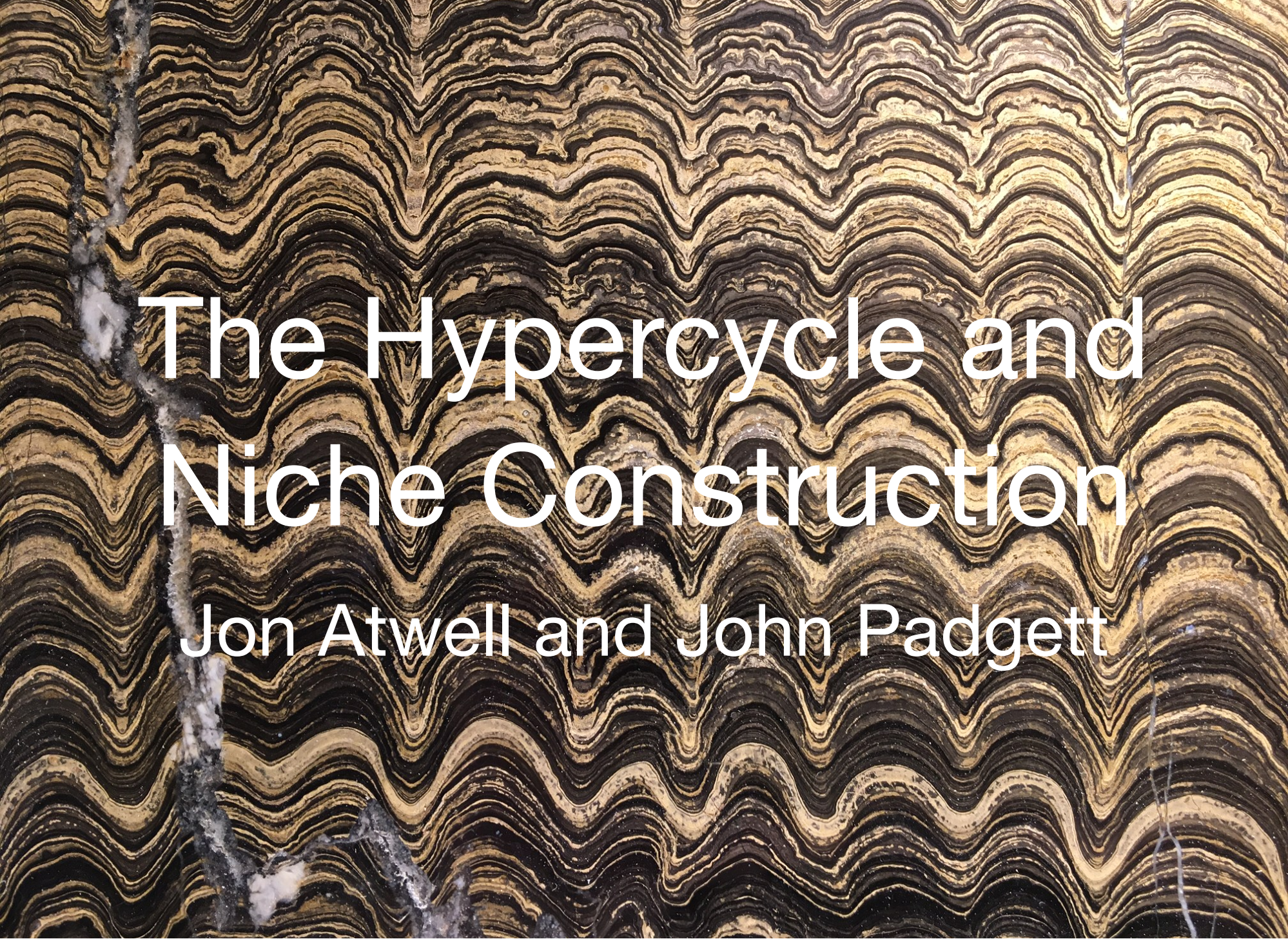


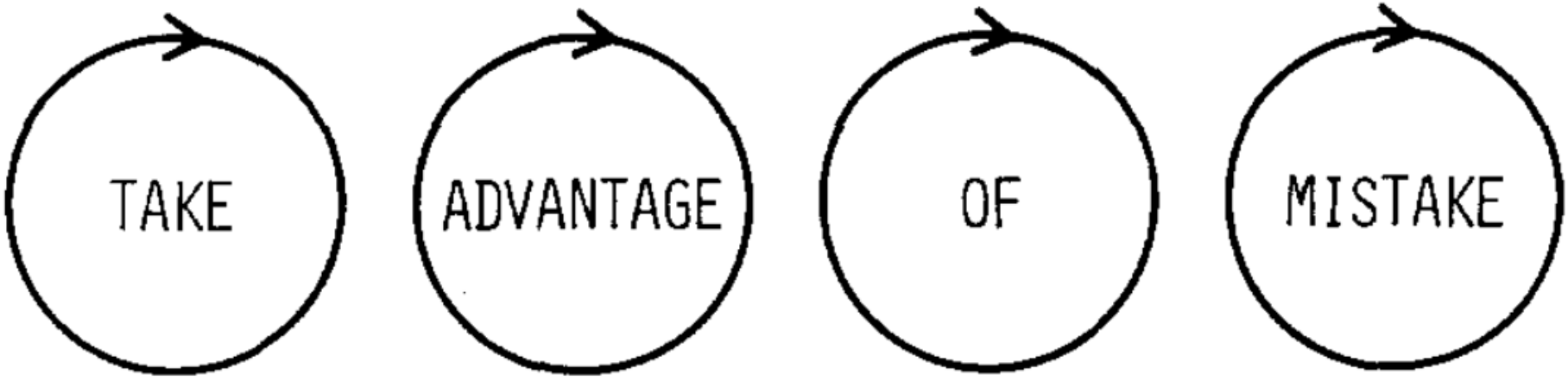
The Hypercycle and Niche Construction

Jon Atwell and John Padgett

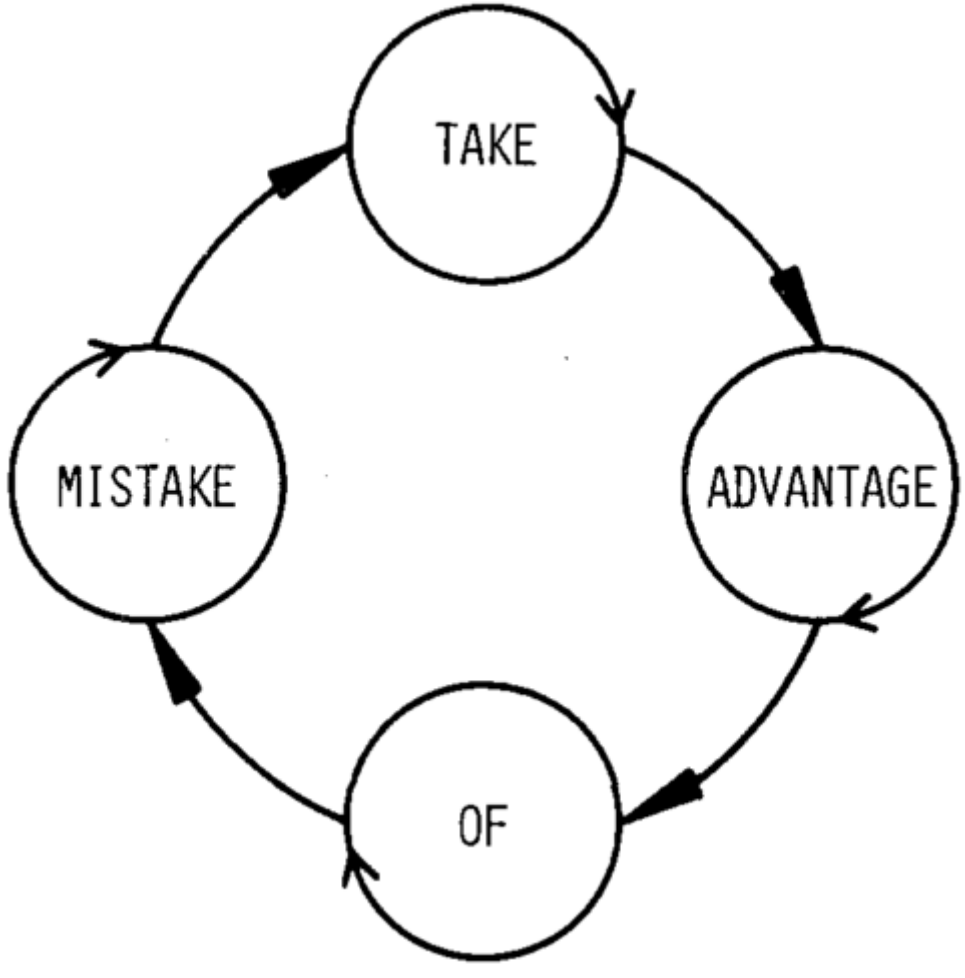
The *information crisis* of *pre-biotic* chemistry

Macro-molecules can create macro-molecules (translation)

But where did the first macro-molecules come from?


TAKE
ADVANTAGE
OF
MISTAKE

T=0	T=1	T=2	T=3	T=4		T=inf.
TAKE	TAKE	TAKE	TO**K**E	MISTAKE		MISTAKE
TAKE	TAKE	TAKE	TAKE	TAKE		MISTAKE
TAKE	TA**X**E	MISTAKE	MISTAKE	MISTAKE		MISTAKE
TAKE	TAKE	TAKE	TAKE	TAKE		MISTAKE
ADVANTAGE	ADVANTAGE	ADVANTAGE	AD**K**ANTAGE	MISTAKE		MISTAKE
ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE		MISTAKE
ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE	...	MISTAKE
ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE	ADVANTAGE		MISTAKE
OF	O**R**	MISTAKE	**N**ISTAKE	MISTAKE		MISTAKE
OF	OF	OF	OF	OF		MISTAKE
OF	OF	OF	OF	OF		MISTAKE
OF	OF	OF	OF	OF		MISTAKE
MISTAKE	MISTAKE	MISTAKE	MISTAKE	MISTAKE		MISTAKE
MISTAKE	MISTAKE	MISTAKE	MISTAKE	MISTAKE		MISTAKE
MISTAKE	MISTAKE	MISTAKE	MISTAKE	MISTAKE		MISTAKE
MISTAKE	MISTAKE	MISTAKE	MISTAKE	MISTAKE		MISTAKE


