California State Polytechnic University
San Luis Obispo

Handbook and Reference for the
ME318 Laboratory

Although care has been taken to make these exercises as safe as possible, the authors cannot take responsibility for any damage or injury which may occur while performing the exercises described in this manual. It is the responsibility of students and instructors to practice safe laboratory procedures at all times. Refer to the Laboratory Risk Assessment (page v) for specific safety information.

This document was compiled on January 16, 2015.

Contents

Vibrations Lab Risk Assessment

- **120-volt power supply** to equipment necessitates that frayed or damaged power cords be repaired immediately.
- **Rotor Kit.** This apparatus has rotating parts and care must be taken as with all rotating machinery. No loose hair or clothing; attach nothing that may fly off; never touch rotating parts. The apparatus has gear teeth around the inboard coupling with a pinch point between housing and gear teeth. Safety glasses are to be worn by *all persons in the lab* while this experiment is in operation.
- **Trip hazard** from beams mounted on concrete blocks and from wires extending from portable equipment.
- **Danger from heavy equipment** placed atop light carts falling over.
- **Hydraulic Shake Table.** Hydraulic fluid can reach 150^{o}F. Hydraulic pressures of about 1000 psi can cause serious injury if leaks occur. Table may move suddenly with possible pinch points between the table and housing. All parts on the test stand must be securely fastened. Stand back from the table when in operation.
- **The hydraulic power source** is in an insulated closet. Prolonged exposure to this noise source could damage hearing. Ear protection available in the lab.
- **Slip Table.** People should not stand near table when in operation. Possible pinch points between slip table and granite slab or housing.
- **Drop table.** Extreme danger of serious injury if safety guards are disabled. An electronic switch deactivates controls when someone steps on the mats around the machine. Also, the machine will not operate unless the safety pins are put in the fixture on the side of the controller. Danger from heavy falling object and pressurized tanks of nitrogen.

Exercise 1

Computer Simulation

1.1 Objectives

The objectives of this exercise are to

1. Familiarize the student with the Matlab™ program, and use it to numerically integrate a differential equation using a fourth-order Runge-Kutta algorithm.

2. Investigate the equilibrium points of a nonlinear system.

3. Investigate the effects of initial conditions on the frequency of response of undamped linear and nonlinear systems.

1.2 Introduction

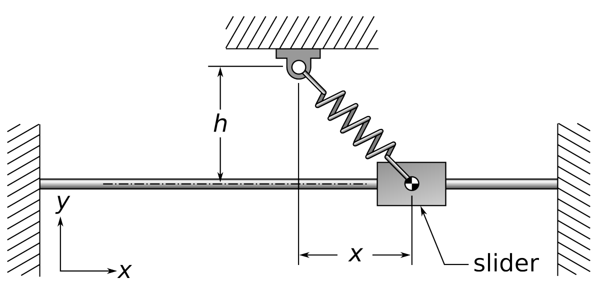


Figure 1.1: Nonlinear system

An interesting non-linear vibration system is shown in Figure 1.1. The system's parameters are as follows:

- Mass: $m = 3$ kg
- Spring: $k = 100$ N/m, $\ell_0 = 5$ m
- Spacing: $h = 3$ m
- Friction: $\mu_S = \mu_K = 0$ for the slider

Notice that the mass is located at $x = 4.0$ meters for one of its static equilibria. The spring acts in both tension and compression with a force $F_S = k(\ell - \ell_0)$.

If the slider is released from rest and the initial displacement from this static equilibrium is small, the mass will oscillate about the equilibrium point with x always greater than zero. For larger initial displacements, still released from rest, the mass will "pop" through the midpoint and will oscillate between positive and negative values of x.

1.3 Analysis

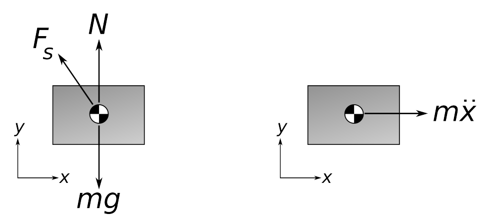


Figure 1.2: Diagrams used in the analysis

The Free Body Diagram and Mass-Acceleration Diagram are shown in Figure 1.2, for the mass at an arbitrary position x. Using these figures, we can write the differential equation of motion in the x direction as

$$m\ddot{x} = \sum F_x = \frac{-kx\,(\ell - \ell_0)}{\sqrt{x^2 + (3\text{ meters})^2}} \tag{1.1}$$

Where the spring's length, at an arbitrary value of x, is given by

$$\ell = \sqrt{x^2 + (3\text{ meters})^2} \tag{1.2}$$

In your writeup, you should identify the **spring** force in Equation 1.1 and also the sine of "the angle" and then show, from the above equations, that the differential equation of motion in the x direction can be expressed as:

$$m\ddot{x} = -kx\left[1 - \frac{\ell_0}{\sqrt{x^2 + (3\text{ meters})^2}}\right] \tag{1.3}$$

1.4 Procedure

(1) Calculate by hand the initial value of x for which the slider should just barely "pop" through the midpoint. You can do this by noting that, from this initial displacement, the slider will reach the midpoint with zero final velocity. If it reaches zero velocity before the midpoint it will always oscillate on the same side of the midpoint, but if it has a nonzero velocity when it reaches the midpoint it will go on through and oscillate between positive and negative values of x. So we want the system to begin with

zero kinetic energy at the initial displacement and end with zero kinetic energy at the midpoint. Note that the slider's path is horizontal, so there is no gravitational potential energy.

(2) It is not practical to solve the differential equation in Equation 1.3 by hand, so you must write a Matlab™ program to solve it. See Appendix E for help with writing Matlab™ code to solve the equations.

(3) Make a plot of the response for each of the following five initial conditions. In each case the slider is released from rest so the initial velocity is zero.

Case 1 $x_0 = 0 + 0.01$ m *Adjust relative tolerance if necessary.* [1]
Case 2 $x_0 =$ 4m + 0.01 m
Case 3 $x_0 =$ -4m - 0.01 m
Case 4 The "pop" through initial displacement less 0.01 m
Case 5 The "pop" through initial displacement plus 0.01 m

Note that cases 1, 2, and 3 are each 0.01 m away from a static equilibrium position. Explain why the response is different in each case. Compare the oscillation frequencies for the three cases. Could you determine that the system is nonlinear based on the plots alone? In a linear single-degree-of-freedom system, there is one natural frequency and the motion is sinusoidal for all initial conditions. How many frequencies does your system exhibit? Is the motion always sinusoidal?

[1]
```
options = odeset ('RelTol', 1E-6, 'AbsTol', 1E-9);
[t, x] = ode45 ('dataname', tspan, x0, options);
```

Exercise 2

Load Cell Calibration

2.0 Prelab

This prelab is due at the beginning of the laboratory period.

1. Given the following experimental data for an apparatus which is set up as shown in Figure 2.1:

   - The total mass carried by the load cell is 0.03 slug. Remember that one slug is one $\mathrm{lb} \cdot \mathrm{s}^2/\mathrm{ft}$.
   - An accelerometer with a calibration constant of 100 mV/g produces a 75 mV signal while the load cell produces a 345 mV signal. Note that 'g' here denotes the acceleration due to gravity.

   Calculate the acceleration in $\mathrm{ft/s}^2$ and the calibration constant of the load cell in mV/lb.

2. Read Appendix F to learn about our oscilloscopes.

3. Read Appendix G about accelerometers and load cells.

2.1 Objectives

The objectives of this lab are to:

1. Experimentally determine the calibration constant of a load cell and compare it to the published value.

2. Check the load cell for linearity and flat frequency response.

2.2 Experimental Overview

We attach an accelerometer on the top of a proof mass and then mount the assembly on the top of a force transducer.

> Note: The force transducer (load cell) must be oriented with its anvil on top, otherwise it will measure the force necessary to accelerate

its own mass along with that of the proof mass and accelerometer. Some of the force transducers are marked TOP and BASE; others require that you look for the gold-colored anvil.

The system (accelerometer, proof mass and force transducer) is then attached to the electromechanical shake table and vibrated sinusoidally. A signal generator provides the sine wave input to the table.

Getting the correct output from the function generator

If you don't set it up properly, the HP33120A function generator displays one half the voltage it is actually producing, *i.e.* setting the generator to 1 V_{PP} (one volt, peak to peak) actually produces 2 V_{PP}. This is because the default setting of the generator is to match a 50 Ω load circuit, and we don't have a 50 Ω load circuit – we have a high impedance ("High Z") circuit. You should change the function generator's setting by entering the following keystrokes:

Shift Enter	Enter menu mode
> > >	The display should say D SYS MENU
∨ ∨	The display should say 50 OHM
>	The display should now say HIGH Z
Enter	Save changes, back to regular operating mode

Now you need to check the function generator's output with an oscilloscope to make sure that the voltage coming out of the generator is what the display says it should be.

The accelerometer and force transducer outputs are sent through the charge amplifier to separate channels on the oscilloscope. The accelerometer signal peak-to-peak amplitude is read on the oscilloscope and converted to units of g's, using the published calibration constant. Now, Newton's Second Law is solved for the load. Using this data with the corresponding voltage output of the load cell we can calculate the load cell's calibration constant in units of mV/lb. Best results are obtained by taking several data points and plotting voltage versus force in lb. The slope of this plot is the calibration constant.

2.3 Procedure

(1) Proof mass and coupling weights are provided on the data sheet. Record the force transducer and accelerometer calibration constants and masses (see chart attached to the side of the cabinet near the equipment room) on the data sheet.

(2) Hook up the equipment as shown in the wiring diagram of Figure 2.1. Please be careful with the thin load cell wire; it is fragile and expensive.

Connect to the signal generator's OUTPUT terminal, **not** to its SYNC terminal (which produces only square waves!).

(3) Set the signal generator to sine wave, 90 HZ, no DC offset and minimum amplitude (initially). Thereafter keep the input voltage below 3 Vpp.

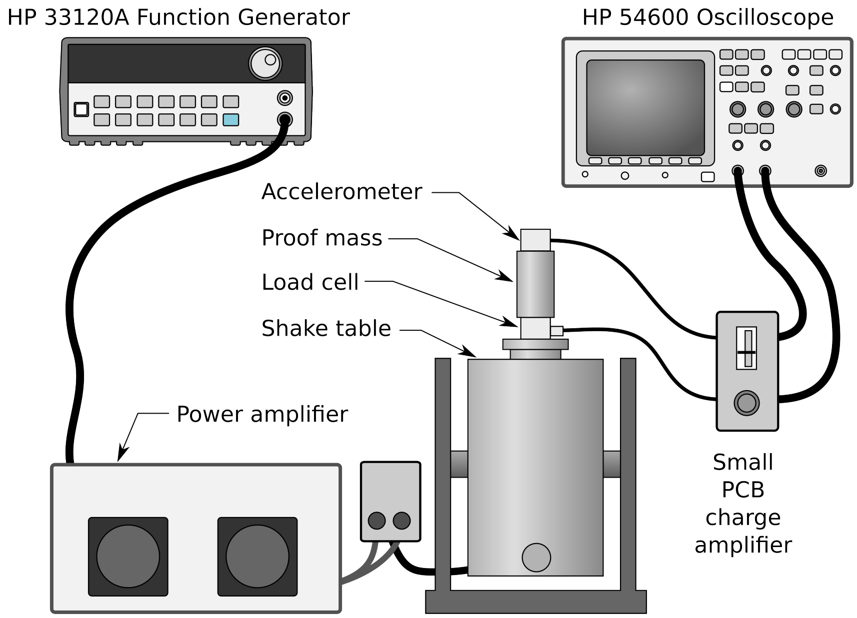


Figure 2.1: Experimental setup and wiring diagram for load cell calibration experiment

(4) Turn on the power to all of the components. Check the PCB power supply meter to ensure that the connections are good (meter reads in the green) for the two channels you are using. Slowly increase the amplitude of the signal at the signal generator, and touch the shake table – you should be able to feel it vibrate.

(5) Check the actual driving frequency with the oscilloscope to make sure it is the same as that you set in step (3).

(6) Set the input voltage to the power amp to one of six different levels up to 3 Vpp. When you choose a new level, check the oscilloscope to make sure the signal remains sinusoidal. At each new level, read and record the voltages of the accelerometer and the force transducer using the oscilloscope's measurement functions. Repeat this procedure for 5 other shake table input voltages up to 3 Vpp.

(7) Repeat steps (5) and (6) for a driving frequency of 900 Hz.

(8) Fill in all the other columns of the data sheet by using the appropriate calibration constants, formulae, and conversion factors.

(9) Use the 90 Hz data and make a plot of load cell output (volts) versus force (lb). Find a linear regression fit for the data. Then write an equation showing the calibration function for the load cell, using appropriate variables and units in your equation. **Always write the date, serial numbers of accelerometer and load cell, and technicians' names on calibration curves.**

(10) Repeat step 9 for the 900 Hz data.

Exercise 3

Introduction to Spectral Analysis

3.0 Prelab

This prelab is due at the beginning of the laboratory period.

1. Read Appendix B about Fourier analysis.
2. Determine a sine wave's Fourier coefficients. Let $f(t) = F \sin \omega t$ in the integrals found in Table B.1. *Note: These integrals define the period as 2T.* Many of the coefficients are zero, but you must evaluate the integrals to prove that they vanish. Use of symbolic math software such as MathCAD™, Mathematica™, or Wolfram.com™ is encouraged as long as you document what you have done.
3. Use the math handbook page located in Appendix B to determine the first five harmonic frequencies of the series and their amplitude coefficients for a triangle wave and a square wave. Modify the equations to show ω (which requires that you relate period to frequency) so you can clearly identify the 1ω, 2ω, 3ω, 4ω and 5ω terms. Also modify the equations to yield a waveform with an amplitude of $A = 0.5$.
4. Enter the coefficients you calculated in steps 2 and 3 onto your data sheet for this exercise. Extract the coefficients from the equations and insert them in the cells marked A_{theory} in the data sheet on Page H-4.
5. Watch the "Spectrum Analyzer" movie on the ME318 PolyLearn site.

3.1 Objective

The objective of this experiment is to find the first five terms in the Fourier series for periodic waveforms, both experimentally and theoretically, and to compare the results.

3.2 Introduction

This experiment will develop your ability to think in the frequency domain and will also introduce you to the laboratory tools used to obtain frequency domain information. We will study the relationship between the time and frequency domain from both an experimental and analytical standpoint, by finding the frequency domain representations of some common periodic waveforms.

3.3 Procedure

(1) Turn in your prelab to your instructor before proceeding.

(2) Connect the function generator to the oscilloscope and signal analyzer as shown in Figure 3.1. Set the HP 33120A function generator for a high impedance output; see the instructions on Page 2-2. Set the function generator output to a sine wave at 300 Hz. Be sure there is no DC bias on the periodic waveform. Set the amplitude so that the signal is about 1 volt peak-to-peak. Measure the peak-to-peak voltage and the frequency with the oscilloscope and record them on the datasheet. Set up the signal analyzer using the instructions given on the following page.

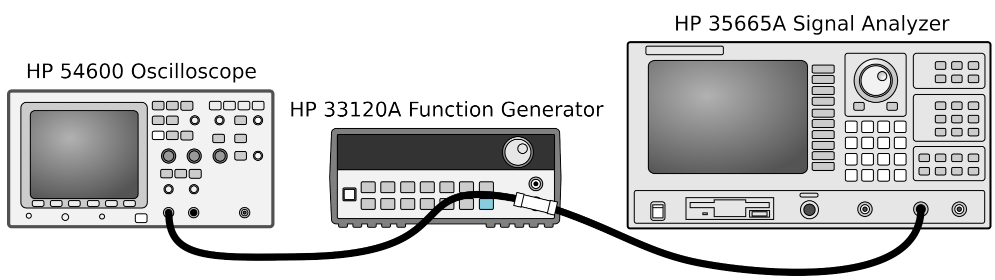


Figure 3.1: Wiring Diagram for Introduction to Spectral Analysis

(3) Take a set of data with the analyzer by pressing the [Start] key. A set of readings will be taken and averaged; the results will be displayed on the screen. Activate automatic scaling with the command sequence

[Trace Coord] (Linear Magnitude) [Scale] (Autoscale On)

You should now have an amplitude versus frequency display on the analyzer; this is the Fourier series representation of a sine wave. Try changing the axes to log magnitude and notice the difference in how data is displayed. Use the marker knob and marker functions to read data from plot; enter your magnitude and frequency data into the data sheet for the frequencies of interest.

General setup for the HP 35665A signal analyzer

Keys are shown as [Key] and menu items, chosen from the screen by pressing keys [F1] through [F10], are shown as (Menu Item).

- Set analyzer to default settings using SYSTEM keys:
  [Preset] (Do Preset)
- Set frequency, channels, and averaging with MEASUREMENT keys:
  [Inst Mode] (1 Channel) (FFT Analysis)
  [Window] (Flat Top)
  [Freq] (Span) 1600 Hz (Start) 0 Hz
  [Avg] (On) (Number Averages) 8
- Set the display mode using DISPLAY keys:
  [Disp Format] (Single)
  [Trace Coord] (Linear Magnitude) *also look at log magnitude*
  [Scale] (Autoscale On)
  [Meas Data] (Lin Spec CH 1)
- Set up printing using the SYSTEM keys:
  [Plot/Print] (More Setup) (Device is Prnt) (Return)
  Then to print, just press [Plot/Print] (Start Plot/Prnt)
- The analyzer has an online help system; for example, try
  [Help] [Window] (Flat Top)
  To exit help push [0] on the numeric keypad

(4) Compare the amplitude data you have measured with the results predicted by Fourier Series Theory from your prelab. At what frequency or frequencies do you have amplitude peaks? How do the amplitudes compare with the theoretical coefficients?

(5) Repeat the above, first for a triangular waveform and then for a square waveform. Record and compare the first five harmonic values with those you obtained analytically.

(6) Set the frequency span to start at 890 Hz and stop at 910 Hz:

[Freq] (Center) 900Hz (Span) 20 Hz

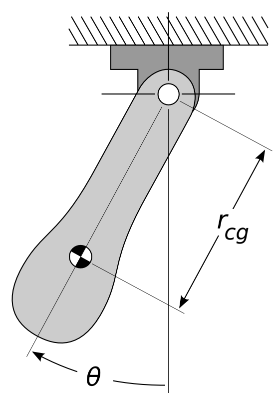
For the square wave measure the amplitude and frequency at the peak within this zoomed in span. We will use this zooming-in technique in future experiments to improve the accuracy of our data.

Exercise 4

Mass Properties of a Rigid Body

4.0 Prelab

On a separate sheet of paper draw a free body diagram and mass acceleration diagram for the compound pendulum shown below, and then write the linearized (*i.e.*, small angle) equation of motion for the compound pendulum as it swings in normal Earth gravity.
Use the equation of motion to find an expression for the natural frequency of free vibration ω_n in radians/sec. Use the proper conversion factor to replace ω_n (in rad/s) with f_n (in Hz). Solve the resulting equation to create two equations which give you:



- The mass moment of inertia $\bar{\mathbf{I}}$ about the center of mass

- The mass moment of inertia $\mathbf{I}_O$ about the pivot point

You will need to use the parallel axis theorem. These equations should be functions of mass, distance r_{cg}, and natural frequency in Hertz. Make a photocopy of your work. You will hand in the original prelab at the beginning of lab and use the copy for your reference.

4.1 Objective

The objective of this experiment is to investigate the mass properties of a rigid body. You will experimentally measure relationships between mass, weight, center of mass, area moments of inertia, and mass moments of inertia.

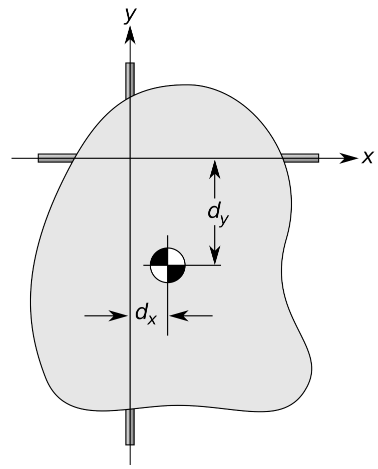


Figure 4.1: Wooden rigid body model, showing coordinate axes

4.2 Procedure

Use the oddly shaped wooden board provided. Record the results on the data sheet. Work in units of kg, m, and sec for all parts of this exercise. Your data sheet and report for this lab will be **due at the end of lab today.**

(1) Find the center of mass. If a rigid body is hung from a single support, the mass center will hang below the support. Hang the plywood sheet from one support point. Make a plumb bob with a length of string and a small weight such as a nut. Tie the plumb line to the same support point from which the board is hanging. After the plumb line stops swinging, tape it to the board and cut off the small weight. Repeat this procedure using a second support point. Where the two strings cross is the center of mass.

(2) Swing the board as a pendulum and time the period of the oscillation. For decent accuracy use a time interval of 30 seconds or more and count the number of swings. You can gently "pump" the board if the oscillations die out too quickly. Find the mass moment of inertia about the x_G and y_G axes (parallel to the "swing" axes but passing through the mass center), using the formula derived in your prelab.

(3) Use the parallel axis theorem to calculate the mass moment of inertia about

1. The x axis marked on the board
2. The y axis marked on the board.

Do these calculations neatly on a clean sheet of paper and show all your work; you will attach these calculations to your lab report for this exercise. Show a diagram of the board, with the axes clearly shown and labeled, along with your calculations.

