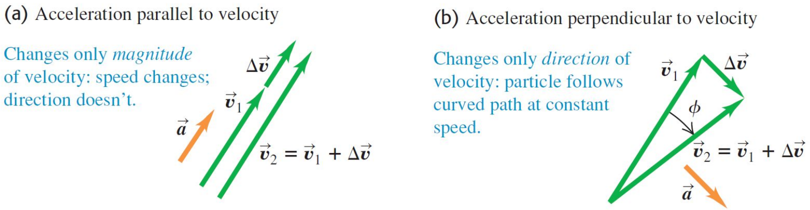

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Velocity and acceleration vectors for a particle moving through a point P on a curved path with (a) constant speed,
ing speed, and (c) decreasing speed.

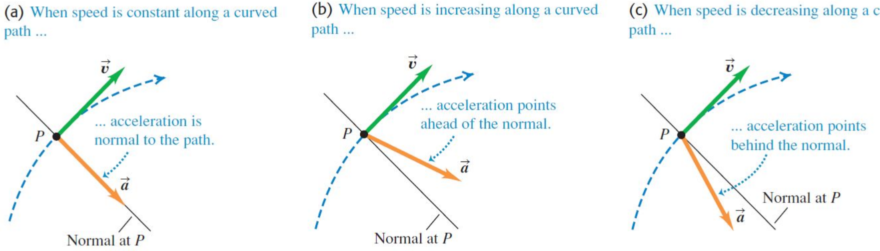
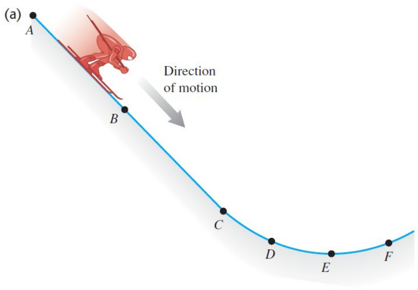


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Conceptual Example:

A skier moves along a ski-jump ramp. The ramp is straight from point A to point C and curved from point C onward. The skier speeds up as she moves downhill from point A to point E, where her speed is maximum. She slows down after passing point E. Draw the direction of the acceleration vector at each of the points B, D, E, and F.



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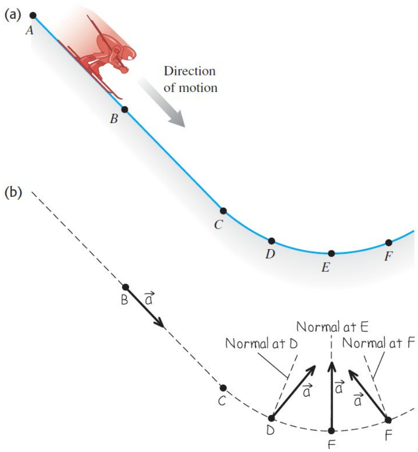

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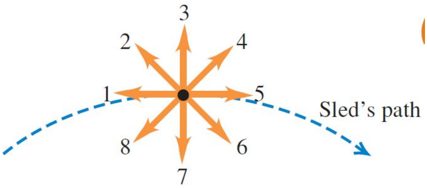
Test Your Undertanding

A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)



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Projectile Motion

A projectile is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

The path followed by a projectile is called its trajectory.

Projectile Motion

- we can treat the x- and y-coordinates separately.
- The x-component of acceleration is zero, and the y-component is constant and equal to $-g$
- So, projectile motion is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

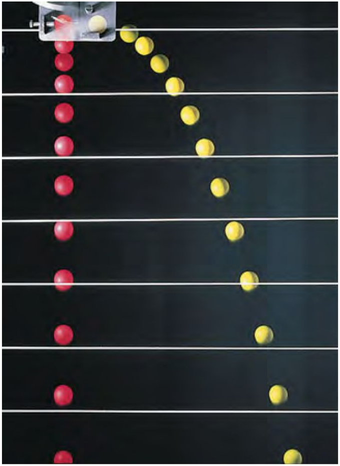
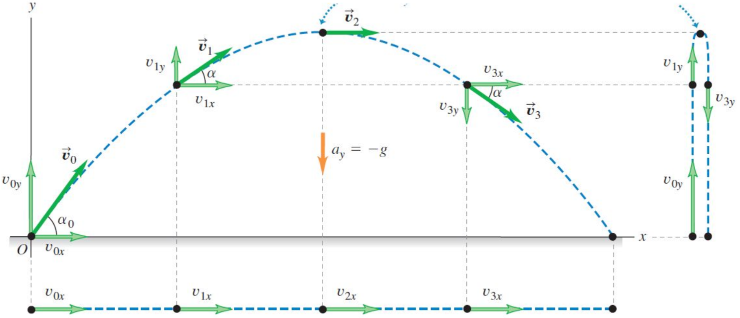