TAKE
ADVANTAGE
OF
MISTAKE

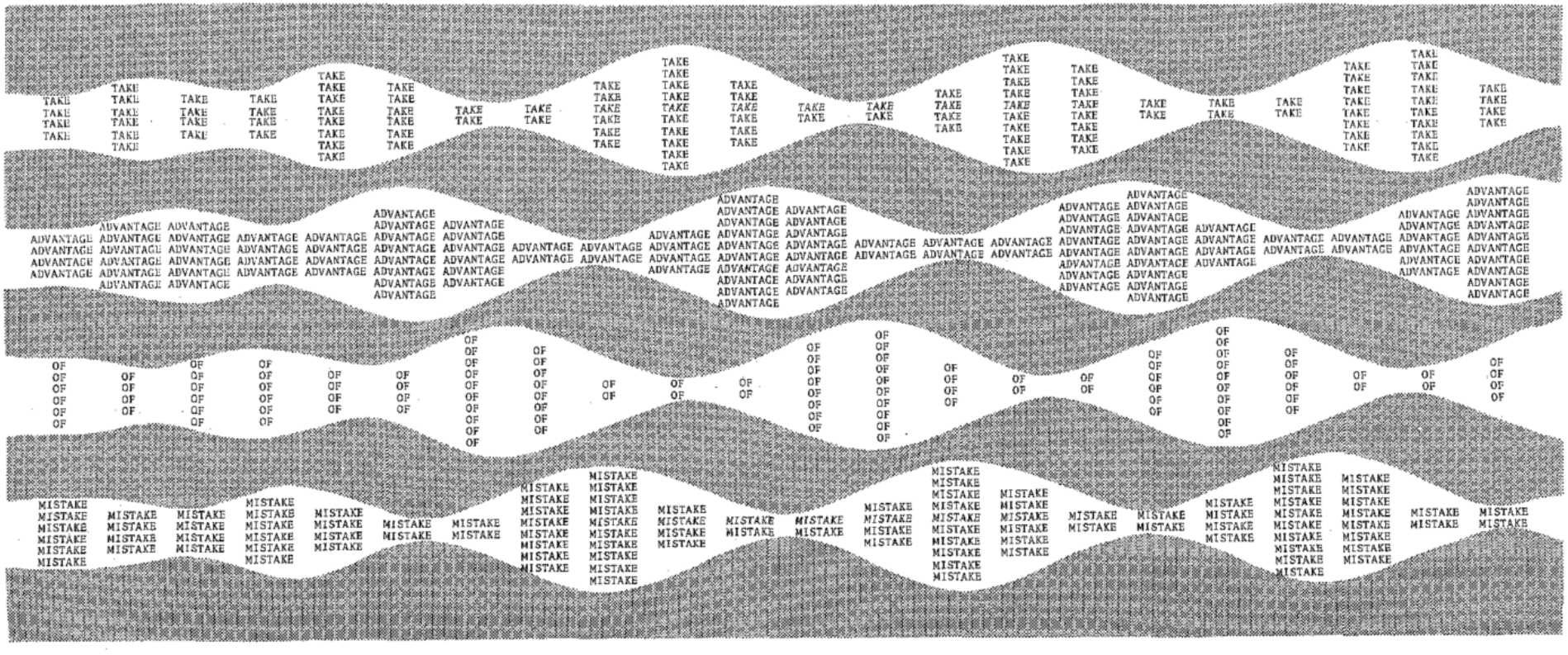


Fig. 16. Each word of the sentence TAKE ADVANTAGE OF MISTAKE represents a self-reproducing unit. The information of the sentence is stabilized by hypercyclic coupling among the words. In the graph, each printed word is representative of 10 copies in the computer store. All words in a vertical row refer to the same time, the intervals of which change discontinuously along the horizontal axis. The oscillation builds up from an initial equipartition of words and maintains a phase shift from word to word. The error distribution for each of the four words is stable and of the same kind as shown in Table 5

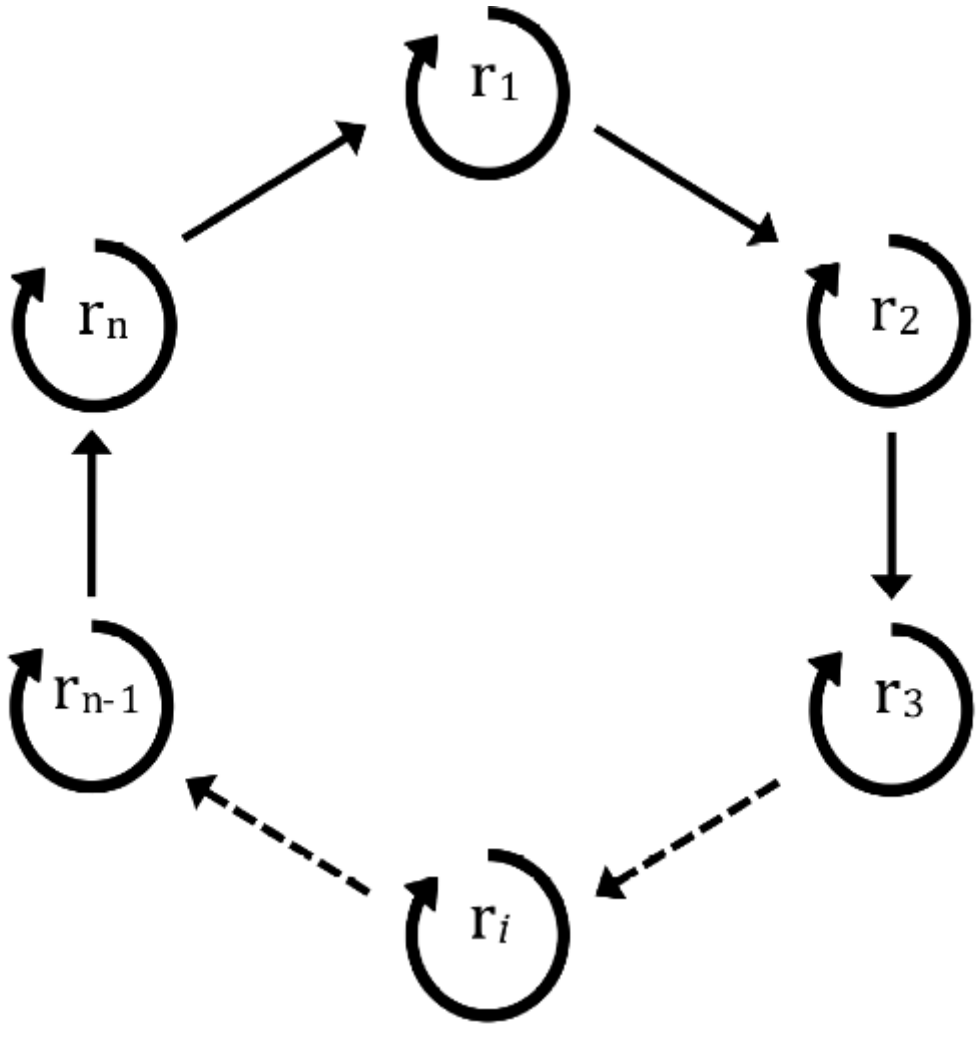
$$TAKE_{t+1} = \frac{TAKE_t}{ALL} * \frac{ADVANTAGE_t}{ALL} - \frac{TAKE_t}{ALL} * totalgrowth$$

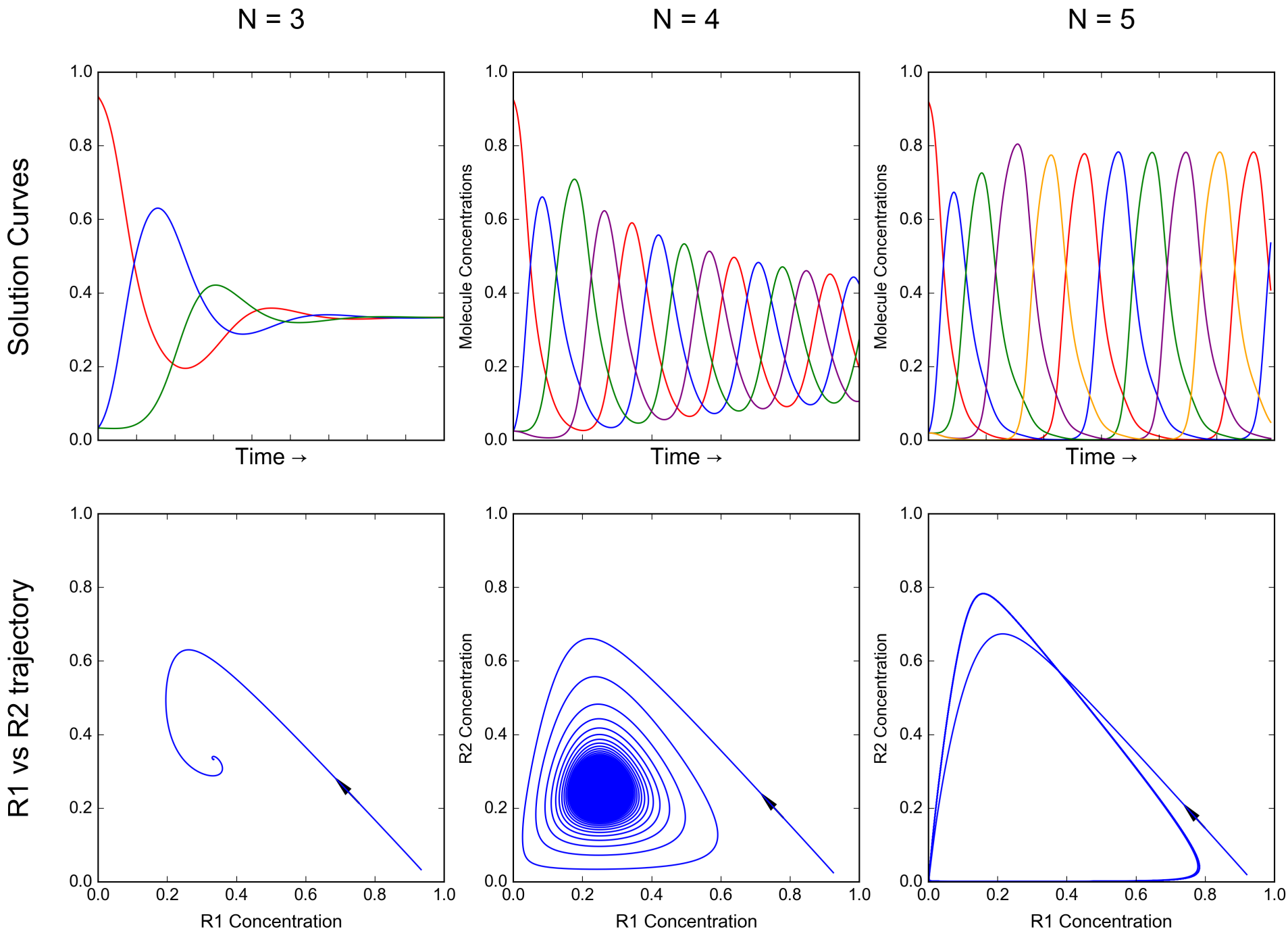
$$ADVANTAGE_{t+1} = \frac{ADVANTAGE_t}{ALL} * \frac{OF_t}{ALL} - \frac{ADVANTAGE_t}{ALL} * totalgrowth$$

$$OF_{t+1} = \frac{OF_t}{ALL} * \frac{MISTAKE_t}{ALL} - \frac{OF_t}{ALL} * totalgrowth$$

$$MISTAKE_{t+1} = \frac{MISTAKE_t}{ALL} * \frac{TAKE_t}{ALL} - \frac{MISTAKE_t}{ALL} * totalgrowth$$

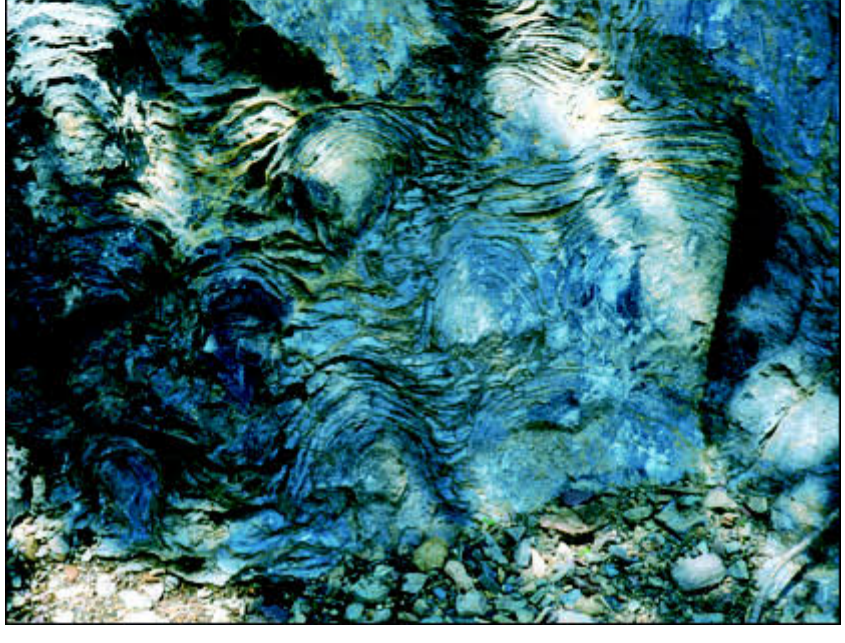
$$\dot{r}_i = r_i * r_{i+1} - r_i \sum_{j=1}^{N} r_j * r_{j+1}$$