(4) Calculate the polar moment of inertia J_0 about an axis perpendicular to the board and passing through the intersection of the x and y axes marked on the board (not the mass center). Remember that, for planar bodies,

$$J_O = I_x + I_y$$

where the x and y axes intersect at point O.

(5) Experimentally determine J_O by swinging the board and using the same formula to calculate an experimental inertia. Compare it with the value found in Step (4).

(6) Check the mass moment of inertia I_x found in Step (3) by finding the area moment of inertia $(I_x)_A$ about the x axis. Remember, for a thin uniform plate:

$$I_x = \rho t (I_x)_A$$

where t is the thickness of the plate and ρ is the mass density of the wood from which the plate is made.

Hint: Find the area of each strip of area ΔA_i which is a distance y_i from the x axis. Then to estimate the total area moment of inertia,

$$(I_x)_A = \int y^2 \, dA \approx \sum_i y_i^2 \, \Delta A_i$$

Use the premarked squares on the board to estimate the area of each strip.

Exercise 5

Single and Double Pendulum

5.0 Prelab

Watch the "Oscilloscope" movie on the course PolyLearn site. Then complete the three prelab steps below. Make a photocopy of your work; you will hand in the original at the beginning of lab and use the copy for your reference.

Prelab step 1: Double pendulum, first mode

Mode shapes can be analyzed by techniques we will study towards the end of this quarter. In the double pendulum's first vibration mode, the two pendula should move together, in the same direction. The spring is neither compressed nor tensioned; it merely goes along for the ride, and the two pendula are parallel to each other. Therefore, first mode vibrations should be the same as those of a single simple pendulum oscillating by itself. We will test this hypothesis in the experiment portion of this lab.

On a separate piece of paper (not your lab manual), draw the free body diagram and mass acceleration diagram of a simple pendulum without a spring, as shown in Figure 5.1(a). Use the sum of moments about the pivot to generate the equation of motion in terms of θ, the angle of the pendulum with respect to its equilibrium position. Use the equation of motion to write an expression for the first modal frequency.

Prelab step 2: Double pendulum, second mode

In the second mode the pendula move in equal but opposite directions, and the middle of the spring does not move. Therefore, the frequency can be calculated by analyzing the one degree of freedom system shown in Figure 5.1(b) where only half of the spring is involved. Think: is the equivalent spring constant k_{eq} of half a spring whose spring constant is k equal to $k/2$, $2k$, or k? Complete the free body diagram and mass acceleration diagram for this spring assisted pendulum. Sum moments at the pivot to derive the differential equation of motion. Use this equation of motion to write an expression for the second modal frequency.

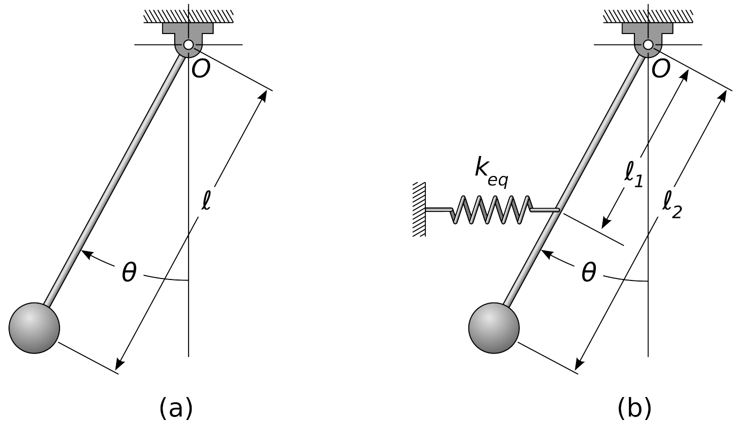


Figure 5.1: A simple pendulum, (a) Acting alone in the first mode and (b) with a spring attached in the second mode

Prelab step 3: Double pendulum general equations of motion

Draw a separate free body diagram and mass acceleration diagram for each pendulum in Figure 5.2. Give θ_1 and θ_2 arbitrary and **unequal** values so that the spring force depends on **both** angles. Sum moments about the pivots to generate the two equations of motion (one for each FBD) for this two degree of freedom system. You will use these two second-order differential equations when you do a computer simulation of the system in lab. **Remember to make a photocopy of your prelab; you will need to refer to it during the lab.**

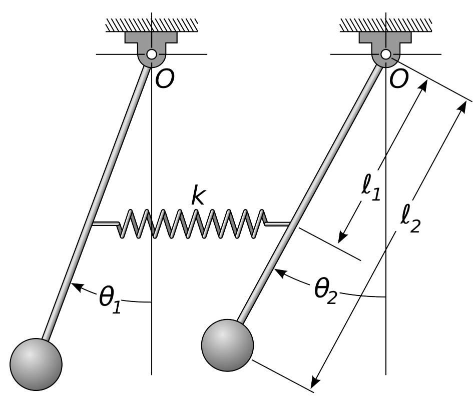


Figure 5.2: Double pendulum system for general vibrations

5.1 Objectives:

The objectives of this experiment are the following:

1. Identify the two modal (natural) frequencies, and their respective mode shapes, for a two degree of freedom system, using both experimental and theoretical techniques.

2. Find the beat frequency of a system with two closely spaced modal frequencies, when both vibration modes are excited.

3. Develop a computer simulation of the system and verify that the model correctly predicts the motion of the physical system.

5.2 Procedure

(1) Note that this exercise includes measurement and testing of the physical system as well as a simulation of that system using Matlab™. You should measure your physical system's behavior *and* get your Matlab™ simulation running during the laboratory period so that you can compare the simulated and measured results and correct any problems before leaving. Refer to Appendix E for help in writing your program if needed.

(2) Measure the system parameters. Use metric units. Disconnect the spring and determine the spring constant two ways:

- Suspend small weights from the spring and measure the static elongation (use just two data points). Make a graph of weight *vs.* displacement and calculate the slope to find the spring constant. Use weights which produce deflections similar to the deflections which the spring will see when it's attached to the moving pendula.
- Suspend one small weight from the spring and measure the natural frequency ω_n of oscillation. Calculate k using the equation of the natural frequency of a spring-mass system.

(3) Wire the output from either potentiometer (left or right pendulum) to Channel 1 of the oscilloscope. Attach the printer cable to the oscilloscope.

(a) **First Mode:** Attach the spring between the pendula. Hold the pendula the same distance and direction from their equilibrium positions. Use the meter stick to estimate the pendulum angles; they should both be about 5^o. Simultaneously release the masses from rest. Record the voltage signal from one potentiometer on the oscilloscope. Print the oscilloscope screen. Find the first modal frequency.

(b) **Second Mode:** Hold the pendula the same distance but *opposite* directions from their equilibrium positions. The pendulum angles should be about $+5^o$ and -5^o when released from rest. Record the voltage signal from one potentiometer on the oscilloscope. Print the oscilloscope screen. Find the second modal frequency.

Using the HP 54600 oscilloscope with the double pendulum

1. Push $\boxed{\text{Setup}}$ key, then Default setup softkey below the screen.
2. Check that Channel 1 is on and its mode is DC coupling.
3. To find the signal adjust the Channel 1 vertical scale (volts/div) to some large value. Twist the Channel 1 Position knob to move the trace down or up to the center of the screen. When the trace is at the center, turn the Volts/div knob clockwise incrementally (zooming in). Keep turning the Position and Volts/div knobs to keep the signal on the screen while making it as large as it can be while allowing the entire signal to be seen.
4. To see the sinusoidal pattern of the signal adjust the horizontal Time/Div knob to a large number, say 5 seconds per division (note that the screen will redraw slowly at this setting). Then adjust the horizontal delay knob until the solid small triangle at top of screen moves from top center to the upper left corner of the screen.
5. Push the $\boxed{\text{Stop}}$ key and $\boxed{\text{Erase}}$ key.
6. Release the pendulums and push the $\boxed{\text{Run}}$ key.
7. Push the $\boxed{\text{Stop}}$ key when the trace reaches the end of the screen.
8. Use the voltage measurement buttons to find the peak-to-peak amplitude and the DC offset. Use the time measurement buttons to find frequency or period. Use the time cursors for other time measurements such as beat frequency.
9. Push the $\boxed{\text{Print/Utility}}$ key, then print screen to get a copy of screen.

(c) **Beat Frequency:** In the previous two experiments the initial conditions were carefully chosen to excite only one modal frequency at a time. In the general case with arbitrary initial conditions, both modes will be excited, and the response will be given by

$$\theta_1(t) = A_1 sin(\omega_{n1} t) + A_2 sin(\omega_{n2} t)$$

and the output will look like that shown in Figure 5.3. If the amplitudes A_1 and A_2, which depend on the initial conditions, are about equal we can use a trigonometric identity to analyze the response.

$$\begin{aligned} \theta(t) &\approx sin(\omega_{n1} t) + sin(\omega_{n2} t) \\ &= 2\cos\left[\frac{(\omega_2 - \omega_1)}{2} t\right] \sin\left[\frac{(\omega_2 + \omega_1)}{2} t\right] \end{aligned} \tag{5.1}$$

When ω_{n1} and ω_{n2} are closely spaced, the cosine term acts as a very low *beat frequency* and forms a non-constant amplitude for the much

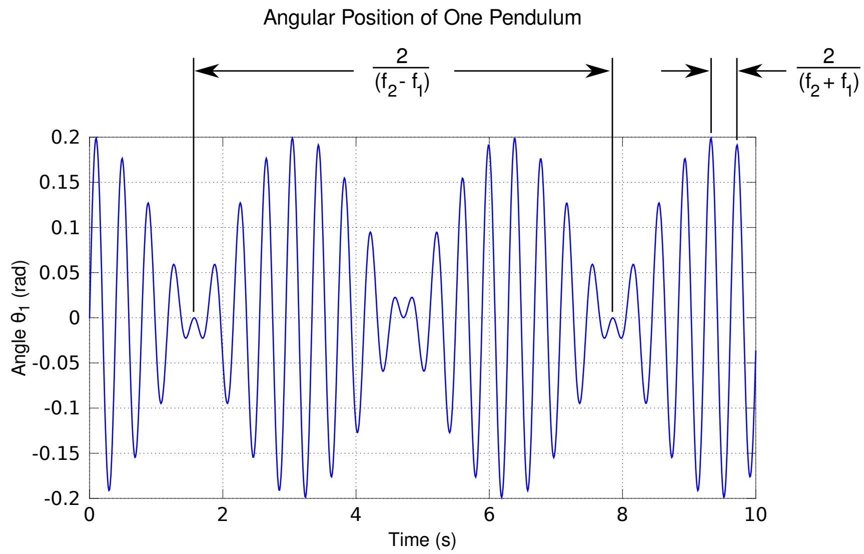


Figure 5.3: Angular position θ_1 as a function of time when both modes are excited

higher average-frequency sine term. This slowly varying amplitude creates an envelope within which the sine term oscillates. If these are audio signals the sound gets louder and softer as the amplitude changes. This is exactly what happens on multi-engine vehicles such as aircraft and boats, often creating a very uncomfortable environment for the passengers. Many modern vehicles use active feedback signals to ensure that the engines run at exactly the same speed just to eliminate this annoyance, though if you have flown in a twin-engine turboprop airliner, you have probably noticed the absence of such a controller.

To create a beat frequency with the pendula, hold one pendulum vertical and the other at about 5°. Release them simultaneously. Record the voltage signal from one potentiometer with the scope. Print the oscilloscope screen. Find the period of the sinusoidal envelope and the period of the sine wave oscillating within the envelope. The envelope period is the beat frequency and the sine wave period is the average modal frequency. Compare these frequencies to their theoretical counterparts using the experimental modal frequencies measured in parts (a) and (b) above.

5.3 Computer Simulation

When we create a digital simulation of a mechanical system, we must carefully validate the simulation before we use it for constructive purposes. It is not enough that the simulation compiles and runs; it must also represent the actual system within the tolerances we as engineers find acceptable. This validation usually consists of two parts:

(a) Validation of parameter values

(b) Validation of the mathematical models used. Did we use linear equations for a nonlinear system? Did we ignore high frequency effects? Did we ignore friction? Discuss parameter validation in your write-up.

We will concentrate our simulation efforts on validating the parameters. Notice that we usually start with the simplest models possible when designing a computer simulation; then, as we go through the validation steps, we increase the model's complexity to match the experimental data we possess. While you will not validate your model in this experiment, you should speculate on its validity in your report.

5.3.1 Simulation Procedure

(4) Write a Matlab™ program using the two equations of motion you derived in the prelab Step 3.

(5) Run your program with the initial conditions of part 3(a), for the first mode. The theory predicts that the frequency found in this step depends only on the pendulum's length, so this test helps us validate the length parameter. What assumptions were made in these models? Include these in your speculations.

(6) Run your program with the initial conditions of part 3(b), the second mode. The theory suggests that this frequency depends on the mass, the two lengths and the spring constant. If your frequency doesn't match the oscilloscope results, try making small changes in these parameters to determine which you may have measured incorrectly.

(7) Run the simulation with the initial conditions in step 3(c). Check the sensitivity of the beat frequency to the spring constant by making small ($\pm 5\%$) changes in the spring constant and seeing how large a difference this causes in the beat frequency.

5.3.2 Example Matlab™ Simulation Code

This code is not exactly what you should write; it is an example which shows some of the elements which should be in your code. In this example, θ_1 is represented by `x(1)`, $d\theta_1/dt$ is `x(2)`, θ_2 is `x(3)`, and $d\theta_2/dt$ is `x(4)`.

Create a file `your_file_name.m` *in which you write:*

```
function dx = your_file_name (t,x)
...
dx(1,1) = x(2);             dθ1/dt = ω1
dx(3,1) = x(4);             dθ2/dt = ω2
dx(2,1) = ...               d²θ1/dt² = ... Your first equation of motion
dx(4,1) = ...               d²θ2/dt² = ... Your second equation of motion
```

In the Matlab™ workspace, or in a script file if you prefer:

```
x0 = [θ1, ω1, θ2, ω2];        # The initial conditions
tspan = [0, 30];
[t, x] = ode45 ('your_file_name', tspan, x0);  Run the simulation
plot (t, x(:,1));             # This plots θ1 only
```

Exercise 6

Sine Sweep Test

6.0 Prelab

1. Read Appendix D regarding Rayleigh's Energy Method. Perform the analysis described on Page D-2 where it says, "It is left as an exercise for the student..."

2. Read Appendix C regarding theoretical bending modes of a cantilever beam. Using Equation C.2 on Page C-1, calculate the first three natural frequencies of a 7 inch long cantilever beam made from aluminum (density 167 lb/ft^3) that is 0.075 inch thick by 1.125 inch wide. Estimates of the discrete values for Lλ can be found from Figure C.1; use a numerical solver in a calculator or Matlab™ to obtain precise values.

6.1 Objectives

The objectives of this experiment are to:

1. Find the first natural frequency of a cantilever beam using both theoretical and experimental techniques and to estimate the first mode's damping.

2. Find the second and third natural frequencies of a cantilever beam by locating its 180° phase shift.

3. Check whether this system obeys the linear equation $f(kx) = kf(x)$.

6.2 Introduction

Every physical body has an infinite number of resonant or natural frequencies. The body will vibrate with a different vibration envelope for each of these resonant frequencies. We refer to the vibration envelope as the *mode* of vibration; hence we refer to the resonant frequencies as *modal* frequencies. You can see the vibration envelope of a meter stick by holding it horizontally by its end and moving it rapidly up and down. The first mode will look like a wedge with the apex at your hand. When the meter stick reaches the edge of the vibration envelope, it must come to a stop so it can reverse its direction. Your eye can see the meter stick when it briefly stops moving, but when it passes through the midpoint it is going very fast and appears as a blur. If you shake the stick faster you will see the second vibration mode. No one is known to have been able to shake a vibrations lab meter stick fast enough to see the third mode.

The first three vibration modes of a cantilever beam are shown in Figure 6.1. Notice that each mode has one more vibration node than the previous mode. The natural (or resonant or modal) frequency associated with each of these vibration envelopes is greater than that associated with the previous envelopes. That is, $\omega_{n1} < \omega_{n2} < \omega_{n3} < ...$

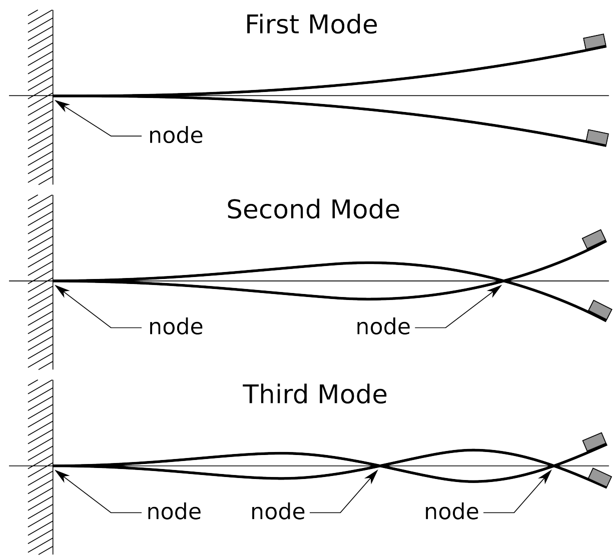


Figure 6.1: First three mode envelopes for the vibrating beam

We will use Rayleigh's method and basic beam theory to find the first natural frequency of a cantilever beam that has both distributed and concentrated mass. We will also obtain this frequency experimentally using an electromechanical shake table, accelerometers and an oscilloscope. We will use the **sine sweep** method, which is usually used to test systems with friction joints (bolted joints, riveted joints, shock absorbers, etc.) since they require a great deal of input energy concentrated at particular frequencies to get satisfactory output data.

6.3 Procedure

(1) Measure the beam's length, width, and thickness using appropriate equipment. Not all the dimensions can be measured precisely; you need to account for the uncertainty. Measure the thickness at several points along the beam and note the maximum and minimum thickness that you measure. Also note the maximum and minimum measured length – how do you define exactly where the base and free ends of the beam are?

Record the accelerometers' model numbers and serial numbers. Find the masses and calibration constants of the accelerometers from the chart located on the side of the big shake table cabinet in the laboratory.

You will use some of the values from this step to make a theoretical calculation of the first natural frequency of the beam. You will do these calculations at the end of the lab period (see Step 13).

(2) Prepare the shake table and the instrumentation according to Figure 6.2. Make sure the square aluminum rod, attached with a security cable, near

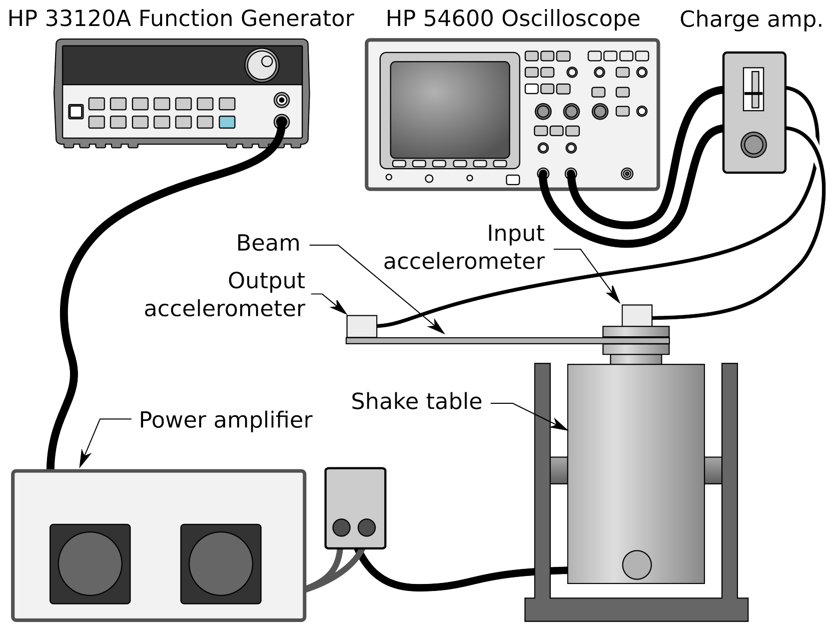


Figure 6.2: Experimental setup and wiring diagram for sine sweep test

the shake table sits at the base of the shake table, not on top.

(3) If one is not already mounted, mount an accelerometer on top of the beam at the shake table's center; this is the input accelerometer. Also mount an accelerometer on the end of the beam; this is the output accelerometer. These need only be hand tight; please do not overtighten them, as this may break the attachment threads.

(4) Carefully connect a cable to each accelerometer. Run these cables to a power supply, then use BNC cables to connect each accelerometer's output to the oscilloscope.

(5) Check to see if the accelerometers are working by tapping them lightly with your finger. If you do not see any wiggle on the oscilloscope screen, then something is wrong. Check the wiring, the accelerometer power box (is it on?), and the oscilloscope.

