PHY115

Momentum, Impulse and Collisions

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For example, when a moving van collides head-on with a compact car, what determines which way the wreckage moves after the collision?

We would need to know how are the forces acting between the colliding bodies.

We will find that we don't have to know anything about these forces to answer questions of this kind!

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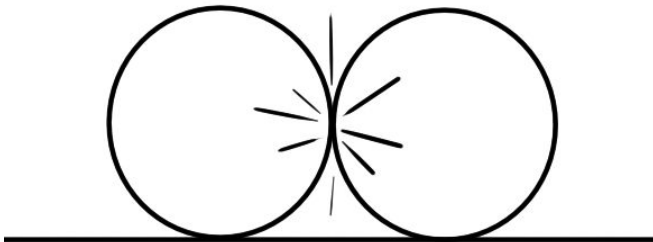
then. . .

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \tag{2}$$

**The net force (vector sum of all forces) acting on a particle
equals the time rate of change of momentum of the particle.**

▶ $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$

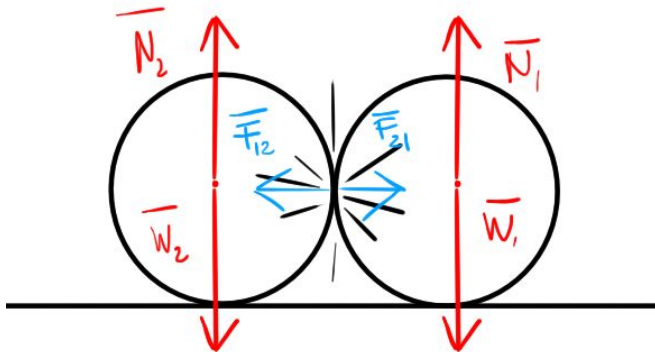- $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$
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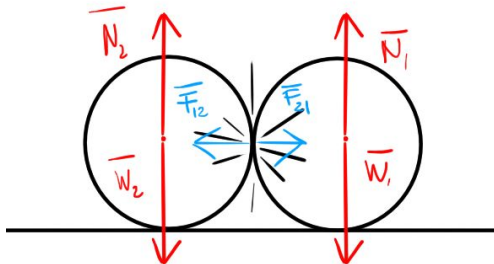▶ $\sum \vec{F} = \vec{N_1} + \vec{W_1} + \vec{N_2} + \vec{W_2} + \vec{F_{12}} + \vec{F_{21}} = 0 \rightarrow \Delta p = 0$

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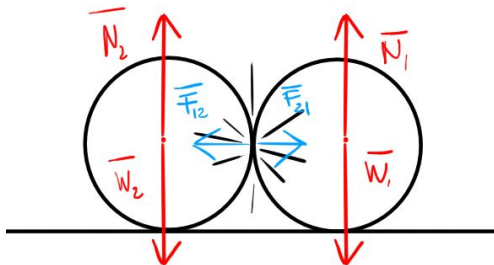
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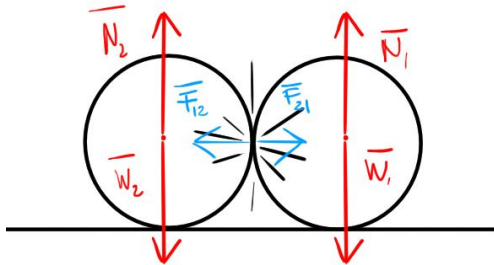► $\sum \vec{F} = 0 \rightarrow \Delta \vec{p} = 0$

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► $\sum \vec{F_1} = \vec{N_1} + \vec{W_1} + \vec{F_{21}} = \frac{\Delta \vec{p_1}}{\Delta t}$

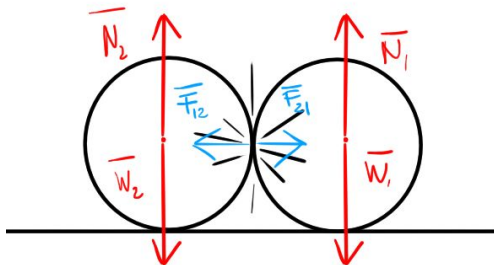► $\sum \vec{F_2} = \vec{N_2} + \vec{W_2} + \vec{F_{21}} = \frac{\Delta \vec{p_2}}{\Delta t}$

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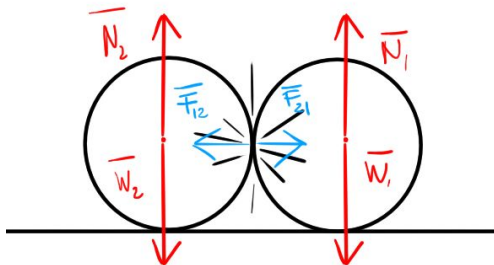- $\rightarrow \vec{p_1} = -\vec{p_2}$

If we consider the two balls as a system...