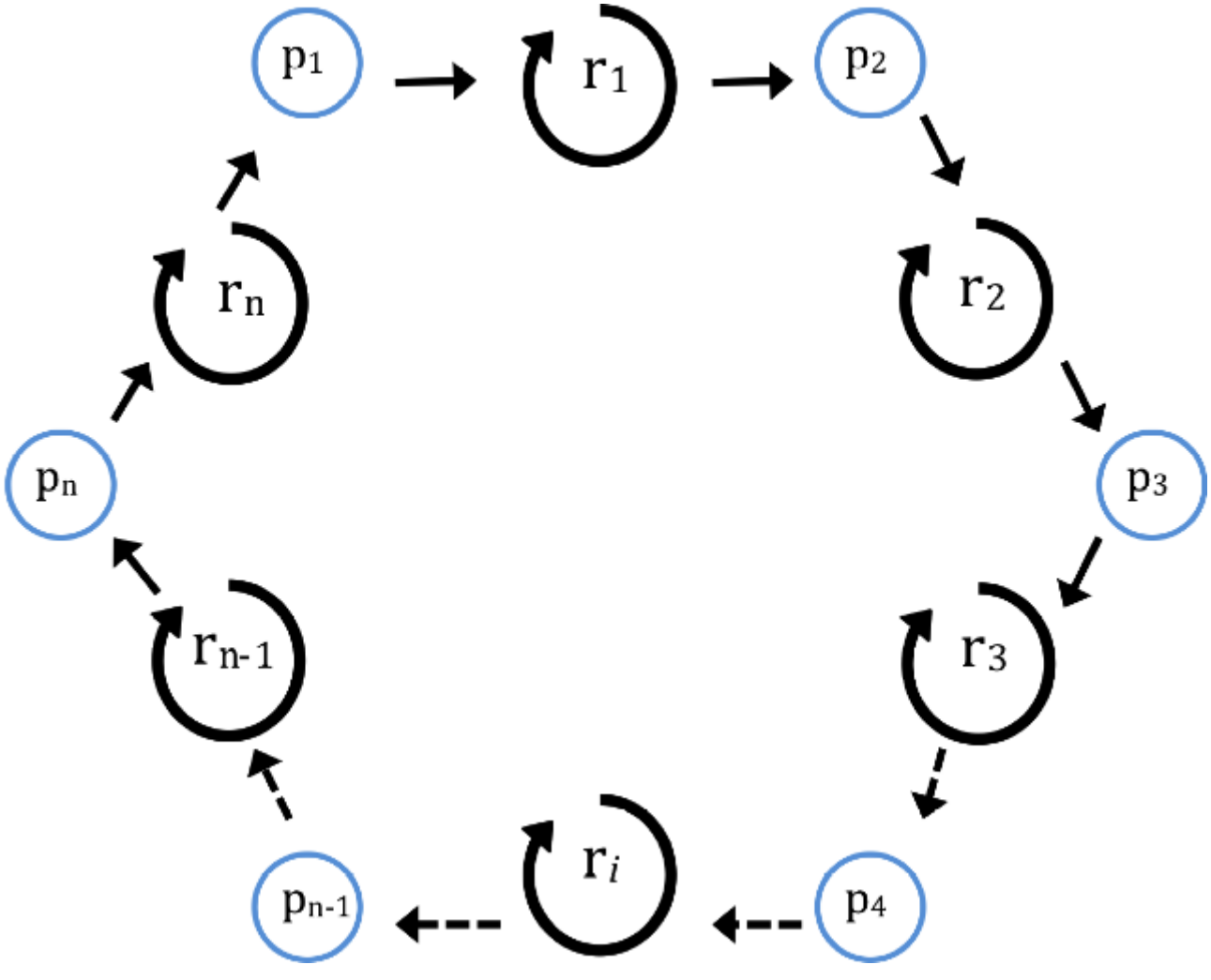

r_1
r_2
r_3
r_i
r_{n-1}
r_n


N = 3
N = 4
N = 5
Solution Curves
R1 vs R2 trajectory
Time →
Molecule Concentrations
R1 Concentration
R2 Concentration
0.0
0.2
0.4
0.6
0.8
1.0

The Emergence of Organizations and Markets

John F. Padgett and Walter W. Powell




p1
r1
p2
r2
p3
r3
p4
ri
pn-1
rn-1
pn
rn

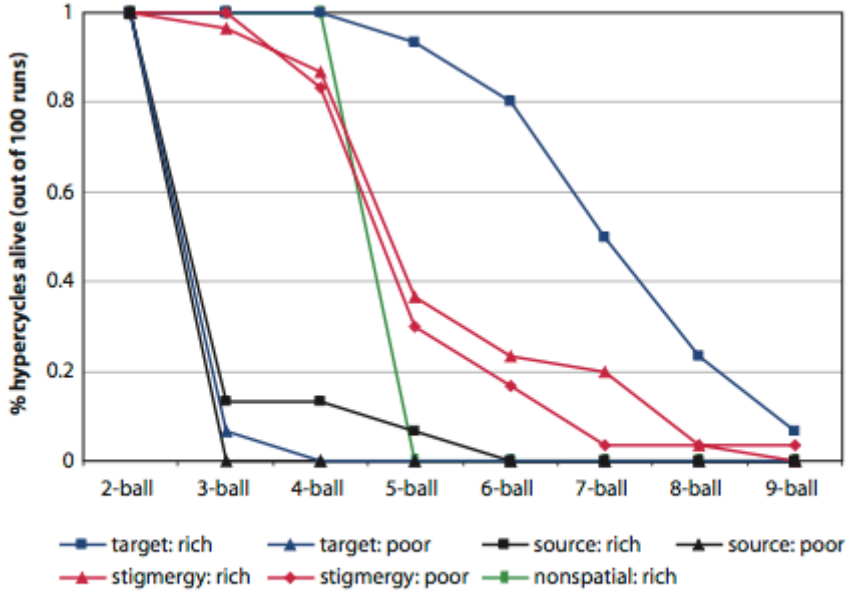


Figure 3.2 Survival of autocatalytic networks: hypercycle (i.e., SOLO H) chemistry.

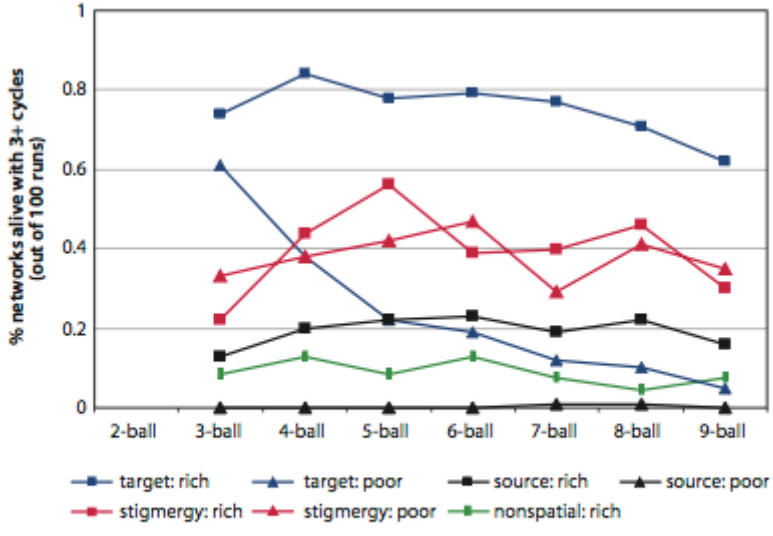


Figure 3.4 Survival of autocatalytic networks: ALL chemistry, 3+ cycles.

Step #1	Step #2	Step #3
$R_{1,k}$	$P_{1,k}$	$R_{2,k}$
$Pr(Type = R_1) = \#R_1/N$	$Pr(Type = P_1) = \#P_1/M$	$Pr(Type = R_2) = \#R_2/N$

$$Pr(R_{i,t+1} = R_{i,t} + 1) \approx \overbrace{\frac{R_i}{N} * \frac{P_i}{M} * \frac{R_{i+1}}{N}}^{\text{growth}}$$

$$\Delta R_i = \overbrace{\frac{R_i}{N} * \frac{P_i}{M} * \frac{R_{i+1}}{N}}^{\text{growth}} - \overbrace{\frac{R_i}{N} \sum_{j=1}^{N} \frac{R_j}{N} * \frac{P_j}{M} * \frac{R_{j+1}}{N}}^{\text{death}}$$

$$R_{i,t+1} = R_{i,t} + \overbrace{\frac{R_{i,t}}{N} * \frac{P_{i,t}}{M} * \frac{R_{i+1,t}}{N}}^{\text{growth}} - \overbrace{\frac{R_{i,t}}{N} \sum_{j=1}^{N} \frac{R_{j,t}}{N} * \frac{P_{j,t}}{M} * \frac{R_{j+1,t}}{N}}^{\text{death}}$$

$$R_{i,t+1} = R_{i,t} + \overbrace{r_i p_i r_{i+1}}^{\text{growth}} - \overbrace{r_i \sum_{j=1}^{N} r_j p_j r_{j+1}}^{\text{death}}$$

$$P_{i,t+1} = P_{i,t} + \overbrace{r_{i-1}p_{i-1}(1 - r_i)}^{\text{incoming}} + \overbrace{r_i p_i (1 - r_{i+1})}^{\text{outgoing}}$$

Full dynamics (i.e. with chains)

$$R_{i,t+1} = R_{i,t} + \frac{r_i * r_{i+1}}{1-\prod_{j=1}^{N} r_j} * \left(p_i + \sum_{k=1}^{N-1} \left[p_{i+k} \prod_{h=i+k}^{N-1+i} r_h \right] \right) - r_i * \bar{f}$$

$$\bar{f} = \sum r_j$$

$$P_{i,t} = P_{i,t} + \frac{r_{i+1}(1-r_i)}{1-\prod_{j=1}^{N} r_j} \left(p_{i+1} + \sum_{k=i}^{N-2+i} \left[p_k \prod_{h=k}^{N-2+i} r_h \right] \right)$$

$$- \frac{r_i(1-r_{i+1})}{1-\prod_{j=1}^{N} r_j} \left(p_i + \sum_{k=i+1}^{N-1+i} \left[p_k \prod_{h=k}^{N-1+i} r_h \right] \right)$$

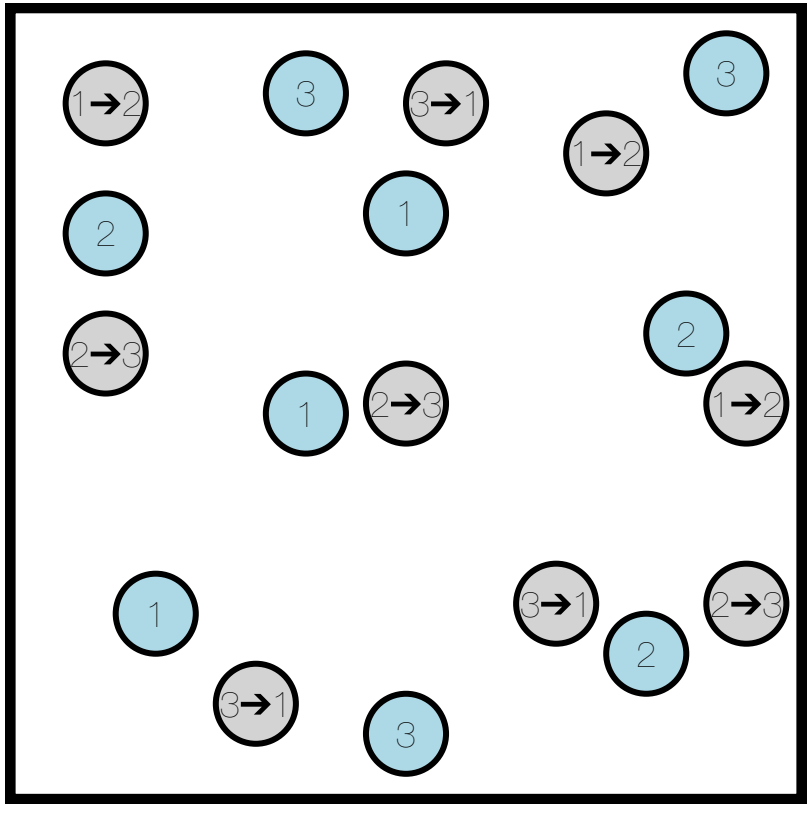
Leading term dynamics (i.e. no chains)