(6) Attach a cable from the function generator to the shake table's power amplifier. This cable is always attached to the power amplifier, so all you need do is fish it out from under the edge of the table (this may entail a fierce battle with the resident dustbunnies) and attach it to the function generator. Make sure you have the correct cable so that you are shaking your own table, not your neighbor's. Set the function generator to high impedance (HIGH Z) mode (see the instructions on Page 2-2) and use the following settings:

- Sine wave
- 100 Hz
- 500mV_{PP} output (increase later)
- Zero DC offset

Turn on the power amplifier and the instrumentation. The power amplifier has two power cords, one for the amplifier itself and one for the fans. You just plug in both and make sure the fans are running to prevent damage to the amplifier. Use the amplitude control of the function generator to adjust the shake table amplitude. It will be necessary for you to adjust this each time you change the frequency so as to maintain a low output amplitude near the natural frequency.

(7) Locate the first three natural frequencies of the beam by "sweeping" the frequency from 20 Hz to 800 hz and watching for two phenomena:

- Near a natural frequency, the amplitude of the signal from the end of the beam will become very large, and you will be able to see the end of the beam moving much more. **Turn down the amplitude near natural frequencies to avoid bad (nonlinear) data, blown fuses, and broken beams.**
- The signals coming from the two accelerometers will switch from being in phase with one another to being 180^{o} out of phase, or *vice versa.*

Record the natural frequencies that you have found on your data sheet.

(8) Now return to the region near the first natural frequency and record data to generate plots (described below). You should tabulate frequency, input amplitude, output amplitude, and the phase between output and input. You can use the measurement keys on the oscilloscope to display the phase difference between the two signals. Ask your instructor if you are unable find the correct keys. Begin at about 70% of ω_{n1} and take about 20 data points up to 130% of ω_{n1}. It may be difficult to get data right at the natural frequency because the input amplitude goes almost to zero. Be sure that the oscilloscope signals are undistorted sine waves; you may have to adjust the function generator's output to maintain sine waves at

all times. It is a good idea to plot your data as you take it; this will help you spot bad data so you can fix it before it's too late.

(9) Make two plots, superimposed on the same graph (use a *secondary axis*).

- The first plot is the ratio of the displacement amplitudes, output displacement over input displacement, *vs.* frequency.
- The second plot is the phase shift between input and output displacements *vs.* frequency.

Will you need to use the calibration constants of the accelerometers when plotting the ratio of the displacement amplitudes? Can you use acceleration/acceleration (rather than calculating displacement/displacement) to obtain the displacement amplitude ratio? Explain the method you use and why you use it in your report.

(10) Estimate the first mode's damping ratio using the approximation,

$$\zeta \approx \frac{1}{2(\text{Peak Amplitude Ratio})}$$

Record the damping ratio.

(11) Hold the frequency constant, at any frequency **not** near the natural frequency, and vary the amplitude of the input signal. Find the phase and amplitude ratio for about five different input signal levels. What happens to the phase and the amplitude ratio under these conditions? What is the meaning and importance of this result? *Hint: See Objective 3 for this exercise.*

(12) Calculate the beam end tip deflection and tip acceleration at the first natural frequency, using only accelerometer data and the fact that the motion is known to be harmonic, $x = A\sin(\omega t)$.

(13) Use your numbers from step 1 to find the beam's first natural frequency using the following analytical techniques.

(a) Compute the active portion of the distributed mass using the fraction found in the preparation section. The beam is made of aluminum whose density is 167 lb/ft^3. Add this active mass to the mass of the accelerometer to find the lumped mass acting at the beam's end.

(b) Find the beam's spring constant by rearranging the deflection formula for a cantilever beam. Recall that spring constant is equal to force divided by deflection.

$$\delta = \frac{PL^3}{3EI} \quad \text{and} \quad k = \frac{P}{\delta}$$

Where P = force at the end of the beam, L = beam length from clamp edge to accelerometer, E = the modulus of elasticity for aluminum

(10×10^6 psi), and I = the cross-sectional area moment of inertia of the beam.

(c) Compute the beam's first natural frequency using the mass value from part (a) above and the spring constant from the formula you derived in part (b). Use your beam measurements from Step 1. Do the calculation 3 times:

(1) Using the minimum beam length found in step 1

(2) Using the maximum beam length

(3) Changing the thickness by 5% (decrease or increase, to bring your calculation into closer alignment with the experimental ω_{n1})

Show all units and conversions in your calculations. Answer these questions in your report:

(1) Which of the 3 calculations was closest to the experimental ω_{n1}?

(2) Why is the calculation especially sensitive to beam length and beam thickness?

(d) Ask your instructor if you should proceed to the exercise in Step 7.3.2 on Page 7-5 at this time.

Automated sine sweep using signal analyzer

If time permits, you can try the automated sine sweep capability loaded on some of the signal analyzers. See the labels on the back of each analyzer.

[Inst Mode] (swept sine)

[Freq] (Start) 30 Hz (Stop) 300 Hz

[Source] (Level) 350 mVrms (Ramp Rate) 1 Vrms/s

[Start]

Exercise 7

Random Noise Test of a Cantilever Beam

7.0 Prelab

1. Watch the "Spectrum Analyzer" movie on PolyLearn.
2. Re-calculate the first natural frequency in Step 13 of Lab 6. Show **all** your units and conversions. Give your answers in both rad/sec and Hz. If your answer is not in the 20 to 50 Hz range you have made an error, most likely in your units. Write your answers here so you can include them in this week's writeup.

7.1 Objective

The objectives of this experiment are to:

1. Find the first three modal frequencies and their associated damping ratios for a simple cantilever beam with concentrated end mass using a random periodic noise test.
2. Find the damping ratio of the first mode vibration using a transient vibration step relaxation test.

7.2 Experimental Overview

A periodic noise excitation, using the HP 35665A Spectrum Analyzer's noise source, drives the shake table. Periodic noise is a random mixture of sine waves in a specific frequency range that is repeated "periodically", at a period matching the analyzer's sampling time. The accelerations at the beam's end and at the shake table will be monitored using the output and input accelerometers as shown in Figure 7.2. The analyzer computes and displays the beam's spectral response by dividing the beam's output signal by its input signal in the frequency domain. Thus the response displayed is the beam's alone. The general form of typical amplitude ratio and phase graphs is shown in Figure 7.1.

A mathematical representation of the response of the beam in the frequency domain is called a *transfer function.*

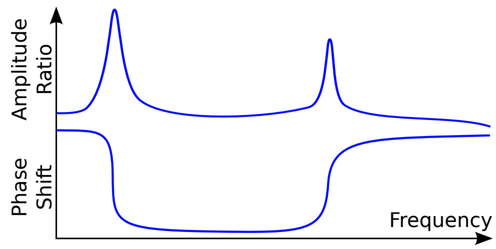


Figure 7.1: Appearance of typical amplitude ratio and phase plots

7.3 Procedure

7.3.1 Random Noise Test Procedure

(1) Wire the HP 35665A as shown in Figure 7.2. Make sure that the channel 1 input to the analyzer is the input accelerometer and that the square aluminum rod, attached with a security cable, near the shake table sits at the base of the shaker, not on top.

(2) Set the analyzer controls according to the following instructions.

Analyzer setup for random noise test

- Set analyzer to default settings using SYSTEM keys:
  $\boxed{\text{Preset}}$ (Do Preset)
- Set frequency, channels, and averaging with MEASUREMENT keys:
  $\boxed{\text{Inst Mode}}$ (2 Channel)
  $\boxed{\text{Window}}$ (Hanning)
  $\boxed{\text{Source}}$ (On) (Random Noise) (Level) 500 mV rms
  $\boxed{\text{Freq}}$ (Span) 800 Hz
  $\boxed{\text{Avg}}$ (On)
- Set the display mode using DISPLAY keys:
  $\boxed{\text{Disp Format}}$ (Bode Diagram) *or* (Frequency Spectrum)
  $\boxed{\text{Scale}}$ (Autoscale On)

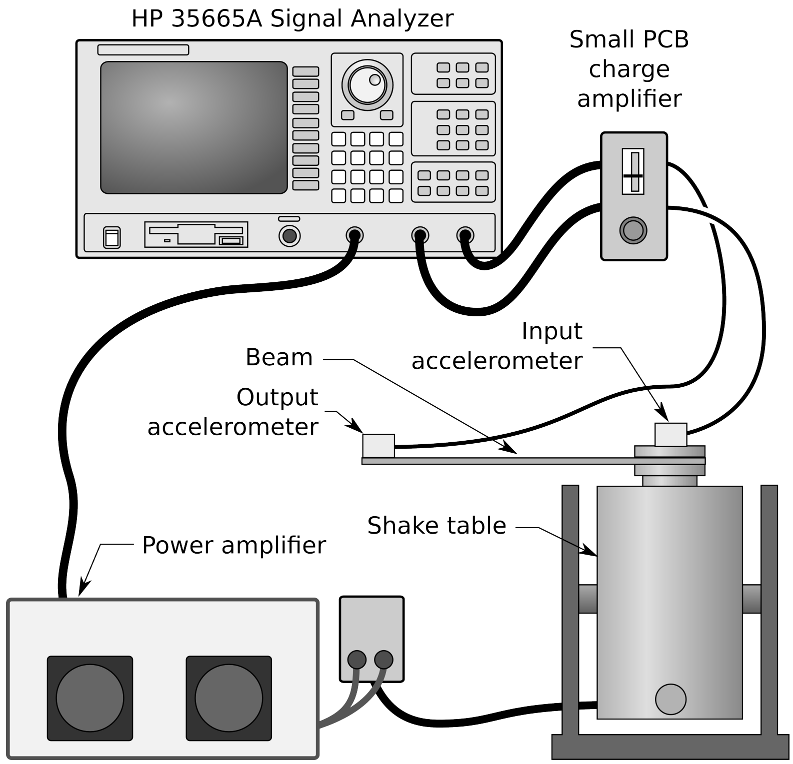


Figure 7.2: Experimental setup and wiring diagram for the periodic random noise test

(3) After setting up the analyzer, press [Start] and the analyzer will take 10 samples of the beam's input and output; the samples are then averaged. Turn autoscaling on (see Page 3-3 for instructions if needed). Press [Source] (Off) to turn off the analyzer's noise source between measurements.

(4) Use the on-screen marker to measure the first three natural frequencies. Also note which peak's amplitude is the tallest, which is the shortest, and which has a medium amplitude.

(5) Use the printer on the analyzer to print out your response plot, showing all three peaks. See Page 3-3 if you need to review the printer setup keys. Make sure to **annotate these plots** properly and completely with captions and the appropriate labels.

(6) Zoom in on the first peak. Try the following key sequence:

[Freq] (Center) (*your value*) (Span) (100 Hz)

[Source] (On)

[Start]

When the 10 averages are complete, press $\boxed{\text{Source}}$ (Off)

(7) Check for the following problems which may affect your data:

- A "notch" in amplitude or phase plots as pictured in Figure 7.3. If this problem occurs, change the frequency span to zoom in closer to the peak as shown in the picture, and retake your data to eliminate the notch.

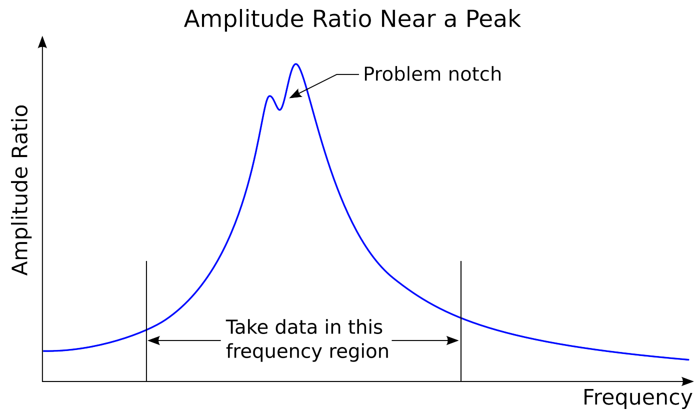


Figure 7.3: Form of analyzer display showing a "notch" problem

- Low coherence. Coherence is checked with the keys
  $\boxed{\text{Meas Data}}$ (Coherence)
  Coherence is a measure of the fraction of the output signal that is caused by the input signal. A coherence of less than about 0.8 indicates bad data. When you have finished checking coherence, return to the frequency response plot by pressing the (Freq Response) screen function key.

If either of these problems occurs, try adjusting the frequency span or increasing the periodic noise source level until good data is obtained. This is a trial and error process. You may also want to use overload rejection, set by the keys
$\boxed{\text{AVG}}$ (Ovld Reject) (On)
Then adjust the full scale range for either channel as needed.

(8) After acquiring a good response plot, use the marker to take data in the region shown in Figure 7.3. This region is where the phase angle is changing rapidly. Record the amplitude ratio, phase, and frequency at each of about 25 points.

Note: To make sure you have time to take all your data, you may need to skip ahead to the transient response of the beam at this time and come back to complete these plots and calculations.

(9) Plot the amplitude ratio and phase vs. frequency, on a single graph. Identify the natural frequency on this graph. It occurs where the amplitude ratio is maximum and the phase is -90°. Which trace yields the most accurate value of ω_n – the amplitude trace or the phase trace?

(10) Estimate the damping ratio from the peak amplitude ratio (you may wish to use the flat top pass band to get this value more accurately) using the approximation,

$$\zeta \approx \frac{1}{2(\text{peak amplitude ratio})} \tag{7.1}$$

(11) Find the "half power" frequencies ω_1 and ω_2, where the phase is -45° and -135° respectively. Use linear interpolation since you are not very likely to have data points at exactly these angles. Also interpolate to find the natural frequency which occurs at a phase of -90°. Calculate the damping ratio ζ, using the half-power points as follows:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} \tag{7.2}$$

(12) Now zoom in on the second peak and take 10 averages. Use the marker and record amplitude ratio, phase and frequency **for six points only:** two near a phase angle of 90°, and two near each of the half power points where the phase angles are ±45° and ±135°. Calculate ζ_2 as in Steps 10 and 11.

(13) Repeat Step 12 for the third peak.

(14) Disconnect the source output of the analyzer from the shake table.

7.3.2 Transient Response of a Beam Test Procedure

In this part of the experiment, you will perform a step relaxation test on the beam. This test consists of "plucking" the beam and measuring the resulting transient response. Then we will use the Log Decrement method to calculate the damping ratio of the first vibration mode.

(15) Make sure the source output of the analyzer has been disconneted from the shake table.

(16) Connect Channel 1 of the HP 54600 oscilloscope to the output accelerometer that is on the free end of the beam.

(17) Find the square aluminum rod that is attached to the base of the shake table with a security cable. Place the rod across the shake table's rim and slide it under the lip at the back edge of the beam's mounting block. This fixes the base of the beam in place.

Using the HP 54600 oscilloscope for the transient response

(a) Push Setup key, then Default setup softkey below the screen.

(b) Press the trigger Source key, then choose the soft key corresponding to your output accelerometer's signal.

(c) Press the trigger Mode key, then select the Single soft key.

(d) Press the trigger Slope/Coupling key, then press the Slope soft key until you have an upward arrow. This indicates positive edge triggering, meaning that the screen will be redrawn when the measured voltage goes from below the trigger level that you're about to set to above the trigger level.

(e) Turn the Volts/Div knob for your output accelerometer's channel until the screen shows 100 mV near the upper left screen corner. This indicates a scale of 100 mV (or 0.1 volts) for each of the large divisions on the screen grid. If the grid is not displayed, press the Display key and then press the Grid soft key until it reads Full.

(f) Slowly adjust the trigger Level knob until the trigger level is slightly above channel 1's ground symbol on the right hand screen edge. The trigger level line will appear on the screen as you move the knob.

(g) Press the Main/Delayed key. Press the Time ref soft key until it reads Lft. This sets the screen's left edge to zero seconds.

(h) Press the Run key. This arms the trigger circuit. As soon as the measured output accelerometer voltage exceeds the trigger level, the oscilloscope will begin capturing data.

(18) Use the oscilloscope to view the **shape of the vibration envelope.** Set the horizontal Time/Div knob to 200 ms. Gently deflect and quickly release the free end of the beam. Adjust the vertical scale (increase the Volts/Div) if the peaks are chopped off. What is shape of the envelope of the decaying vibration? What does the shape indicate about the form of damping in the system?

(19) View the **early cycles of vibration** in the step test. Set the horizontal Time/Div knob to 20 ms to view about 5 to 10 oscillations. Gently deflect and quickly release the free end of the beam. Adjust the Volts/Div knob if the peaks are chopped off. Deflect and release again. If the triggering worked you should be able to see the beginning of the trace on the screen.

You should notice some "noise" near the tip of each peak which appears to "clean up" after a few cycles. Use the time cursors and measure the frequency of this "noise." You can change the time scale (decrease the Time/Div) to zoom in and get a better measurement.

What is this "noise"? Think about the fact that a step input excites all frequencies. How many vibration modes does the beam have? If the beam's vibration modes all have about the same damping ratio (ζ) and are all excited at the same time, which mode would you expect to damp out last? *Hint: does a vibration damp out in a fixed time interval or in a fixed number of cycles?*

(20) View the **later cycles** of the step test to **calculate damping**. Turn the horizontal Delay knob until you have a 200 millisecond delay (displayed at the top of the screen in the center).

Repeat the step relaxation test. Adjust the vertical scale and repeat as needed to show the peaks at a reasonable size. Notice the time delay has omitted the first half-second of the beam's response and you should now see smooth peaks (without the aforementioned "noise").

Press the key Cursors. Press the soft key to Clear cursors. Toggle the soft key Active until V_1 is selected.

Use the knob under the Cursors key to position the cursor line on the first peak. Toggle the soft key Active to V_2. Position the second cursor on the last peak. Read and record V_1 and V_2. Count the number of cycles between the first peak and the last peak.

Compute the damping ratio using the log decrement theory shown below. Read the damped natural frequency from the oscilloscope (use the Time key.) Then use your damping ratio and damped natural frequency ω_d to compute the natural frequency ω_n.

Using the damping ratio you have computed, how many cycles would occur before an oscillation is reduced to 1% of its initial value? How long would it take for your second mode to reduce to 1% if it has a similar damping ratio? Do your calculations support your observations of the "noise" in the previous step?

(21) Remove the aluminum rod so that the shaker is free to move up and down.

7.4 Reference: Log Decrement

The following are basic equations for log decrement, damping ratio, damped frequency of oscillation, and natural frequency.

$$\delta = \frac{1}{n} \, ln \, \frac{X_0}{X_n} \tag{7.3}$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \approx 2\pi\zeta \quad \text{(for small } \zeta\text{)} \tag{7.4}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{7.5}$$

Where:	δ	=	log decrement
	n	=	number of cycles
	X_0	=	amplitude at start
	X_n	=	amplitude after n cycles
	ζ	=	damping ratio

Exercise 8

Modal Analysis of a Cantilever Beam

8.0 Prelab

1. Read Appendix C, "Theoretical Bending Modes of a Cantilever Beam." Verify that the second and third bending mode frequency constants are

$$L\lambda_2 = 4.694 \quad \text{and} \quad L\lambda_3 = 7.855$$

   by showing that they satisfy Equation C.5.

2. Draw graphs of the first, second and third mode shapes by using these frequency constants in Equation C.11. Use a spreadsheet or Matlab™ to create the graphs. They should look similar to the mode shape example graphs in Appendix C.

3. Calculate the first three modal frequencies of a 0.5 inch thick, 1.5 inch wide, 58 inch long aluminum beam. Use Equation C.2 and the three bending mode frequency constants. You will recalculate these numbers with accurate dimensions you measure in lab when you submit the final report, so keep a copy of your calculations.

8.1 Objectives

The objectives of this experiment are to:

- Investigate the vibration theory of distributed mass systems.
- Compare the first three theoretical frequencies and mode shapes of a cantilevered beam to those found experimentally with a spectrum analyzer.

8.2 Background

Every body has an infinite number of natural frequencies, and each frequency has a mode shape which describes the deflection envelope.

The history of rocket development gives an example of the mode shape's importance. Early rockets such as the V-2 were relatively short and stiff, so they had high bending frequencies. The V-2 was guided by an attitude gyro which moved vanes in the exhaust plume to steer the rocket. As rocket technology developed, two major changes occurred. First, the rockets got longer and more flexible, lowering the bending frequencies into the flight controller's range. Secondly, damping was added to the flight control system to augment aerodynamic damping. This was done by adding a rate gyro (which measures angular velocity) and sending its signal to the flight control system, which then positioned the nozzles to develop a damping moment to oppose the angular velocity. In one early rocket, the rate gyro was inadvertently mounted near the node of the first bending mode. While there is no displacement at a node, the bending slope is usually a maximum there. The poor rate gyro could not tell the difference between the angular velocity due to the bending slope and that due to the rigid body. It sent the combined signal to the flight control system, which dutifully gimbaled the nozzles, but this further excited the bending slope. The rate gyro sensed an increased angular velocity and asked the flight controller for more damping force, which further excited the bending mode. This continued until the rocket broke in two. Since then, the careful definition of bending modes has been one of the primary tasks in any new rocket design.

8.3 Procedure

(1) Attach an accelerometer to the beam to measure the motion that results from tapping the beam with an instrumented hammer. Two problems affect our choice of a suitable location for the accelerometer:

- If the accelerometer is located at an anti-node, its mass will have a large effect on the natural frequencies and mode shapes of the beam. Since our accelerometer's mass is small compared to the mass of the beam, we can safely ignore this problem.
- If the accelerometer is located near a node, there is very little acceleration, so the signal to noise ratio (coherence) is low. This is a serious problem, so we locate the accelerometer away from the nodes.

