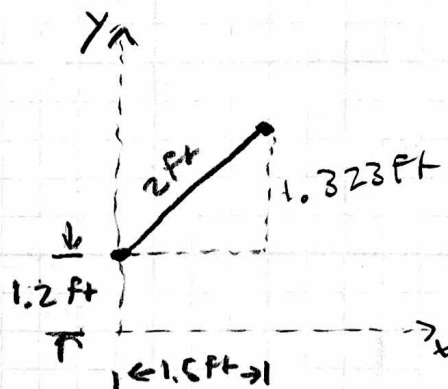
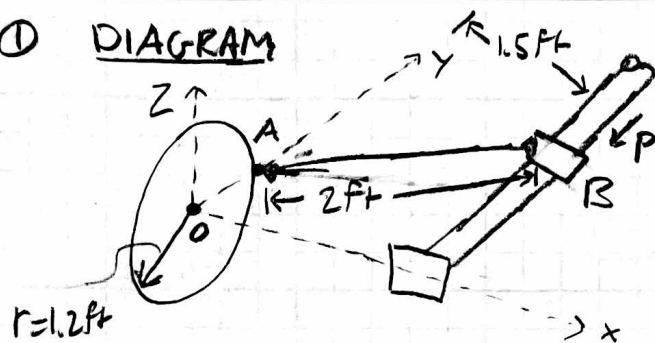


① DIAGRAMGIVEN

$$\vec{\omega}_A = (5 \text{ rad/s}) \hat{k}$$

FIND

$$\vec{\omega}_{AB} = ?$$

$$\vec{v}_B = ?$$

SOLVE

$$\vec{v}_A = \vec{\omega}_A \times \vec{r}_{AO} = (5 \frac{\text{rad}}{\text{s}}) \hat{k} \times (1.2 \text{ ft}) \hat{j} = 6 \frac{\text{ft}}{\text{s}} \hat{i}$$

$$\vec{\omega}_{AB} \cdot \vec{r}_{B/A} = (\omega_{ABx} \hat{i} + \omega_{ABY} \hat{j} + \omega_{ABz} \hat{k}) \cdot (1.5 \hat{i} + 1.323 \hat{j}) \text{ ft} = 0$$

$$(\omega_{ABx})(1.5) + (\omega_{ABY})(1.323) = 0 \quad (1)$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$(-P) \hat{j} = (6 \frac{\text{ft}}{\text{s}}) \hat{i} + (\omega_{ABx} \hat{i} + \omega_{ABY} \hat{j} + \omega_{ABz} \hat{k}) \times (1.5 \hat{i} + 1.323 \hat{j}) \text{ ft}$$

$$(-P) \hat{j} = (6 \frac{\text{ft}}{\text{s}}) \hat{i} - (1.323 \omega_{ABx} \hat{k} - 1.5 \omega_{ABY} \hat{k} + 1.5 \omega_{ABz} \hat{j} - 1.323 \omega_{ABz} \hat{i})$$

$$\hat{i}) \quad 0 = -1.323 \omega_{ABz} \Rightarrow \omega_{ABz} = 0 \text{ rad/s}$$

$$\hat{j}) \quad -P = 1.5 \omega_{ABz} \Rightarrow P = 0 \text{ ft/s}$$

$$\hat{k}) \quad 0 = 6 \frac{\text{ft}}{\text{s}} + 1.323 \omega_{ABx} - 1.5 \omega_{ABY} \quad (2)$$