$$R_{i,t+1} = R_{i,t} + r_i p_i r_{i+1} - r_i \sum_{j=1}^{N} r_j p_j r_{j+1}$$

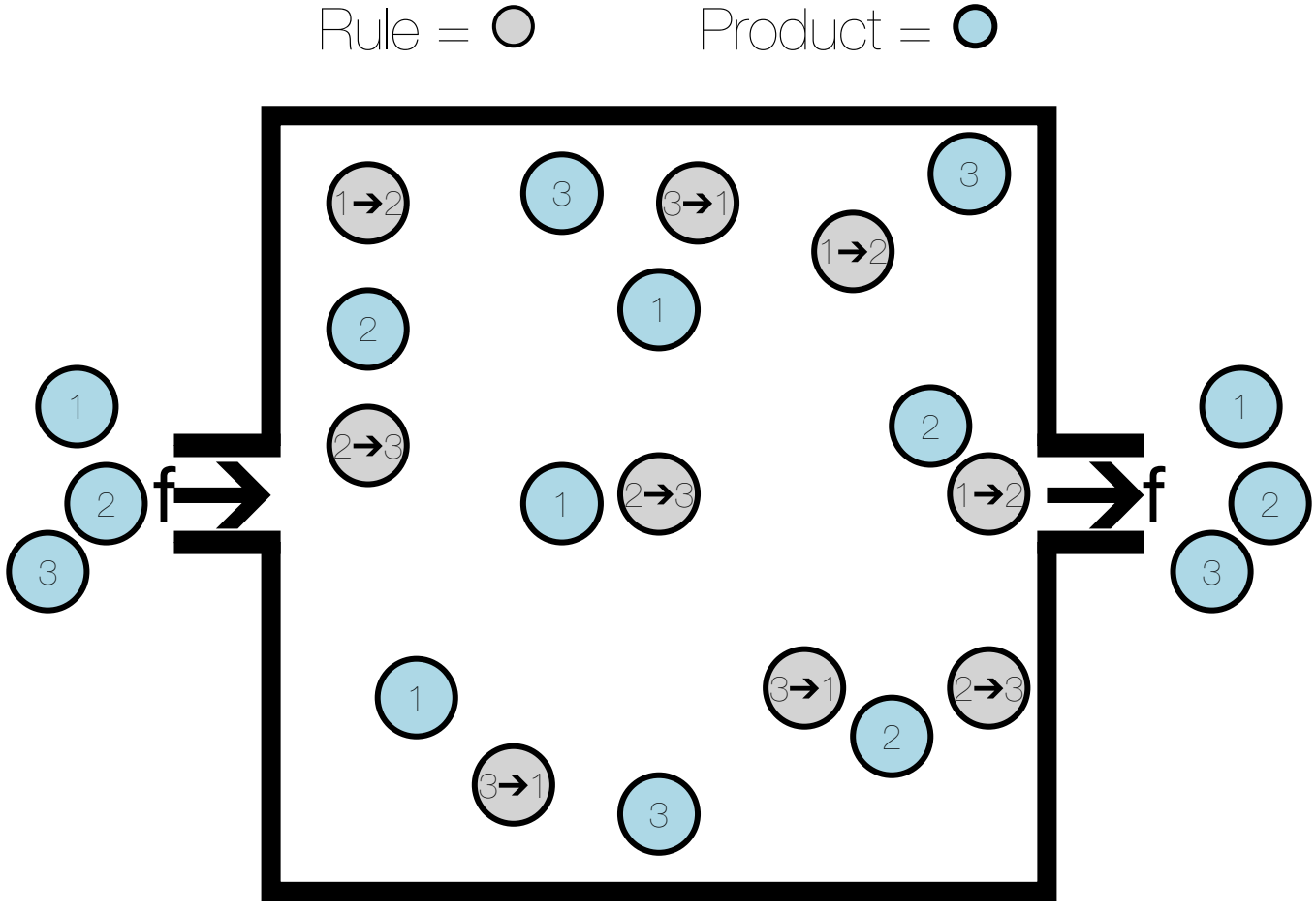
$$P_{i,t+1} = P_{i,t} + r_{i-1} p_{i-1} + r_i p_i$$

Closed Reactor

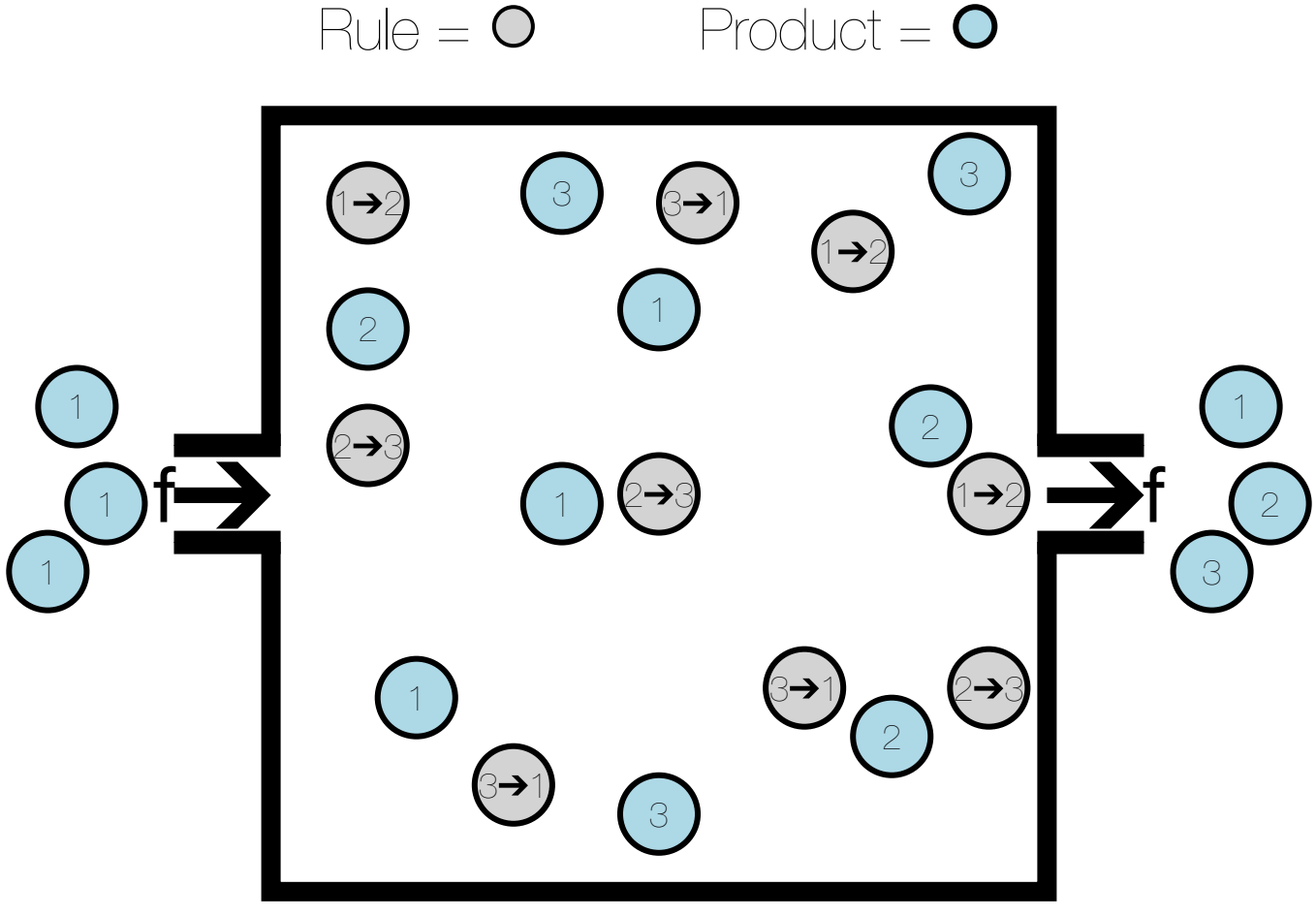
Rule = Product =



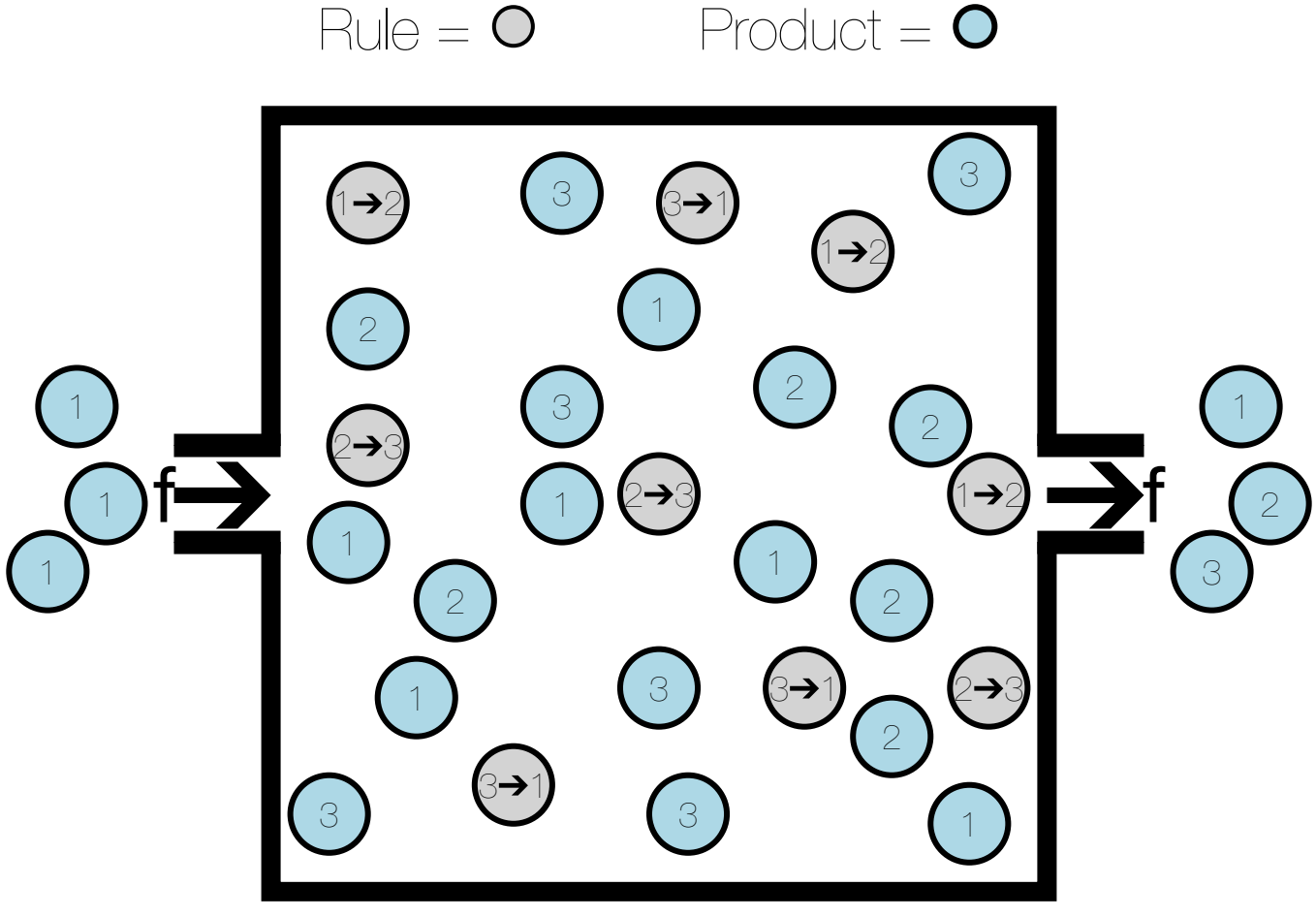
Flow Reactor



Flow Reactor



Flow Reactor



Final Difference Equations

$$R_{i,t+1} = R_{i,t} + r_{i,t}p_{i,t}r_{i+1,t} - r_{i,t}\sum_{j=1}^{N} r_{i,t}p_{i,t}r_{i+1,t}$$

$$P_{i,t+1} = P_{i,t} + (1-F)[r_{i-1,t}p_{i-1,t} - r_{i,t}p_{i,t}] - F[p_{i,t} - p_{i,0}]$$