Mount the accelerometer under the free end of the beam, because the free end is an anti-node of the first few modes.

(2) Wire the transducer outputs through a PCB amplifier into the two input channels of the spectrum analyzer. Make sure you measure output (beam accelerometer) divided by input (force hammer). Refer to Exercise 7 if you've forgotten how to set this up.

(3) Set up the spectrum analyzer for this test according to the following instructions.

Analyzer setup for cantilever beam test

- Set analyzer to default settings using SYSTEM keys:
  $\boxed{\text{Preset}}$ (Do Preset)
- Set frequency, channels, and averaging with MEASUREMENT keys:
  $\boxed{\text{Inst Mode}}$ (2 Channel)
  $\boxed{\text{Freq}}$ (Span) *above third natural frequency* (Start) 0 Hz
  $\boxed{\text{Window}}$ (Uniform)
  $\boxed{\text{Trigger}}$ (Channel 1 Trigger) (Trigger Setup) (Level) 3% (Slope Neg)
  $\boxed{\text{Avg}}$ (On) (Number of Averages) 5 (Enter)
  $\boxed{\text{Input}}$ (Channel 1 Range) 1 Vrms (Channel 2 Range) 1 Vrms
  Note: These channel ranges will need to be adjusted if either channel overloads during testing. Increase the range, but don't make it larger than necessary or you will get poor data or have trouble with triggering. Retake data for any set of averages where an overload occurs.
- Activate overload rejection to prevent corrupted data:
  $\boxed{\text{Avg}}$ (Ovld Rej) On
- Set the display mode using DISPLAY keys:
  $\boxed{\text{Disp Format}}$ (Bode Diagram)
  $\boxed{\text{Trace Coord}}$ (Log Magnitude) (x Axis Log)
  $\boxed{\text{Active Trace}}$ *Toggles between the two plots displayed* (x Axis Log)
  $\boxed{\text{Scale}}$ (Autoscale On)
  $\boxed{\text{Active Trace}}$ *Toggle back*
- You will need to switch between the coherence plot and the imaginary part of the frequency response as you take data. Here's how:
  $\boxed{\text{Meas Data}}$ (Coherence) *or,* $\boxed{\text{Meas Data}}$ (Frequency Response)
  $\boxed{\text{Trace Coord}}$ (Real Imag) *or* (Linear Magnitude)

(4) Set up a spreadsheet with the following format:

Strike Point	First Peak		Second Peak		Third Peak	
	$f_{n1} =$ Hz		$f_{n2} =$ Hz		$f_{n3} =$ Hz	
	Imag Part	Coherence	Imag Part	Coherence	Imag Part	Coherence
1						
2						
⋮						
10						

(5) Begin with a display of the frequency response "Linear Magnitude" in order to locate the values of f_{n1}, f_{n2}, and f_{n3}. Do 5 hits at strike point 1 (hit. . . wait. . . stop the vibration. . . repeat.) If the signal analyzer won't trigger, adjust the percent level downwards. After each hit, check to see if channel 1 or channel 2 overloaded by monitoring the OV1 and OV2 indicators on the screen. If OV1 or OV2 is highlighted, reset the channel sensitivity (range) to a higher value and retake that set of averages. When your 5 averages are complete for strike point 1, identify the first three natural frequency values and enter them in your data sheet below the labels "First Peak", "Second Peak", and "Third Peak."

(6) Now switch the display to "imaginary part" and be sure to read the data at exactly the same values of f_{n1}, f_{n2}, and f_{n3} you found in Step (5), whether you see a "peak" or not.

(7) Following the same procedure as in the previous step, conduct 5 hits at strike point 2 and fill in row 2 of data table. Repeat this process for all 10 strike points.

Inspect the numbers in the columns of your data table as you go. Since you know the mode shapes, you can predict the trend of the values in the columns.

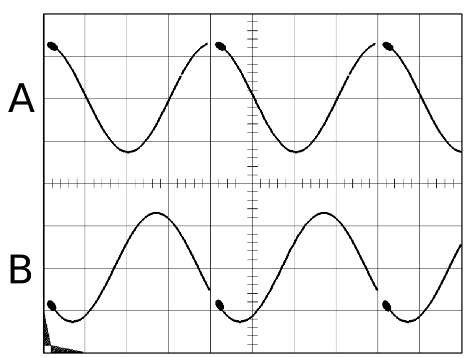
(8) To interpret your data, make three plots – one of each mode shape. Plot the amplitude of the imaginary part of the transfer function (vertical axis) versus strike point (horizontal axis). Superimpose the theoretical mode shape on each using your prelab results. Normalize all your mode plots so that each has a maximum amplitude of 1.

Exercise 9

Single Plane Balancing and Phase Measurement

9.0 Prelab

- Read the introduction below and complete the prelab worksheet, which is on the course PolyLearn site. Use the data in the table below.
- Watch the "Rotor" movie on PolyLearn.
- Expect a quiz at the beginning of lab.



Signals from two proximity sensors. The small gap in the signal is the location of the Keyphasor® pulse. This data is used to determine phase angles.

Frequency rpm	Amplitude mil pp	Phase degrees
280	0.9	23.5
1670	4.7	38.1
1920	8.9	56.3
1980	12.0	72.0
2020	13.7	85.5
2120	14.9	104.5
2150	15.0	111.3
2190	14.8	120.3
2250	13.4	137.0
2300	10.0	158.4
2340	7.5	169.9
2470	5.1	180.2
2950	1.4	194.5

Rotor shaft whirl data for a test run from 0 to 3000 rpm.

9.1 Introduction

Rotor imbalance is a frequent cause of vibration in turbomachinery. Imbalance occurs when the center of mass does not lie on the axis of rotation or the principal axis of inertia is not coincident with the rotation axis, thus producing a rotating couple.[1] The state of balance varies with shaft speed due to rotor

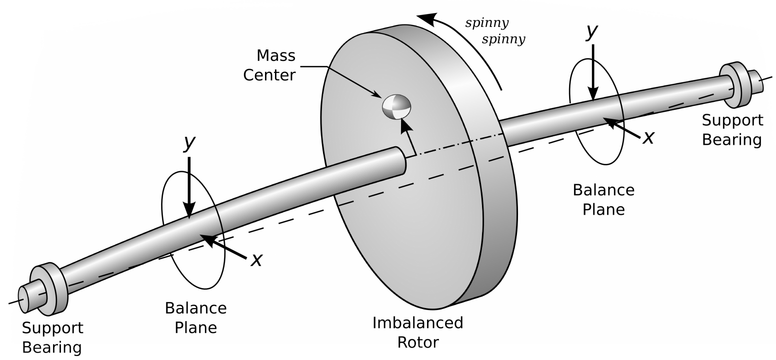


Figure 9.1: Imbalanced rotor on a flexible shaft

flexibility. This causes a shaft that is balanced at one speed to not be balanced at another. If two correction planes are used, a flexible rotor can only be balanced for one speed. Ideally the number of balancing planes should equal the number of critical speeds traversed while accelerating to operating speed. In practice, accessibility of the rotor governs the number of balancing planes. Most high-speed rotors are balanced before assembly in balancing machines. However, it is often necessary to trim balance a rotor in place after installation due to differences in alignment and stiffness between the balance machine and actual machine housing. Rotors with most of the mass concentrated near a single plane can sometimes be balanced by using only one balance correction plane and one plane for vibration measurements (not necessarily the same). The inherent assumption is that the couple imbalance is negligible so that the imbalance is primarily due to the offset of the center of mass.

9.1.1 Phase Angle Measurements

Phase angle is a timing relationship, measured in degrees, between two periodic events. Phase measurement can be either relative or absolute:

- Relative phase means that the delay between two events is measured in terms of a vibration cycle angle. The angle goes from 0^o to 360^o in one cycle of vibrational motion.

- Absolute phase means that one event is measured with respect to some signal which is taken as an absolute zero degree measurement.

[1] **Acknowledgement:** Most of the following information, figures, plots, and procedures included herein were developed at Bently Nevada Corporation, who have graciously granted us permission for our educational use.

Relative Phase

Relative phase is the timing relationship, measured in degrees, from a point on one signal to the nearest corresponding point on another signal. To measure

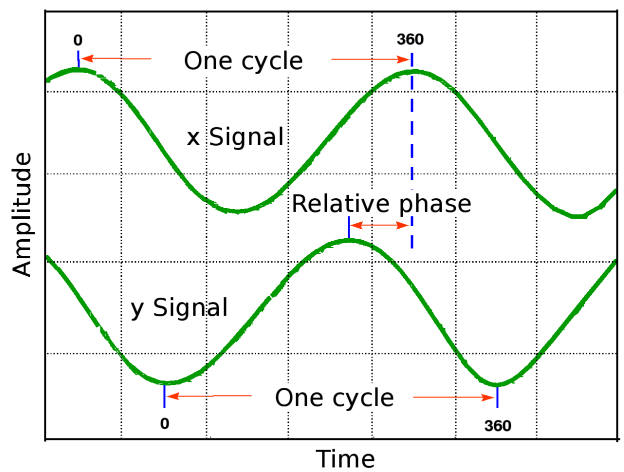


Figure 9.2: Oscilloscope screen showing how relative phase is measured

relative phase, two vibration signals of the identical frequency must be compared. Either can be used as the reference signal. Relative phase is measured between 0 and 180 degrees, leading or lagging. By knowing the relative phase relationship between two orthogonal (x and y) transducers and their location on the machine, one can determine the direction of precession, which is the direction of rotation of the bent rotor plane. From the plot in Figure 9.2, one could deduce that the precession is from x to y, since x is leading. We can look at the relative phase along the length of the shaft to give us an idea of the deflection shape of the shaft. In Figure 9.3, probes 2 and 3 are in phase for one of the deflected shapes and out of phase for the other.

Absolute Phase

Absolute phase is the number of degrees of vibration cycle from when a once-per-turn reference pulse fires to the first positive peak in the vibration signal. The vibration signal must be filtered to the same frequency as the shaft speed. The reference signal can come from a variety of instruments:

- Keyphasor® probe (sensing a projection or notch).
- Optical pickup (with reflective tape on the shaft).
- Strobe light (with match marks on the shaft and casing). Beware of aliasing at multiples or submultiples of running speed.

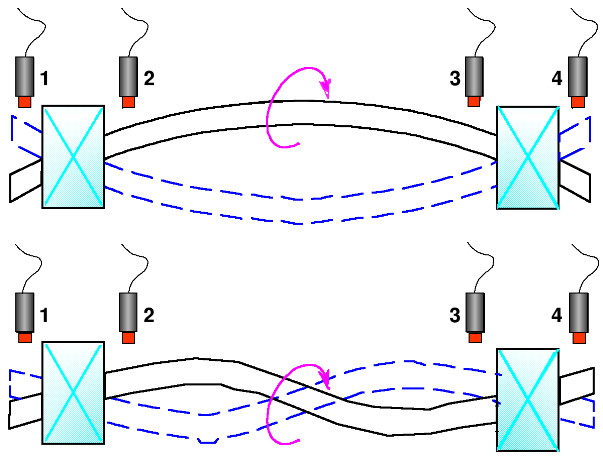


Figure 9.3: Deflections of shaft for two shaft vibration modes

- A magnetic pickup (sending changes in EMF with rotation).

It is important to note that in all cases, the reference pulse must be a once-per-turn signal.

In Figure 9.4 the vibration frequency is “1X”, or one times the rotational frequency of the rotor. This is evident because one vibration cycle matches one rotation of the shaft (Keyphasor® pulse to the next Keyphasor® pulse). The phase angle, measured in degrees of vibration cycle, is measured from the leading edge of the blank spot to the first positive peak of vibration. The measurement

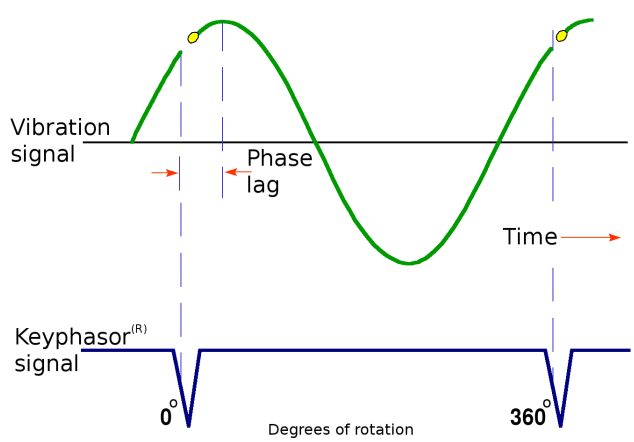


Figure 9.4: Vibration and Keyphasor® signals for 1X vibration

is from the reference signal to the positive peak of vibration. Absolute phase, therefore, is always a lag angle. The absolute phase of the vibration signal shown above is 45 degrees phase lag. There are other vibrational modes in which two vibration cycles occur per rotor rotation (“2X”), or three, and so on.

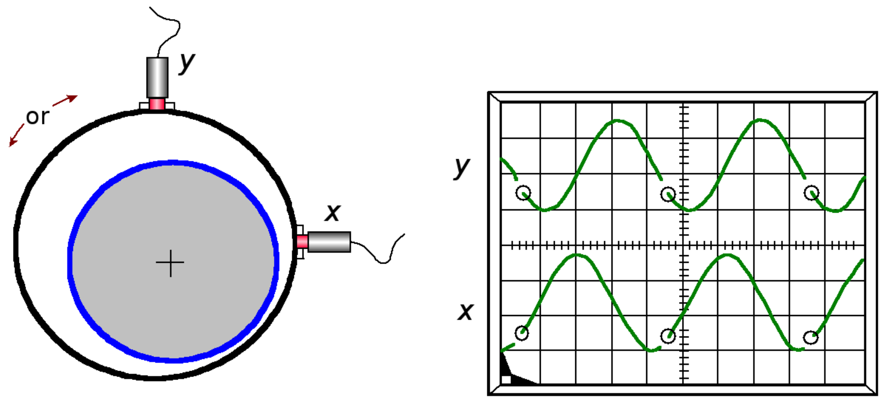


Figure 9.5: Vibration signals for counterclockwise vibration with x and y components

In Figure 9.5 you can see that the direction of vibration is x to y (counterclockwise), but there is insufficient data to determine the direction of rotation. If the direction of rotation is known to be clockwise, for example, then there would be a state of reverse precession since shaft rotation would then be in the opposite direction to shaft whirl (the bend in the shaft going around in circles).

In the oscilloscope image of Figure 9.6 an orbit display is shown. The two proximity probes when displayed in X-Y mode give a picture of shaft centerline motion in the bearing clearance. Individually the x or y probe would display a sinusoidal variation for this shaft motion which is shown on the oscilloscope with DC offset. The shaft positions numbered 1 through 8 represent the location of the shaft at various times in an orbit. Can you mark the location of the Keyphasor® notch on the appropriate shaft when the Keyphasor® pulse occurs?

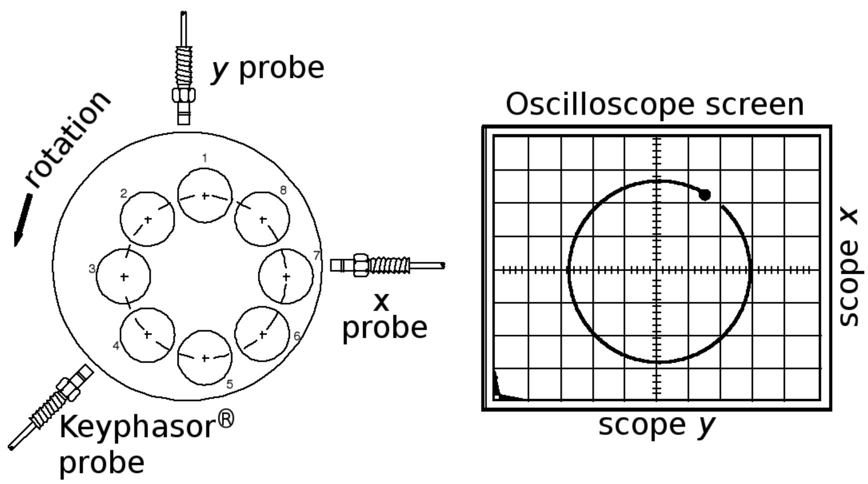


Figure 9.6: Vibration signals for counterclockwise vibration with x and y components

9.2 Background

In Figure 9.7 is a picture of the rotor kit. Take time to investigate the hardware. Notice the proximity probes at two planes. Standard convention is to name the plane closest to the driver the inboard plane and the farther plane the outboard plane. Shaft rotation is specified as CW or CCW as seen by an observer looking outboard from the motor. Each of the kits is configured so that the shaft will rotate CCW or said another way from the X or horizontal probe to the vertical or Y probe. The probes are set up so that Y is at 0 degrees and X is 90 degrees right. Notice the keyphasor probe. It is adjusted so that the leading edge of the keyslot is in the middle of the keyphasor probe when 0 degrees, as marked on the rotor mass wheel, is adjacent to the vertical Y probe. Trace the keyphasor lead (yellow). Trace the inboard vertical (white) and inboard horizontal (red) signals.

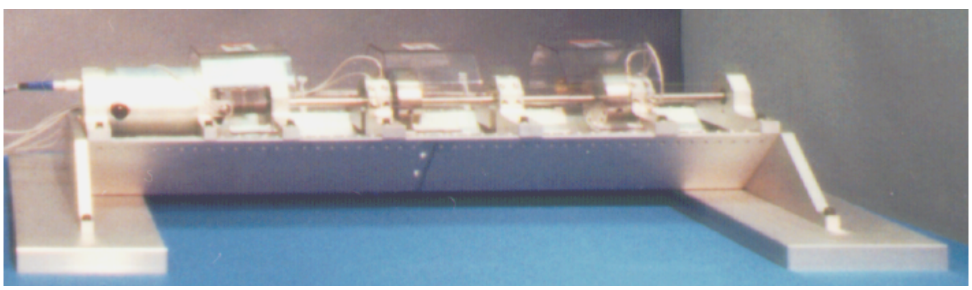


Figure 9.7: The Bently Nevada rotor kit

9.3 Objectives

- Determine location and magnitude of imbalance using polar plot data
- Use trial calibration weight to determine influence vector in order to balance the rotor using a single plane

Warning: Do not turn anything on until you have read this overview completely. Follow the instructions on the next page to perform the laboratory experiment. Wear safety glasses while you or any other group is working on this experiment.

9.4 Overview

The rotor can be balanced in as few as three runs using the following technique.

Run 1 Original configuration test run: Run the rotor up to 3000 rpm while the system collects data. Correct for shaft bow, electrical runout and magnetic runout by choosing an appropriate slow-roll vector $\overline{Y_{SR}}$. You will notice that the polar plots shift to the correct position (passing through the origin of the plot) when you select $\overline{Y_{SR}}$. Using the corrected polar plot (called the 1X compensated polar plot), determine angular location of heavy spot by drawing the low speed tangent line on the plot. Go to the "operating speed" specified by your lab instructor and record the magnitude and angle at this speed. This is the $\overline{O}$ vector.

Run 2 Calibration run: Put a calibration weight, $\overline{W_2}$, across from the heavy spot (after making a very rough calculation for its mass). Run the rotor up to 3000 rpm while the system collects data. Go to the same operating speed as in Run 1 and record the magnitude and angle at this speed. This is the $\overline{O + C_2}$ vector. Subtract $\overline{O}$ (from Run 1) from $\overline{O + C_2}$ to get the change vector, $\overline{C_2}$. Calculate the influence vector $\overline{H}$, which can be thought of as the effect divided by the cause. That is: $\overline{H} = \overline{C_2}/\overline{W_2}$.

Run 3 Correction run: Use $\overline{H}$ to determine the amount and location of a correction weight, $\overline{W_3}$. Remove the calibration weight (from Run 2) and add the correction weight. Run the rotor up to 3000 rpm while the system collects data. Go to the operating speed and record the magnitude and angle at this speed. With luck the amplitude of the response at the operating speed should now be close to zero. Recalculate the influence vector $\overline{H} = \overline{C_2}/\overline{W_3}$ to verify that you get the same value as in run 2.

If you are not satisfied with the result, do another run. Try using the newer influence vector.

9.5 Rotor Kit Workstation Setup Checklist

Safety

- [ ] Wear safety glasses and follow procedures for safe operation of rotating equipment. No loose clothing, or jewelry is allowed. Long hair must be tied back so it cannot get close to the rotor.

- [ ] Before power is turned on, slowly rotate the rotor through several revolutions to check for possible interference.

- [ ] Ensure that safety covers are in place and all lockdown bolts are in place and tightened.

Power

- [ ] All instruments on: two TK21 interfaces, motor speed controller (after making sure toggle switch is in the stopped position), computer monitor, computer, and 208 Data Acquisition Unit (under lab table). Power to the table is controlled by an outlet near the front of the table. This is a total of 6 – 7 switches.