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

$$x-motion: \quad x = v_{x0}t + x_0 \tag{18}$$

$$y-motion: \quad y = -\frac{1}{2}gt^2 + v_{y0}t + y_0 \tag{19}$$



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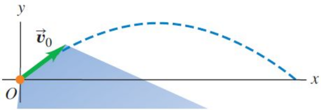

$v_{x0} = ? \quad v_{y0} = ?$

$$v_{x0} = v_0 cos\alpha_0, \quad v_{y0} = v_0 sin\alpha_0$$

$$tan\alpha_0 = \frac{v_{x0}}{v_{y0}}$$

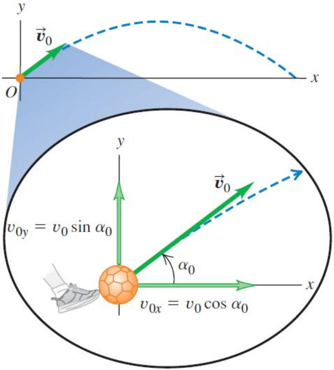

Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

$$x = (v_0 \cos\alpha_0)t \qquad \text{(projectile motion)}$$

$$y = (v_0 \sin\alpha_0)t - \tfrac{1}{2}gt^2 \qquad \text{(projectile motion)}$$

$$v_x = v_0 \cos\alpha_0 \qquad \text{(projectile motion)}$$

$$v_y = v_0 \sin\alpha_0 - gt \qquad \text{(projectile motion)}$$

Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Motion Path?

Motion Path?

$$y(x) = (tan\alpha_0)x - \frac{g}{2v_0^2 cos^2\alpha_0}x^2$$

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PARABOLA

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$$y(x) = (tan\alpha_0)x - \frac{g}{2v_0^2 cos^2\alpha_0}x^2$$

PARABOLA

$$\rightarrow y(x) = bx - cx^2 \quad parabola$$

What if we include air resistance?

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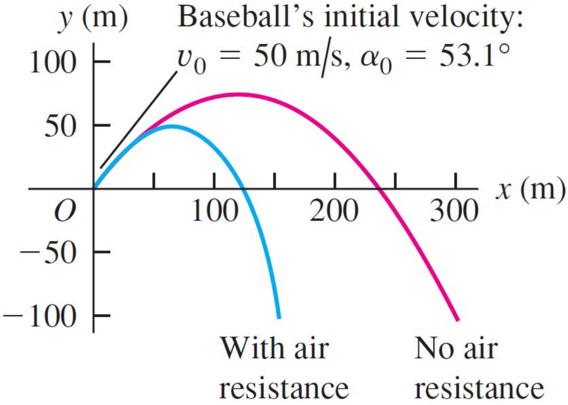

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HEIGHT AND RANGE OF A PROJECTILE

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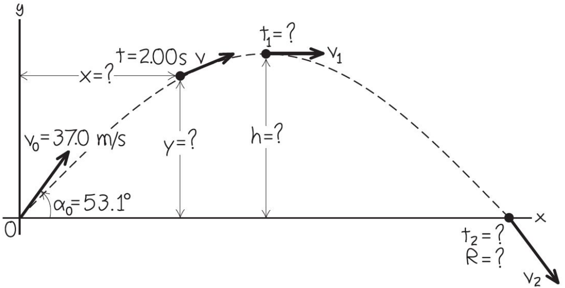
A batter hits a baseball so that it leaves the bat at speed $v_0 = 37\ m/s$ at an angle $\alpha_0 = 53.1^\circ$. Find:

1. the time for the highest point, and its height h at this times
2. the horizontal range R



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HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 sin\ \alpha_0}{g}$$

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$$h = v_{0y}t - \frac{1}{2}gt^2$$

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$$\rightarrow h = \frac{v_0^2 sin\ \alpha_0^2}{2g}$$