With initial conditions

$$R_{i,0} = [R_{1,0}, R_{2,0}, \ldots, R_{n,0}]$$

$$P_{i,0} = [\sigma * \sum_{i=1}^{n} R_{i,0}, 0, \ldots, 0]$$

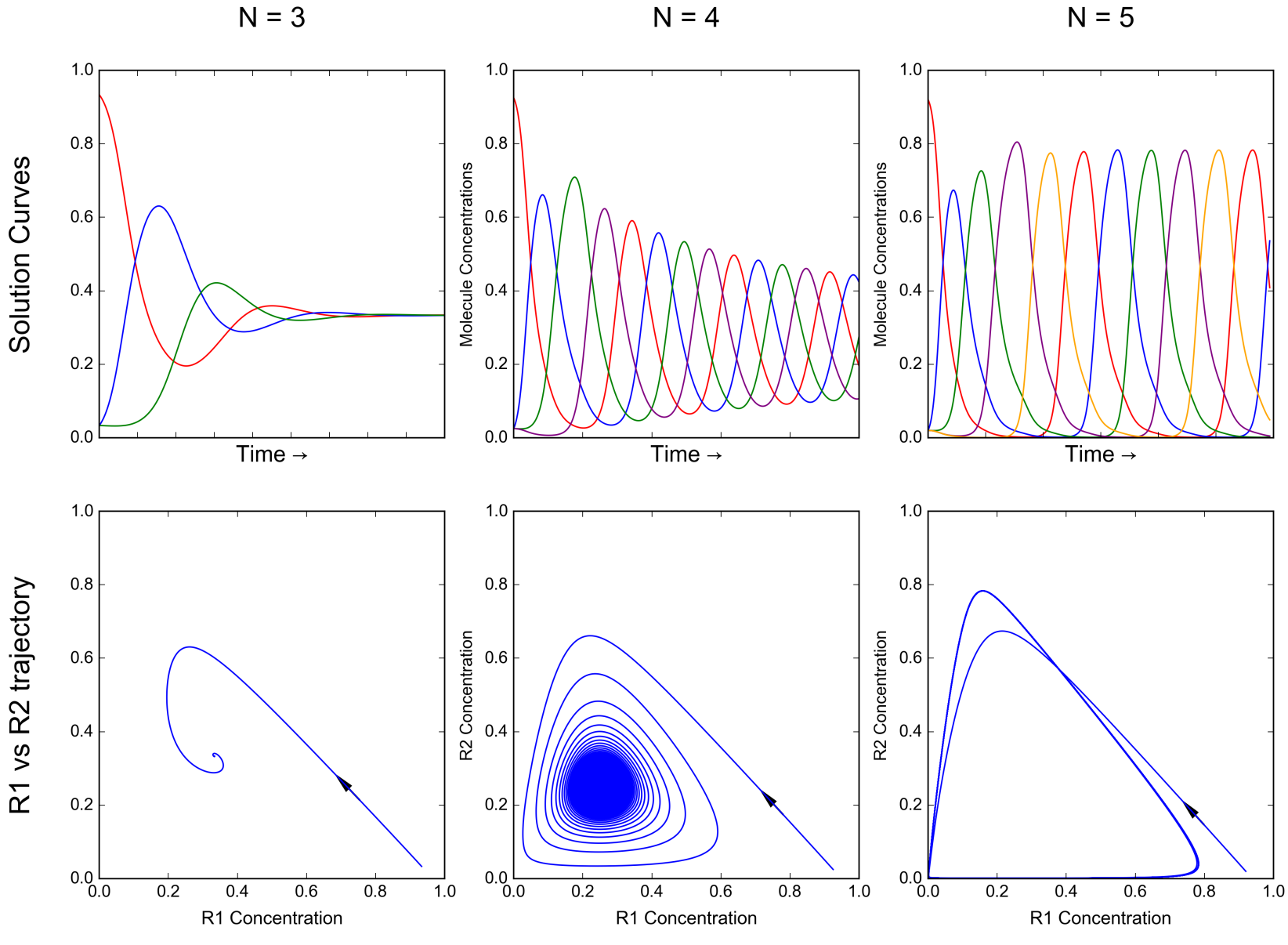
The Niche Construction Model

Laland, Odling-Smee, and Feldman (1999)

$$R_t = \lambda_1 R_{t-1}(1 - \gamma p) + \lambda_2 p + \lambda_3$$

$$0 \leq \gamma, \lambda_1, \lambda_2, \lambda_3 \leq 1$$

$$0 < \lambda_1 + \lambda_2 + \lambda_3 < 1$$


N = 3
N = 4
N = 5
Solution Curves
R1 vs R2 trajectory
Molecule Concentrations
Time →
R2 Concentration
R1 Concentration
0.0
0.2
0.4
0.6
0.8
1.0

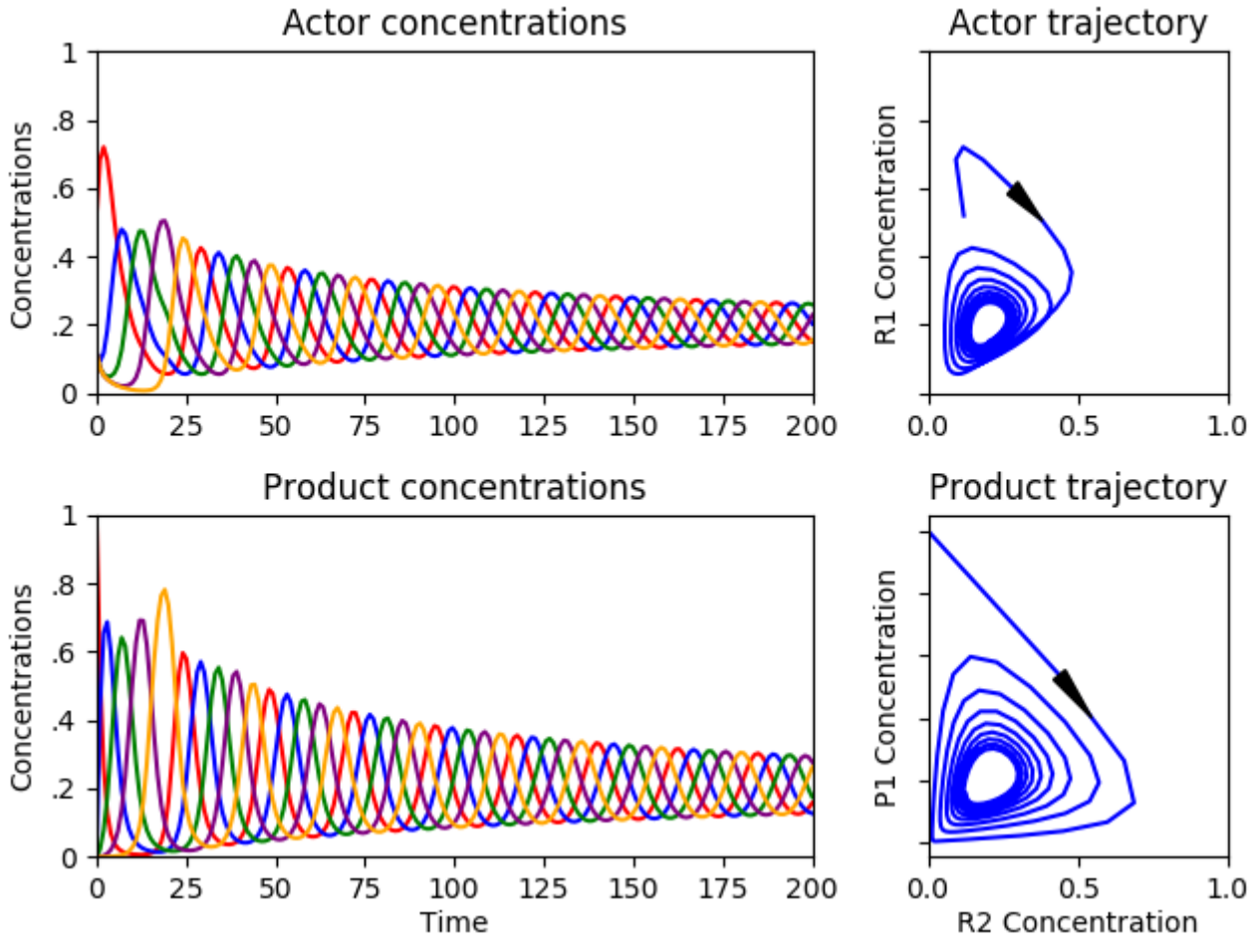
Conditions for Stability Analysis

$$r_{i,0} = [Total - (N-1) * 50, 50, \ldots, 50]$$

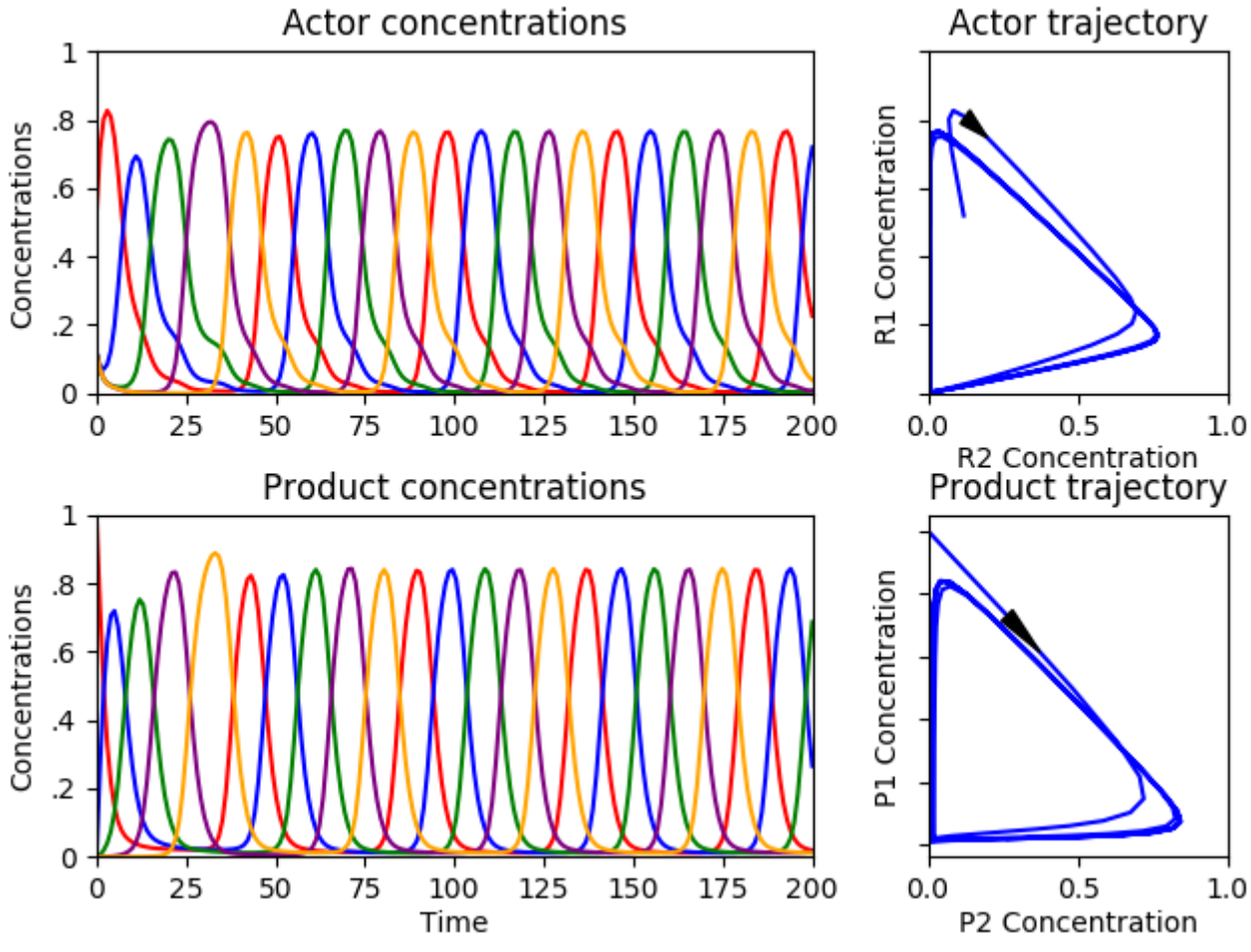
$$p_{i,0} = [\sigma * Total, 0, \ldots, 0]$$