Check Wiring

- [ ] KPH to KPH1 on the 208 (DAIU) *Note: KPH = Keyphasor®*
- [ ] Inboard Y to 208 (DAIU) Channel 1 (white)
- [ ] Inboard X to 208 (DAIU) Channel 2 (red)

Rotor kit speed controller

- [ ] Check motor is stopped
- [ ] Ramp Up
- [ ] Ramp Rate 3 (3000 rpm/minute)
- [ ] Manual Speed Control Set to 300 (3000 rpm)

ADRE® for Windows Program

This setup uses the "SINGLE" file configuration:

- [ ] Click the ADRE® for Windows icon on the desktop
- [ ] Click on "FILE" on the ADRE® main menu.
- [ ] Click on "Open"
- [ ] Double click on "SINGLE." This will load the proper ADRE® configuration into the computer. Typical path: `c:\adrewin\database\SINGLE`

9.6 Balancing Procedure

Run 1: Original Configuration

(1) Check if there are any weights in the rotor kit. If there are, record the magnitude and location of any such weights, but do **not** remove them; these have been put in the rotor by the instructor to ensure that the rotor is in the correct starting configuration. When you are finished with the exercise, you must return the rotor to its starting configuration.

(2) Start the rotation of the rotor by placing the right toggle switch of the speed controller to SLOW ROLL.

(3) Click on Store Enable from the ADRE® main menu. The real time Orbit/Timebase plot and the Current Values plot will be displayed.

Note: At various times when moving between windows within ADRE®, a dialogue box may appear with the statement "…Proceeding will erase your existing WAVEFORM and VECTOR data. . . ” Always choose No, Continue without saving.

(4) Switch right toggle to RAMP. Do not adjust the speed controller potentiometer for the remainder of this exercise.

If a yellow warning box indicates a “range” error, click on Clear error.

(5) Click on Stop in the ADRE® menu (so that it stops collecting data). Then slow the rotor by flipping the toggle switch on the speed controller to Ramp Down. After it has slowed to “slow roll” speed, you can toggle to Stop. Click on Close in the ADRE® menu.

(6) View the polar plot: In the main menu, **click** on the **icon** that looks like a Polar plot. Notice that the data at slow speeds, which should sit at the origin of the plot, may not. The reason it doesn’t is a combination of non-dynamic effects such as rotor bow, mechanical runout, and electrical inaccuracy. We can subtract these effects by specifying a slow roll vector. The ADRE® software then subtracts this slow roll vector from all the data. The result is that the polar plot is shifted to the origin giving us a polar plot of the dynamic effects only, as we desire. Choose your slow roll vector as follows:

Click on Edit in the main menu. Click on Reference Data. Next click on Vector Reference. You may have to “unfreeze” the samples. Then view the samples with the Up/Down arrow buttons in the sample box in the upper part of the window. Select a sample near 250 rpm that is representative of samples near this speed (amplitude and phase nearly constant). Then Freeze the sample by clicking on the button in the first row in the lower half of the window.

(7) Record the 1X slow roll vector (amplitude, phase, and speed).

(8) Click on OK. Return to the polar plot. You will see that the plot has shifted such that the “Vector Reference” data point has been placed at the origin of the polar plot. If you don’t like the result try a different sample.

(9) In the main menu, **click** on the **icon** that looks like a Bode plot. Have your instructor help you select an “operating speed”. This will be the speed for balancing the rotor (the speed at which you try to reduce the vibration amplitude to zero). Record this operating speed and the vector (amplitude and phase) at this speed. This is the $\overline{O}$ vector. The arrow buttons can be used to move the cursor.

(10) Choose File, Print, and Print to get a copy of the Bode plot. Close the window and the do the same to obtain a copy of the polar plot. On the polar plot, click on the data point at the balance speed so that the value of the $\overline{O}$ vector is displayed.

(11) Mark the location of the heavy spot on the polar plot. Recall that the heavy spot lies on the low-speed tangent line and leads the peak response by 90 degrees.

(12) Draw the $\overline{O}$ vector on the polar plot.

Run 2: Calibration Run

(13) Make a rough calculation of the appropriate size of the calibration mass. As a "rule of thumb", not knowing anything about the response characteristics of this rotor, the maximum size of this calibration mass should generate a dynamic force that does not exceed 10% of the **static** weight of the rotor when spinning at maximum speed. Use $F = mrw^2$ and solve for m. Let F be 10% of the rotor weight. For the single mass rotor kit:

- The mass of the rotor is 810 grams
- Radius from center axis to weight = 1.2 inches
- The test rotational speed is 3000 rpm
- No matter what you calculate, **do not unbalance the rotor by more than 2 grams** for safety reasons
- When you install the calibration weight setscrew, make sure the setscrew **does not protrude beyond the outside of the disk**

Setscrew locations are available every 22.5° around the disk. If "weight splitting" is considered, make sure the *vector sum* is what you desire.

(14) Record your calibration mass, then insert your calibration mass opposite the "heavy spot" on the rotor.

(15) Run the rotor up to 3000 rpm with the trial calibration mass in place by repeating Steps 2 through 5. Print the polar plot.

(16) From the Polar plot, record the response **at the operating speed you selected.** This is the $\overline{O + C_2}$ vector. Draw $\overline{O + C_2}$ on the polar plot.

From the plotted $\overline{O}$ and $\overline{O + C_2}$ vectors, you can find $\overline{C_2}$, the effect that was caused by $\overline{W_2}$. Find $\overline{C_2}$ in two ways: graphically on the plot by drawing the vectors head to tail, and trigonometrically as in the pre-lab. Make sure they agree.

(17) Calculate the influence vector $\overline{H}$ (the effect divided by the cause):

$$
\begin{aligned}
\left|\overline{H}\right| &= \frac{|\overline{C_2}|}{|\overline{W_2}|} \\
\angle\overline{H} &= \angle\overline{C_2} - \angle\overline{W_2}
\end{aligned}
\tag{9.1}
$$

The influence vector is an important characteristic of machine response that should be documented for the particular rotor speed and machine operating conditions. For a machine that responds linearly, the influence vector tells you how the 1X vibration vector changes in response to any weight added to the rotor. It should be the same value for all runs. It can be used for future calibrations.

Run 3: Correction Run

- Determine $\overline{W_3}$, the correction weight, using the following equation:

$$\overline{W_3} = \frac{-\overline{O}}{\overline{H}} \tag{9.2}$$

- Remove the calibration weight $\overline{W_2}$ and insert the correction weight $\overline{W_3}$.
- Run the rotor up to 3000 rpm with the correction weight in place by repeating Steps 2 through 5.
- **After ramping down,** stop the rotor kit by placing the motor speed control to Stop.
- Print the Polar plot. From the Polar plot, record the response **at the operating speed you selected.** This is the $\overline{O + C_3}$ vector and your final vibration amplitude for your balanced rotor (so it should be significantly smaller than $\overline{O}$. Draw $\overline{O + C_3}$ on the polar plot.
- Calculate the influence vector $\overline{H}$ (yes, again):

$$\begin{gathered} \left|\overline{H}\right| = \frac{|\overline{C_3}|}{|\overline{W_3}|} \\ \angle\overline{H} = \angle\overline{C_3} - \angle\overline{W_3} \end{gathered} \tag{9.3}$$

- This new influence vector should be nearly identical to one from the calibration run. If it is not, or if you are not satisfied with the reduction in vibration, try a fourth run. Consider weight splitting to create a more accurate correction weight.
- When you are finished remove any weights which you have added to the rotor (but leave the weights which were there you began the exercise) so the system is ready for the next lab group.

Appendix A

Reporting

A.1 Formal Reports

Formal reports are concise and carefully written. Your engineering as well as your writing will be graded. You are reporting to your "boss" on the results of an experiment and their significance. You may assume your "boss" is familiar with the experiment, so it is not necessary to duplicate information given in the lab manual. However, you will still need a brief introduction to the objective. The major goal is to convey the most important information quickly, so you'll want to put the results up front.

Format

The following sections should be present in a formal report of an engineering experiment.

1. **Objective:** What are you trying to determine, measure, etc., and how does the procedure allow you to reach the objective? Give a short introduction to the purpose of the test.

2. **Procedure:** Describe your testing methodology. You can paraphrase procedures detailed in lab manual; reference the lab manual as a place to look for further details. Show a sketch of the equipment setup and give a complete, detailed list of equipment, including serial numbers where available. Include a specification of the apparatus tested.

3. **Results:** Begin with a summary of quantifiable results supported by measurements presented in tabular and/or graphical form. Present every table or graph in such a way that its significance is understood without reference to the text. Plots must be completely labeled and include a descriptive caption.

4. **Discussion/Conclusion:** What do the results indicate? Are they as expected? Are the errors within reason or did something go wrong? Do not cite a cause of error without assessing the magnitude and significance of its effect.

5. **Computational Procedures:** This section should contain clearly done sample calculations. Each sample calculation is to be treated as a miniature report: begin with an objective followed by the final result. Next

include a sample calculation that details each step of the calculation. Reference all formulas used unless you are willing to derive them.

6. **Appendix:** Supporting material which does not belong in the main body of the report goes here. This is *not* a garbage dump; everything you include should be neat and of interest to the "boss."

A.2 Technical Writing Tips

Active Voice

Use *active voice* whenever possible. The active voice emphasizes the subject of the sentence: "Jones measured the refractive index of the liquid." The passive voice emphasizes the object: "The refractive index of the liquid was measured." The writer may add a "by" clause, "...by Jones."

Deadwood Expressions

Try to avoid expressions such as these that don't convey much meaning:

- for the purpose of
- in the case of, in the area of, in the field of
- in connection with
- on the part of
- through the use of
- as far as ______ is concerned
- due to the fact that (just write *because*)
- at this point in time (just write *now*)
- it has been observed that

Redundant Phrases

Redundant phrases and words such as those shown below *in italics* do not add value and should generally not be seen in technical reports:

- because *of the fact that*
- large *in size*, green *in color*
- *free* gift, *solid* brick wall, *past* history
- *rather* unique, *more* complete

Implied Antecedents of Pronouns

Pronouns should clearly refer to a specific antecedent.[1] In the following example, the use of "this" is ambiguous:

> "A mathematical analysis and computer simulation will be used in the investigation of air drilling. This will provide a practical approach…"

What will provide a practical approach – analysis, simulation, investigation, or the process as a whole? This problem is easy to fix; just supply a copy of the antecedent:

> "This simulation will provide a practical approach…"

Smothered Verbs

It may be tempting to write a sentence such as "Evaporation of the liquid takes place." This a poor structure; the words "takes place" convey no useful meaning and distract from the point you should be getting across: "The liquid evaporates."

Tables and Figures

- Each and every table or figure is to be presented in such a way that its significance can be understood without reference to other text in the report.

- Tables are numbered consecutively; each contains a short title above the table. The title should include the name of the experiment.

- Figures are numbered consecutively. Each contains a long descriptive caption below the figure. The axes must be labeled with the variable names and units. Significant points must be identified. The caption must state the significance of the figure. Avoid extensive discussion of problems or errors in the captions.

[1] *Little, Brown Handbook,* 5th ed., 1992, 12c

A.3 Guidelines for Writing Technical Memoranda

M E M O R A N D U M

To:	ME 318 Vibrations Students Mechanical Engineering Department	**Date:** December 30, 2012 File: ME-1-97
From:	Jim Meagher, Professor	

Subject: GUIDELINES FOR WRITING TECHNICAL MEMORANDA

Technical memoranda (memos) are typically used to convey technical results between departments within a company. Memoranda of Record are used to create a record, within a department's filing system, of technical results. Every company will have its own numbering system for memoranda so that they can be filed and retrieved.

The header formats for memoranda are fairly standard between companies. An example of a memorandum header is shown above.

The first paragraph after the header should introduce the reader to what you did and to the organization of your memorandum. The body of your memorandum should present and discuss your results and recommendations. You may think of this section as a "brief report" or "short discussion" as used in our lab reporting.

You may attach graphs, drawings, data sheets, etc., to your memorandum. Each attachment must have an attachment number in the upper right hand corner, placed so that these numbers can be easily seen when the pages of a stapled memo are flipped. It is good practice to number your attachments in the following form: **Attachment 2 of 3.**

If an attachment has more than one page, each page of that attachment should bear the same attachment number. Remember, if an item is worth attaching to your memorandum, then it is worth mentioning in the body of your memorandum. Any mention of an attachment in the body of your memorandum should refer to the attachment by number, for example: "The new experimental data, shown in Attachment 2, is provided for your review and analysis."

A.4 Guidelines for Writing Technical Letters

Mechanical Engineering CAL POLY

December 30, 2012 File No. ME-2-97

Recipient's name
California Polytechnic State University
San Luis Obispo, CA 93407

Dear *Recipient*:

Technical letters are typically used to convey results between companies or between consultants and their clients. If results require a letter of more than 2 – 3 pages, they should be documented in a formal report and introduced by a brief cover letter.

As in any business letter, the header should show the date and full name the address of the addressee. A salutation follows such as: "Dear sirs" or "Dear Mr. Jones." Except for the header and the signature block, the format of a technical letter is fairly loose. The first paragraph should briefly describe what it is that you have done for the client and express your gratitude for the chance to be of service. The following paragraphs should present and discuss your results and recommendations. In the closing paragraph you should again express your gratitude for the opportunity to be of service and your hope for future opportunities.

You may attach graphs, drawings, data sheets, etc., to your letter. These do not count toward your maximum pages. Each attachment must have an attachment number in the upper right hand corner. Number your attachments in the form: **Attachment 3 of 5.** If an attachment has more than one page, each page of that attachment bears the attachment number. Every attachment must be referenced within the letter by number, for example: "The graph of Attachment 3 shows..."

Your letter should close with a "complimentary close" followed by your name, title, address and phone number. The address and phone number may be omitted if your address and phone number is on letterhead.

Sincerely,

Professor Jim Meagher
Mechanical Engineering Department
California Polytechnic State University
San Luis Obispo, CA. 93407

Appendix B

Fourier Analysis

B.1 Introduction

This Appendix contains a brief discussion of Fourier series analysis. You have used the Fourier series method of representing periodic functions in your math classes; in ME 318, you will use the Fourier series and the associated Fourier transform to represent periodic functions which are measured from physical systems.

B.2 Fourier Series

For a periodic function $f(t)$ we can obtain a Fourier series of constants A_n and B_n such that

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \ cos\left(\frac{\pi n t}{T}\right) + \sum_{n=1}^{\infty} B_n \ sin\left(\frac{\pi n t}{T}\right) \tag{B.1}$$

and

$$A_n = \frac{1}{T}\int_0^{2T} f(t) \ cos\left(\frac{\pi n t}{T}\right) dt \tag{B.2}$$

$$B_n = \frac{1}{T}\int_0^{2T} f(t) \ sin\left(\frac{\pi n t}{T}\right) dt \tag{B.3}$$

where 2T is the period of the periodic waveform.

The Fourier series can be used to approximate a periodic waveform by a sequence of sine waves; we merely truncate the summation after a few terms of the series, *i.e.* we do not let n go all the way to infinity. As we add each new sine wave to the series, the approximation comes closer to the desired periodic waveform. For example, Figure B.1 shows several Fourier series approximations to a square wave in the time domain. The square wave is difficult to approximate because it is discontinuous in time and has infinite derivatives at the steps; yet the approximation with 19 terms is reasonably close to a square shape.

The frequency domain representation can be thought of as the coefficients of the Fourier series. These are shown on the amplitude versus frequency plot in Figure B.2.

Table B.1 shows the theoretical Fourier series for several waveforms which are commonly seen in a laboratory.

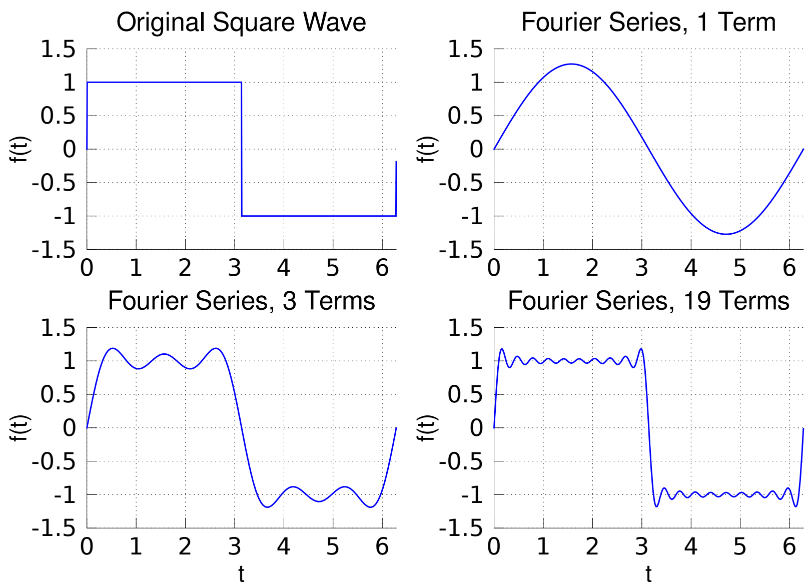


Figure B.1: Progression of Fourier approximations

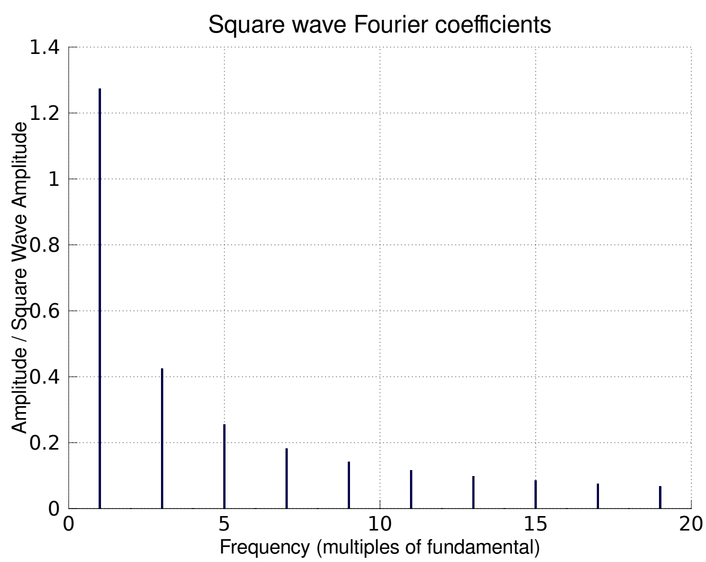


Figure B.2: Fourier series coefficients for square wave

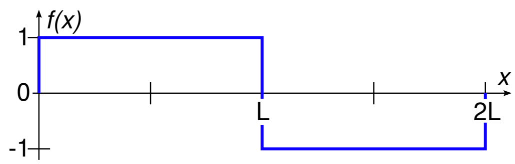
B.3 The Fourier Transform

The Fourier transform can be thought of as a way to determine the coefficients of a Fourier series which represents a particular signal. The Fourier transform is a bit different from the Fourier series above because the series is only defined for specific frequencies ω_1, $2\omega_1$, $3\omega_1$, $\ldots$, while the transform is defined for all frequencies. The Fourier transform equations are shown below.

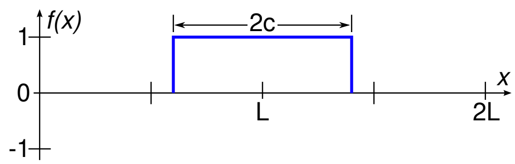
$$P(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt \tag{B.4}$$

$$p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(\omega)e^{-j\omega t}d\omega \tag{B.5}$$

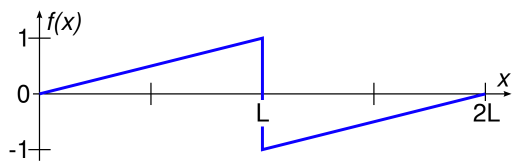
You can think of $P(\omega)$ as representing the Fourier coefficients of a series of sine waves of various amplitudes and phases which, when combined, make up the periodic function $p(t)$.



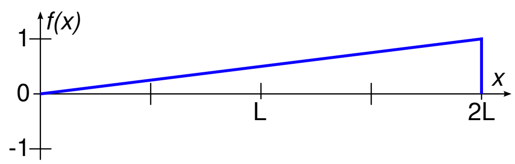
$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin \frac{n\pi x}{L}$$



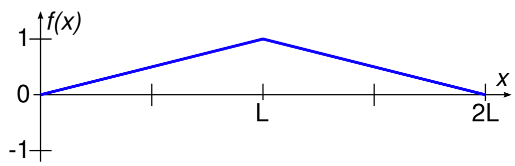
$$f(x) = \frac{c}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi c}{L} \cos \frac{n\pi x}{L}$$



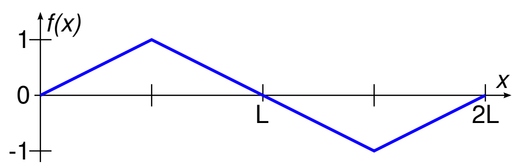
$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{L}$$



$$f(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{L}$$



$$f(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1,3,5,\ldots} \frac{1}{n^2} \cos \frac{n\pi x}{L}$$



$$f(x) = \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots} \frac{(-1)^{(n-1)/2}}{n^2} \sin \frac{n\pi x}{L}$$

Table B.1: Table of common waveforms with their Fourier transforms

Appendix C

Theoretical Bending Modes of a Cantilever Beam

It can be shown that the vibrations of a simple beam, made of a linearly elastic material and whose cross section is constant with length, are described by the following equation:

$$y(t, x) = \phi(x)\ \xi(t)$$

Where $y(t, x)$ is the deflection of the beam at position x (measured from one end of the beam) and time t. The time function $\xi(t)$ is not important to us except for its natural frequency and damping ratio; but, for example, it might be $\sin(\omega_n t)$. The function $\phi(x)$ is the mode shape function and is independent of time; it forms an envelope within which the time function $\xi(t)$ oscillates.

$$\phi(x) = C_1\ \cosh(\lambda x) + C_2\ \sinh(\lambda x) + C_3\ \cos(\lambda x) + C_4\ \sin(\lambda x) \tag{C.1}$$

where

$$\lambda^4 = \frac{m}{EIL}\omega_n^2$$

in which m is the total mass of the beam, L is its length, I is its cross-sectional area moment of inertia, and E is the elastic modulus of the material from which the beam is made.