HEIGHT

$$v_y = v_{0y} - gt = 0 \rightarrow t = \frac{v_0 sin\ \alpha_0}{g}$$

$$h = v_{0y} t - \frac{1}{2} g t^2$$

$$\rightarrow h = \frac{v_0^2 sin\ \alpha_0^2}{2g}$$

EXTRA-CREDIT: PROOF IT

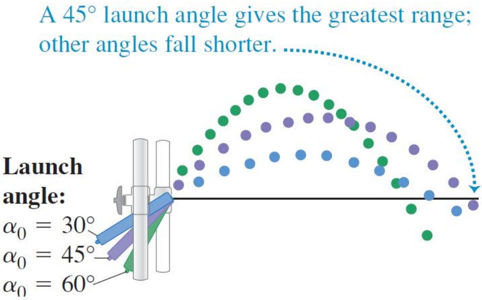
RANGE

$$t_l = 2\frac{v_0 sin\ \alpha_0}{g}$$

$$R = v_{0x}t = 2v_{0x}\frac{v_0 sin\ \alpha_0}{g} = 2v_0 cos\ \alpha\frac{v_0 sin\ \alpha_0}{g}$$

$$\rightarrow R = \frac{v_0^2 sin\ 2\alpha_0}{g}$$

VARIATION OF RANGE WITH INITIAL INCLINATION



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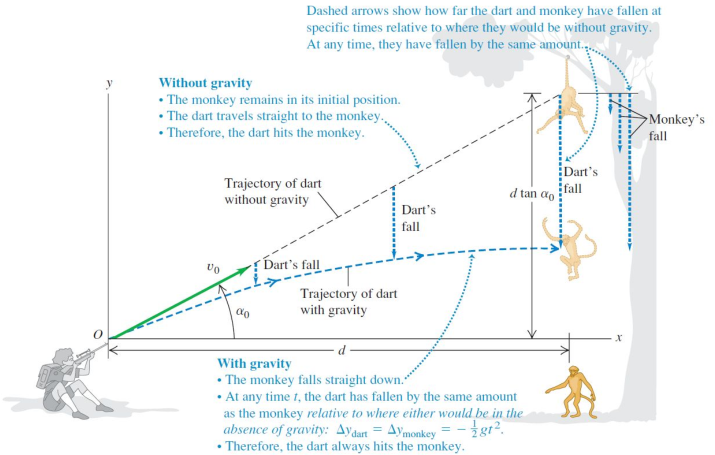
A classic Physics problem: "The zookeeper and the monkey"



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DART:

$$x_d = v_{0x} t$$

$$y_d = v_{0y} t - (\frac{1}{2}) g t^2$$

MONKEY:

$$x_m = D$$

$$y_m = -(\frac{1}{2}) g t^2 + H$$

$$\rightarrow tan\ \alpha = \frac{H}{D}$$

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The dart is going to hit the monkey as long the zookeeper points right to the monkey at the begining.

$$\rightarrow \mathit{tan}\ \alpha = \frac{H}{D}$$

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EXTRA-CREDIT: PROOF IT

Suppose the tranquilizer dart has a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.

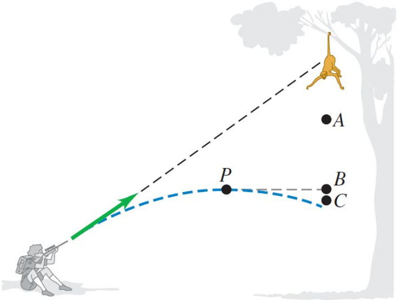
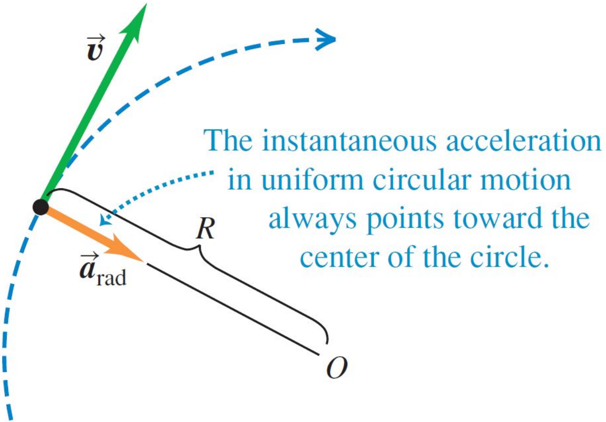


Figure: Figure from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Uniform Circular Motion

- Motion in a circle with constant speed.
- There is no component of acceleration parallel (tangent) to the path.
- The acceleration vector is perpendicular to the path and hence directed inward.
- This causes the direction of the velocity to change without changing the speed



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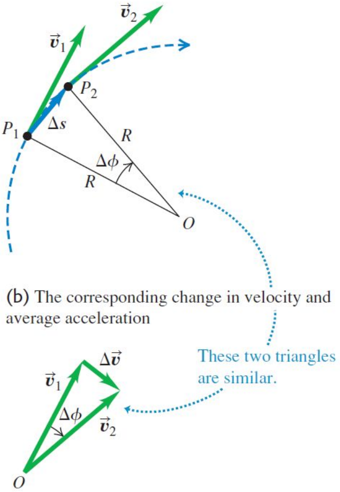
Magnitude of $\vec{a}$?