$$\text{Boundary } \delta = .001$$

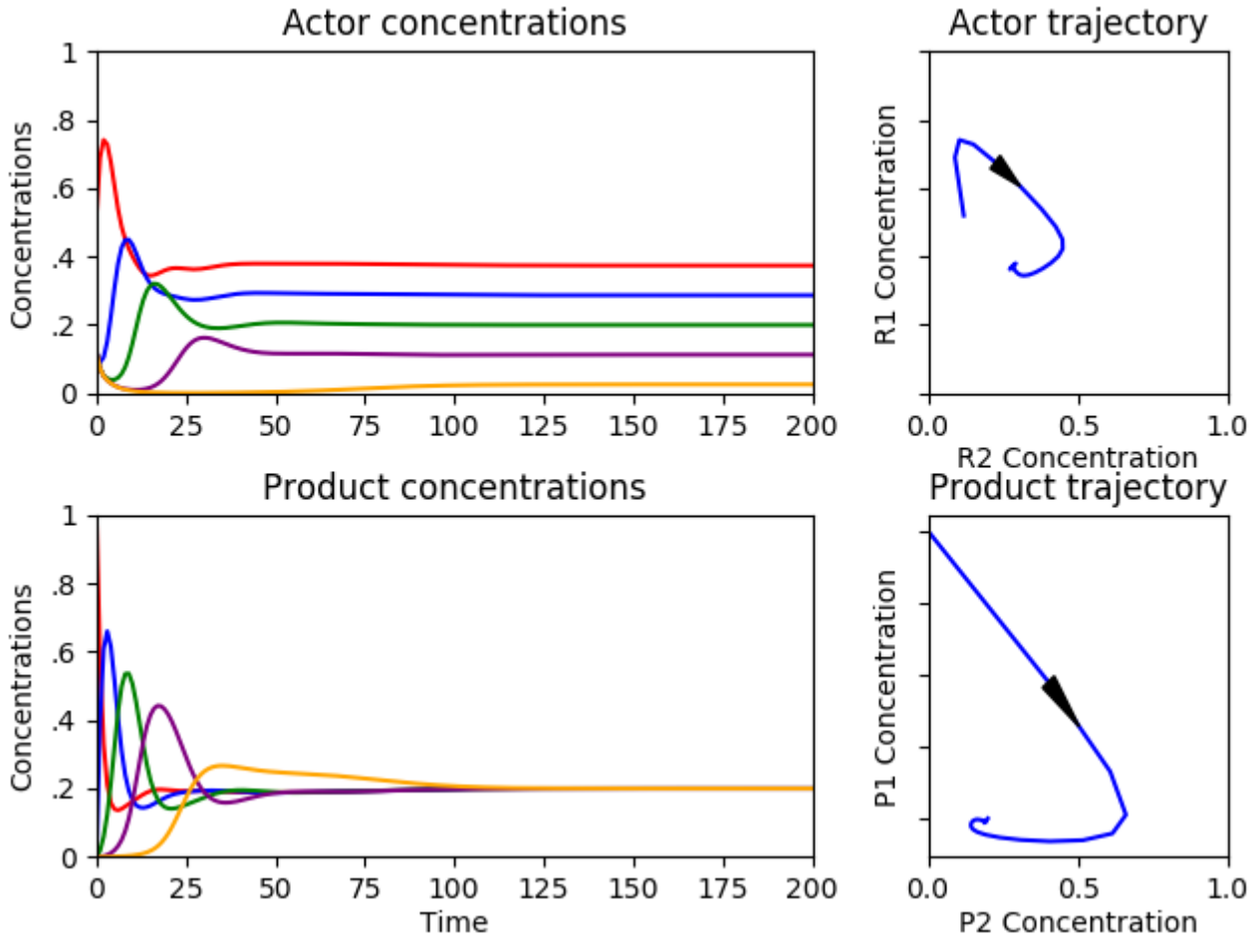
$$N = 5, \sigma = 1, F = 0$$


Actor concentrations
Concentrations
Actor trajectory
R1 Concentration
Product concentrations
Concentrations
Time
Product trajectory
P1 Concentration
R2 Concentration