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If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

Example: https://www.youtube.com/watch?v=4IYDb6K5UF8

More than two particles?

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$$\rightarrow \Delta \vec{p}_x = 0$$

$$\rightarrow \Delta \vec{p}_y = 0$$

External force...

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\rightarrow \Delta \vec{p}_x = F_x$$

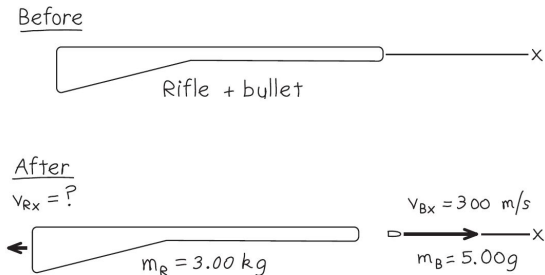$$\rightarrow \Delta \vec{p}_y = F_y$$

External force...

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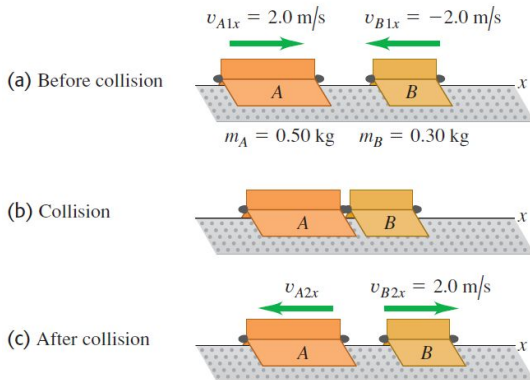$$\rightarrow \Delta \vec{p}_y = \sum_i m_i \vec{g}_i$$

Example

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum of the bullet and rifle?

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Example

Two gliders with different masses move toward each other on a
frictionless air track. After they collide, glider B has a final velocity
of $+2.0$ m/s. What is the final velocity of glider A? How do the
changes in momentum and in velocity compare?

(a) Before collision $v_{A1x} = 2.0$ m/s $v_{B1x} = -2.0$ m/s
                       $m_A = 0.50$ kg $m_B = 0.30$ kg

(b) Collision

(c) After collision v_{A2x} $v_{B2x} = 2.0$ m/s

Example

Test Your Understanding

A spring-loaded toy sits at rest on a horizontal, frictionless surface.
When the spring releases, the toy breaks into three equal-mass
pieces, A, B, and C, which slide along the surface. Piece A moves
off in the negative x-direction, while piece B moves off in the
negative y-direction.

1. What are the signs of the velocity components of piece C?
2. Which of the three pieces is moving the fastest?

Example

Two kind of collisions

▶ Elastic Collisions: the total kinetic energy of the system is the same after the collision as before (billiard balls).

▶ Inelastic Collisions: the total kinetic energy of the system is not conserved (collision between 2 pieces of clay).

Example

Example of inelastic collision in 2D: An automobile collision

A 1000 *kg* car traveling north at 15 m/s collides with a 2000 *kg* truck traveling east at 10 m/s. The occupants, wearing seat belts, are uninjured, but the two vehicles move away from the impact point as one. The insurance adjuster asks you to find the velocity of the wreckage just after impact. What is your answer?
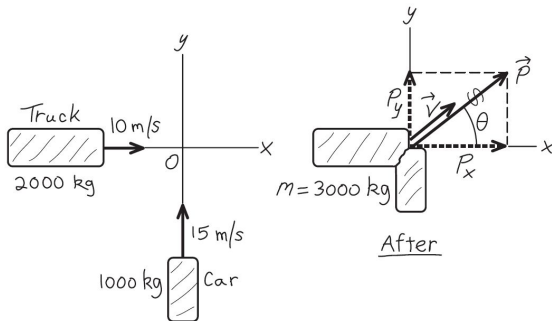
Testing you understanding about Linear Momentum.

(a) When a large car collides with a small car, which one undergoes the greater change in momentum: the large one or the small one? Or is it the same for both? (b) In light of your answer to part (a), why are the occupants of the small car more likely to be hurt than those of the large car, assuming that both cars are equally sturdy?

Testing you understanding about Linear Momentum.

A woman holding a large rock stands on a frictionless, horizontal sheet of ice. She throws the rock with speed v_0 at an angle above the horizontal. Consider the system consisting of the woman plus the rock. Is the momentum of the system conserved? Why or why not? Is any component of the momentum of the system conserved? Again, why or why not?

