The Dot Dirichlet Model differs crucially from the Standard NK when we observe the Local Optima Networks. We notice that the Dot Dirichlet models will, for a given number of local maxima, contain fewer clusters. We see that for K=3 in the Standard NK model, for example that there are on average only 15 local maxima but over 4 clusters on average. The Dot Dirichlet model however, for K=5, maintains similarly only 15 maxima yet less than 2 clusters on average. At the higher end, for K=6 the standard NK model has 48 optima and about 6 clusters, but the dot Dirichlet model maintains, when it has 41 optima (at k=2) only 2 clusters. This shows that, in the standard NK model, search may not lead to one ever discovering the global maxima whereas search in the Dot Dirichlet model will.

Standard NK

Table

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Dot Dirichlet

Table

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Furthermore, we notice that consistently about 90% of all local optima are in the same cluster as the global optima for the Dot Dirichlet model; yet in the standard NK model roughly one third of maxima are in the global optima’s cluster.

We may now reconsider some of the past literature in this light. Rivkin (2000) examined how firms may be able to copy strategies from other firms. His key finding was that as a strategy increases in complexity that it will be harder for other firms to copy a strategy. His model assumed that a strategy, a location on a Kauffman N-K landscape, is imperfectly imitated and then firms will make hill-climbing incremental improvements. In this model the copying firms make improvements until they reach a local maxima. We can see his model results in figure 2 copied from his paper below. It is notable to note that as K increases the correlation between the distance from the highest peak and finesses (or firm values) of other reached peaks tends towards zero. This illustrates a property of the Kauffman NK model which the dot model does not imitate, owing to the dot model’s higher levels of autocorrelation. Indeed, should a firm make a minor mistake while copying in the standard NK model, disastrous outcomes are likely owing to the low autocorrelation, a global minimum and maximum could be only a few steps away and one may get ensnared in a relatively low local optima traversing between the two.

Indeed the LON table illustrates this point most clearly. If one supposes a firm gets stuck while copying and tries to find the global optima by making larger steps, it is more probable in the dot model as the global optima’s cluster is large, but no so for the standard NK model. Incremental improvement through somewhat broad search is thus more probable to lead to the global optima in the dot model, and the local optima values one gets stuck in if close to the global optima are likely to not wildly vary from the global optima’s value.

Gavetti and Levinthal (2000) found that in complex Kauffman landscapes that reducing dimensionality aided in search, however this aid in search arises from the ruggedness in Kauffman landscapes not found in the dot model, and their proposed each strategy is somewhat effective insofar as it does not consider LONs. In their model, representations of a landscape are formed by choosing a subset of N decisions to search over. This representation smooths the landscape as the number of peaks are necessarily reduced. What matters, they write, is that this representation leads to larger basins of attraction; as they cite Kauffman (1993) “The extent of a basin of attraction is positively correlated with the height of the local peak with which it is associated”. That is they find that their method of search through cognitive representations increases the basins of attraction. However for high K, these basins of attraction become negligibly small. Indeed as we have already discussed what matters are the local optima networks. The LONs are essentially basins of attraction, but of a higher step size. In the standard NK the LONs demonstrate that one may become stuck (never reach a global optima) when K is high. Indeed it seems that what Gavetti and Levinthal found is that representations help one shift to a larger basin of attraction, but in essence they are creating with different macro-level properties.

Similar work by Csaszar and Levinthal (2016) similarly illustrated how new landscapes may be created that have better properties for search. However their representation of the landscape (reproduced below) is not very illustrative of NK models, their model shows a landscape that is rugged in the sense that it has many local peaks, but has the macro property of a clear global optima. In the NK model, when ruggedness is high there is actually little variation with regard to local optima fitness, it more closely resembles the first inverted egg crate model below. Indeed, if anything, Csaszar and Levinthal’s model on the left hand side resembles the dot Dirichlet model – a landscape of a form the literature has yet to examine – given the clear existence of a global optima and a gradient of optima finesses that approach it.A picture containing shape

Description automatically generated

Chart

Description automatically generatedChart, surface chart

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The existing literature has been plagued by often taking as a baseline the heuristic search of local improvement/myopic (i.e. improve one at a time until a maximum is reached), and then proposing more complex strategies (Levinthal 1997; Rivkin 2000; Gavetti and Levinthal 2000; Summer and Loch 2004 Jain and Kogut 2014)

Lenox, however can be more easily explained. With high K, mior hanges to a landscape can lead to large changes in fitness even when there are many different maximas.

in their paper by noting the importance of basins of attraction and the correlation between basin size and a peak’s fitness. However, as we have demonstrated, as K increases these basins of attraction become smaller in the Kauffman NK model. Indeed their paper’s contribution is a strategy whereby firms may evaluate positions of greater than one step from

Csaszar and Levinthal similarily developed a model of cognitive representations. Such representations smooth the ruggedness of the landscape in terms of peaks. However, under the Kauffman model, when there are many peaks, the variation in fitness values in the landscapes is locally high relative to global variation, and so smoothing procedures would flatten a landscape. The image below shows how smoothing works on a landscape with many peaks but still a clear global maximum, in essence we can see two LON clusters. However the standard NK model presents a tradeoff whereby increasing the number of peaks reduces the size of the LON, that is there will be fewer hills in extended basins of attraction (search steps of size 2.) The dot model preserves ruggedness in terms of a high number of local maxima, but crucially it retains autocorrelation.

Motivating much of the past research utilizing fitness landscapes is the examination of optimal strategy with regards to exploration vs. exploitation (Levitt and March, 1988;March 1991; papers in this framework: Vuculescu 2016). Much research has used NK models or similar to study search strategies of firms, how they adapt, and implications for population dynamics with regards to variation and selection firms in an industry (Levinthal 1997; Levinthal and Posen 2007; McKelvey 1999; Rivkin 2000; Lenox et al. 2006; Lenox et al. 2007)