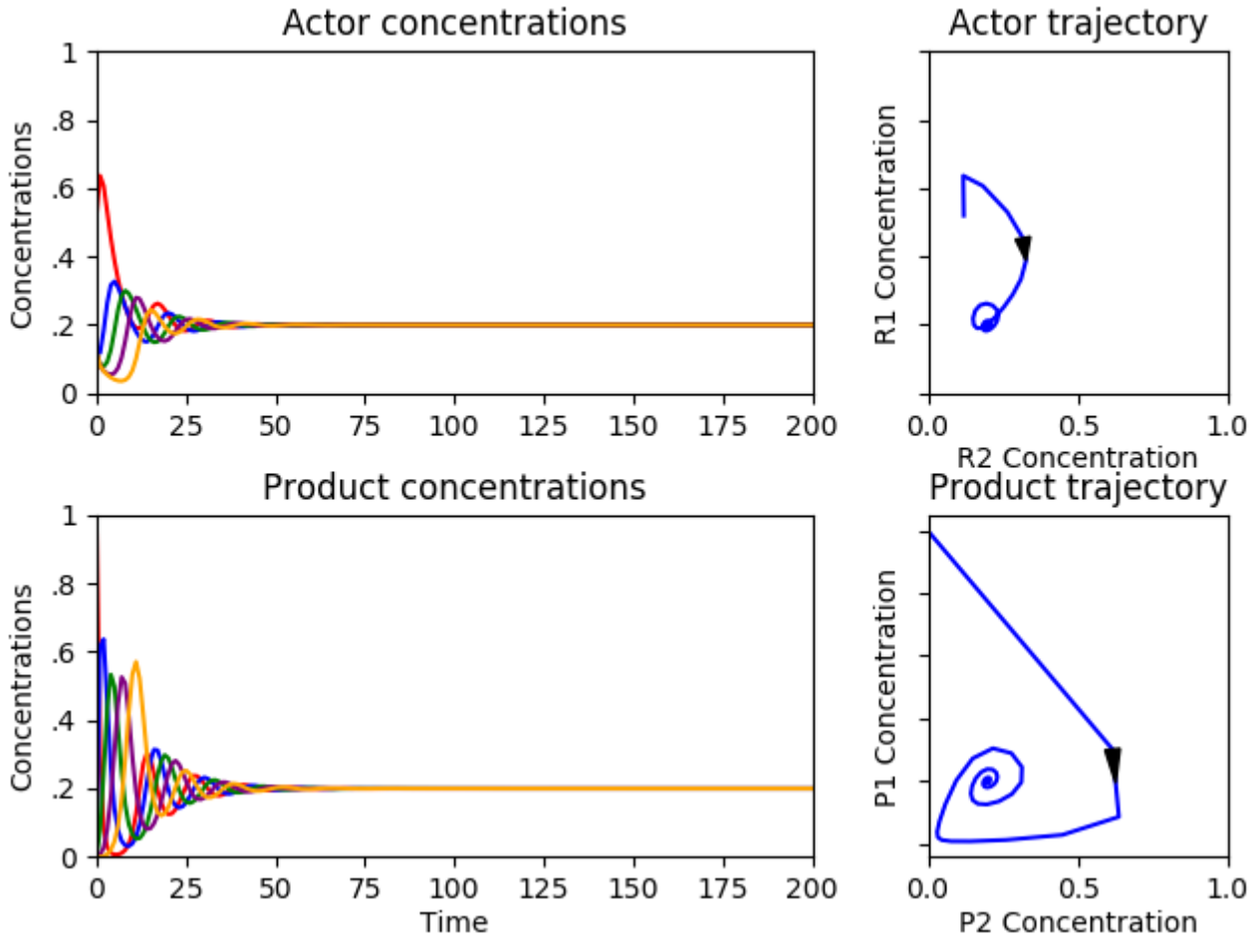
$$N = 5, \sigma = 1, F = 0.5$$



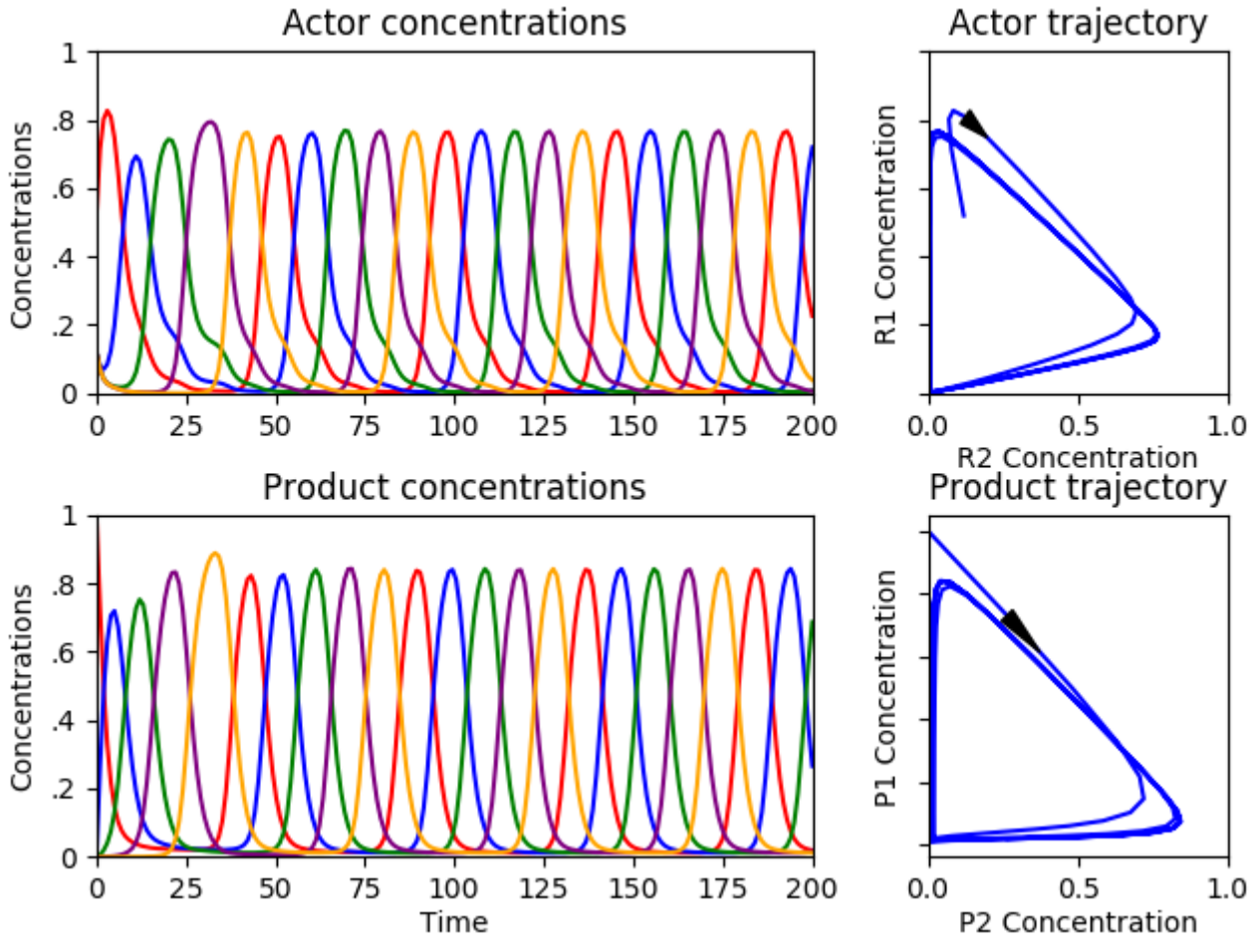
$$N = 5, \sigma = 1, F = 0.08$$



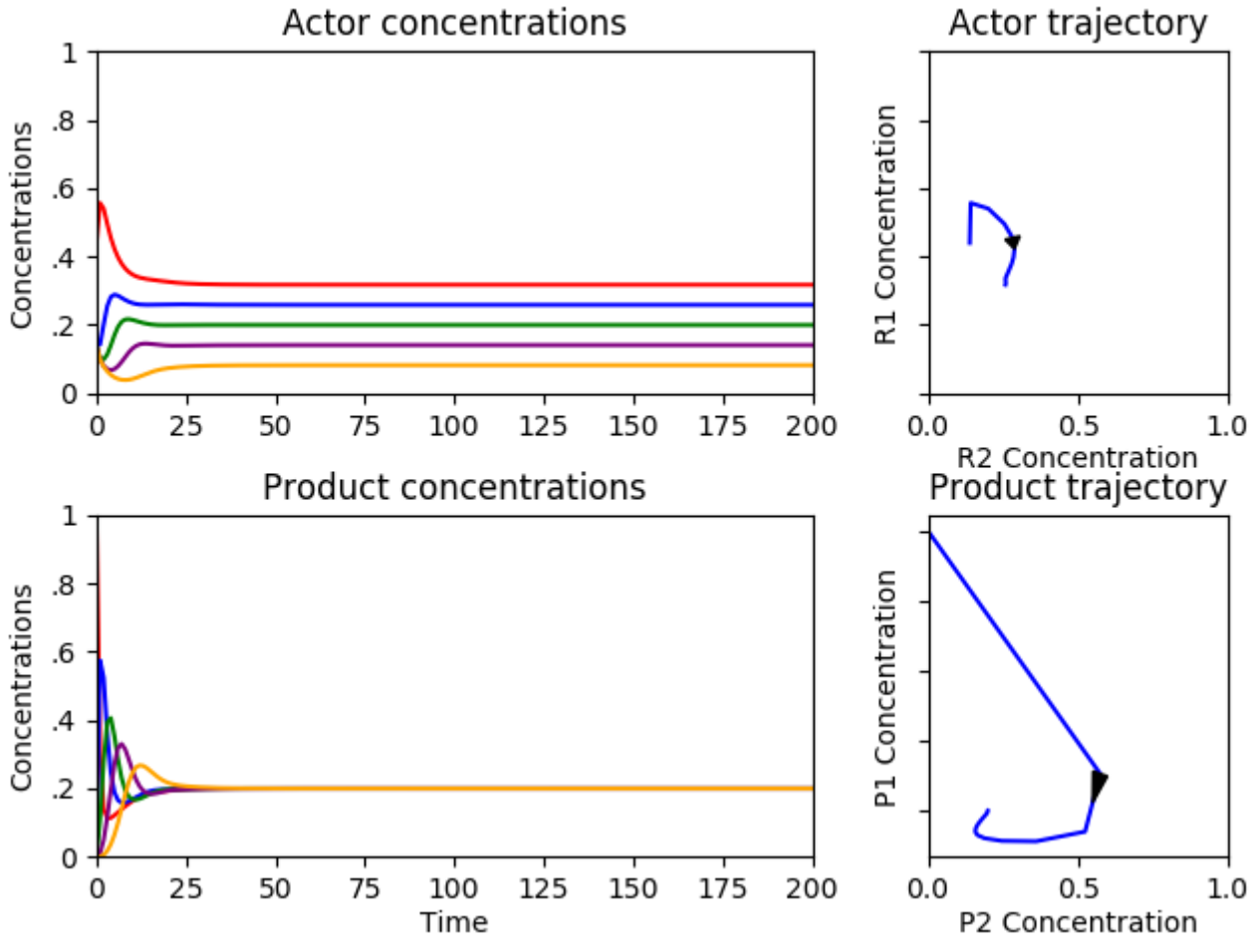
$$N = 5, \sigma = 2, F = 0$$


Actor concentrations
Concentrations
Time
Actor trajectory
R1 Concentration
R2 Concentration
Product concentrations
Concentrations
Time
Product trajectory
P1 Concentration
P2 Concentration

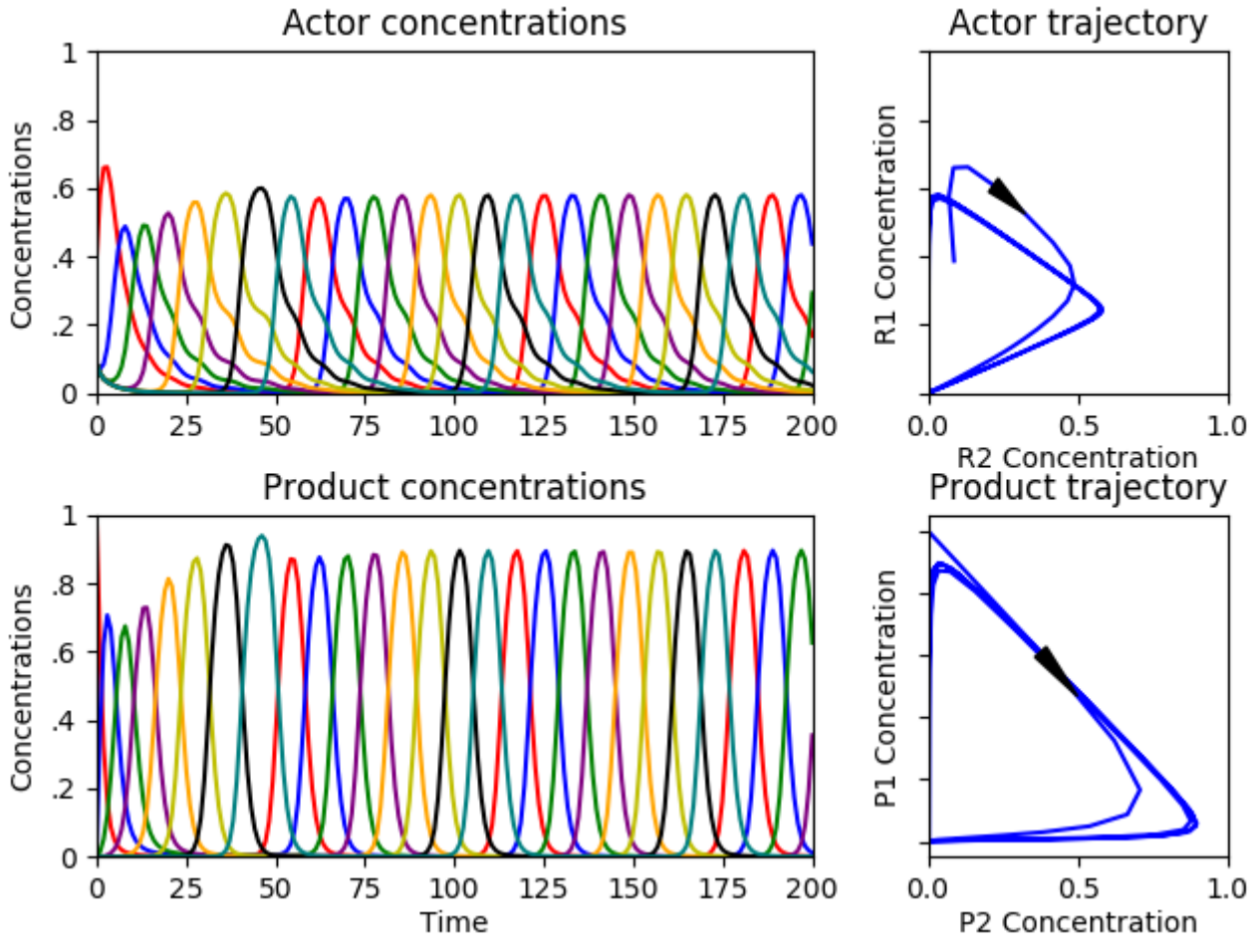
$$N = 5, \sigma = .5, F = 0$$


Actor concentrations
Concentrations
Time
Actor trajectory
R1 Concentration
R2 Concentration
Product concentrations
Concentrations
Time
Product trajectory
P1 Concentration
P2 Concentration