EXANPLE: https://www.youtube.com/watch?v=Kf0bBxmNeec

Testing you understanding about Linear Momentum.

A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why?

CENTER OF MASS
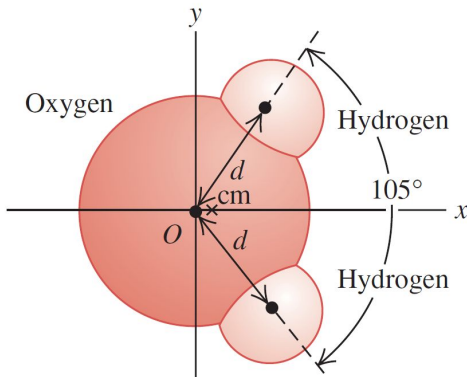
The total mass of the system is: $M = \sum_i m_i$

We define the **Center of Mass** of the system as,

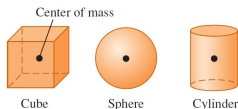$$\vec{R} = \frac{1}{M} \sum_i m_i \vec{r}_i \qquad (4)$$

CENTER OF MASS

EXAMPLE: Center of mass of a water molecule

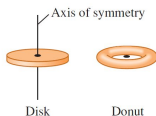$d = 9.57 \times 10^{-11}$ m, $m_o = 16$ u, $m_H = 1$ u

CENTER OF MASS

Center of mass

Cube   Sphere   Cylinder

If a homogeneous object has a geometric center, that is where the center of mass is located.

Axis of symmetry

Disk   Donut

If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

For solid bodies, in which we have (at least on a macroscopic level) a continuous distribution of matter, the sums in eq. 4 have to be replaced by integrals.

MOTION OF THE CENTER OF MASS

We can show that,
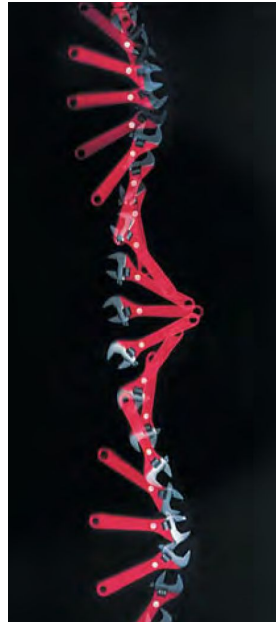
$$\vec{P} = M\vec{V}_{CM} \tag{5}$$

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} \tag{6}$$

MOTION OF THE CENTER OF MASS

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.
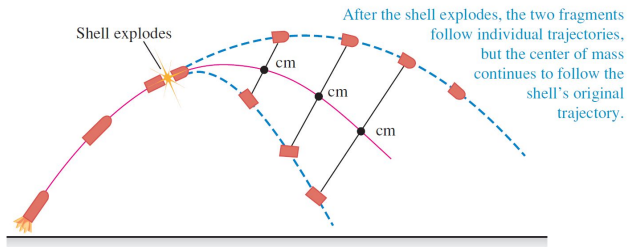
This is a very impressive
result!!

EXAMPLE

Shell explodes

cm

cm

cm

After the shell explodes, the two fragments
follow individual trajectories,
but the center of mass
continues to follow the
shell's original
trajectory.

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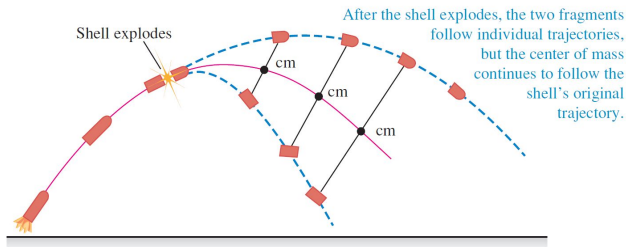continues to follow the
shell's original
trajectory.

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Will the center of mass in the figure continue on the same
parabolic trajectory even after one of the fragments hits the
ground? Why or why not?

EXAMPLE

Explain the motion of a Newton Cradle

https://www.youtube.com/watch?v=8dgyPRA86K0

Rotation of Rigid Bodies

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▶ We can not represented adequately their motion as a moving point.

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► Rotations in 3D have a much more complex mechanics.

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$$\omega = \frac{\Delta\theta}{\Delta t}, \ \alpha = \frac{\Delta\omega}{\Delta t} \ (\alpha \ \textit{constant})$$

▶ But rotations can have two different sense, so to specify a rotation, we need a number and a sense.