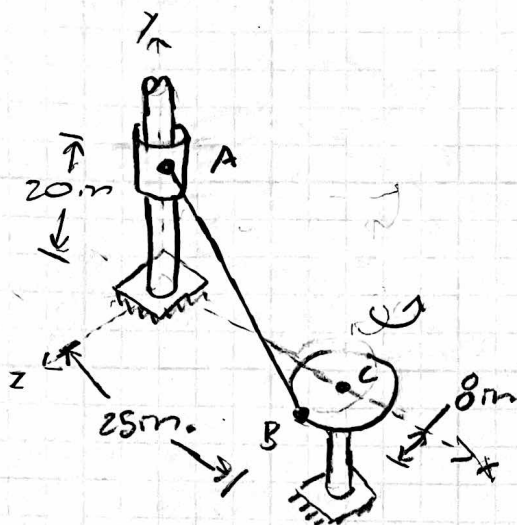
EQ. 1 + 2 GIVE THE RESULT:

$$\omega_{ABx} = -1.98 \text{ rad/s} \quad \omega_{ABY} = 2.25 \text{ rad/s}$$

$$P = 0 \text{ ft/s}$$

$$\vec{\omega} = (-1.98 \hat{i} + 2.25 \hat{j}) \text{ rad/s}$$

② DIAGRAM



GIVEN

$$\omega_{BC} = \left(-3 \frac{\text{rad}}{\text{s}}\right) \hat{j}$$

FIND

$$v_A = ?$$

SOLVE

$$\vec{v}_B = \vec{\omega}_{BC} \times \vec{r}_{B/C} = (-3\hat{j}) \times (8\hat{k}) = -24\hat{i} \text{ m/s}$$

$$\vec{\omega}_{AB} \cdot \vec{r}_{A/B} = 0 = (\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}) \cdot (-25\hat{i} + 20\hat{j} + 8\hat{k})$$

$$0 = -25\omega_x + 20\omega_y - 8\omega_z \quad (\text{eq 1})$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$-24\hat{i} = v_A\hat{j} + (\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}) \times (-25\hat{i} + 20\hat{j} + 8\hat{k})$$

$$-24\hat{i} = v_A\hat{j} + 20\omega_x\hat{k} + 8\omega_y\hat{i} + 25\omega_z\hat{k} - 8\omega_y\hat{i} - 25\omega_z\hat{j} - 20\omega_x\hat{j}$$

$$\hat{i}) -24 = -8\omega_y - 20\omega_z \quad (\text{eq 2})$$

$$\hat{j}) 0 = v_A + 8\omega_x - 25\omega_z \quad (\text{eq 3})$$

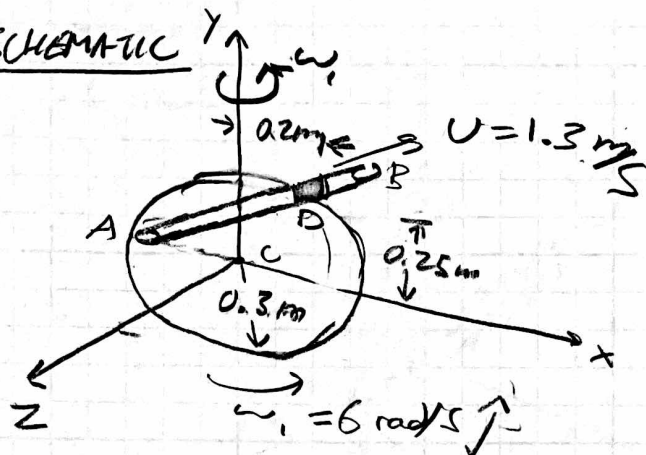
$$\hat{k}) 0 = 20\omega_x + 25\omega_y \quad (\text{eq 4})$$

SOLVE EQUATIONS 1-4 WITH COMPUTER

$$v_A = -30 \text{ m/s}$$

2)

SCHEMATIC



FIND a) $U_D = ?$
b) $a_D = ?$

SOLVE

ATTACH COORD SYSTEM TO A

$$\vec{v}_A = \vec{\omega}_1 \times \vec{r}_{A/C} = (6 \text{ rad/s} \hat{j}) \times (-0.3 \text{ m} \hat{i}) = 1.8 \hat{k} \text{ m/s}$$

$$\vec{\omega}_2 = \vec{\omega} = (6 \text{ rad/s}) \hat{j}$$

$$\vec{v}_D = \vec{v}_A + \vec{\omega}_2 \times \vec{r}_{D/A} + \vec{v}_{D/A}$$

$$\vec{v}_D = (1.8 \hat{k} \text{ m/s}) + (6 \text{ rad/s} \hat{j}) \times (0.5 \hat{i} + 0.208 \hat{j}) \text{ m} + U (\cos(22.6) \hat{i} + \sin(22.6) \hat{j})$$