Turning the equation around, if we know λ (or more precisely, λL), we can find ω_n from:

$$\omega_n = \sqrt{\frac{EI}{m}\frac{(L\lambda)^4}{L^3}} = \sqrt{\frac{EI}{mL^3}}(L\lambda)^2 \tag{C.2}$$

Where ω_n is a resonant frequency of the system. Since the mass of the beam is distributed over its length and is not "lumped", the system will have an infinite number of resonant frequencies. These are called the *modal frequencies* of the beam. Each modal frequency will result in a new $L\lambda$ and, consequently, a new mode shape function $\phi(x)$. The constants C_i in the mode shape function depend on the boundary conditions of the beam. Since there are four constants, four boundary conditions are required. For example, for a cantilevered beam:

$$\begin{aligned}
&\phi(0) = 0 \\
&\phi'(0) = 0 \\
&M(L) = EI\phi''(L) = 0 \\
&V(L) = M'(L) = EI\phi''(L) = 0
\end{aligned}$$

where M is the moment on the beam at a particular location and V is the shear at a given location. Applying the boundary conditions to Equation C.1 and writing the results in matrix notation yields:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \cosh L\lambda & \sinh L\lambda & -\cos L\lambda & -\sin L\lambda \\ \sinh L\lambda & \cosh L\lambda & \sin L\lambda & -\cos L\lambda \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{C.3}$$

If this matrix has an inverse, then the solution of these boundary equations would be $C_1 = C_2 = C_3 = C_4 = 0$. Therefore, we conclude that the matrix must not have an inverse (*i.e.* it is singular). The only way this can be true is if the determinant of the matrix is 0. This occurs when:

$$\cosh L\lambda \;\cos L\lambda = -1 \tag{C.4}$$

This is called the frequency equation, since we can solve it for $L\lambda$ and then use Equation C.2 to determine ω_n. Equation C.4 is a transcendental equation which has an infinite number of solutions. This equation will be solved when:

$$\cos L\lambda = \frac{-1}{\cosh L\lambda} \tag{C.5}$$

Figure C.1 shows graphs of the two sides of Equation C.5, which can be used to find solutions, marked here with circles. The lowest frequency value of $L\lambda_1$

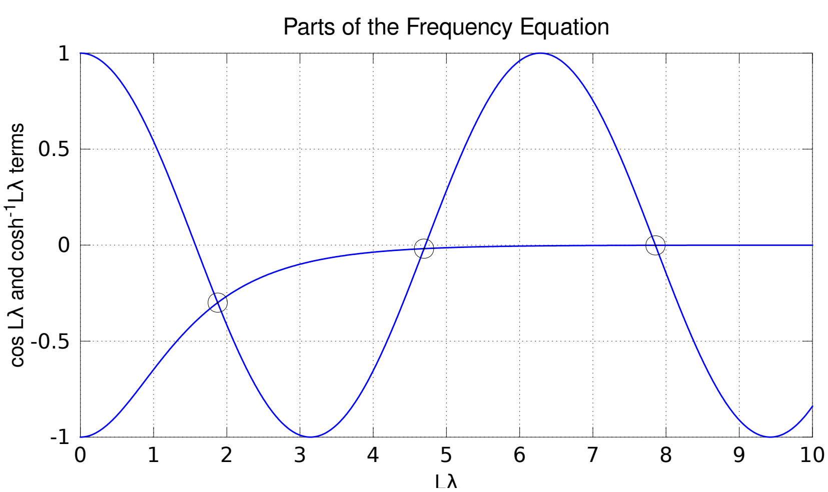


Figure C.1: Graphical solutions of the frequency equation

corresponds to the *fundamental* frequency or mode.

As an exercise, calculate the first three $L\lambda$ values. Simply use trial and error (or a convenient numeric solver) until your value satisfies Equation C.5. Find the values to an accuracy of 3 decimal places. You can use Figure C.1 as a “sanity check” to make sure that your answers are reasonable.

We can substitute each $L\lambda_i$ into Equation C.3 and solve for the i-th mode shape. Remember that the matrix in Equation C.3 is singular, so we can solve for at most three coefficients in terms of the fourth; this is why the mode shape is only known within a constant. Lets try for C_1, C_2, and C_3. Throw away the last row of the matrix. Then:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \cosh L\lambda & \sinh L\lambda & -\cos L\lambda & -\sin L\lambda \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{C.6}$$

and

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ \cosh L\lambda & \sinh L\lambda & -\cos L\lambda \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 1 \\ -sin\ L\lambda_i \end{Bmatrix} C_4 \tag{C.7}$$

Premultiplying by the matrix inverse yields:

$$\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = \frac{\begin{bmatrix} -\cos L\lambda_i & \sinh L\lambda_i & -1 \\ 0 & -\cos L\lambda_i - \cosh L\lambda_i & 0 \\ -\cosh L\lambda_i & -\sinh L\lambda_i & 1 \end{bmatrix}}{-\cos L\lambda_i - \cosh L\lambda_i} \begin{Bmatrix} 0 \\ 1 \\ -\sin L\lambda_i \end{Bmatrix} C_4$$

We can choose C_4 to be anything we wish, so let's choose it to simplify the analysis:

$$C_4 = \cos L\lambda_i + \cosh L\lambda_i \tag{C.8}$$

Then,

$$\begin{aligned} C_1 &= \sinh L\lambda_i + \sin L\lambda_i \\ C_2 &= -\cos L\lambda_i - \cosh L\lambda_i \\ C_3 &= -C_1 \\ C_4 &= -C_2 \end{aligned} \tag{C.9}$$

and by substituting the equations above into Equation C.1

$$\begin{aligned} \phi(x) = {} & (\sinh L\lambda_i + \sin L\lambda_i)(\cosh \lambda_i x - \cos \lambda_i x) \\ & - (\cosh L\lambda_i + \cos L\lambda_i)(\sinh \lambda_i x - \sin \lambda_i x) \end{aligned} \tag{C.10}$$

Now let

$$\Lambda_i = L\lambda_i \quad \text{and} \quad \rho = x/L$$

Then,

$$\begin{aligned} \phi(\rho) = {} & (\sinh \Lambda_i + \sin \Lambda_i)(\cosh \Lambda_i\rho - \cos \Lambda_i\rho) \\ & - (\cosh \Lambda_i + \cos \Lambda_i)(\sinh \Lambda_i\rho - \sin \Lambda_i\rho) \end{aligned} \tag{C.11}$$

$\phi(\rho)$ is plotted versus ρ in Figure C.2 for the first and second modes.

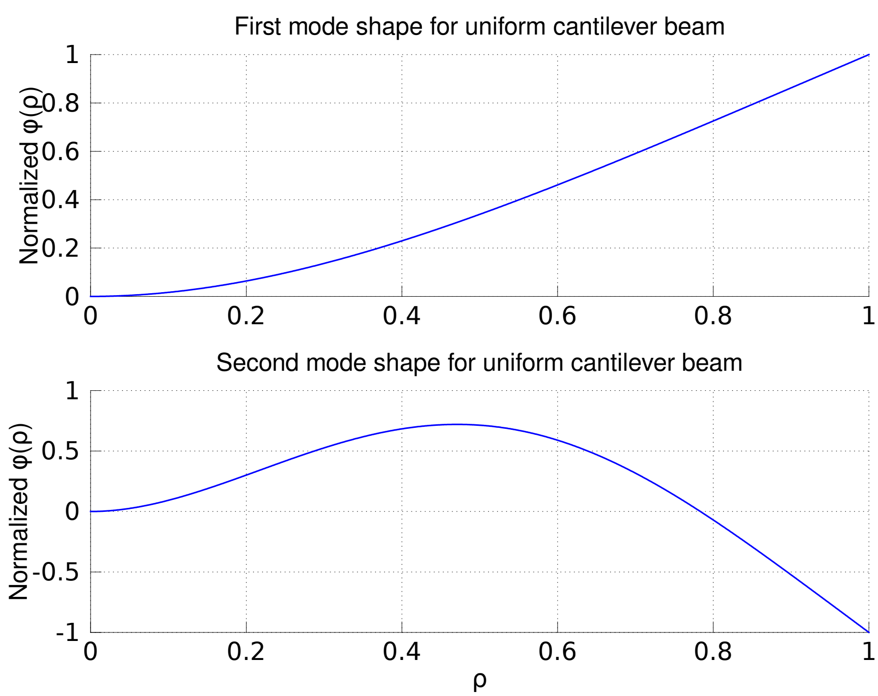


Figure C.2: Mode shapes for cantilever beam

Appendix D

Rayleigh's Energy Method Introduction

Your vibrations text discusses Rayleigh's method for finding an approximate first natural frequency for a distributed mass system. When the system's mass is not concentrated at one location, it is necessary to determine how much of the mass is actively vibrating. Rayleigh's method does this by using energy methods. Rayleigh's method enables us to add a portion of the beam's mass to the mass at the end of the beam and then to treat the system as a simple spring mass problem.

The beam's deflection, with a unit load at the end of the beam, yields the reciprocal of the spring constant. The mass for the vibration model is the sum of the concentrated mass and that portion of the beam's mass that is active. See the text for more details about Rayleigh's method.

D.1 Theory

Let us look at the kinetic energy T stored in a cantilever beam vibrating about its static equilibrium. Let ℓ be the length of the beam, δ the deflection at the end of the beam, ρ the mass per unit length, and v the velocity of a point along the beam. Then

$$T = \frac{1}{2}\int_{0}^{\ell} v^2 \mathrm{d}m \tag{D.1}$$

In order to find T by integration of Equation D.1, we will first assume a uniform beam so that $\mathrm{d}m = \rho \mathrm{d}x$, then we must describe v as a function of x. We assume that the velocity of any point x is proportional to the maximum y displacement at that point. If we make the crude approximation that $y = (x/\ell)\delta$, which means that the vibration envelope is assumed to be proportional to x, then we have that

$$\dot{y}_{max} = \left(\frac{x}{\ell}\right)\dot{\delta}_{max}$$

where $\dot{\delta}_{max}$ is the maximum velocity at the end of the beam, which is where the lumped mass is assumed to be mounted. Notice that this envelope does not have zero slope at $x = 0$, so we do not expect this to be a very good approximation.

Substituting into Equation D.1,

$$\begin{aligned} T &= \frac{1}{2}\int_0^\ell v^2 \mathrm{d}m \\ &= \frac{1}{2}\int_0^\ell \frac{x^2}{\ell^2}\dot{\delta}^2_{max}\rho \mathrm{d}x \\ &= \frac{1}{2}\dot{\delta}^2_{max}\left(\frac{\rho\ell}{3}\right) \\ &= \frac{1}{2}\dot{\delta}^2_{max} m_{eff} \end{aligned} \tag{D.2}$$

The above expression tells us to add an effective mass, equal to 1/3 of the beam's mass, to the mass at the end of the beam and then to neglect the beam's mass in finding the natural frequency.

What has just been done is useful as an example, but the assumption that $\dot{y} = (x/\ell)\dot{\delta}$ is very poor. A much better assumption would be to use the equation for the deflection of a cantilever beam with a load at the end to describe the vibration envelope,

$$\begin{aligned} y &= \frac{x^2}{2\ell^3}(3\ell - x)\delta \\ v = \dot{y} &= \frac{x^2}{2\ell^3}(3\ell - x)\dot{\delta} \end{aligned} \tag{D.3}$$

It is left as an exercise for the student to repeat the above procedure using this v and add the portion of the beam's mass found by this procedure to your concentrated mass.

Appendix E

Solving ODE's with Matlab™

Some tips are given in this section for solving ordinary differential equations numerically with Matlab™. It is assumed that the reader is already familiar with basic use of Matlab™. If this is not the case, you can review your previous coursework or use references on the Internet to practice your programming skills.

E.1 Example

This example demonstrates the use of Matlab™ to solve an initial value problem.

Given: The equation of motion is

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + c\frac{\mathrm{d}x}{\mathrm{d}t} + kx = A\sin(\omega t)$$

The initial conditions are

$$\frac{\mathrm{d}x}{\mathrm{d}t}(0) = 0, x(0) = 1.45$$

Other constants are $m = 3$ slugs, $k = 100$ lb/in, $c = 6.93$ lb/in/sec

Find and plot: The position and velocity as functions of time.

Matlab™ requires that the model equations be written in the state space format. In state space format, a set of first-order differential equations replaces the single higher-order differential equation which we usually use in ME318. To convert to state space format, we designate the highest order derivative as a *state variable* and solve the system equation for that variable.

$$\ddot{x} = \frac{A\sin(\omega t) - c\dot{x} - kx}{m}$$

Then we choose a set of *state variables* for our system. For a second order system with one degree of freedom, position x and velocity v are chosen. We write equations for the derivatives of the state variables, noting that $\ddot{x} = \dot{v}$:

$$\begin{aligned} \dot{v} &= \frac{A\sin(\omega t) - c\dot{x} - kx}{m} \\ \dot{x} &= v \end{aligned} \tag{E.1}$$

The state space equations must be entered into a Matlab "m-file." The file must be named with extension `.m` and the file name must match the function name in the first line of the file. The directory in which the file resides must be in the Matlab™ path (or be in the current directory in which Matlab™ is running).

You can create an m-file in any text editor or in a Matlab™ window. Create a new file, then type in the following program. Notice that there are only six lines of code in this program; all of the other lines are comments.

```
function dx = sdof (t, x)
% This matlab m-file runs a simulation with an ODE solver.
% Written by Jim Meagher 08-04-00. Last revision 09-26-13.
% Note the file name must be sdof.m to match function name.
% We expect you to copiously use comments in your coding!
k = 100.0;         % Spring constant in lb/ft
m = 3.0;           % Mass in slugs
c = 6.93;          % Damping coefficient in lb/ft/sec

% Ending a line with a semicolon is not required but reduces
% screen output. The syntax and order (constants, single
% variable on left-hand side, etc.) must be strictly followed.
dx(1,1) = x(2);
dx(2,1) = -k * x(1) / m - c * x(2) / m;

% A single second order differential equation is represented
% by two first order differential equations.
% Here, x(1) is position and x(2) is velocity.
```

These equations are to be integrated numerically by the toolbox function `ode45()`, where the `ode` stands for ordinary differential equation solver. If needed, place the directory that holds `sdof.m` on the Matlab™ path using the path browser. Finally, in the workspace, enter the following commands to run the program. These commands could also be placed into a second m-file and run with a single command.

```
>> type sdof.m     % The 'type' command displays the m-file and
                   % verifies that the Matlab path has been set

>> tspan = [0,10];          % Set the time span 0 to 10 sec.
>> x0 = [1.45 0];           % Set initial position and velocity

>> [t, x] = ode45 ('sdof', tspan, x0);     % Run the ODE solver
```

Notice that the parameter list must be in the same order here as it was in the .m file. Notice also that this is a lot of typing to get right. If you find a typing mistake after you press enter you can correct it by pressing the up arrow, this will bring in the last command, which you can then edit. Pressing the up arrow more than once steps back through the history file and you can see all of your previous commands one at a time. If you want to run the `ode45` file a second time just press the up arrow until it shows up in the command line and then press enter.

```
>> plot (t, x)            % Plot x and dx/dt
>> plot (t, x(:,1))       % Plot displacement only
>> plot (t, x(:,2))       % Plot velocity only
>> plot (t, x, 'k')       % Forces lines to be black (for printing)
```

Use the Matlab™ functions `title`, `xlabel`, and `ylabel` to add a title and axis labels (**never** submit a graph without axis labels!). The figure can be copied and then pasted into a word processor, where you can type a caption. Here are some additional hints:

- Type `help ode45` to get more information on the equation solver `ode45`.
- If you suspect that the time step is causing inaccurate results you will want to decrease its value. Type `help odeset` and read about `RelTol`, its default value and how to change it. You will need to add options to the string of parameters in the `ode45` command in order to activate your new tolerance value.
- There is a **free and open source** program called Octave which you can use as an alternative to Matlab™ in ME318. Unfortunately, Octave doesn't have an analogous package to Simulink™, so it cannot be used in ME422. The code for the ME318 examples has been tested in Octave and works fine, with one change from the Matlab™ code on the line that calls the `ode45` solver routine:

`[t, x] = ode45 ('sdof', tspan, x0);`	Matlab™ version
`[t, x] = ode45 (@sdof, tspan, x0);`	Octave version

  You need the Octave package `odepkg`, which is freely available along with the main Octave program at `http://www.octave.org`. For most Linux users, `sudo apt-get octave octave-odepkg` is all it takes to install what you need.

Appendix F

The HP 54600 Oscilloscope

The HP 54600 oscilloscope in the vibrations lab is a digital, two-channel instrument. Figure F.1 shows the front panel of the oscilloscope. There are three types of controls:

- **Hard Keys:** These keys never change their mode of operation. They select particular menus, which are displayed on the screen. They are shown as $\boxed{\textsf{HardKey}}$ in this manual.
- **Soft Keys:** These keys change their mode of operation depending on which hard key has been selected. The definitions of these keys are displayed directly above them on the screen. They are shown as (SoftKey).
- **Knobs:** These are used to change scale factors and move cursors.

The oscilloscope will capture and display either periodic or transient signals, but in this appendix the discussion is limited to periodic signals. The trigger functions necessary for displaying transient signals are left to the experiment descriptions which require them. The oscilloscope has an Autoscale function, but this function only works for some periodic signals at frequencies above 50 Hz. You **must** learn to use manual scaling to succeed in the vibrations lab.

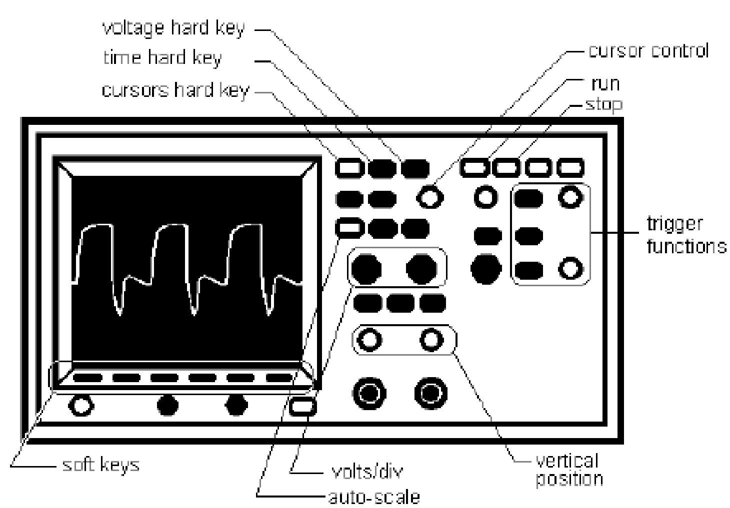


Figure F.1: The front panel of the HP 54600

F.1 Operation for Periodic Signals Over 50 Hz

The instructions in this section are for a very brief practice exercise by which you can familiarize yourself with some of the oscilloscope's controls.

- Connect a signal generator to the oscilloscope's inputs. Connect the HI output to channel 1 and the LO output to channel 2, then set the frequency to about 70 Hz; this is your driving frequency.
- Press the hard keys [1] and [2] several times. Each of these keys selects input channel 1 or 2 and sets the soft keys to act on that channel. When you press [1] or [2] again, it will turn the corresponding input off or on. Use these keys so that both channels are active (traces for channels 1 and 2 are displayed on the screen at the same time).
- Press the hard key [Autoscale]. This causes the oscilloscope to scale the amplitude and time axes of the screen according to whatever high-frequency signal it finds. Both channels should be displayed, showing about two cycles of each. ***Warning:*** *If you are looking for a low-frequency or periodic signal, the autoscale function will find and display electrical noise and prevent you from seeing the signal that you need to see.*
- Use the [2] key to select channel 2 if necessary, then press the soft key (Invert). Notice that this changes the sign of channel 1.
- Press [Display], then press (Grid) several times to see its effect.
- Rotate the TIME/DIV knob. Watch the indicator at the top of the screen to see how many seconds, milliseconds, microseconds, or nanoseconds per division are being displayed. Rotate the position knob under key [1]. It moves the channel 1 trace up and down.
- Rotate the knob under HORIZONTAL. Notice that it shifts the display along the time axis.
- Rotate the Volts/Div knob above hard key 2. Notice that it changes the amplitude scale for channel 2.
- Press [Erase], then press [Autoscale].
- Unplug the signal cables from the signal generator but leave them plugged into the oscilloscope. Press [Autoscale] again. What does the oscilloscope display now? What have the time and voltage scales been set to; in other words, how large and at what frequency is the signal now being displayed?

