

$$① T = \frac{1}{2} \int_0^L \left(\frac{x^2}{2e^3} (3e-x) \dot{\delta} \right)^2 \rho dx$$

$$T = \frac{1}{2} \dot{\delta}^2 \int_0^L \frac{x^4 (x-3e)^2}{4e^6} \rho dx$$

$$T = \frac{\dot{\delta}^2 \rho}{8e^6} \int_0^L x^4 (x-3e)^2 dx$$

$$T = \frac{\dot{\delta}^2 \rho}{8e^6} \left(\frac{83e^7}{85} \right)$$

$$T = \dot{\delta}^2 \rho \frac{33e}{280} = \dot{\delta}^2 \left(\frac{99}{280} \right) \left(\frac{\rho e}{8} \right)$$

$$T = \dot{\delta}^2 \left(\frac{99}{280} \right) m_{eff}$$

$$② \omega_h = \sqrt{\frac{EI}{mL^3}} (L\lambda)^2$$

FIND $L\lambda$ VALUES

$$\cos(L\lambda) = \frac{-1}{\cosh(L\lambda)}$$

SOLVE WITH MATLAB TO GET

$$L\lambda = +.8751, 4.6941, 7.8548$$

$$E = 10 \times 10^6 \text{ psi}$$

$$m = (0.075 \text{ in}) (1.125 \text{ in}) (0.1 \frac{\text{lbm}}{\text{in}^3}) \times \left(\frac{32.174 \text{ slug}}{32.174 \text{ lbm}} \right)$$

$$(m) = 0.00184 \text{ slug}$$

Ans m

$$I = \frac{1}{12} \omega_h^3 = 0.008894 \text{ in}^4$$

$$\omega_1 = 1320.3 \frac{\text{rad}}{\text{s}}$$

$$\omega_2 = 8274.12 \frac{\text{rad}}{\text{s}}$$

$$\omega_3 = 23,187.8 \frac{\text{rad}}{\text{s}}$$