$$\vec{v}_D = (1.8 \hat{k} \text{ m/s}) - (3 \text{ m/s} \hat{k}) + 1.2 \hat{i} + 0.5 \hat{j}$$

$$\boxed{\vec{v}_D = (1.2 \hat{i} + 0.5 \hat{j} - 1.2 \hat{k}) \text{ m/s}}$$

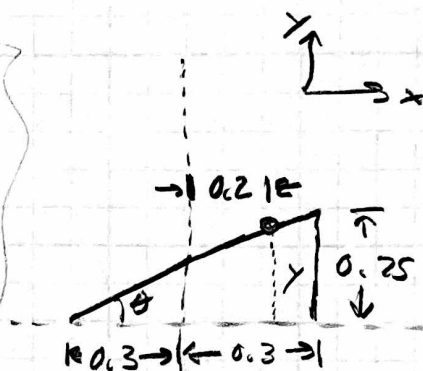
$$\vec{a}_A = \vec{\omega}_1 \times \vec{\omega}_1 \times \vec{r}_{A/C} = 6 \hat{j} \times 6 \hat{j} \times -0.3 \hat{i} = 10.8 \hat{i}$$

$$\vec{a}_D = \vec{a}_A + \vec{\omega}_2 \times \vec{r}_{D/A} + \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{r}_{D/A}) + 2 \vec{\omega}_2 \times \vec{v}_{D/A} + \vec{a}_{D/A}$$

$$\vec{a}_D = 10.8 \hat{i} + 6 \hat{j} \times 6 \hat{j} \times (0.5 \hat{i} + 0.208 \hat{j}) + 2 (6 \hat{j}) \times 1.3 \text{ m/s} (\cos(22.6) \hat{i} + \sin(22.6) \hat{j})$$

$$\vec{a}_D = 10.8 \hat{i} + 18 \hat{i} - 14.4 \hat{k}$$

$$\boxed{\vec{a}_D = -7.2 \hat{i} - 14.4 \hat{k}}$$



$$\theta = \tan^{-1}\left(\frac{0.25}{0.6}\right) = \tan^{-1}\left(\frac{y}{0.5}\right) = 22.6199^\circ$$

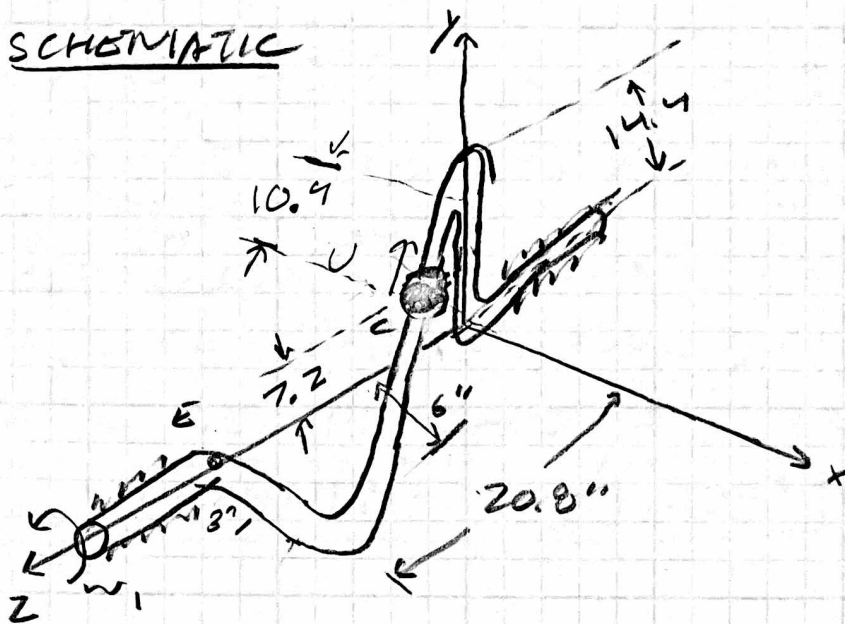
$$y = 0.5 \left(\frac{0.25}{0.6}\right)$$