F.2 Operation for Periodic Signals Under 50 Hz

When setting up the oscilloscope for operation under 50 Hz, you cannot rely on the autoscale feature. Instead, you must first get the oscilloscope to show some trace of your signal, then adjust the settings so that your signal is shown as clearly as possible. The following procedure will help guide you through the process of finding your signal, displaying it properly, and taking measurements from the signal.

- Connect your signal source to the channel 1 input.
- Press hard key Setup, then soft key (Default Setup).
- Set the voltage scale to a high number such as 5 volts/division using the knob just above the 1 key. You should see a bright horizontal trace, showing the input voltage, across the screen. If not, turn the vertical position knob (under the 1 key) until the trace moves into view. Center the trace vertically on the screen.
- Set the time per division to a starting value. Think about the signal you are measuring. What frequency do you expect? Set the Time/Div knob so that 10 divisions will show about two full cycles. For example, for a 20 Hz signal, you have 50 ms per cycle, or 100 ms for two cycles; this gives you 10 ms per division.
- Turn the Volts/Div knob by one click so that the vertical scale changes to the next lower number (for example, from 5 volts per division to 2 volts per division). Re-center the trace vertically if it has moved away from the center. Repeat this process until the signal you're trying to measure fills most the screen vertically.
- If necessary, readjust the time scale with the Time/Div knob so that you see between one and a half to three full waves of your signal on the screen.
- If your signal is not completely stable, you can use the Stop key to freeze your signal to take measurements. After you're done, use Run to take measurements again.

F.3 Taking Data for Periodic Signals

The oscilloscope can be used to measure periodic signals using automatic or manual methods.

Automatic Measurement

For clean sine waves, the automatic method works well; however, real mechanical measurements often contain noise and distortion, and you will need to use the manual method for those signals.

- Make sure that you have at least one full sine wave displayed on the screen. The wave must be large enough so that you can clearly see its shape, and the top and bottom must not be outside the bounds of the screen.

- To measure frequency, press $\boxed{\text{Time}}$ then (Freq).
- To measure voltages, press $\boxed{\text{Voltage}}$ and then choose the soft key corresponding to the measurement you need to take:
  - (V_{P-P}) shows the peak-to-peak voltage. Remember that this is two times the amplitude (amplitude is sometimes called "zero-to-peak" voltage).
  - (V_{AVG}) shows the average value. This is the voltage at the center of the signal, or the offset away from zero of the sine wave.
  - (V_{RMS}) shows the Root-Mean-Square voltage, which is a measure of the average power in a signal. Note that the RMS voltage is affected by the offset. You only get what you really want from RMS if the signal is centered about 0 volts.

Manual Measurement

- Make sure that you have at least one full sine wave displayed on the screen. The wave must be large enough so that you can clearly see its shape, and the top and bottom must be within the bounds of the screen.
- Time measurements:
  - Press the hard key $\boxed{\text{Cursors}}$.
  - Press soft key (t1/t2) to highlight t1 on the screen.
  - Use the cursor control knob (it's near the $\boxed{\text{Cursors}}$ key) to position the vertical cursor to a zero crossing of the trace.
  - Press (t1/t2) to highlight t2 on the screen.
  - Use the cursor control knob to move the second vertical cursor to another zero crossing of the trace which is exactly one or two waves from the first zero crossing.
  - The screen will display the time at each vertical cursor line as well as a time difference Δt between the cursors. It also shows a frequency corresponding to Δt. This frequency is calculated from Δt assuming one full wave, so if you have measured the period of two or three waves, you must correct your measurements, dividing or multiplying by the number of waves.
- Voltage measurements:
  - Press the hard key $\boxed{\text{Cursors}}$.
  - Press the soft key (v1/v2) to light up v1 on the screen.
  - Use the cursor control knob to position the horizontal cursor at the peak of a trace. When the signal is jagged due to noise, you may need to estimate the peak of the signal you need to measure by eye. Your

eye (and brain) are usually better than the oscilloscope at estimating the right place to put a cursor.

- Press the soft key (v1/v2) to light up v2 on the screen.
- Use the cursor control knob to position the second horizontal cursor at a negative peak, estimating the center of a noisy signal as necessary.
- The screen will display the voltages of both peaks and the peak-to-peak voltage $V_{P\text{-}P}$ between them.

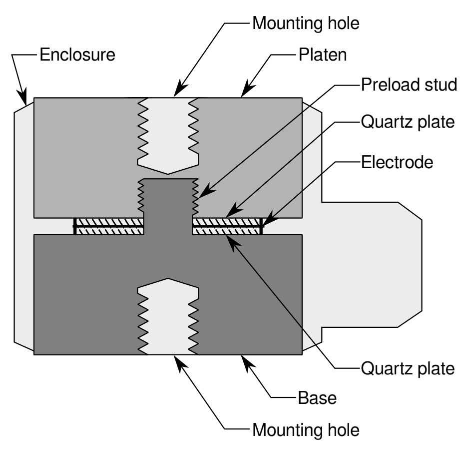
Appendix G

Accelerometers and Load Cells

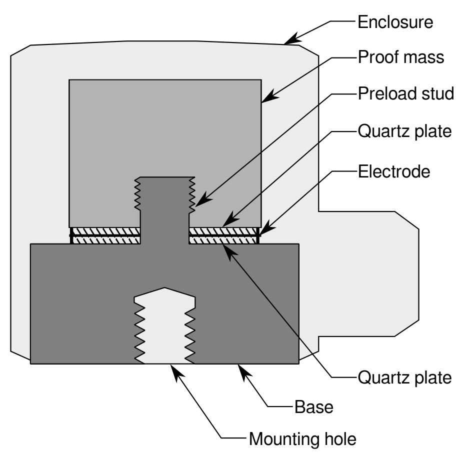
There are many different designs for force transducers (also called load cells) and accelerometers. Some designs are based on strain gauges, some use electrostatic force balancing, and some use the piezoelectric effect.

G.1 Piezoelectric Transducers

When a crystal made of a material such as quartz is placed under stress, a voltage across the crystal is created which is proportional to the level of stress in a particular sensitive direction. A diagram of a piezoelectric force transducer is shown in Figure G.1(a). The platen is screwed to the base using the preload stud, which is tightened so it is in tension. This compresses the quartz plates between the platen and base. The force to be measured is applied vertically down onto the top of the platen, pushing the platen down toward the base and increasing the compression on the quartz plates. This causes a voltage output measured between the electrode and "ground" (which is the voltage of the base and platen). If a force pulls the platen upward, the preload compression on the quartz plates is reduced, and this results in a negative voltage at the electrode.



(a) Force Transducer



(b) Accelerometer

Figure G.1: Piezoelectric force transducer and accelerometer

The output of a piezoelectric transducer has a high voltage but very small current, so a special charge coupled amplifier ("charge amp") must be used to amplify the voltage so it can be measured with an oscilloscope or other instrument. The "blue box" amplifiers in the vibrations lab are used for this purpose. Even the cables used with piezoelectric transducers must be of a special design to allow accurate measurements. Piezoelectric transducers cannot measure static forces well, since their charge tends to leak away in a short time due to the physical properties of the crystal; they can only measure changing force or acceleration.

A piezoelectric accelerometer is a force transducer with a proof mass instead of a platen on top, as shown in Figure G.1(b). An acceleration along the sensitive axis (vertical as shown) causes a reaction force which follows Newton's Second Law, $F = ma$ where m is the mass of the proof mass. This force changes the stress in the crystal just as the applied force on the force transducer does. The accelerometer is designed so that accelerations normal to the sensing axis cause only a very small output; for an ideal accelerometer, this *transverse sensitivity* would be zero.

Figure G.2 shows a typical data sheet for a piezoelectric accelerometer. Of particular interest are the range, sensitivity, resonant frequency, frequency range, and linearity.

Model Number **353B03**	**ACCELEROMETER, ICP®**	Revision G ECN #: 35369

	ENGLISH	SI	
Performance			
Sensitivity (±5 %)	10 mV/g	1.02 mV/(m/s²)	[2]
Measurement Range	±500 g pk	±4905 m/s² pk	
Frequency Range (±5 %)	1 to 7000 Hz	1 to 7000 Hz	
Frequency Range (±10 %)	0.7 to 11000 Hz	0.7 to 11000 Hz	
Frequency Range (±3 dB)	0.35 to 20000 Hz	0.35 to 20000 Hz	
Resonant Frequency	≥38 kHz	≥38 kHz	
Broadband Resolution (1 to 10000 Hz)	0.003 g rms	0.03 m/s² rms	[1]
Non-Linearity	≤1 %	≤1 %	[3]
Transverse Sensitivity	≤5 %	≤5 %	[4]
Environmental			
Overload Limit (Shock)	±10000 g pk	±98100 m/s² pk	
Temperature Range (Operating)	-65 to +250 °F	-54 to +121 °C	
Base Strain Sensitivity	≤0.0005 g/με	≤0.005 (m/s²)/με	[1]
Electrical			
Excitation Voltage	18 to 30 VDC	18 to 30 VDC	
Constant Current Excitation	2 to 20 mA	2 to 20 mA	
Output Impedance	≤100 Ohm	≤100 Ohm	
Output Bias Voltage	8 to 12 VDC	8 to 12 VDC	
Discharge Time Constant	0.5 to 2.6 sec	0.5 to 2.6 sec	
Settling Time (within 10% of bias)	<5 sec	<5 sec	
Spectral Noise (1 Hz)	2800 μg/√Hz	27468 (μm/sec^2/√Hz	[1]
Spectral Noise (10 Hz)	700 μg/√Hz	6867 (μm/sec^2/√Hz	[1]
Spectral Noise (100 Hz)	180 μg/√Hz	1766 (μm/sec^2/√Hz	[1]
Spectral Noise (1 kHz)	64 μg/√Hz	628 (μm/sec^2/√Hz	[1]
Physical			
Size (Height)	0.81 in	20.6 mm	
Weight	0.38 oz	10.5 gm	[1]
Sensing Element	Quartz	Quartz	
Size (Hex)	0.50 in	12.7 mm	
Sensing Geometry	Shear	Shear	
Housing Material	Titanium	Titanium	
Sealing	Welded Hermetic	Welded Hermetic	
Electrical Connector	10-32 Coaxial Jack	10-32 Coaxial Jack	
Electrical Connection Position	Side	Side	
Mounting Thread	10-32 Female	10-32 Female	

CE[5]

All specifications are at room temperature unless otherwise specified.
In the interest of constant product improvement, we reserve the right to change specifications without notice.
ICP® is a registered trademark of PCB group, Inc.

Optional Versions (Optional versions have identical specifications and accessories as listed for standard model except where noted below. More than one option maybe used.)

B - Low bias electronics		
Output Bias Voltage	4.5 to 7.5 VDC	4.5 to 7.5 VDC
Excitation Voltage	12 to 30 VDC	12 to 30 VDC
Constant Current Excitation	1 to 20 mA	1 to 20 mA
Measurement Range	±300 g pk	±2943 m/s² pk
J - Ground Isolated		
Frequency Range (±5 %)	1 to 5000 Hz	1 to 5000 Hz
Frequency Range (±10 %)	0.7 to 9000 Hz	0.7 to 9000 Hz
Resonant Frequency	≥22 kHz	≥22 kHz
Electrical Isolation (Base)	≥10^8 Ohm	≥10^8 Ohm
Q - Extended discharge time constant		
Frequency Range (±5 %)	0.1 to 7000 Hz	0.1 to 7000 Hz
Frequency Range (±10 %)	0.07 to 11000 Hz	0.07 to 11000 Hz
Discharge Time Constant	>10 sec	>10 sec
Settling Time (within 10% of bias)	45 sec	45 sec
Supplied Accessory: Model ACS-4 Single-axis, low frequency phase and amplitude response calibration from 0.5 to 10 Hz		
W - Water Resistant Cable		
Electrical Connector	Sealed Integral Cable	Sealed Integral Cable
Electrical Connection Position	Side	Side

Notes

[1] Typical.
[2] B and Q options supplied with a sensitivity tolerance of ± 10 %.
[3] Zero-based, least-squares, straight line method.
[4] Transverse sensitivity is typically <= 3%.
[5] See PCB Declaration of Conformance PS023 for details.

Supplied Accessories
080A Adhesive Mounting Base (1)
080A109 Petro Wax (1)
081B05 Mounting Stud (10-32 to 10-32) (1)
ACS-1 NIST traceable frequency response (10 Hz to upper 5% point). (1)
M081B05 Mounting Stud 10-32 to M6 X 0.75 (1)

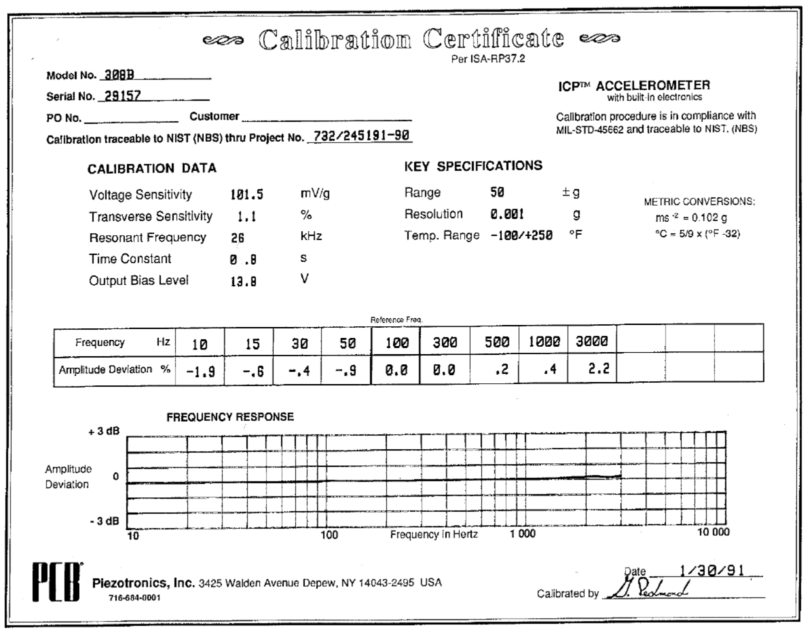
Entered: DMW	Engineer: BAM	Sales: WDC	Approved: ECB	Spec Number:
Date: 03/25/2011	Date: 03/25/2011	Date: 03/25/2011	Date: 03/25/2011	**353-2030-80**

PCB PIEZOTRONICS VIBRATION DIVISION

3425 Walden Avenue
Depew, NY 14043
UNITED STATES
Phone: 800-828-8840
Fax: 716-684-0987
E-mail: info@pcb.com
Web site: www.pcb.com

Figure G.2: Data sheet for a piezoelectric accelerometer

Most piezoelectric accelerometers are high precision instruments (with a high price to match), and each is individually calibrated and supplied with a calibration certificate from the factory. An example of such a certificate is shown in Figure G.3.


Calibration Certificate
Per ISA-RP37.2

Model No. 308B
Serial No. 29157
PO No. ____ Customer ____
Calibration traceable to NIST (NBS) thru Project No. 732/245191-90

ICP™ ACCELEROMETER
with built-in electronics
Calibration procedure is in compliance with MIL-STD-45662 and traceable to NIST. (NBS)

CALIBRATION DATA

Voltage Sensitivity	101.5	mV/g
Transverse Sensitivity	1.1	%
Resonant Frequency	26	kHz
Time Constant	0 .8	s
Output Bias Level	13.8	V

KEY SPECIFICATIONS

Range	50	±g
Resolution	0.001	g
Temp. Range	-100/+250	°F

METRIC CONVERSIONS:
ms^{-2} = 0.102 g
°C = 5/9 x (°F -32)

Reference Freq.

Frequency Hz	10	15	30	50	100	300	500	1000	3000			
Amplitude Deviation %	-1.9	-.6	-.4	-.9	0.0	0.0	.2	.4	2.2			

FREQUENCY RESPONSE

PCB Piezotronics, Inc. 3425 Walden Avenue Depew, NY 14043-2495 USA
716-684-0001

Date 1/30/91
Calibrated by G. Redmond

Figure G.3: Calibration certificate for a piezoelectric accelerometer

An accelerometer does not sense angular acceleration directly; it senses only translational acceleration. If the accelerometer is not on the axis of rotation, it can sense the normal and tangential components of the translational acceleration induced by the angular acceleration. The accelerometer senses that component of an acceleration vector which lies along the accelerometer's sensitive axis.

G.2 MEMS Accelerometers

During the 1990's, accelerometers made using Micromachined ElectroaMechanical Systems (MEMS) technology replaced piezoelectric devices as the dominant technology for measurement of acceleration. The principal reason for the dominance of MEMS accelerometers is cost; a typical 3-axis MEMS accelerometer costs less than the *cable* needed to connect a piezoelectric accelerometer to its charge amplifier. Today, most portable computers (tablets, smartphones, and similar devices) contain MEMS accelerometers; modern vehicles use them for stability control and airbag deployment. One, two, and three-axis accelerometers are fabricated inexpensively from silicon wafers. Integrated circuit manufacturing techniques are used to create these sensors; therefore, amplifiers, calibration circuitry, and even analog to digital converters can be built into the sensors. The result is small (on the order of 2 – 3 mm square), cheap (as little as a dollar in large quantities) sensors which have analog or digital outputs that can be connected directly to an oscilloscope or computer.

Several methods are used to measure acceleration in MEMS devices, though all MEMS accelerometers use a proof mass suspended on springs as the primary transducer. Some MEMS accelerometers have small silicon strain gauges built into the springs to measure deflection. Others measure deflection of the springs by measuring the capacitance between a conductive proof mass and stationary components located near to it. Yet other MEMS accelerometers use electrostatic forces to hold the proof mass stationary; the force needed to prevent the proof mass from deflecting its springs is proportional to the acceleration of the base.

A specification sheet for a three-axis, analog output MEMS accelerometer is shown in Figure G.6. As a low-cost, mass-produced device this accelerometer is not supplied with an individual calibration from the factory; such a calibration would cost more than the manufacture of the sensor. However, for accelerations close to $1g$, calibration by the user is remarkably easy for a MEMS accelerometer. Unlike piezoelectric accelerometers, MEMS accelerometers accurately measure static accelerations such as the acceleration due to gravity. If you know the acceleration due to gravity at your location, simply holding a MEMS accelerometer so that it measures the acceleration of gravity and then inverting it subjects the accelerometer to a known $2g$ change in acceleration. It is recommended to perform this check frequently when using MEMS accelerometers.

G.3 Theory

MEMS accelerometers use complex and varied techniques for measuring acceleration, and often the dynamics of the motion of the proof mass are coupled with the dynamics of the electronic control system which interacts with the mass. Therefore, it is often not feasible to use a simple vibrational model to explain the motion of a MEMS accelerometer nor find its resonant frequency. However, some older MEMS accelerometer designs use simple proof masses supported by springs without using electronics to push the proof mass around; and piezoelectric accelerometers use proof masses supported by a structure consisting of a quartz crystal and a preload bolt. Such accelerometers can be analyzed using the methods of ME 318.

The support structure for the proof mass (be it a quartz crystal with preload bolt or a micromachined silicon structure) acts as a linear spring with some amount of damping. We therefore analyze a mass-spring-damper system subject to base excitation. To make the analysis simpler, we begin by ignoring the damping. From the free body diagram and kinetic diagram in Figure G.4 we have:

$$\sum F_y = m\ddot{y}$$

$$m\ddot{y} = k(y - y_0)$$

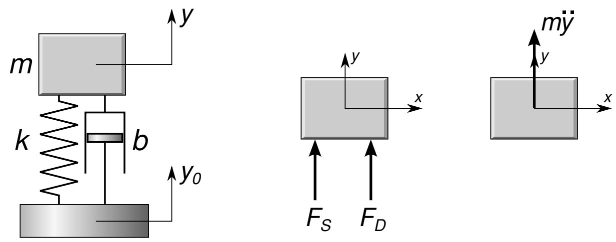


Figure G.4: Simplified model for an accelerometer. The base position is y_0 and the proof mass position is y.

If we let $z = y - y_0$ represent the relative motion between the crystal's two surfaces, then

$$m(\ddot{z} + \ddot{x}) = -kz$$

or

$$\ddot{z} + \omega_n^2 = -\ddot{x}$$

Using Laplace transform techniques, we can find the transfer function for this system. The transfer function gives the amplitude $Z(s)$ of relative motion of the top and bottom of the spring for a given amplitude of base acceleration A_{y0}:

$$G_z(s) = \frac{Z(s)}{A_{y0}(s)} = \frac{-1}{s^2 + \omega_n^2}$$

If we include damping in the derivation and let the damping ratio be ζ, the transfer function becomes

$$G_z(s) = \frac{-1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The voltage transfer function specifies the voltage output $V(s)$ of the transducer for a given amplitude of base acceleration A_{y0}:

$$G_V(s) = \frac{V(s)}{A_{y0}(s)} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where K is a constant relating the voltage output to the amplitude of relative motion of the ends of the piezoelectric crystal.