$$\Delta\Phi R = \Delta s$$



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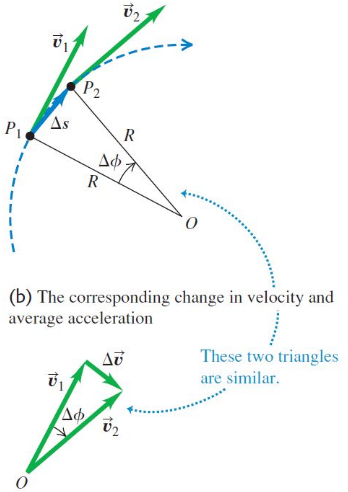
Magnitude of $\vec{a}$?

$$\Delta\Phi R = \Delta s$$

$$\Delta\Phi v_1 = |\Delta\vec{v}| \rightarrow \frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R}$$



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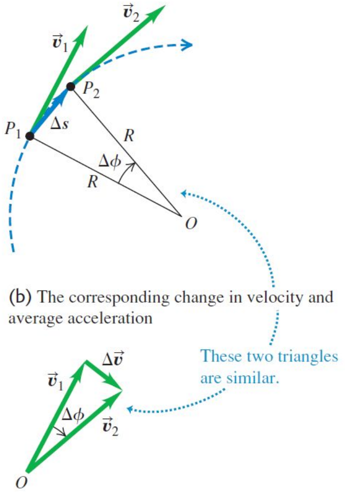

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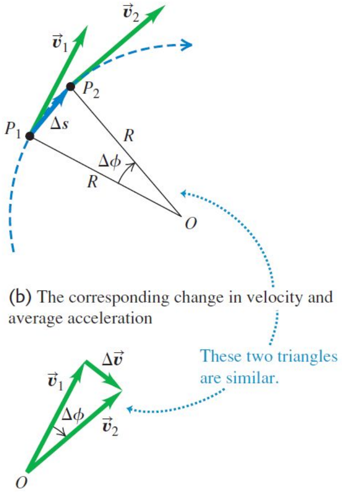
$$|\Delta\vec{v}| = v_1\frac{\Delta s}{R} \rightarrow a_{av} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

Taking the limit for $\Delta t \rightarrow 0$

$$\boxed{a = \frac{v^2}{R}}$$



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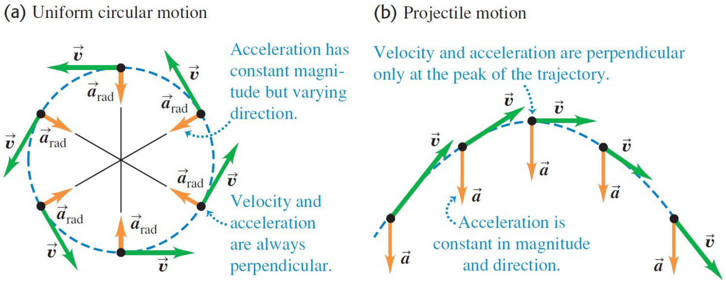
Uniform Motion vs. Projectile Motion

1. In projectile motion, the acceleration is the same at all times.
2. In uniform circular motion the direction of a always points toward the center of the circle.

Uniform Motion vs. Projectile Motion



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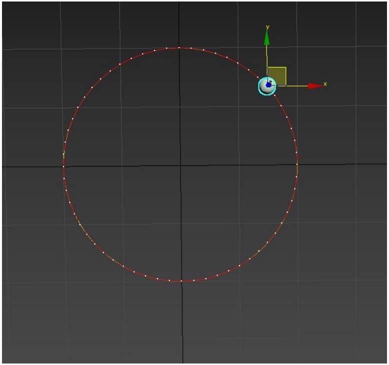
Coordinates in circular motion?



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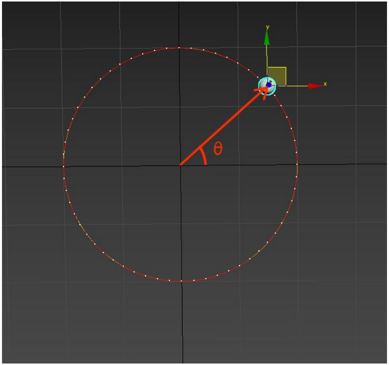

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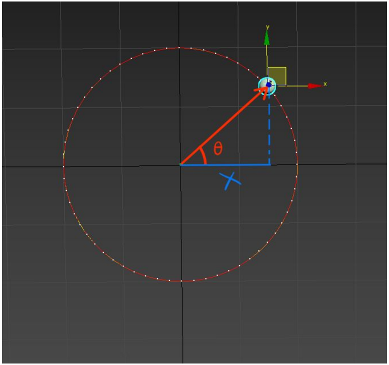