$$y = 0.2083$$

$$\vec{r}_{D/A} = (0.5 \hat{i} + 0.208 \hat{j}) \text{ m}$$

PROBLEM 4

SCHEMATIC



GIVEN

$$U = 39 \text{ m/s}, \quad \vec{\omega}_1 = 5 \text{ rad/s } \hat{k}$$

FIND

$$v_c, a_c$$

SOLVE

$$\vec{r}_{C/E} = (3\hat{i} + 7.2\hat{j} - 10.4\hat{k}) \text{ m}$$

ATTACH ROTATING SYSTEM TO E

$$\vec{\Omega} = \vec{\omega}_1$$

$$\vec{v}_c = \vec{v}_e + \vec{\Omega} \times \vec{r}_{C/E} + \vec{r}_{xyz}$$

$$\vec{v}_c = \vec{\omega}_1 \times \vec{r}_{C/E} + \vec{r}_{xyz}$$

$$\vec{r}_{B/O} = (-6\hat{i} + 14.4\hat{j} - 20.8\hat{k}) \text{ m}$$

$$|\vec{r}_{B/O}| = 26$$

$$\vec{r}_{xyz} = \frac{\vec{r}_{B/O}}{|\vec{r}_{B/O}|} U = 39 \frac{\text{m}}{\text{s}} (-0.231\hat{i} + 0.554\hat{j} - 0.8\hat{k})$$

$$\vec{r}_{xyz} = (-9\hat{i} + 21.61\hat{j} - 31.2\hat{k}) \frac{\text{m}}{\text{s}}$$

$$\vec{v}_C = (5 \text{ rad/s}) \hat{k} \times (3\hat{i} + 7.2\hat{j} - 10.4\hat{k}) + (-9\hat{i} + 216.1\hat{j} - 31.2\hat{k})$$

$$\vec{v}_C = 15\hat{j} - 36\hat{i} - 9\hat{i} + 216.1\hat{j} - 31.2\hat{k}$$

$$\boxed{\vec{v}_C = (-45\hat{i} + 36.61\hat{j} - 31.2\hat{k}) \text{ m/s}}$$

$$\vec{a}_C = \cancel{\vec{a}_e} + \cancel{\dot{\vec{\omega}} \times \vec{r}_{C/e}} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{C/e} + 2 \vec{\omega} \times \cancel{\dot{\vec{r}}_{C/e}} + \cancel{\ddot{\vec{r}}_{C/e}}$$

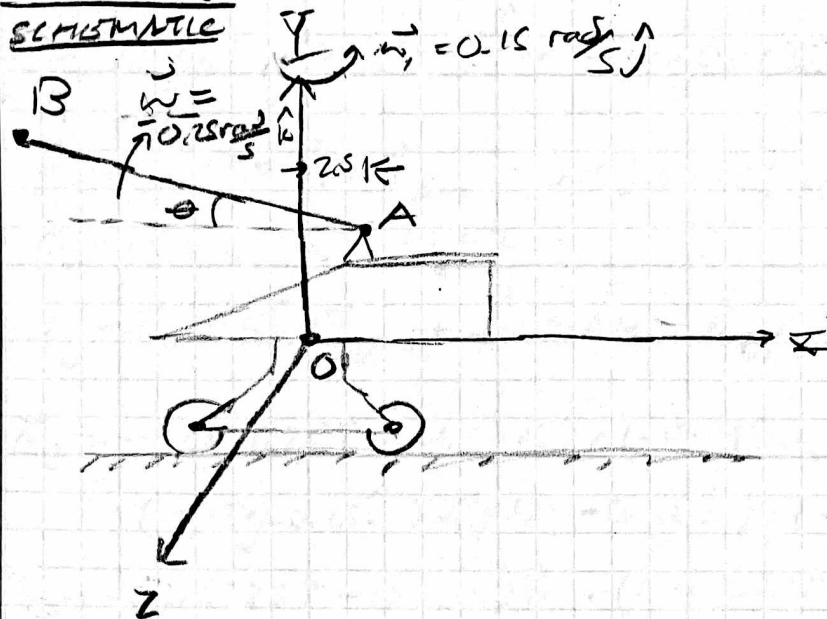
$$\vec{a}_C = 5\hat{k} \times 5\hat{k} \times (3\hat{i} + 7.2\hat{j} - 10.4\hat{k}) + 2(5\hat{k}) \times (-9\hat{i} + 216.1\hat{j} - 31.2\hat{k})$$