The amplitude ratio is the amplitude of the voltage output divided by the amplitude of the base acceleration, which of course is what we are trying to measure:

$$\frac{A_V}{A_{y0}} = \frac{K_{DC}}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta\omega/\omega_n)^2}} \tag{G.1}$$

In this equation, K_{DC} is the accelerometer's "DC gain" which gives the amplitude ratio at very low frequencies. This is a measurable gain for MEMS accelerometers, but it cannot be measured directly for piezoelectric accelerometers, as discussed on page G-2. Figure G.5 shows the form of the frequency

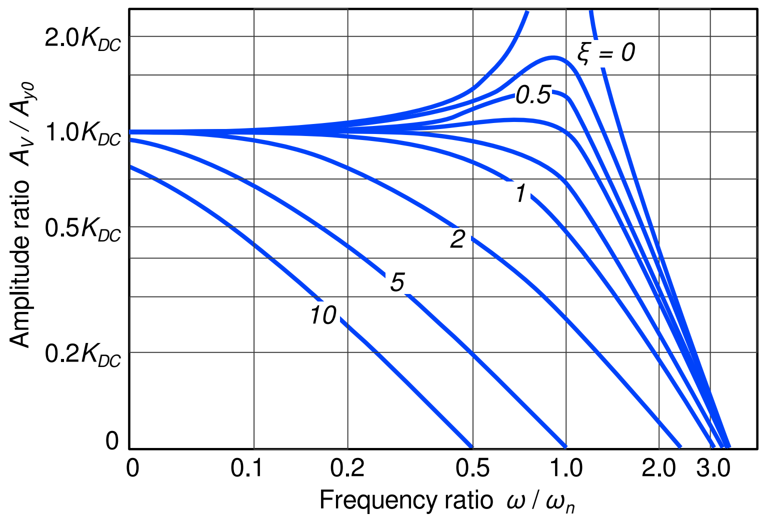


Figure G.5: Typical frequency response of an accelerometer

response for a typical accelerometer; it is a plot of Equation G.1 for varying values of ζ. In order for an accelerometer to give you trustworthy data, you must have a constant amplitude ratio over the entire range of frequencies of signals you intend to measure. Looking at this figure, you should be able to see the desired relationship between the frequency ω of any motion you are trying to measure and the natural frequency ω_n of the accelerometer. This figure also helps explain why piezoelectric accelerometers, with their very stiff internal structures, are preferred for many measurements involving shock and high frequency vibrations.

SPECIFICATIONS

$T_A = 25°C$, $V_S = 3$ V, $C_X = C_Y = C_Z = 0.1$ µF, acceleration = 0 *g*, unless otherwise noted. All minimum and maximum specifications are guaranteed. Typical specifications are not guaranteed.

Table 1.

Parameter	Conditions	Min	Typ	Max	Unit
SENSOR INPUT	Each axis				
Measurement Range		±3	±3.6		*g*
Nonlinearity	% of full scale		±0.3		%
Package Alignment Error			±1		Degrees
Interaxis Alignment Error			±0.1		Degrees
Cross-Axis Sensitivity[1]			±1		%
SENSITIVITY (RATIOMETRIC)[2]	Each axis				
Sensitivity at X_{OUT}, Y_{OUT}, Z_{OUT}	$V_S = 3$ V	270	300	330	mV/*g*
Sensitivity Change Due to Temperature[3]	$V_S = 3$ V		±0.01		%/°C
ZERO *g* BIAS LEVEL (RATIOMETRIC)					
0 *g* Voltage at X_{OUT}, Y_{OUT}	$V_S = 3$ V	1.35	1.5	1.65	V
0 *g* Voltage at Z_{OUT}	$V_S = 3$ V	1.2	1.5	1.8	V
0 *g* Offset vs. Temperature			±1		m*g*/°C
NOISE PERFORMANCE					
Noise Density X_{OUT}, Y_{OUT}			150		µ*g*/√Hz rms
Noise Density Z_{OUT}			300		µ*g*/√Hz rms
FREQUENCY RESPONSE[4]					
Bandwidth X_{OUT}, Y_{OUT}[5]	No external filter		1600		Hz
Bandwidth Z_{OUT}[5]	No external filter		550		Hz
R_{FILT} Tolerance			32 ± 15%		kΩ
Sensor Resonant Frequency			5.5		kHz
SELF-TEST[6]					
Logic Input Low			+0.6		V
Logic Input High			+2.4		V
ST Actuation Current			+60		µA
Output Change at X_{OUT}	Self-Test 0 to Self-Test 1	−150	−325	−600	mV
Output Change at Y_{OUT}	Self-Test 0 to Self-Test 1	+150	+325	+600	mV
Output Change at Z_{OUT}	Self-Test 0 to Self-Test 1	+150	+550	+1000	mV
OUTPUT AMPLIFIER					
Output Swing Low	No load		0.1		V
Output Swing High	No load		2.8		V
POWER SUPPLY					
Operating Voltage Range		1.8		3.6	V
Supply Current	$V_S = 3$ V		350		µA
Turn-On Time[7]	No external filter		1		ms
TEMPERATURE					
Operating Temperature Range		−40		+85	°C

[1] Defined as coupling between any two axes.
[2] Sensitivity is essentially ratiometric to V_S.
[3] Defined as the output change from ambient-to-maximum temperature or ambient-to-minimum temperature.
[4] Actual frequency response controlled by user-supplied external filter capacitors (C_X, C_Y, C_Z).
[5] Bandwidth with external capacitors = $1/(2 \times \pi \times 32\ k\Omega \times C)$. For C_X, C_Y = 0.003 µF, bandwidth = 1.6 kHz. For C_Z = 0.01 µF, bandwidth = 500 Hz. For C_X, C_Y, C_Z = 10 µF, bandwidth = 0.5 Hz.
[6] Self-test response changes cubically with V_S.
[7] Turn-on time is dependent on C_X, C_Y, C_Z and is approximately $160 \times C_X$ or C_Y or C_Z + 1 ms, where C_X, C_Y, C_Z are in microfarads (µF).

Figure G.6: Specification sheet for a MEMS accelerometer
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Appendix H

Data Sheets

Load Cell Calibration Data Sheet

A version of this data sheet as a spreadsheet file (MS-Excel™ format) is available on PolyLearn.

Load Cell Calibration Data Sheet

Names: ______________________

____

	Model #	Serial #	Calibration Constant
Force Transducer			
Accelerometer			

Mass of Proof Mass and mounting studs = 246 + 2(3) grams

Mass of Accelerometer = _________

Total Mass (Proof Mass+Accelerometer) = _________ $lb_f \cdot sec^2/ft$

		1	2	3	4	5	6	7	8	9	10
		Accelerometer		**Force Transducer**		**Acceleration**				**Calculated Force F=ma**	
		O'scope	Voltmeter	O'scope	Voltmeter						
		V_{pp}	V_{rms}	V_{pp}	V_{rms}	g_{pp}'s	$ft/s^2{}_{pp}$	g's$_{rms}$	$ft/s^2{}_{rms}$	lbf_{pp}	lbf_{rms}
Driving Frequency	~90 hz: ____										
	~900 hz: ____										

	O'scope	Voltmeter
Slope from 90 hz plot		
Slope from 900 hz plot		

H-3 *Data Sheets*

Spectral Analysis Data Sheet

Date ______________ Names ____

Lab Section ______________ ____

Prelab results: Fourier series expansion of waveforms with an amplitude of 0.5 volts and a fundamental frequency of $f = 300$ Hz (note $\omega = 2\pi \cdot 300$ rad/s).

Sine wave	
Triangle wave	
Square wave	

		1ω component	2ω component	3ω component	4ω component	5ω component
Sine Wave	f_{Desired}	300 Hz	600 Hz	900 Hz	1200 Hz	1500 Hz
	f_{Measured}					
	A_{Theory}					
	A_{Analyzer}					
Triangle Wave	f_{Desired}	300 Hz	600 Hz	900 Hz	1200 Hz	1500 Hz
	f_{Measured}					
	A_{Theory}					
	A_{Analyzer}					
Square Wave	f_{Desired}	300 Hz	600 Hz	900 Hz	1200 Hz	1500 Hz
	f_{Measured}					
	A_{Theory}					
	A_{Analyzer}					

Note: All voltages must be written as zero-to-peak amplitudes.

H-5 *Data Sheets*

Mass Properties Data Sheet

Date ________________ Names ____

Lab Section ________________ ____

Notes:
1. Work in units of **kg**, **m**, **Hz**, and **sec**.
2. **Show units** of **every** quantity on this page and in your calculations.
3. Numbers (n) refer to the step in Procedure of Section 4.2 on Page 4-2.

	Board number	$N =$ ____________
(1)	Distance from CG to x axis	$d_y =$ ____________
	Distance from CG to y axis	$d_x =$ ____________
	Distance from CG to origin	$d_O =$ ____________
(2)	Oscillation frequency about x axis	$f_x =$ ____________
	Mass moment of inertia about x_G axis through CG	$\bar{I}_x =$ ____________
	Oscillation frequency about "some" y axis .... (Use the only possible y axis...)	$f_y =$ ____________
	Mass moment of inertia about y_G axis through CG	$\bar{I}_y =$ ____________
(3)	Mass moment of inertia about x axis marked on board	$I_x =$ ____________
	Mass moment of inertia about y axis marked on board .... (Attach **complete** calculations with diagram for Step 3)	$I_y =$ ____________
(4)	Calculated polar moment of inertia	$J_O =$ ____________
(5)	Oscillation frequency f_z about z axis through origin	$f_z =$ ____________
	Polar moment of inertia calculated using f_z	$J_O =$ ____________
(6)	Board thickness	$t =$ ____________
	Total board area (estimate by counting squares)	$A =$ ____________
	Mass of board (use triple beam balance)	$m =$ ____________
	Mass density of board	$\rho =$ ____________

Strip area dA_i	Distance from x axis y_i	$(y_i^2)(dA_i)$

Estimated **area** moment of inertia about x axis . . . . $I_{xA} =$ ____________

Estimated **mass** moment of inertia about x axis . . . . $I_x =$ ____________

Percent difference between $I_x = \rho t I_{xA}$ and I_x . . . . $\%\Delta =$ ____________
calculated from f_x on previous page

Turn in the Exercise 4 datasheets and calculations for Step 3, neatly done by hand, showing units with all numerical quantities, **BEFORE YOU LEAVE LAB TODAY.**

Cantilever Beam Data Sheet

Date ____________ Names ______________________________

Lab Section ____________ ______________________________

First three natural frequencies of cantilever beam **from prelab:**

$\omega_{n1} =$	$\omega_{n2} =$	$\omega_{n3} =$

Beam length .... $L =$ ____________

Beam width .... $w =$ ____________

Beam thickness .... $t =$ ____________

Input accelerometer calibration constant .... $K_{IA} =$ ____________

Output accelerometer mass .... $m_{OA} =$ ____________

Output accelerometer calibration constant .... $K_{OA} =$ ____________

First mode's damping ratio, estimated from sine sweep test measurement of peak amplitude ratio .... $\zeta_1 =$ ____________

First three natural frequencies of cantilever beam from **sine sweep test:**

$\omega_{n1} =$	$\omega_{n2} =$	$\omega_{n3} =$

First three natural frequencies of cantilever beam from **random noise test:**

$\omega_{n1} =$	$\omega_{n2} =$	$\omega_{n3} =$

Damping ratios for first three natural frequencies from half-power frequencies:

$\zeta_1 =$	$\zeta_2 =$	$\zeta_3 =$

H-9 *Data Sheets*

Exercise 9 **PRELAB WORKSHEET** Single Plane Balancing

Name: ____________________ Lab Section: ________

1) Use the figure in the Prelab to determine the phase angles in degrees:

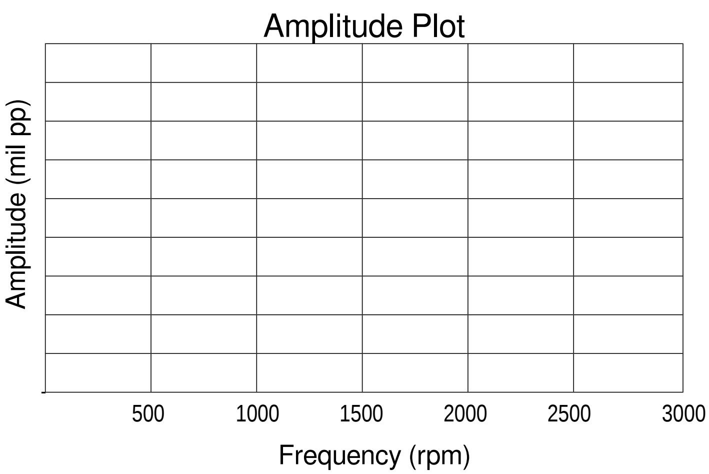
Absolute phase of A ____________

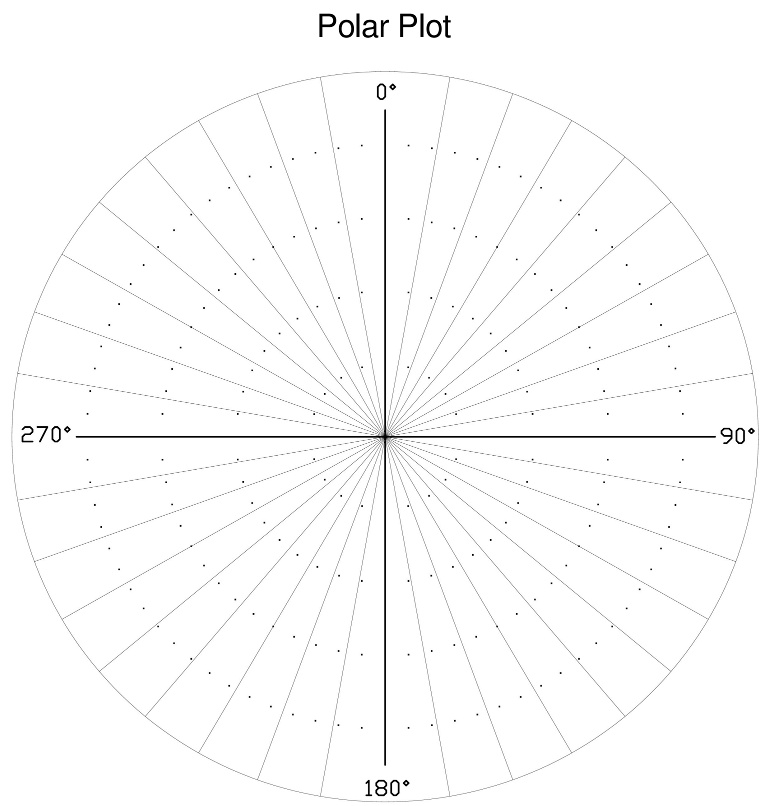
Absolute phase of B ____________

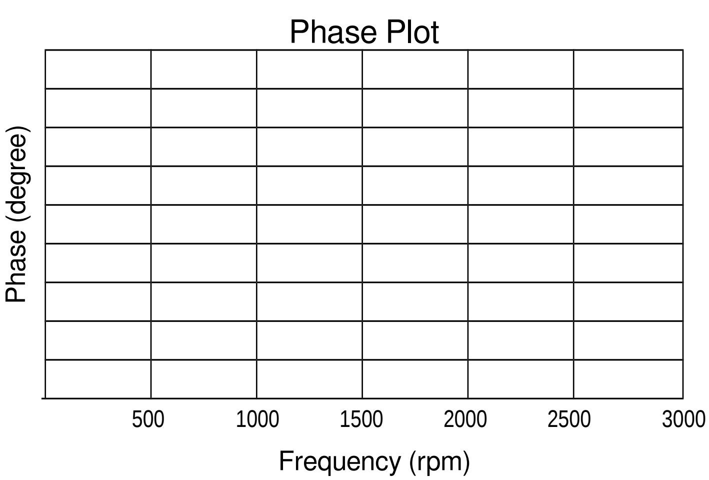
Relative phase: B leads A by ____________

2a) Use the table in the Prelab to draw the Bode amplitude and phase diagrams and the polar plot.

2b) On the Bode and polar plots identify the frequency where force leads the response by 90 degrees. On the Polar plot construct a tangent line through the origin of the grid and identify which end of the tangent line is the heavy side of the rotor.







3) Given the following vectors: $\boldsymbol{O} = 13.4$ mil @ 137 ° $(\boldsymbol{O} + \boldsymbol{C}) = 8.2$ mil @ 187 ° $\boldsymbol{W}_{cal} = 0.2$ grams @ 250 °

a) Graphically determine $\boldsymbol{C}$ by drawing the vector triangle on the above polar plot. $\boldsymbol{C} =$ ____________

b) Analytically determine $\boldsymbol{C}$ $\boldsymbol{C} =$ ____________

c) Calculate $\boldsymbol{H} = \boldsymbol{C} / \boldsymbol{W}_{cal}$ $\boldsymbol{H} =$ ____________

d) Calculate $\boldsymbol{W}_{cor} = -\boldsymbol{O} / \boldsymbol{H}$ $\boldsymbol{W}_{cor} =$ ____________

Attach your calculations to this worksheet before submitting it.

Speed: ____________ rpm

Technicians ___________________________________

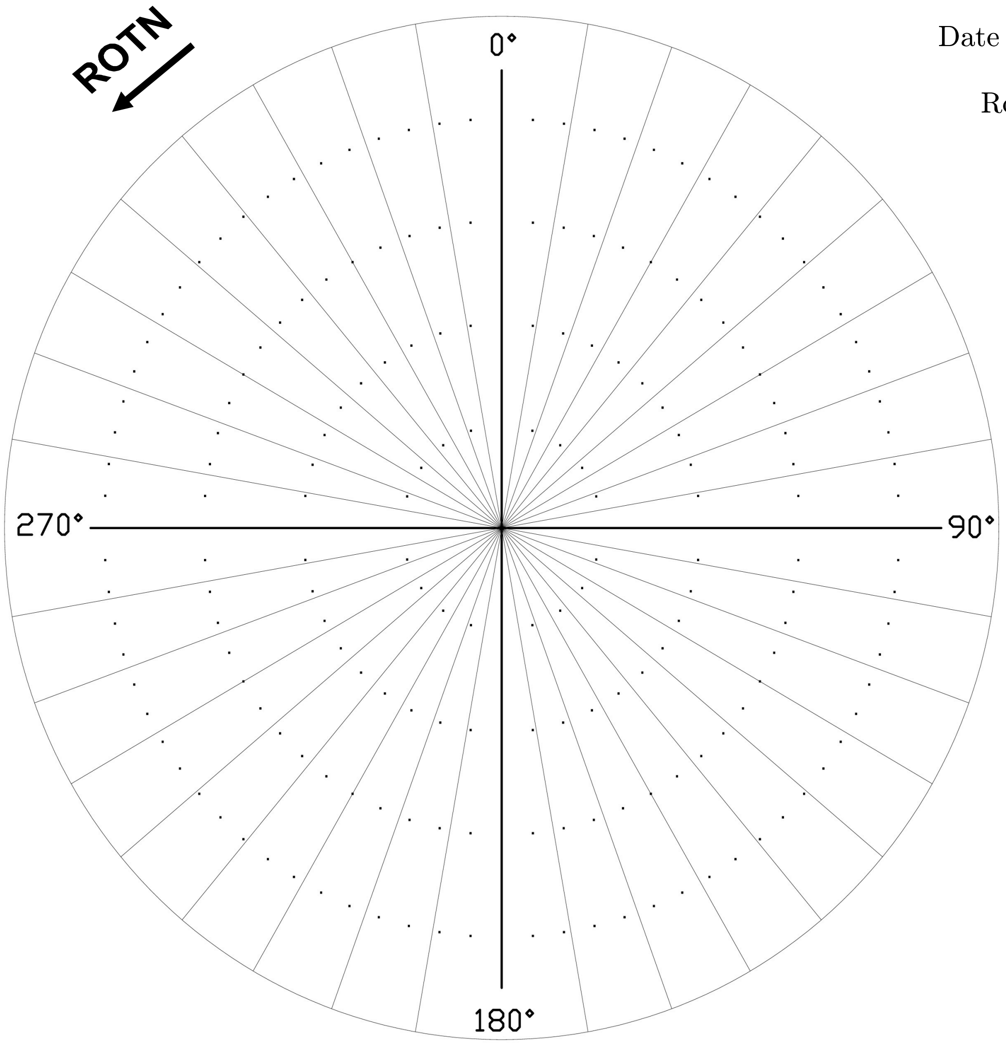
____

Date ________________

Rotor Kit (circle one):

RK1 RK2

RK3 RK4



RUN 1	**RUN 2** calibration	**RUN 3** correction	**RUN 4** optional, with weight splitting
$\boldsymbol{O}$ ______________	$\boldsymbol{W_2}$ ____________ calculated	$\boldsymbol{W_3}$ ____________ calculated	
	$\boldsymbol{W_2}$ ______________ actual	$\boldsymbol{W_3}$ ______________ actual	$\boldsymbol{W_4}$ ______________ actual
	$\boldsymbol{O + C_2}$ ____________	$\boldsymbol{O + C_3}$ ____________	$\boldsymbol{O + C_4}$ ____________
	$\boldsymbol{C_2}$ ________________	$\boldsymbol{C_3}$ ________________	$\boldsymbol{C_4}$ ________________
	$\boldsymbol{H}$ ________________	$\boldsymbol{H}$ ________________	$\boldsymbol{H}$ ________________

On the plot show the vectors $\boldsymbol{O}$, $\boldsymbol{O + C_2}$, and $\boldsymbol{C_2}$.

Draw a box at the rim to show location and size of $\boldsymbol{W_2}$ and $\boldsymbol{W_3}$.