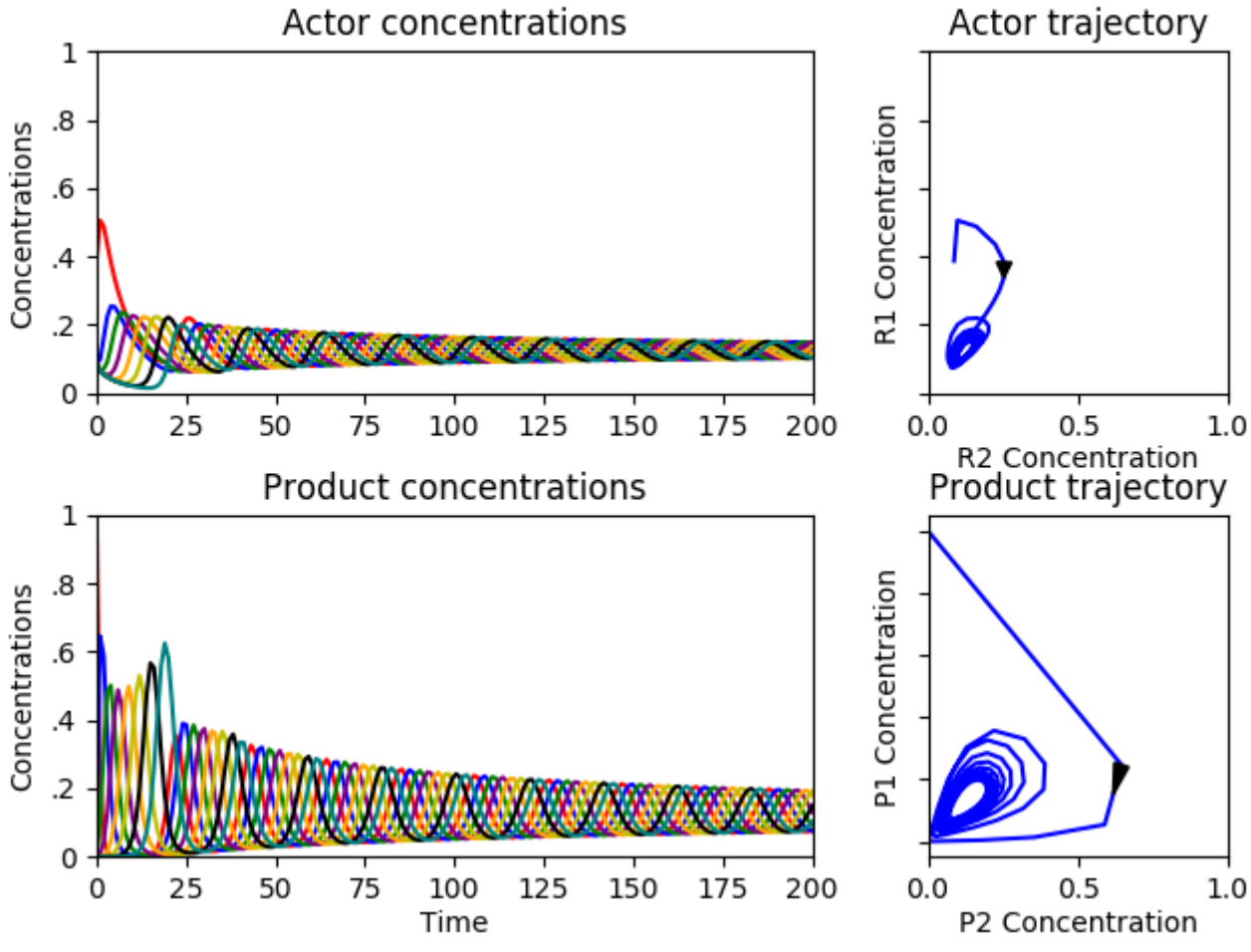
$$N = 5, \sigma = 3, F = 0.15$$



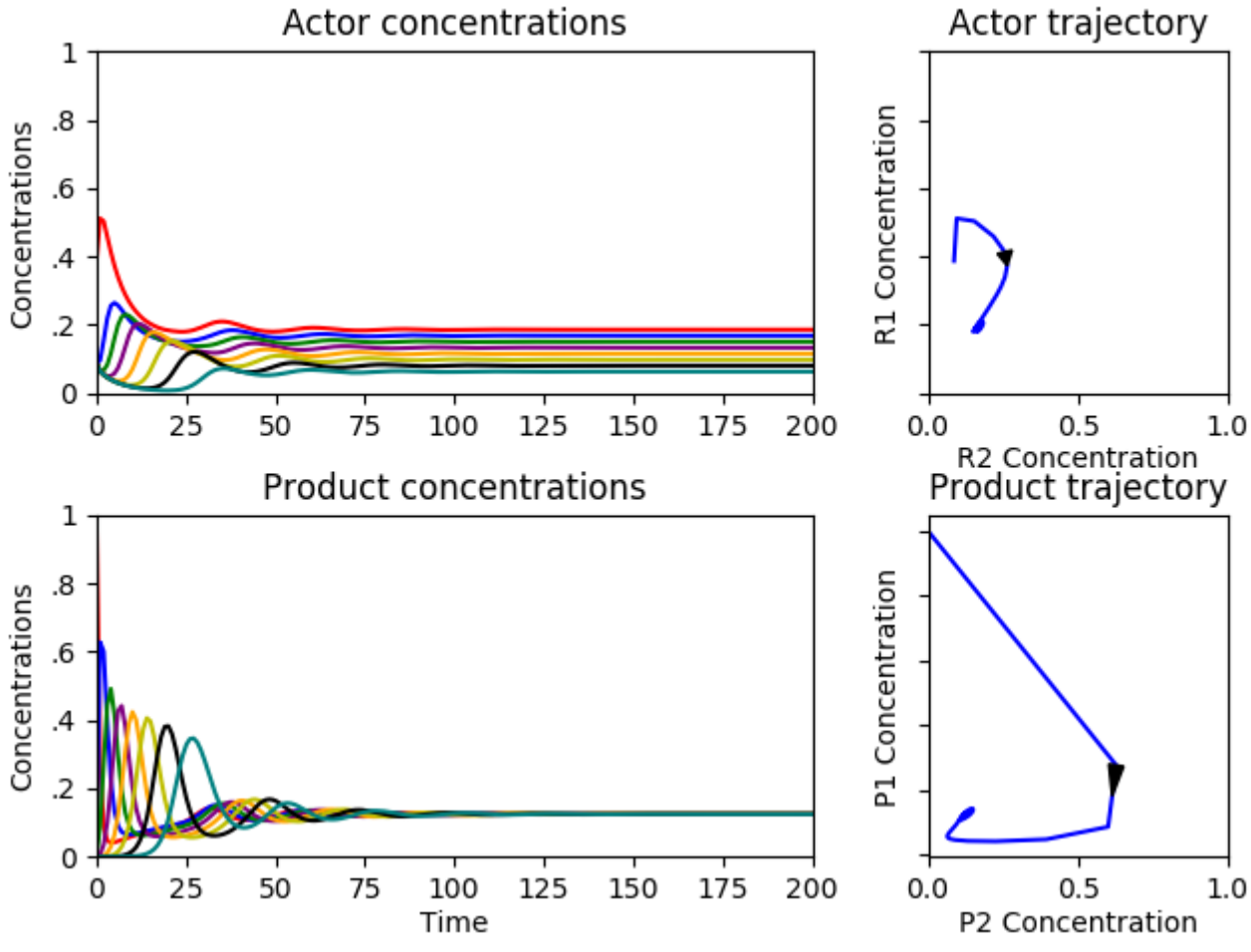
$$N = 8, \sigma = 1, F = 0$$

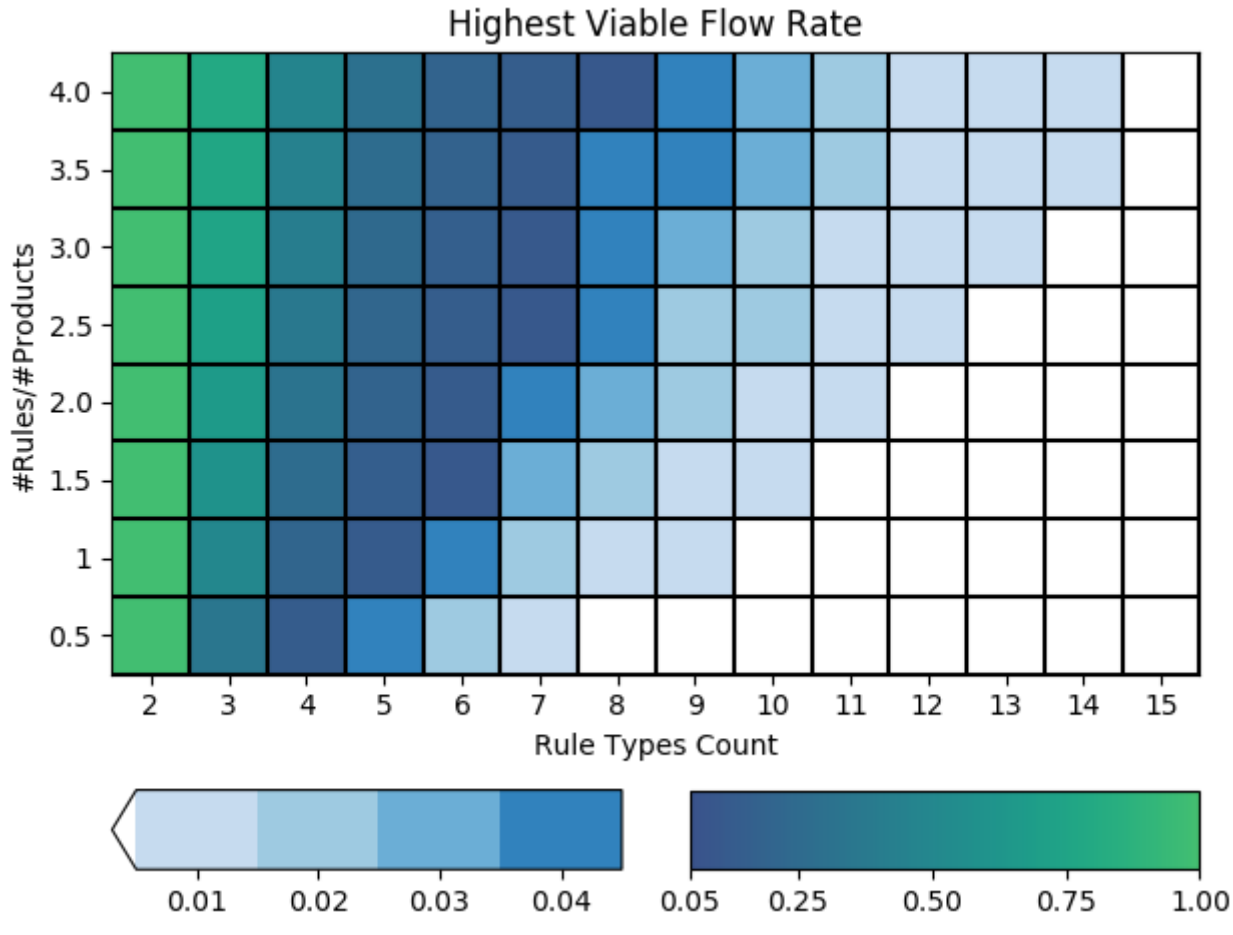

Actor concentrations
Concentrations
Time
Actor trajectory
R1 Concentration
R2 Concentration
Product concentrations
Concentrations
Time
Product trajectory
P1 Concentration
P2 Concentration

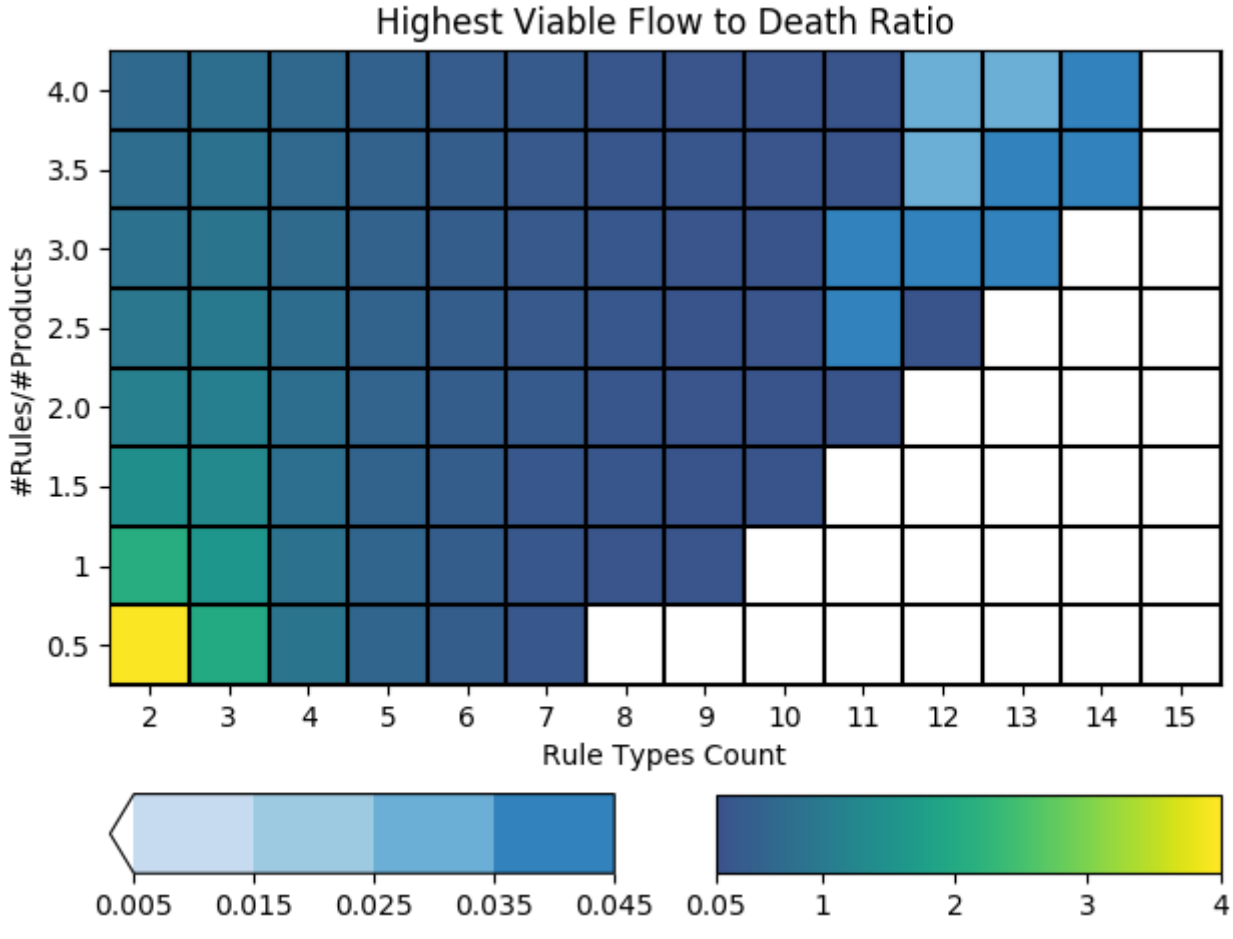
$$N = 8, \sigma = 3, F = 0$$


Actor concentrations
Concentrations
Actor trajectory
R1 Concentration
R2 Concentration
Product concentrations
Concentrations
Time
Product trajectory
P1 Concentration
P2 Concentration

$$N = 8, \sigma = 3, F = 0.05$$




Highest Viable Flow Rate
#Rules/#Products
4.0
3.5
3.0
2.5
2.0
1.5
1
0.5
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Rule Types Count
0.01
0.02
0.03
0.04
0.05
0.25
0.50
0.75
1.00


Highest Viable Flow to Death Ratio
#Rules/#Products
4.0
3.5
3.0
2.5
2.0
1.5
1
0.5
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Rule Types Count
0.005
0.015
0.025
0.035
0.045
0.05
1
2
3
4

Thank you!

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atwell@northwestern.edu