▶ Angular velocity and acceleration are represented by vectors!

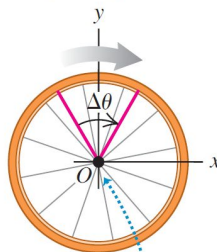**Counterclockwise rotation positive:**
$\Delta\theta > 0$, so
$\omega_{\text{av-}z} = \Delta\theta/\Delta t > 0$
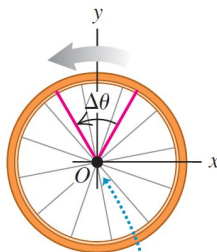
Clockwise rotation negative:
$\Delta\theta < 0$, so
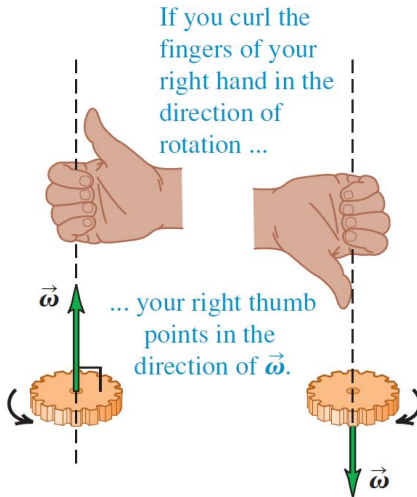$\omega_{\text{av-}z} = \Delta\theta/\Delta t < 0$

Axis of rotation (z-axis) passes through origin and points out of page.

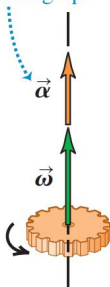Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Angular Velocity As a Vector

(a)

If you curl the fingers of your right hand in the direction of rotation ...

... your right thumb points in the direction of $\vec{\omega}$.

$\vec{\omega}$

$\vec{\omega}$

Angular Acceleration As a Vector

$\vec{\alpha}$ and $\vec{\omega}$ in the **same** direction: Rotation speeding up.

$\vec{\alpha}$ and $\vec{\omega}$ in the **opposite** directions: Rotation slowing down.

$\vec{\alpha}$

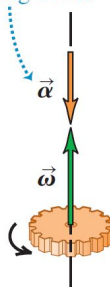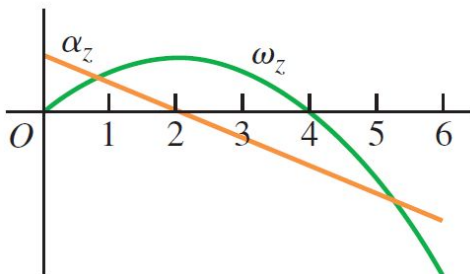$\vec{\omega}$

$\vec{\alpha}$

$\vec{\omega}$

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Test Your Understanding The figure shows a graph of ω_z and α_z

versus time for a particular rotating body. (a) During which time
intervals is the rotation speeding up?

Figure: Figures from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Comparison of Linear and Angular Motion with Constant Acceleration

| Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$

Figure: Table from Sears and Zemansky's University Physics with Modern Physics, 13th Edition.

Rotation with constant angular acceleration

You have finished watching a
movie on Blu-ray and the disc
is slowing to a stop. The disc's
angular velocity at $t = 0$ is
27.5 rad/s, and its angular
acceleration is a constant
$-10\ rad/s^2$. A line PQ on the
disc's surface lies along the at
$t = 0$. (a) What is the disc's
angular velocity at $t = 0.3\ s$
(b) What angle does the line
PQ make with the $+x - axis$
at this time?
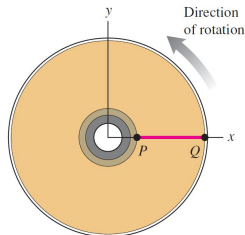
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Relating Linear and Angular Kinematics
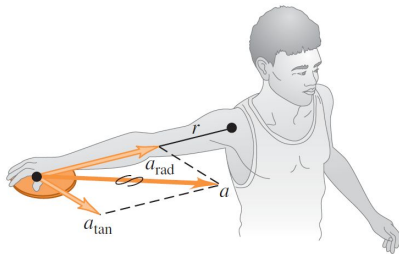
$$v = r\omega, \ a_{rad} = \omega^2 r$$

Relating Linear and Angular Kinematics

$$v = r\omega, \ a_{rad} = \omega^2 r, \ a_{tan} = r\alpha$$

Throwing a discus

An athlete whirls a discus in a circle of radius 80.0cm. At a certain instant, the athlete is rotating at 10 rad/s and the angular speed is increasing at 50 rad/s^2. At this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

(a)

(b)