3.3.2

Suppose $x_n \to q$ as $n \to \infty$. By rearranging the secant method, we have $f(x_n) = (x_n - x_{n+1})\left[\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}\right]$ Taking the limit of this expression gives $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}(x_n - x_{n+1})\left[\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}\right]$. By the Mean Value Theorem, we know $\lim_{n\to\infty}\left[\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}\right] = f'(q)$. From the problem, we know $f'(q) \neq 0$, so we arrive at $\lim_{n\to\infty} f(x_n) = f(q) = \lim_{n\to\infty}(x_n) - x_{n-1}) * f'(q) = (q-q) * f'(q) = 0$. Thus, q is a zero.

3.3.3

Let us begin by finding the Taylor expansions of $f(x+h)$ and $f(x+k)$. We find $f(x+h) \approx f(x) + f'(x) * h + /frac12 f''(x)h^2$. Likewise, $f(x+k) \approx f(x) + f'(x) * k + /frac12 f''(x)k^2$. Rearranging the second expansion gives $\frac{f(x+k)-f(x)-f'(x)k}{k^2} \approx f''(x)$. We plug this in to the first expansion to find $f(x+h) \approx f(x) + f'(x) * h + \left[\frac{f(x+k)-f(x)-f'(x)*k}{k^2}\right] * h^2$. Rearranging this gives $f(x+h) * k^2 - f(x) * k^2 - f'(x) * hk^2 - f(x+k)h^2 + f(x)h^2 + f'(x) * h^2 k = 0$.