$$\vec{a}_C = -75\hat{i} - 180\hat{j} - 90\hat{j} - 216.1\hat{i}$$

$$\boxed{\vec{a}_C = (-291.1\hat{i} - 270\hat{j}) \text{ m/s}^2}$$

PROBLEMS

SCHEMATIC



GIVEN

$\theta = 20^\circ$
 $|r_{BA}| = 18 \text{ ft}$
SOLVE

FIND

$v_B = ?$ $a_B = ?$

ATTACH ROTATING COORD SYSTEM TO A

$$\vec{v}_A = \vec{\omega}_Y \times \vec{r}_{A/O}$$

$$\vec{r}_{A/O} = (2.5\hat{j} + 4\hat{k})$$

$$\vec{v}_A = 0.15\hat{j} \times (2.5\hat{j} + 4\hat{k}) = -0.375\hat{k}$$

$$\vec{\omega} = \vec{\omega}_Y + \vec{\omega}_{AB} = 0.15\hat{j} - 0.25\hat{k}$$

$$\vec{r}_{B/A} = 0, \vec{r}_{A/Z} = 0$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} + \vec{r}_{B/Z}$$

$$\vec{v}_B = (-0.375\hat{k}) + (0.15\hat{j} - 0.25\hat{k}) \times 18(-\cos\theta\hat{j} + \sin\theta\hat{k})$$

$$\vec{v}_B = -0.375\hat{k} + (0.15\hat{j} - 0.25\hat{k}) \times (-15.04\hat{j} + 5.472\hat{k})$$

$$\vec{v}_B = -0.375\hat{k} + 2.256\hat{k} + 3.76\hat{j} + 1.368\hat{j}$$

$$\boxed{\vec{v}_B = 1.37\hat{j} + 3.76\hat{j} + 1.881\hat{k} \text{ ft/s}}$$

$$\vec{\omega} = 0.15\hat{j} \times (-0.25\hat{k}) = -0.0375\hat{i} \text{ rad/s}^2$$

$$\vec{a}_a = \vec{\omega}_1 \times \vec{r}_{A/O} = 0.15\hat{j} \times 0.1\hat{j} \times (2.5\hat{i} + 4\hat{j})$$

$$\vec{a}_a = -0.05625\hat{i} \text{ ft/s}^2$$

$$\vec{a}_B = \vec{a}_a + \vec{\omega} \times \vec{r}_{B/A} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{B/A} + 2\vec{\omega} \times \vec{v}_{B/A}$$

$$\vec{a}_B = -0.05625\hat{i} + -0.0375\hat{i} \times (-15.04\hat{i} + 5.472\hat{j}) +$$

$$(0.15\hat{j} - 0.25\hat{k}) \times (0.15\hat{j} - 0.25\hat{k}) \times (-15.04\hat{i} + 5.472\hat{j})$$