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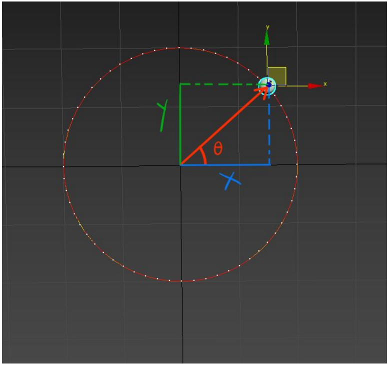

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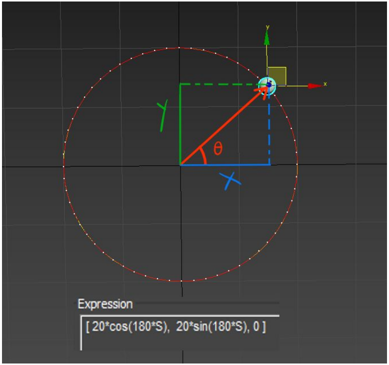
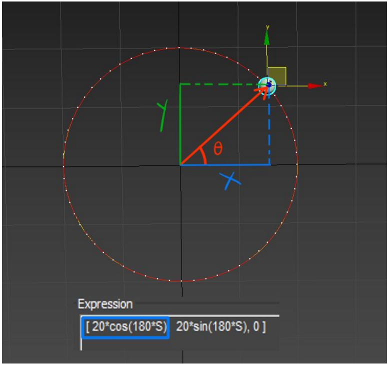


Coordinates in circular motion?

$$x = R\ cos\ \theta(t)$$



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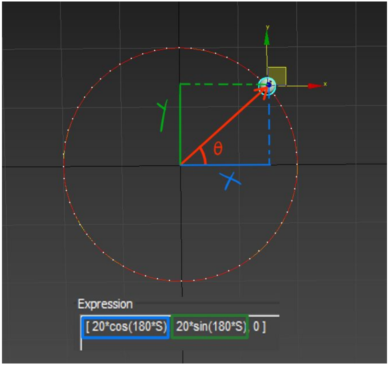

What is $\theta(t)$?

What is $\theta(t)$?

$$\theta(t) = \omega t + \theta_0 \tag{20}$$

where ω is the angular velocity.

Let's define the angular velocity:

$$\boxed{\omega = \frac{d\theta}{dt}} \tag{21}$$

ω counts how many turns per unit time.

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ω counts how many turns per unit time.

Let's define the period of rotation:

$$\boxed{T = \frac{2\pi}{\omega}} \tag{22}$$

T counts how long a single turn takes.

Example, what is the angular velocity if the Space station in 2001: A Space Odyssey?

https://www.youtube.com/watch?v=0ZoSYsNADtY

When ω is constant,

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}} \tag{23}$$

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Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

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Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

$$\rightarrow v = \frac{\Delta S}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

When ω is constant,

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}} \tag{23}$$

Relation of ω with the speed ?

$$\Delta S = R\Delta\theta$$

$$\rightarrow v = \frac{\Delta S}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

$$\rightarrow v = R\omega$$

Non uniform Circular Motion

Non uniform Circular Motion $\rightarrow$ there is tangential acceleration

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$$\theta(t) = \frac{1}{2}\alpha t^2 + \omega t + \theta_0 \tag{24}$$

where α is the angular acceleration.

Non uniform Circular Motion

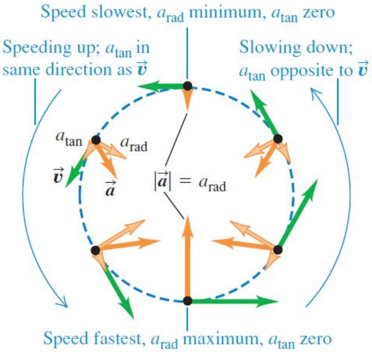

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Nonuniform Circular Motion

1. The speed varies.
2. the radial component of acceleration is $a_{rad} = \frac{v^2}{R}$.
3. v changes $\rightarrow$ a changes.
4. ω also changes.
5. There is also a tangential component, $a_{tan} = \frac{d|\vec{v}|}{dt}$

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Let's define the angular acceleration:

$$\alpha = \frac{d\omega}{dt} \tag{25}$$

quantity	spacial	angular
coordinates	x	θ
velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Motion with constant angular acceleration:

$$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \tag{26}$$