

Motivation: Why we need to solve linear system (large scale)

$$-u'' = f$$

$$u(0) = 1$$

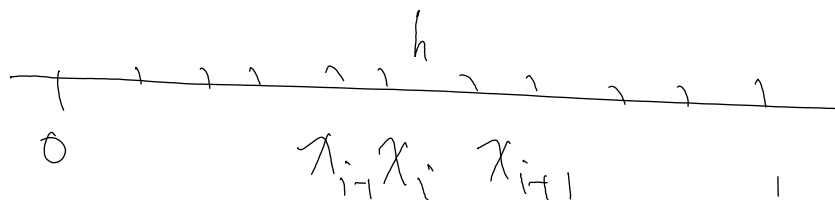
$$u(1) = 2$$

$$x \in (0, 1)$$

Given f , find u ,

finite difference scheme

$$u'' \approx \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1}))}{h^2}$$



error $(O(h^2))$

$$-\frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1}))}{h^2} \approx f_i$$

$$\vec{u} = \begin{bmatrix} u(x_0) \\ \vdots \\ u(x_N) \end{bmatrix}$$

$$A \vec{u} = \vec{f} \quad \vec{f} = \begin{bmatrix} f(x_0) \\ \vdots \\ f(x_N) \end{bmatrix}$$

$$A = \frac{1}{h^2} \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & -1 \end{bmatrix}$$

$$h^2 \left[\begin{array}{ccc|cc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ 0 & & & -1 & 2 \end{array} \right]$$