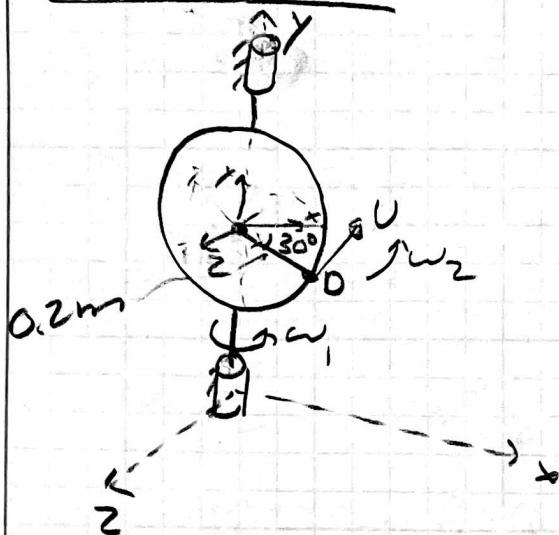
$$\vec{a}_B = -0.05625\hat{i} - 0.2052\hat{k} + (0.15\hat{j} - 0.25\hat{k}) \times (2.256\hat{i} + 3.76\hat{j} + 1.368\hat{k})$$

$$\vec{a}_B = -0.05625\hat{i} - 0.2052\hat{k} + (0.3384\hat{i} - 0.2052\hat{k} + 0.44\hat{i} - 0.342\hat{j})$$

$$\vec{a}_B = (-1.22\hat{i} - 0.342\hat{j} - 0.408\hat{k}) \text{ ft/s}^2$$

PROBLEM #6

SCHEMATIC



GIVEN

$$\begin{aligned}\omega_1 &= 10 \text{ rad/s } \uparrow \\ \dot{\omega}_1 &= -25 \text{ rad/s } \uparrow \\ U &= 1.5 \text{ m/s} \\ \dot{U} &= -3 \text{ m/s}^2\end{aligned}$$

FIND

- a) v_D
- b) a_D

SOLVE

ATTACH SYS AT C

$$v_C = 0$$

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$$

$$U = r\omega_2$$

$$\omega_2 = \frac{1.5 \text{ m/s}}{0.2 \text{ m}} = 7.5 \text{ rad/s } \uparrow$$

$$\vec{\omega} = 10 \uparrow + 7.5 \hat{k}$$

$$\dot{\vec{\omega}} = \vec{\omega}_1 \times \vec{\omega}_2 + \dot{\vec{\omega}}_{2 \text{ rel}} = 10 \uparrow \times 7.5 \hat{k} + \left(\frac{-3 \text{ m/s}^2}{0.2 \text{ m}} \right) \hat{k}$$

$$\dot{\vec{\omega}} = 75 \uparrow - 15 \hat{k}$$

$$\vec{\omega}_2 = (-25 \text{ rad/s } \uparrow) + 75 \uparrow - 15 \hat{k}$$

$$\vec{\omega}_2 = 75 \uparrow - 25 \uparrow - 15 \hat{k}$$

$$\vec{v}_b = \vec{v}_C + \vec{\omega} \times \vec{r}_{b/c} + \vec{v}_{b/2}$$

$$\vec{v}_b = (10\hat{j} + 7.5\hat{k}) \times 0.2(\cos(20)\hat{i} - \sin(20)\hat{j})$$

$$= (10\hat{j} + 7.5\hat{k}) \times (0.173\hat{i} - 0.1\hat{j})$$

$$\boxed{\vec{v}_b = -1.73\hat{k} + 1.248\hat{i} + 0.75\hat{j}}$$

$$\vec{a}_b = \vec{a}_C + \vec{\omega} \times \vec{r}_{b/c} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{b/c} + \vec{a}_{b/2}$$

$$\vec{a}_b = (25\hat{i} - 25\hat{j} - 15\hat{k}) \times (0.173\hat{i} - 0.1\hat{j}) + (10\hat{j} + 7.5\hat{k}) \times (0.75\hat{i} + 1.248\hat{j} - 1.73\hat{k})$$

$$\vec{a}_b = (-7.5\hat{k} + 4.325\hat{k} - 2.595\hat{j} - 1.5\hat{i}) + (-7.5\hat{k} - 17.3\hat{i} + 5.625\hat{j} - 9.735\hat{j})$$

$$\boxed{\vec{a}_b = (-29.54\hat{i} + 3.03\hat{j} - 10.68\hat{k}) \text{ m/s}^2}$$